ASEN 5090
Introduction to GNSS

Lecture 18: Ionosphere Errors

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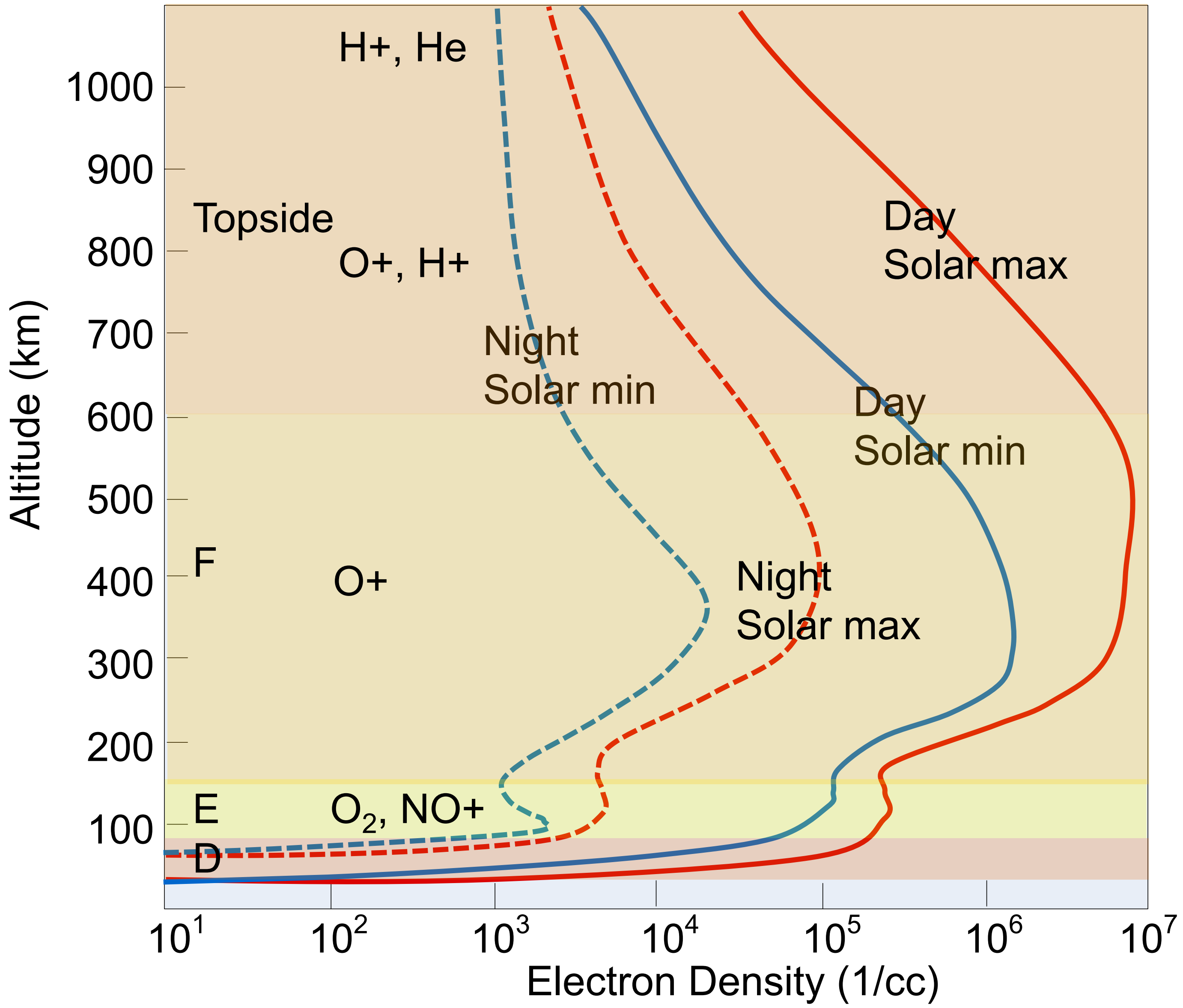
Questions for This Lecture

- What parameters determine the refractive index in the ionosphere?
- What is the single parameter that determine the ionosphere range error?
- How to obtain that parameter?
- What is the typical ionosphere-induced range error?
- How to estimate the ionosphere range error?

