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MECH 5970 – Project 2: Analog Control

3/4/2022

Design Process

The design process for our analog control circuit began with a consideration of our constraints. The control circuit needed a 1 V to 1 degree relation prior to the control gains for the position side of the circuit. Given the Midori position sensor had a 0-350 degree rotational range, and we were inputting 5 V into the Midori, we knew this meant a gain of 70. We split this between two Op-Amps for gains of 7 and 10. After this point, we used coefficient matching based on Eq. 1 to determine the mechanical system control gains.

$$s^2 + 2\zeta\omega_n s + \omega_n^2 \tag{1}$$

In Eq. 1, ζ is the damping ratio, ω_n is the natural frequency, and s is the Laplace domain variable. Given the system control gains, we used dimensional analysis to get the control gains into the form of Op-Amp gains, and created one last amplification stage for this in the position side of the circuit.

The velocity side of the circuit was a touch more difficult as we needed to a use a practical differentiator. This meant that we weren't concerned with the input voltage, but the change in input voltage which isn't as easy to find. We began solving for this using the project's desired damping ratio and settle time. Using Eq. 2, we found the natural frequency from the settle time and made a transfer function based on Eq. 1 and our solved values.

$$t_{s,2\%} = \frac{4}{\zeta\omega_n} \tag{2}$$

We took the max of this transfer functions step response and used this as our maximum change in voltage into the differentiator.

In the design of the differentiator, we knew that the maximum speed of the pendulum wouldn't be the natural frequency, but the damped natural frequency. Given this, we set the cutoff frequency of the differentiator to the natural frequency as it was higher than it's damped variant, meaning it would be included in the "band pass" section of the differentiator's frequency response. We then found the gain based on the cutoff, and some nominal capacitor and resistance values, and computed what it's max output voltage would be. It was less than the 10V requirement, so we added a secondary amplification to get the output to the 10V mark. With the voltage corrected, we applied the same dimensional analysis to get the control gain in the form of an Op-Amp gain and added the final amplification stage to the velocity side of the circuit.

At the end of our circuit, we have a summing Op-Amp with a gain of 1, just because we felt it cleaner and more easily understandable to have the control gains separated. Specific values and calculations have been left out of the above section as they are more easily understood from our written work and MATLAB code.

Testing & Revisions

Our circuit burnt up an Op-Amp on it's first test. Through rigorous testing of each resistor and a complete rebuilding of the circuit, it was found that a resistor we thought to be a 100 kΩ was in fact a 100 Ω. This created a gain 1000x larger than we needed and burnt up the Op-Amp; specifically, the differentiator. We also believe at some point during our first test, we somehow burnt out the LM336 regulator as well. We attributed this to a misunderstanding of the pinout.

Given a new Op-Amp and a new regulator, the circuit was rebuilt and we actually updated some of the resistors so that we would have less in series. During the construction of this circuit, a 9V was used to test the regulator to ensure that we didn't have another incident with that component. After the construction, this circuit was tested without the inverted pendulum apparatus, and all the voltages seemed to be in line with all of our constraints, except for the velocity circuit. It was believed that because we were testing with a potentiometer that was not the Midori, that could have been throwing off the results, but either way, the velocity side of the circuit was rebuilt once again.

