Re: BY13030  
    Ferromagnetism and spin excitations in topological Hubbard models with a flat band  
    by Xiao-Fei Su, Zhao-Long Gu, Zhao-Yang Dong, et al.  
  
Dear Dr. Gu,  
  
The above manuscript has been reviewed by one of our referees. Comments from the report appear below for your consideration.  
  
When you resubmit your manuscript, please include a summary of the changes made and a brief response to all recommendations and criticisms.  
  
Yours sincerely,  
  
Yonko Millev  
Associate Editor  
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P.S. Another referee we consulted was not able to review your manuscript.  
  
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Report of the Referee -- BY13030/Su  
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This manuscript presents the spin-1 excitation spectrum of two topological Hubbard models with a flat band, obtained by projecting the Hamiltonian to a flat band. Flat bands are known to lead to interesting phases of matter due to the enhanced effect of interactions and correlations when kinetic energy is vanishing. While ground states are known already for several interesting cases, a lot remains, however, to be understood about the excitations and stability of these phases. This manuscript addresses the question about how stable the predicted ferromagnetic flat band state is when the band is not completely flat.  
  
The question is certainly of interest to researchers working on topological and flat band physics, as well as for understanding ferromagnetism and strongly correlated phases in lattice systems. The manuscript considers the pi-flux square lattice which provides both a Chern and Z2 Hubbard models, both paradigmatic models of interacting topological systems. Both models feature nearly flat bands. The authors solve the spin-1 excitation spectrum by first projecting to the (nearly) flat band of the system, and then solving the resulting excitation problem numerically. They find dispersions that correspond to collective spin waves, together with a continuum that corresponds to individual spin-flips. They show that the spin-flip process may utilize the kinetic energy available in a non-flat band in such a way that the continuum reaches quite low energies. These low-energy continuum excitations couple with the spin-waves and cause a roton-type feature in the dispersion. Softening of this roton leads to break-up of the ferromagnetic phase when the band-width becomes comparable to half of the interaction energy.   
  
The results are of interest because they give a prediction, together clear physical understanding, with when can one expect the predicted ferromagnetic ground state to appear for a nearly flat band. The results seem very reliable and thoroughly understood. The manuscript is extremely well written and clear. The only concern I have is about broadening the discussion of previous and related literature: Flat bands lead to interesting phenomena also for attractive interactions: it has been predicted that the pairing gap and superfluid weight in a flat band are linearly proportional to the interaction energy, and due to the particle-hole symmetry in bipartite lattices these results have a connection to ferromagnetism, see e.g. S. Peotta and P. Törmä, Nat. Comm. 6, 8944 (2015), A. Julku et al., Phys. Rev. Lett. 117, 045303 (2016), L. Liang et al., Phys. Rev. B 95, 024515 (2017). In particular, recently it was shown that, in a flat band, two-body physics produces many of the essential results predicted by mean-field and beyond-mean-field approaches for flat band superconductivity, see P. Törmä et al., arXiv:1810.09870 (2018). In this preprint, the dispersion of the pair is investigated in an approach very similar (see also the supplementary material) to how the dispersion of the spin wave was obtained in the present manuscript. There are of course many differences, such as that the present manuscript considers the many-particle case (as obvious from using the ground state in eq(14)) and the non-flatness of the bands. The connections of the work presented in the manuscript to the results on the flat band physics with attractive interactions, such as in the works mentioned here, should be discussed in the manuscript. After that, I would be willing to recommend the article to be published in Physical Review B.