Lecture 18 Outline

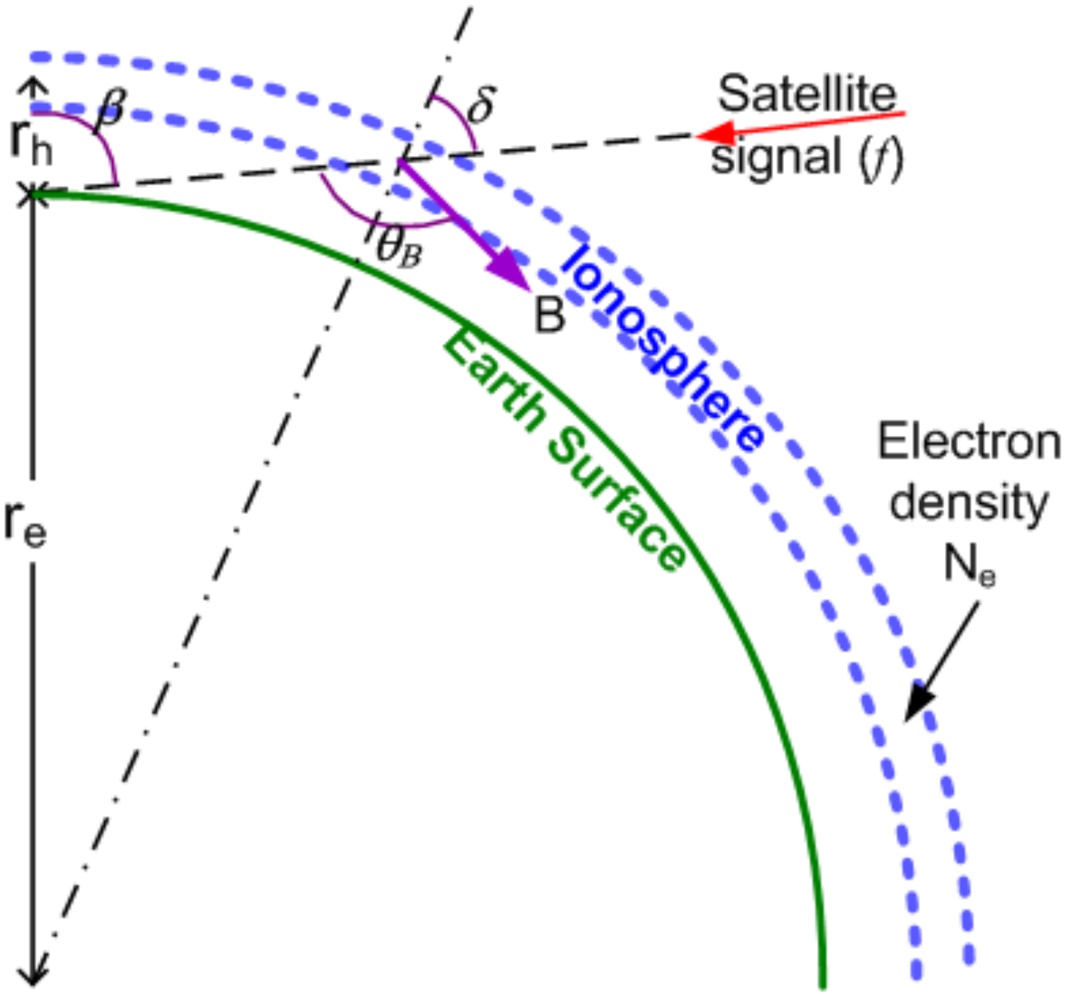
- Basic Ionosphere Properties
  - Vertical profile, gyro frequency, plasma frequency, refractive index
- Wave Propagation:
  - Period, wave number, phase velocity, group velocity, carrier-code divergence
- Ionosphere Errors
  - Phase advance, group delay, obliquity factor, TEC, ionosphere-free range
  - Broadcast TEC model
  - Dual-frequency TEC estimation

Ionosphere Profiles



Ionosphere refractive index

EM wave propagating through weakly ionized plasma immersed in magnetic field



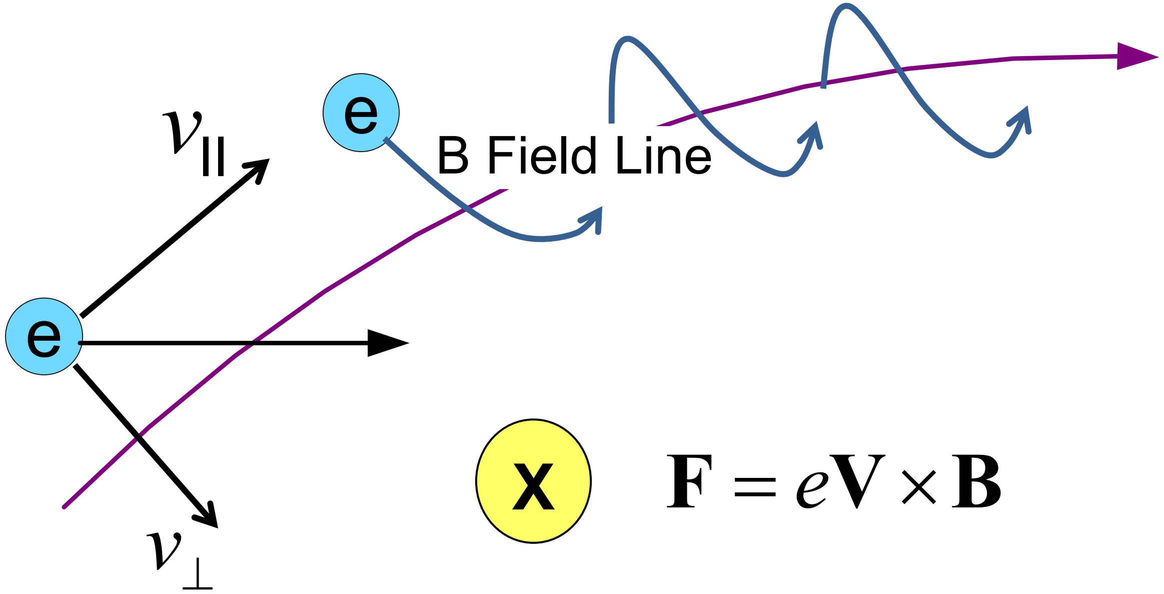
$$\Delta\tau = \tau - \tau_{vacuum} = \frac{1}{c}\int_s^R \left(n(l) - 1\right) dl$$