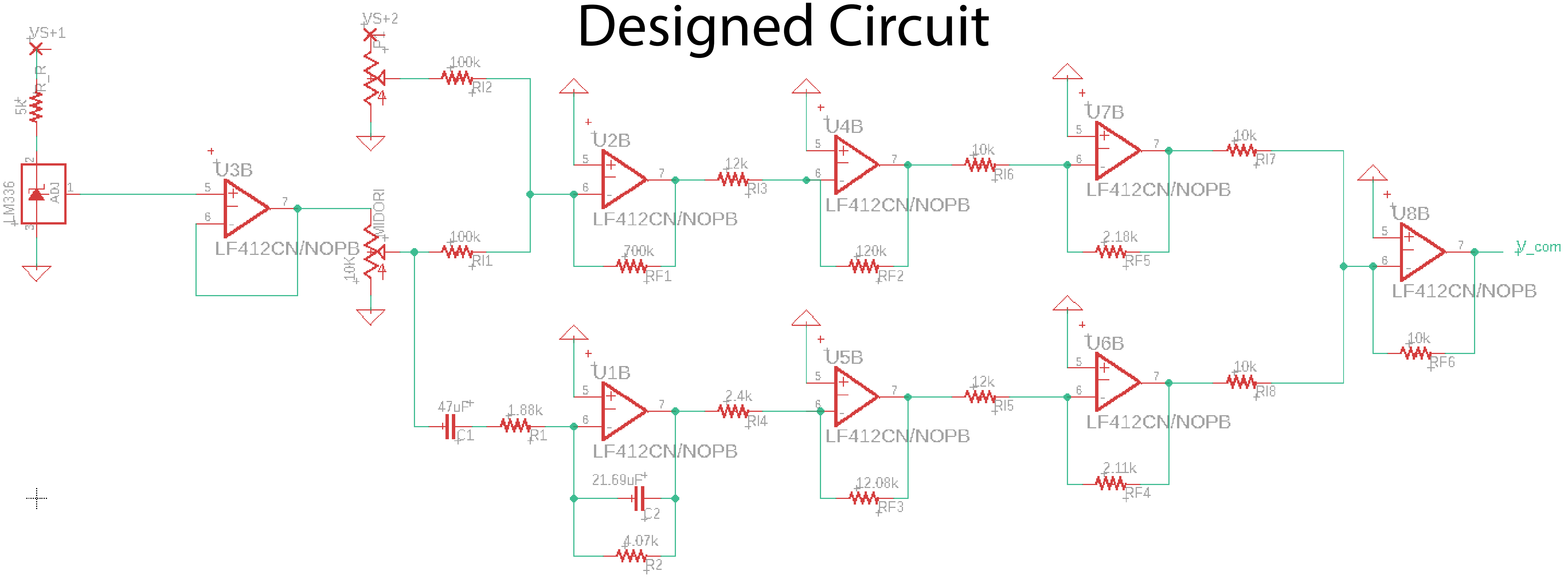
When the inverted pendulum was available, we tested again and our position circuit worked near perfectly, it just had no damping; this being due to an inaccuracy in the function of the velocity side. A video of this test is provided in this report. The velocity side had the differentiator Op-Amp constantly saturated at ~14.5 V. We ended finding that a resistor was not plugged into the correct pin on the Op-Amp and this had burnt up another Op-Amp.

Giving the findings from our second pendulum test, the affected Op-Amp was replaced, it's corresponding circuit rebuilt, and both team members went over the circuit with a multimeter and the circuit diagram multiple times to ensure proper wiring. In our final test of the circuit, the pendulum did reach it's steady-state, but oscillated a fair bit around the point. We had a potentiometer for adjusting the control gain of the velocity circuit, but were unable to source a second potentiometer for the control gain on the positional circuit. Given this fact, we believe that we had the derivative, or velocity gain, fairly tuned, but the proportional, or position, gain seemed to be slightly incorrect. Oscilloscope readings as well as videos of the final performance are both available in this report.

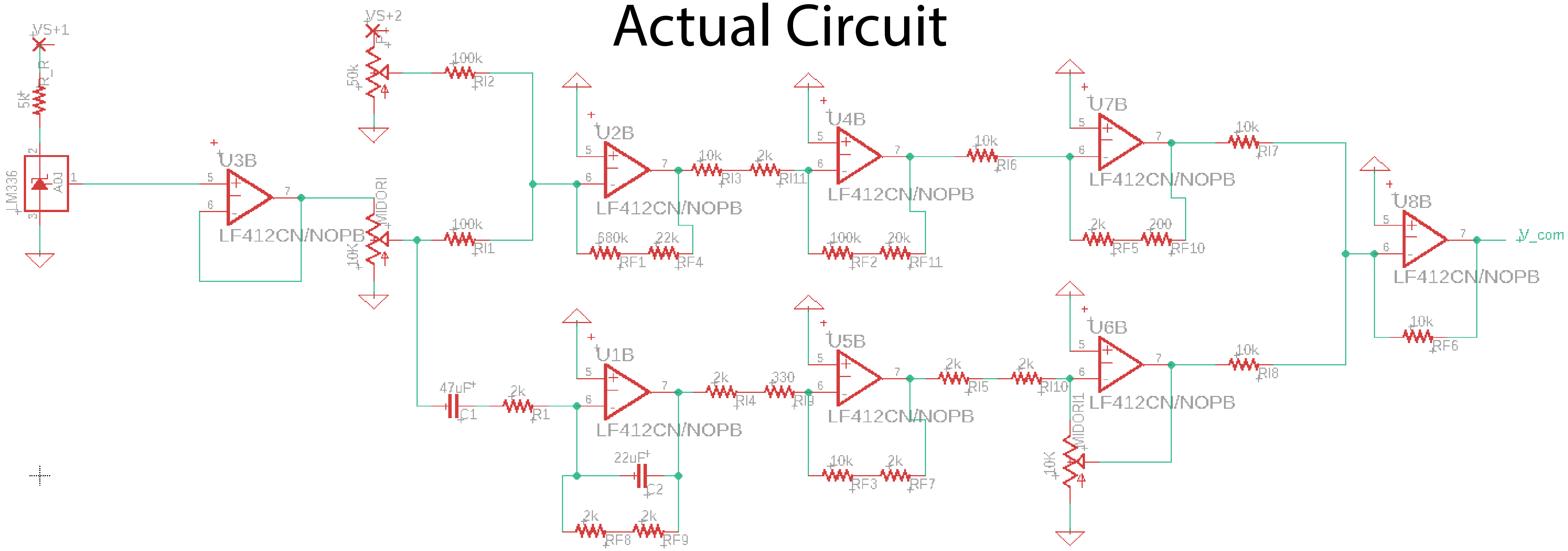
A few more points about our circuit's performance include our output voltages. They were slightly offset at about ±14 V. We believe this is due to the adjustments we had to make to our voltage offset due to the impedance matching. Our offset potentiometer was a 50 kΩ not a 10kΩ and this led to the offset needing to be adjusted. The output voltage of the entire control circuit was more in the range of 3.68 V which is the approximate output voltage we expected.

