Tutorial: Rejection Sampling

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Sampling

- Sampling is one of the most common tasks in statistical data analysis and in particular Monte Carlo simulations.

- The task of sampling is to draw samples of a random variable, given a probability density/mass function.

Sampling

- For example, how do you generate samples from a triangular distribution given its probability density function $p(x) = 1 - |x - 1|, x \in [0, \; 2]$?
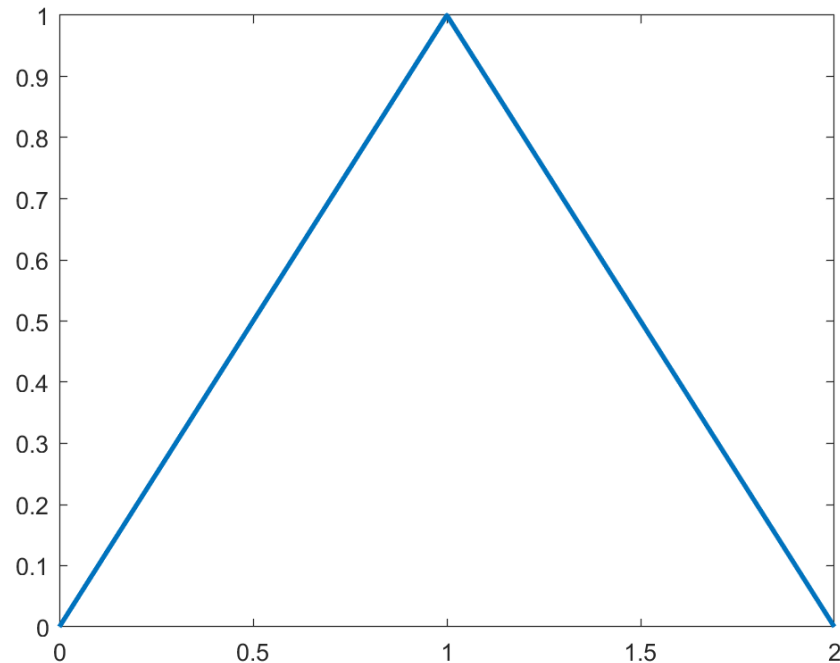
Rejection Sampling

- Suppose we have a uniform sampler that samples observations from $U(0,1)$ and a basic sampler for a known distribution with density function $q(x)$

- Target: sample $n_{\max}$ observations from a distribution with density function $p(x)$.

- Algorithm:
  - Draw a sample x from $q(x)$ and a uniform sample $u \sim U(0,1)$
  - If $u < \frac{p(x)}{Mq(x)}$, accept sample x.
    - If # accepted samples equals to $n_{\max}$, quit.
  - Repeat the algorithm.

Rejection Sampling

- M in the previous algorithm is a constant, which needs to satisfy $\frac{p(x)}{q(x)} \leq M$.

- When ``accepting'' a sample, we save it to a vector, so the vector later contains only accepted samples.

Vector Appending

- If `a` is a length-k vector, `a[k + 1] = c` will append another element `c` at the end of `a`.

```
a <- c(1,2,3,4)
a[5] <- 5
print(a)
[1] 1 2 3 4 5
```

Code Skeleton

```r
# how many samples do we want?
n_max = 1000

pbar <- function(x){
  # the PDF of distribution, from which you want to sample.
  return((1-abs(x-1)))
}

acc <- c() # create an empty vector
n <- 0 # how many samples have we already obtained?
M <- 200 # M = 200
while(n < n_max){
  u <- runif(1) # generate a uniform sample
  x <- runif(1,0,2) # in this example, q(x) is U(0,2)

  # TODO: complete the algorithm here
  # Think: What is M in our case?

}
hist(acc) #plot the histogram of accepted samples
```

Questions:

1. Complete the above code skeleton according to the rejection sampling algorithm.

2. What output is expected when the algorithm is implemented correctly?

3. Fill out the blank: The bigger M is, the ___ (faster/slower) the algorithm becomes.