$$n = 1 - \frac{X}{1 - \dfrac{Y^2 \sin^2\theta_B}{2(1-X)} \pm \sqrt{\dfrac{Y^4 \sin^4\theta_B}{4(1-X)^2} + Y^2\cos^2\theta_B}}$$

$$X = \left(\frac{f_p}{f}\right)^2 \qquad Y = \frac{f_g}{f}$$

$$f_g = \frac{eB_0}{m_e} \qquad f_p = \sqrt{\frac{N_e e^2}{m_e \varepsilon_0}}$$

Electron Gryo Frequency



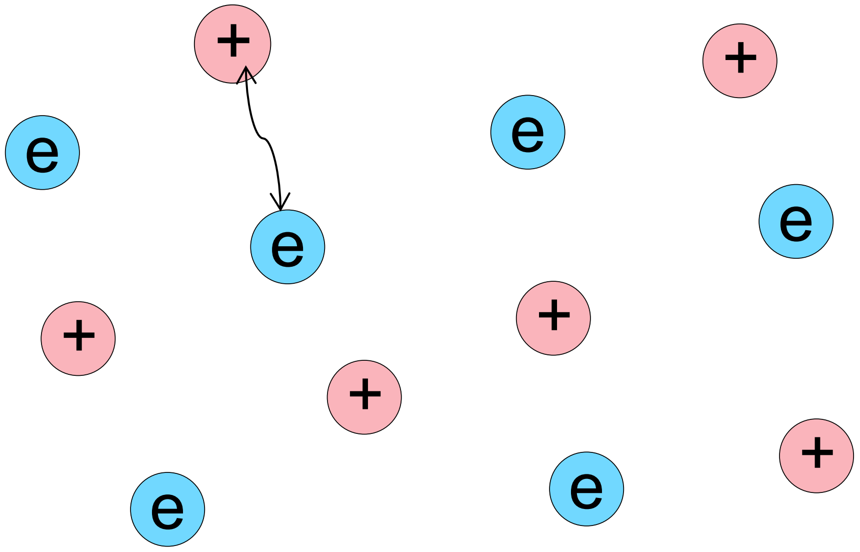
$$\mathbf{F} = e\mathbf{V} \times \mathbf{B}$$

f_g: Rate of electrons cycling along the **B** field line

$$f_g = \frac{|e|B_0}{2\pi m_e} \approx 28 \times 10^9 B_0 \approx 1MHz$$

30,000nT ~ 3x10^{-5} T

Plasma Frequency



f_p: Rate of electron oscillations in a plasma

$$f_p = \frac{1}{2\pi}\sqrt{\frac{N_e e^2}{m_e \varepsilon_0}} \approx 9\sqrt{N_e} \approx 9MHz$$

Peakionosphere N_e value: 1 million/cc

Ionosphere Refractive Index

$$f_g \leq 1MHz \qquad f_p \leq 10MHz$$

At GPS frequency (f ~ GHz): $X = \left(\frac{f_p}{f}\right)^2 \ll 1 \qquad Y = \frac{f_g}{f} \ll 1$

$$n \approx \underbrace{1}_{\text{Vacuum}} - \underbrace{\frac{X}{2}}_{\frac{40.3N_e}{f^2}} \pm \underbrace{XY\left|\cos\theta\right|}_{2^{nd}\text{ order}} - \underbrace{\frac{1}{4}X\left(\frac{X}{2} + Y^2\left(1+\cos^2\theta\right)\right)}_{3^{rd}\text{ order}}$$