Overall, this control circuit was a success. It was not quite to specification which is failure on the part of the circuit and its design, but it did successfully get us to the setpoint. Given another potentiometer for the adjustment of the proportional gain, and more availability of power supplies and testing apparatus, this control circuit would meet specification.

Designed Circuit



Actual Circuit



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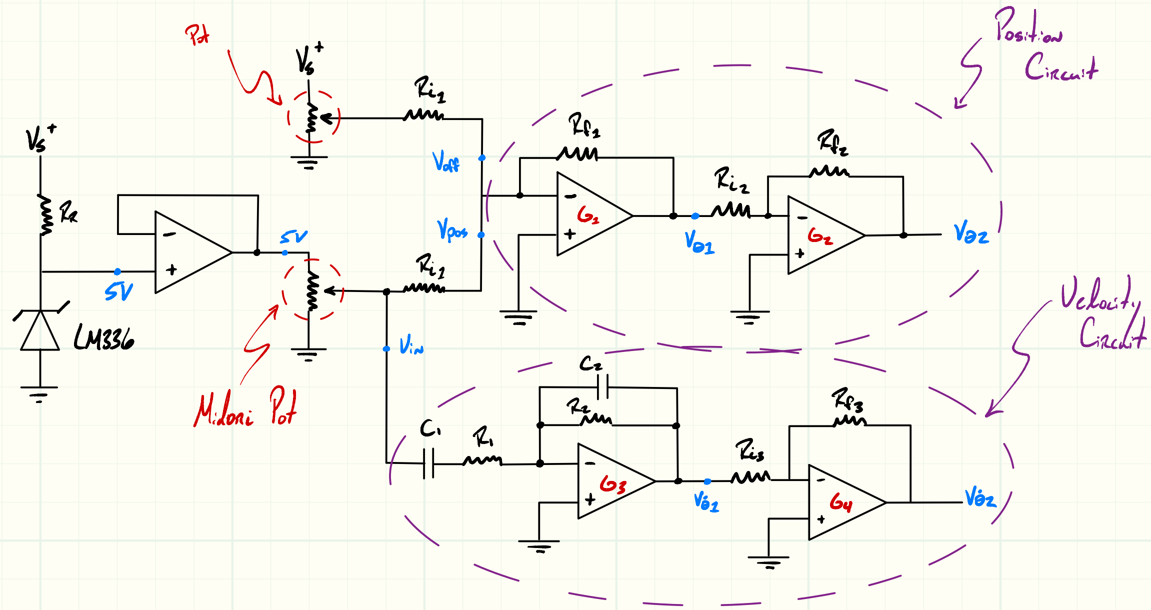
Analog Control

2/8/22



Position Circuit Calculations

$$V_{\theta 1} = -\left(\frac{R_{f1}}{R_{i1}} V_{pos} + \frac{R_{f1}}{R_{i1}} V_{off}\right) \qquad V_{\theta 2} = -\frac{R_{f2}}{R_{i2}} V_{\theta 1} = \frac{R_{f2}}{R_{i2}}\left(\frac{R_{f1}}{R_{i1}} V_{pos} + \frac{R_{f1}}{R_{i1}} V_{off}\right)$$

Velocity Circuit Calculations

$$\frac{V_{\dot{\theta}}}{V_{in}} = \frac{-C_1 R_2 s}{(1+C_1R_1 s)(1+C_2R_2 s)} \qquad \omega_c = \frac{1}{C_1R_1} = \frac{1}{C_2R_2}$$

(Eqns sourced from prdiff.m)

$$V_{\dot{\theta}2} = V_{\theta 1}\frac{R_{f3}}{R_{i3}}$$

$\zeta = 0.707$ $\qquad t_{2\%} = \frac{4}{\zeta\omega_n}$ $\qquad \omega_n = \frac{4}{\zeta t_{2\%}} = 11.3154\, rad/s$