For most analysis: $\boxed{n \approx 1 - \frac{40.3N_e}{f^2}}$

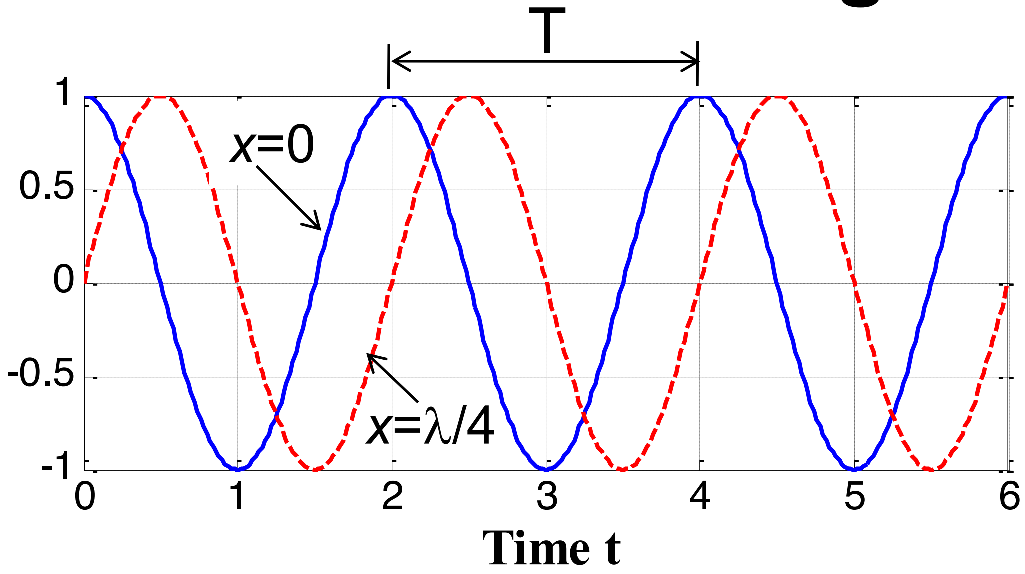
Propagating Sinusoids: Period and Wavelength

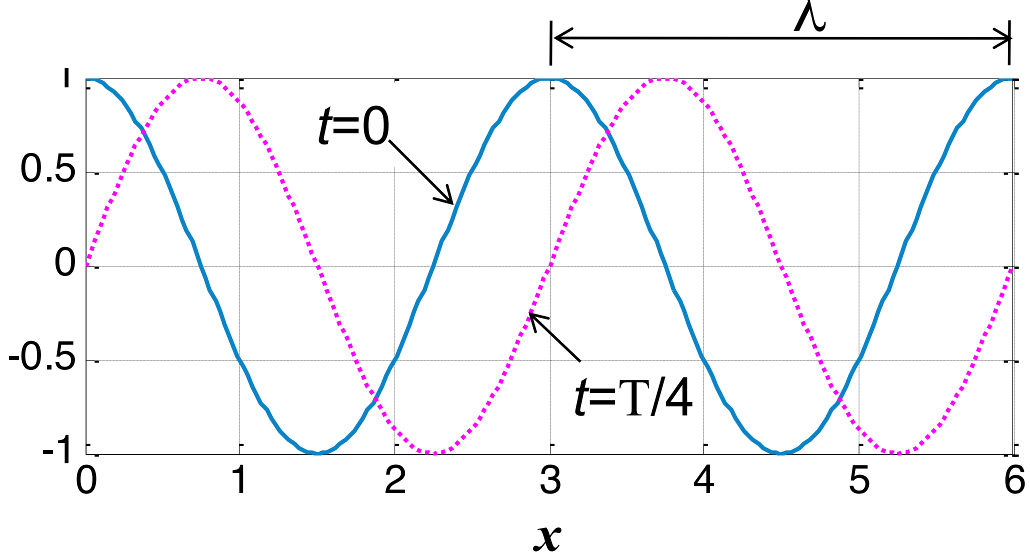
$$s(x,t) = s_0 \cos(\omega t - kx + \phi_0)$$

s_0: amplitude
ω: circular frequency
(the number of cycles in 1s)
k: wave number
(the number of cycles in 1m)
ϕ_0: initial phase

$$\omega = \frac{2\pi}{T} \quad \longrightarrow \text{period}$$

$$k = \frac{2\pi}{\lambda} \quad \longrightarrow \text{wavelength}$$





Propagating Sinusoid Phase Velocity v_p

What is a sinusoid Phase velocity?

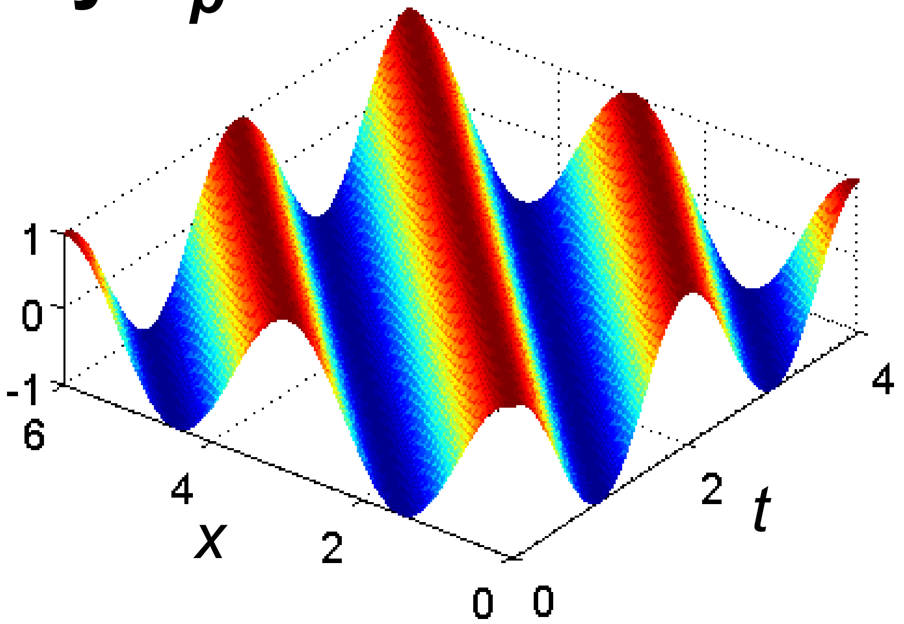
$$s(x,t) = s_0 \cos(\underbrace{\boxed{\omega t - kx}}_{\text{Phase}})$$

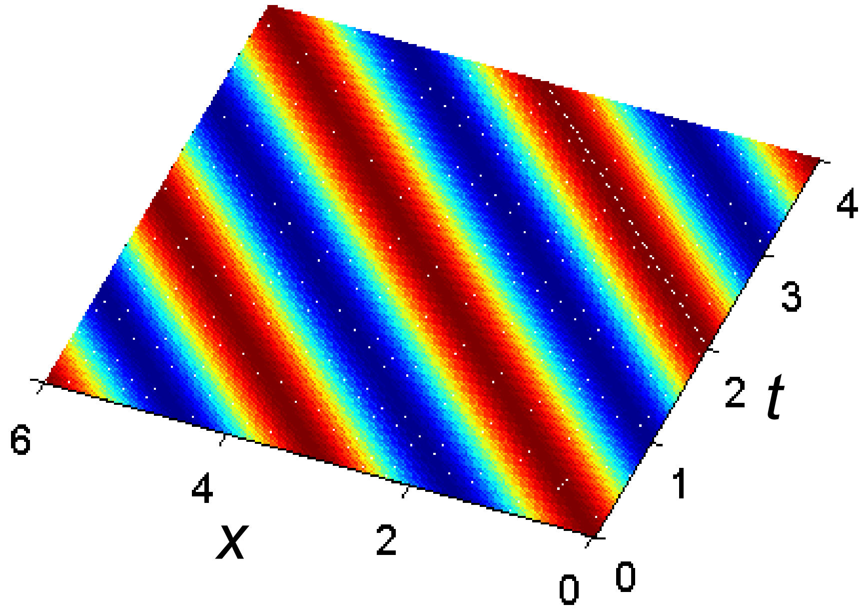
The velocity of the constant phase plane:

$$\omega t - kx = const.$$

$$\frac{dx}{dt} = \frac{\omega}{k} = \frac{\lambda}{T} = v_p$$

$$x = v_p t + const.$$

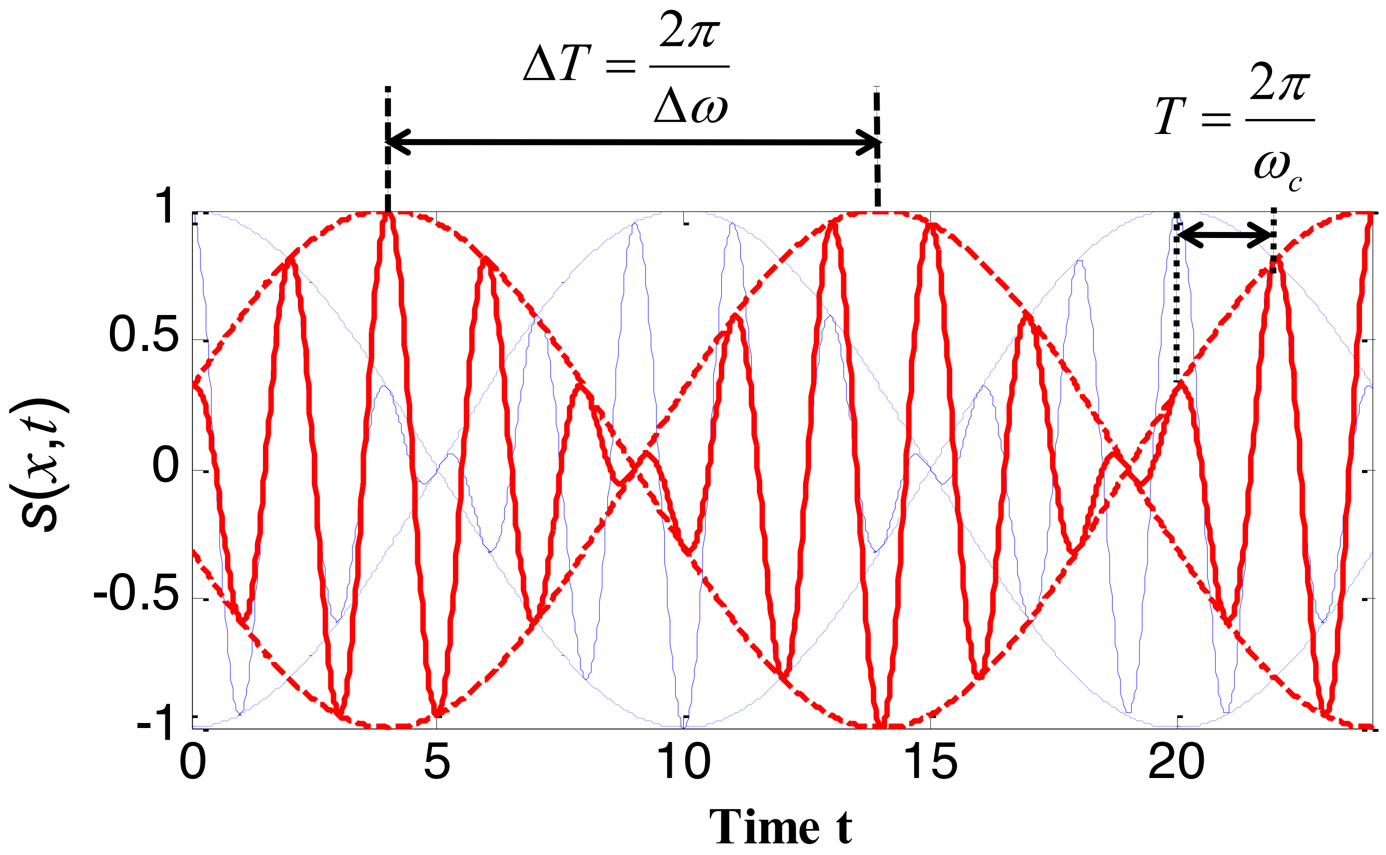




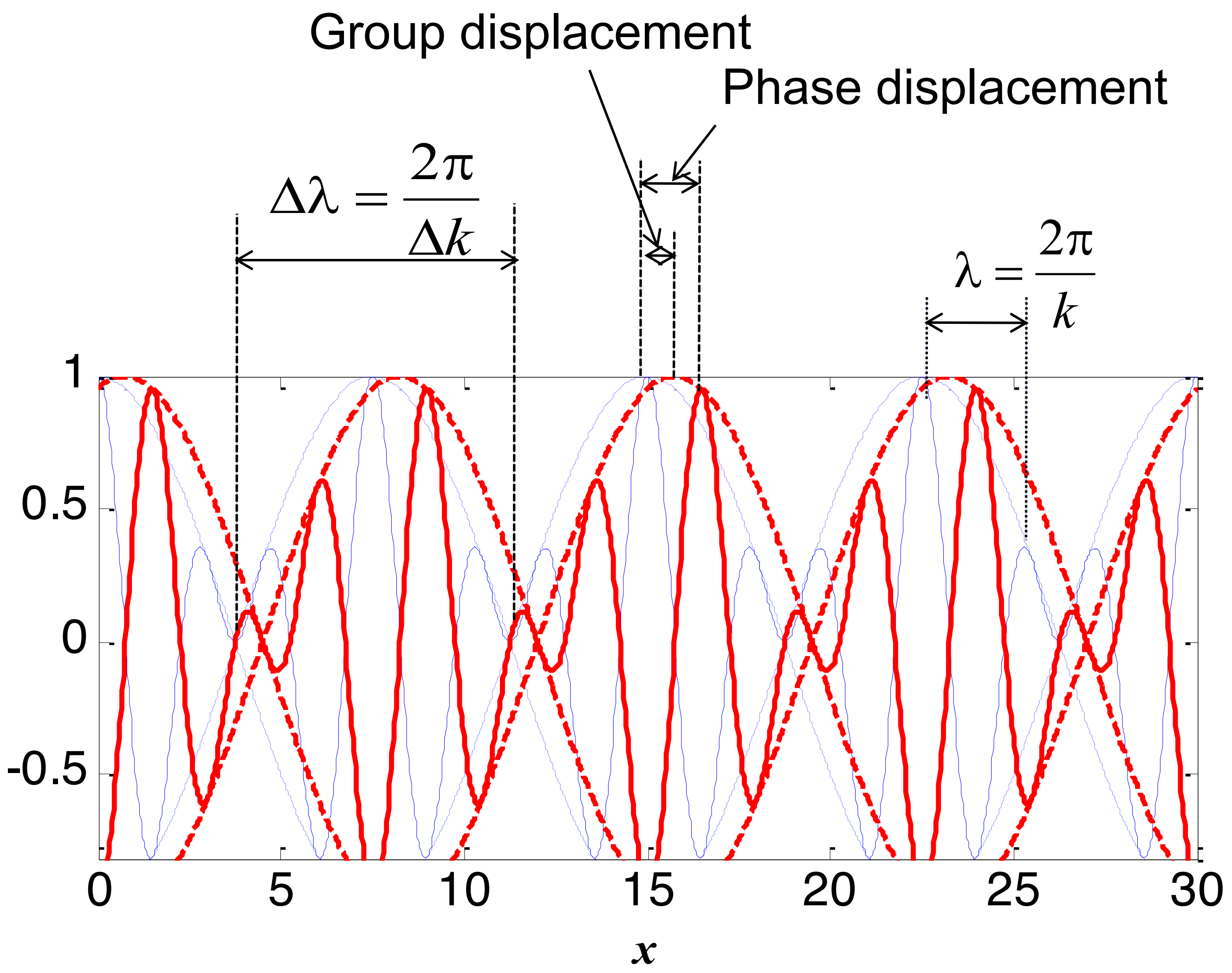
Two Combined Sinusoids at Fixed Point in Space