$t_{2\%} = 0.5$

Ideal Position Transfer Fxn: $\frac{1}{s^2+2\zeta\omega_n s+\omega_n^2}$

Ideal Velocity Transfer Fxn: $\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$

Max velocity found in MATLAB to be $\sim 1.9 \frac{rad}{s}$

Required Voltage Range: $-10V \leq V_{\dot{\theta}} \leq 10V$

Determined Velocity Range: $-1.9\frac{rad}{s} \leq \dot{\theta} \leq 1.9\frac{rad}{s}$

Controls

$$J\ddot{\theta} - m_2 g l \theta = K_t i \rightarrow \text{FSF} \rightarrow s^2 + \left(K_{\dot{\theta}} \frac{K_t}{J}\right)s + \left(K_\theta \frac{K_t}{J} - \frac{m_2 g l}{J}\right) = 0$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$K_\theta = \frac{\omega_n^2 J + m_2 g l}{K_t} \; [A] \quad \rightarrow K_\theta \cdot \frac{pi}{180} \cdot \frac{1}{K_A} = G_\theta$$

$$K_{\dot{\theta}} = \frac{2\zeta\omega_n J}{K_t} \; [A \cdot s] \quad \rightarrow K_{\dot{\theta}} \cdot \frac{pi}{180} \cdot \frac{1}{K_A} \cdot \frac{10V}{1.9445 \text{ rad/s}} = G_{\dot{\theta}}$$

```
clear; clc; close all
```

POSITION CIRCUIT

```
% Impedance Matching Resistors
Ri1 = 100e3; % Ohm
Ri2 = 12e3; % Ohm

% Desired First Op-Amp Gain
% This was arbitrarily chosen to get nominal values
G1 = 7; % Must be less than or equal to 10

% Max Voltages
V_theta2_max = 350/2; % V
V_pos_max = 5; % V
V_off = -2.5; % V
% Because this is a summing amp and R1 = R2 = Ri1
% so it acts as an inverting amp with V_in_max as input voltage
V_in_max = V_pos_max + V_off; % V

% Solving for Filter Resistor 1
Rf1 = G1*Ri1; % Ohm
V_theta1_max = V_in_max*G1; % V

% Solving for Filter Resistor 2
Rf2 = (V_theta2_max/V_theta1_max)*Ri2; % Ohm
G2 = Rf2/Ri2;

% Returning Values
G1
```

```
G1 = 7
```

```
G2
```

```
G2 = 10
```

```
Ri1
```

```
Ri1 = 100000
```

```
Ri2
```

```
Ri2 = 12000
```

```
% 680K and 22K in series
Rf1
```

```
Rf1 = 700000
```

```
Rf2
```

```
Rf2 = 120000
```

```
```

SYSTEM

```
% Givens
g = 9.81; % m/s/s
L_r = 101.6/1000; % m
m_r = 18.14/1000; % kg
m_b = 32/1000; % kg
r_b = 6.35/1000; % m

% Given Damping Ratio (Zeta) and Settling Time (ts)
zeta = 0.707;
ts = 0.5; % s

% Moments of Inertia
J_b = 1/2 * m_b*r_b^2 % kg.m^2
```

```
J_b = 6.4516e-07
```

```
J_r = 1/3 * m_r*L_r^2 % kg.m^2
```

```
J_r = 6.2417e-05
```

```
J_m = 33.5/(1000*100^2) % kg.m^2
```

```
J_m = 3.3500e-06
```

```
J = J_r + J_b + m_b*(L_r + r_b)^2 + J_m % kg.m^2
```

```
J = 4.3931e-04
```

```

% Combined Mass
m = (m_b + m_r/2) % kg
```

```
m = 0.0411
```

```

% Motor Constants
K_e = 1/369 * 30/pi; % V/rad/s
K_t = 25.9/1000; % N.m/A

% The constants are equal to one another so we unify them
K = K_t; % N.m/A

% Motor Values
R = 0.611; % Ohm
L = 0.119/1000; % H
b = 0;

% Calculation of Natural and Damped Natural Frequency
w_n = 4/(zeta*ts); % rad/s
f_n = w_n/(2*pi); % Hz
w_d = w_n*sqrt(1-zeta^2); % rad/s
```

```
f_d = w_d/(2*pi); % Hz

% Desired Step Characteristics
M_percent = exp((-zeta*pi)/sqrt(1-zeta^2))*100
```

```
M_percent = 4.3255
```

```
tp = pi/w_d
```

```
tp = 0.3926
```

```

% Plant based on Coefficient Matching
ideal_plant = tf(K/J,[1 2*zeta*w_n w_n^2]); % radians

% Derivative of the Plant (Plant = Position; Derivative = Velocity)
s = tf('s');
p_deriv = ideal_plant*s;

% Maximum Velocity of the Plant
max_vel = max(step(p_deriv)) % rad/s
```

```
max_vel = 2.3757
```

```

% Maximum dVin/dt; Input to Differentiator
dVin_dt_max = max_vel * 180/pi * 5/350 % V/s
```

```
dVin_dt_max = 1.9445
```

```
% Velocity range is equal to dVin/dt for our model
vel_range = dVin_dt_max;

% Returning Values
f_n
```

```
f_n = 1.8009
```

```
f_d
```

```
f_d = 1.2736
```

```
dVin_dt_max
```

```
dVin_dt_max = 1.9445
```