$$s(x,t) = \cos\big((\omega+\Delta\omega)t - (k+\Delta k)x\big) + \cos\big((\omega-\Delta\omega)t - (k-\Delta k)x\big)$$

$$s(x,t) = \cos(\Delta\omega t - \Delta k x)\cos(\omega t - kx)$$



Two Combined Sinusoids at A Fixed Time Instant



Group Velocity

$$s(x,t) = \boxed{\cos(\Delta\omega t - \Delta k x)} \cos(\omega t - kx)$$

Modulating envelope

Group velocity: *modulating envelop* propagation speed

$(\Delta\omega t - \Delta k x)$ = Constant

$$\frac{dx}{dt} = \frac{\Delta\omega}{\Delta k} = \frac{\Delta\lambda}{\Delta T} = v_g$$

Group and Phase Velocity Relationship

Phase velocity: constant *carrier phase* propagation speed
Group velocity: constant *modulating envelop phase* propagation speed

$$v_p = \frac{\omega}{k} \qquad\qquad v_g = \frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk}$$

Phase and group refractive index:

$$n_p = \frac{c}{v_p}$$

From Appleton-Hartree equation: $n_p = 1 - \dfrac{40.3 n_e}{f^2}$

$$n_g = \frac{c}{v_g} \Rightarrow n_g = n_p + f\frac{dn_p}{df} \Rightarrow n_g = 1 + \frac{40.3 n_e}{f^2}$$

Ionosphere Refractive Index & GPS Signal Propagation

Define: Total Electron Content $TEC = \int_S^R n_e dl$

$$\Delta\tau_p = \frac{1}{c}\int_S^R (n_p - 1) dl = \frac{1}{c}\int_S^R \left(-\frac{40.3 n_e}{f^2}\right) dl = -\frac{40.3}{cf^2} TEC$$

$$\Delta\tau_g = \frac{1}{c}\int_S^R (n_g - 1) dl = \frac{40.3}{cf^2} TEC$$

$$I_p = c\Delta\tau_p = -\frac{40.3}{f^2} TEC \qquad I_g = c\Delta\tau_g = \frac{40.3}{f^2} TEC = -I_p$$

Define 1 TEC unit = 10^{16} electrons/m^2, its corresponding $I_g = -I_p = 0.16$(m)

Obliquity Factor

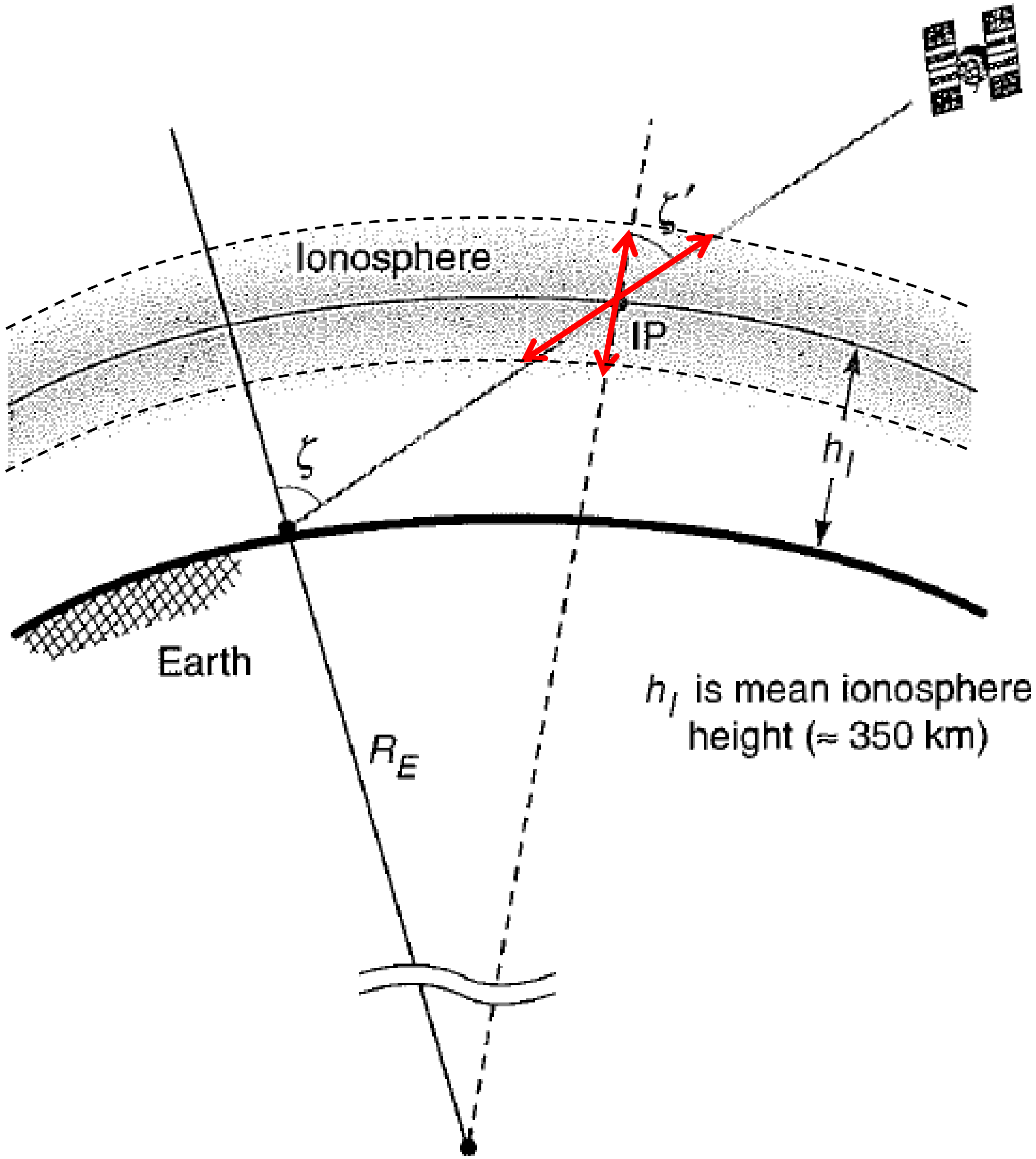
$$TEC(\zeta)=\frac{1}{\cos\zeta'}TEC_{vertical}$$