VELOCITY CIRCUIT

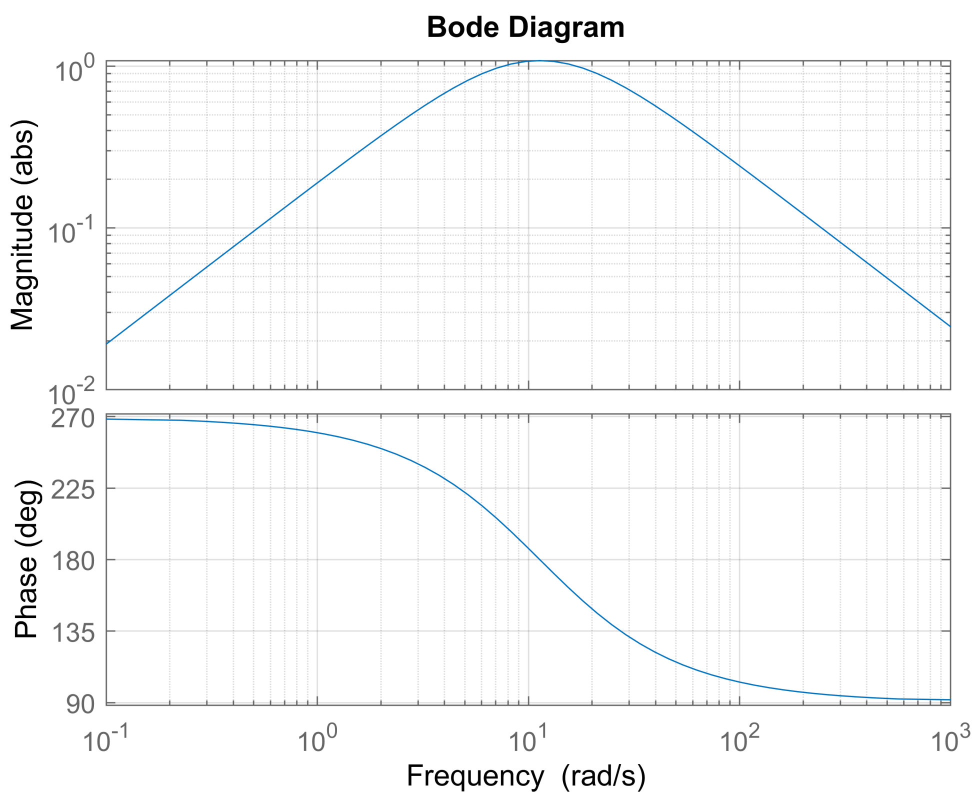
```
% Cutoff Frequency
w_c = 2*pi*f_n; % rad/s

% Desired Gain and First Capacitor
% Needs to be less than 1
% Chosen to obtain nominal capacitor and resistor values
G = 0.19;  % this is gain (10^0 = 1) @ 1 rad/sec
C1 = 47e-6; % F or 1 pico F
```

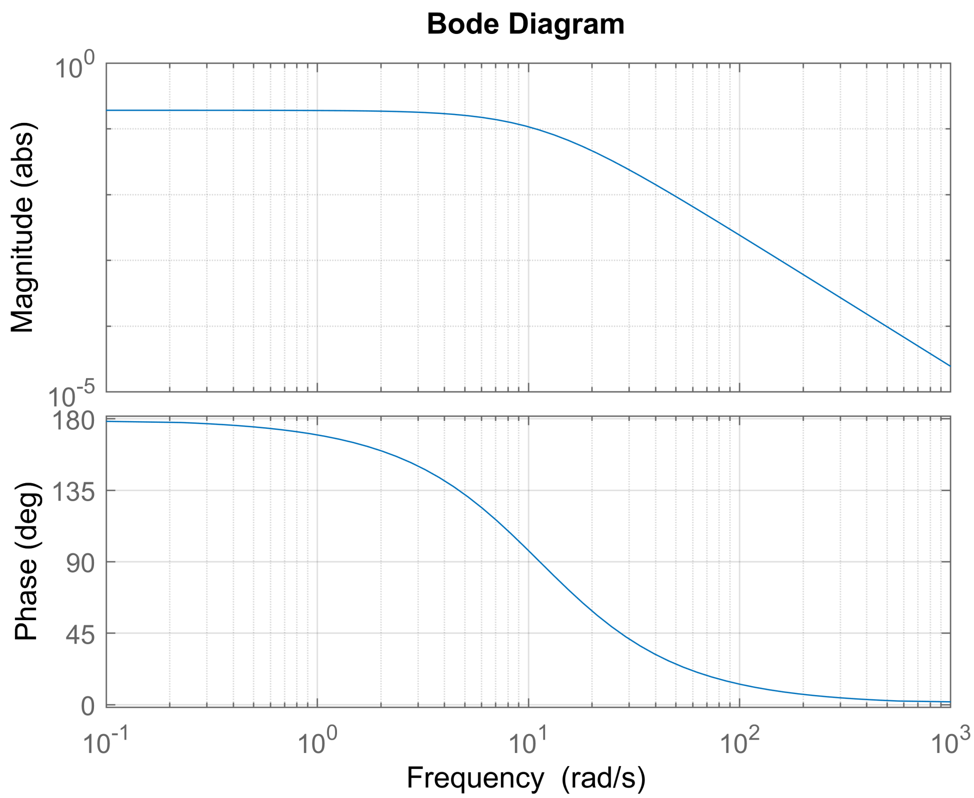
```
% Calculate Component Values
    % equations from prdiff.m
    % w_c = 1/(C1*R1) = 1/(C2*R2)
    % Gain at 1 rad/sec = C1*R2 / (1 + 1/w_c^2)
R1 = 1/(C1*w_c); % Ohm
R2 = (1/C1)*G*(1+1/w_c^2); % Ohm
C2 = 1/(R2*w_c); % F

% Calculate Transfer Function
num = [ -C1*R2, 0 ];
den = [ C1*C2*R1*R2, (C1*R1) + (C2*R2), 1 ];
sys1 = tf(num,den);

% Plotting Vo/Vi Bode
figure
bode(sys1)
grid on
setoptions(gcr,'MagUnits','abs')
setoptions(gcr,'MagScale','log')
```




```
% Plotting Vo/dVin Bode
figure
sys2 = sys1/s;
bode(sys2)
grid on
setoptions(gcr,'MagUnits','abs')
setoptions(gcr,'MagScale','log')
```




```
% Gain and Maximum Velocity Calculations
G3 = abs(freqresp(sys1,w_d)) % Must be less than 10
```

```
G3 = 1.0215
```

```
V_theta_dot1_max = dVin_dt_max*G3 % Must be less than 10
```

```
V_theta_dot1_max = 1.9863
```

```
% Inverting Op amp
% Required gain to meet 10V requirement
G4 = 10/V_theta_dot1_max;
% Arbitrary Impedance Matching Resistor
Ri3 = 2.4e3; % Ohm
% Calculation of Filter Resistor
Rf3 = G4*Ri3; % Ohm

% Max voltage output of Velocity Circuit
% (Pre Control Gains)
V_theta_dot2_max = G4*V_theta_dot1_max;

% Returning Values
R1
```

```
R1 = 1.8803e+03
```

```
R2
```

```
R2 = 4.0741e+03
```

```
C1
```

```
C1 = 4.7000e-05
```

```
C2
```

```
C2 = 2.1692e-05
```

```
Ri3
```

```
Ri3 = 2400
```

```
Rf3
```

```
Rf3 = 1.2083e+04
```

```
R1_error = 100*abs((2e3 - R1)/2e3)
```

```
R1_error = 5.9840
```

```
R2_error = 100*abs((4e3 - R2)/4e3)
```

```
R2_error = 1.8532
```

```
C2_error = 100*abs((22e-6 - C2)/22e-6)
```

```
C2_error = 1.4011
```

```
Rf3_error = 100*abs((12e3 - Rf3)/12e3)
```

```
Rf3_error = 0.6915
```

```
V_theta_dot2_max
```

```
V_theta_dot2_max = 10
```

CONTROLS

```
% Servo Amp Gain
K_A = 0.3; % A/Vc
% Degree to Radian Conversion Factor
K_deg_rad = 180/pi; % deg/rad
% Velocity Circuit Gain
K_vel = 10/vel_range; % rad/s/V

% Calculation of System Control Gains
K_theta = (J*w_n^2 + m*g*L_r)/K;
K_theta_dot = 2*zeta*w_n*J/K;

% Conversion to Op-Amp Gains
G_theta = K_theta / (K_deg_rad * K_A);
G_theta_dot = K_theta_dot / (K_vel * K_A);

% Input Resistors
Ri_theta = 10e3;
```

```
Ri_theta_dot = 12e3;
Ri_C = 10e3;
Rf_C = Ri_C;

% Filter resistor calculations
Rf_theta = G_theta*Ri_theta
```

```
Rf_theta = 2.1830e+03
```

```
Rf_theta_dot = G_theta_dot*Ri_theta_dot
```

```
Rf_theta_dot = 2.1109e+03
```

```

% Max voltage for specific systems
V_theta_max = 10*G_theta
```

```
V_theta_max = 2.1830
```

```
V_theta_dot_max = V_theta_dot2_max*G_theta_dot
```

```
V_theta_dot_max = 1.7591
```

```

% Max Output Voltage of Analog Control System
V_com = (V_theta_max + V_theta_dot_max);

% Returning Values
K_theta
```

```
K_theta = 3.7523
```

```
G_theta
```

```
G_theta = 0.2183
```

```
K_theta_dot
```

```
K_theta_dot = 0.2714
```

```
G_theta_dot
```

```
G_theta_dot = 0.1759
```

```
Ri_theta
```

```
Ri_theta = 10000
```

```
Ri_theta_dot
```

```
Ri_theta_dot = 12000
```

```
Rf_theta
```

```
Rf_theta = 2.1830e+03
```

```
Rf_theta_dot
```

```
Rf_theta_dot = 2.1109e+03
```

```
Rf_theta_error = 100*abs((2.2e3 - Rf_theta)/2.2e3)
```

```
Rf_theta_error = 0.7738
```

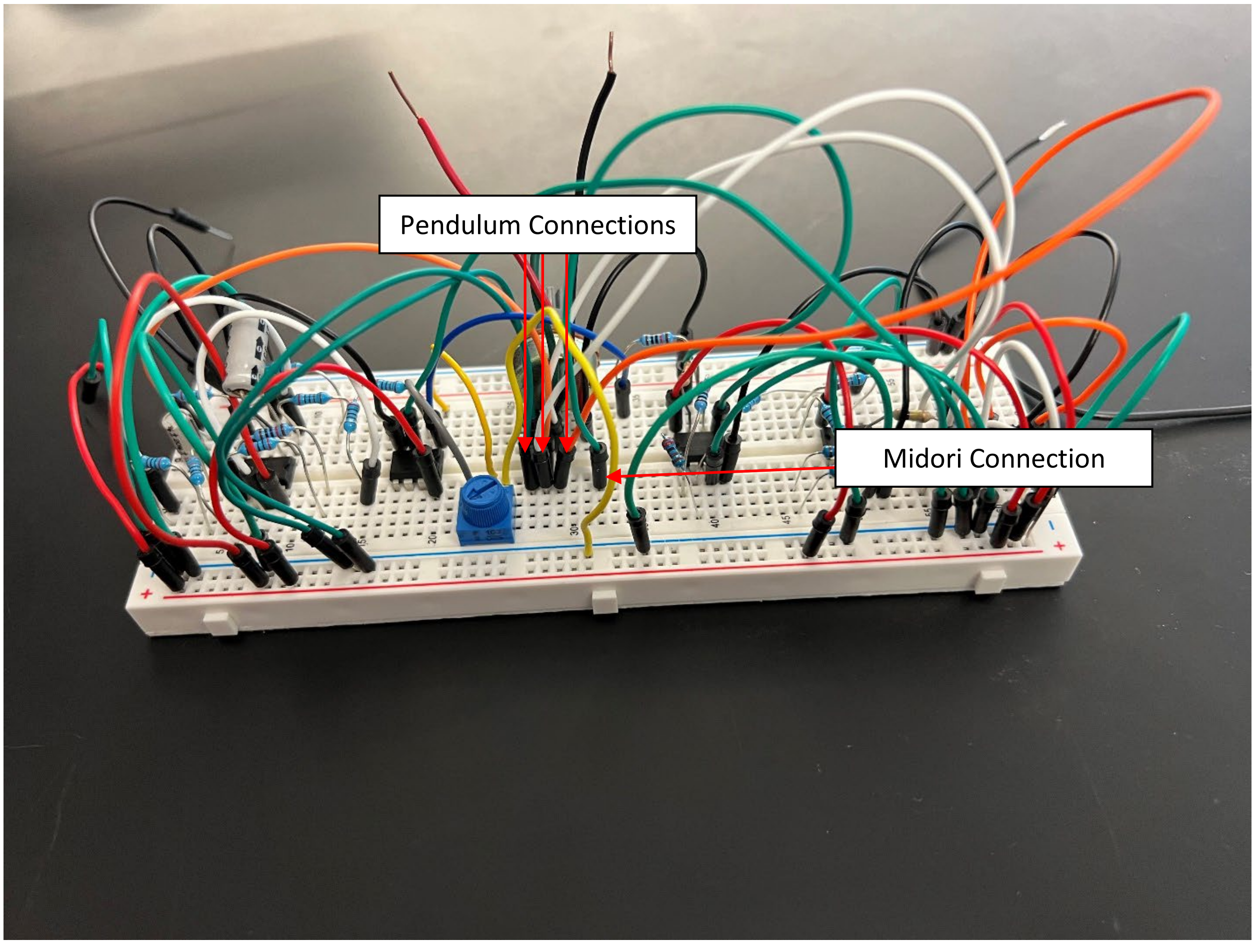
```
Rf_theta_dot_error = 100*abs((2.1e3 - Rf_theta_dot)/2.1e3)
```

```
Rf_theta_dot_error = 0.5207
```

```
V_com
```

```
V_com = 3.9421
```

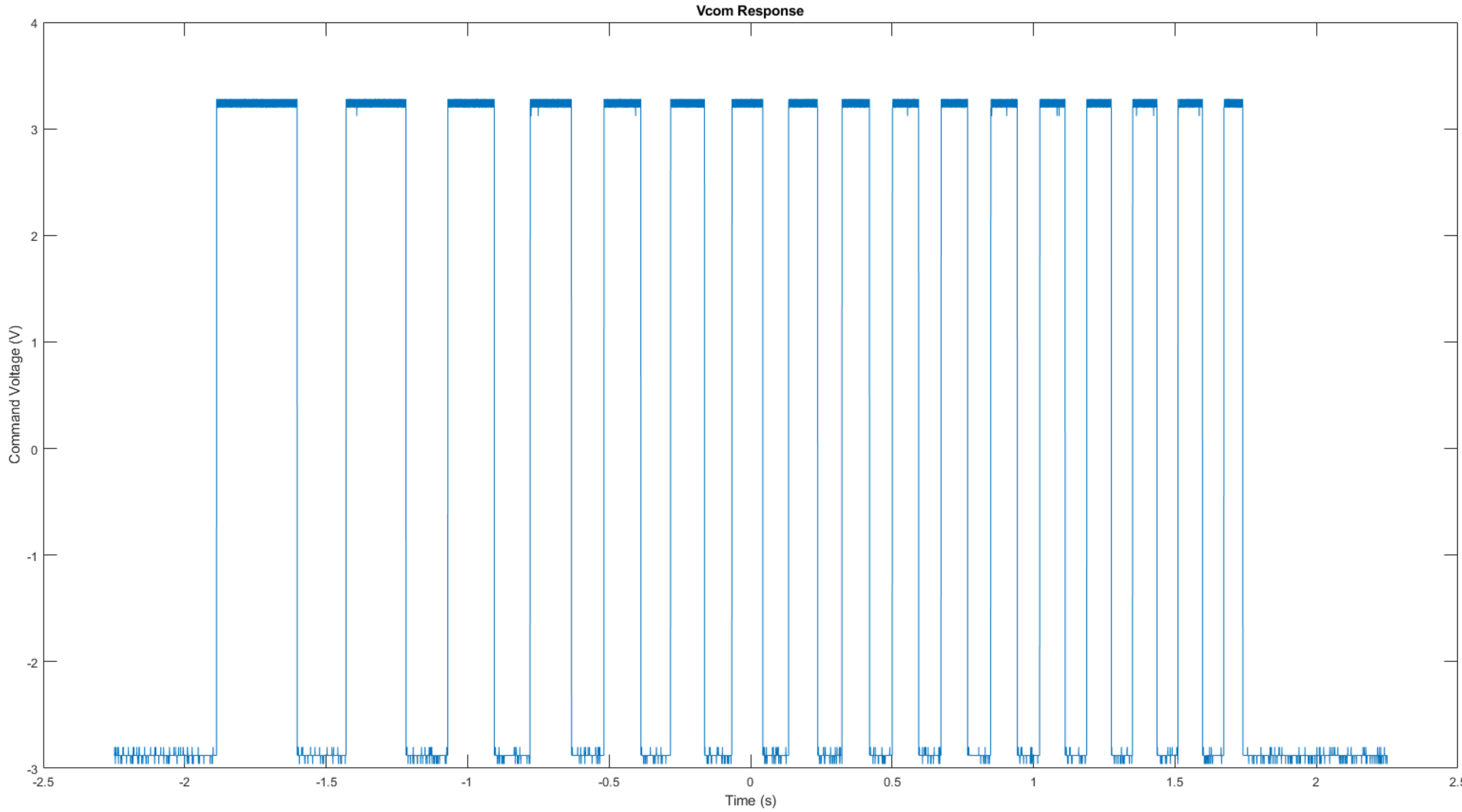
Circuit Picture and Connection to Pendulum



Video Links

- https://youtu.be/LRjNv5_xP1Q
- https://youtu.be/xHAjFVZcSTY
- https://youtu.be/nD8CPAMmjfM
- https://youtu.be/UPd1VenimzA

Oscilloscope Graph



Perspective Photos