$$\frac{\sin\zeta}{R_E+h_I}=\frac{\sin\zeta'}{R_E}$$

$$OF_I=\frac{1}{\cos\zeta'}=\frac{1}{\sqrt{1-\left(\frac{R_E\sin\zeta}{R_E+h_I}\right)^2}}$$

$$I(\zeta)=I_z\times OF_I(\zeta)$$

I_z: Zenith delay



Ionosphere Error Correction Broadcast (Klobuchar) Model

$$\frac{I_{L1,z}}{c} = \begin{cases} A_1 + A_2 \cos\left(\frac{2\pi(t - A_3)}{A_4}\right) & if \left|t - A_3\right| < \frac{A_4}{4} \\ A_1 & otherwise \end{cases}$$

t: Local time at IP

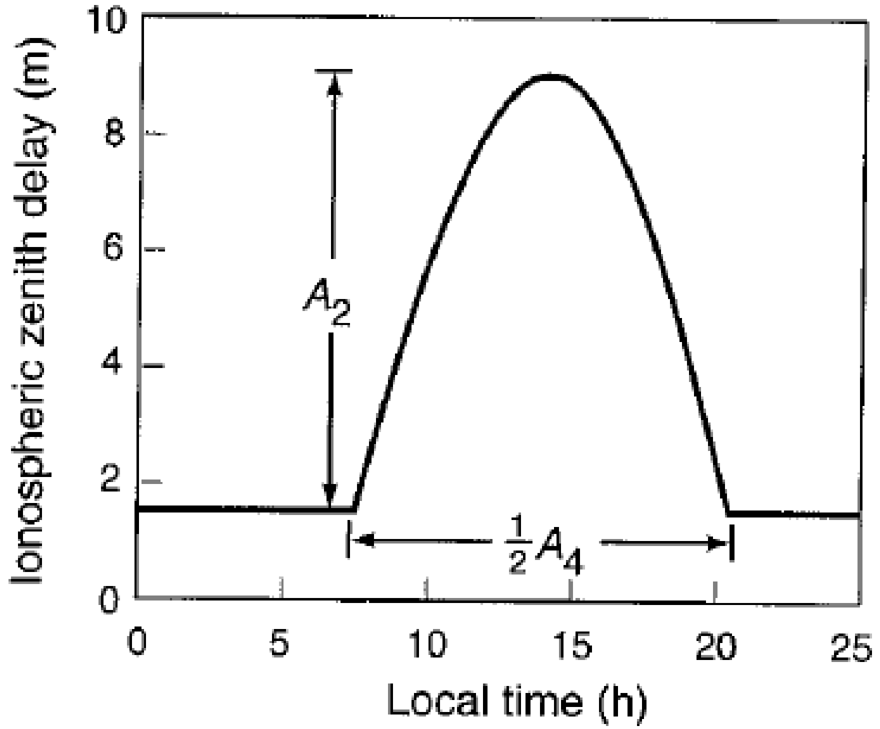
$A_1 = 5\times10^{-9}s$ Nighttime zenith delay

A_3 = 50,400 s (LT) Phase of the cosine peak

$$A_2 = \alpha_0 + \alpha_1\Phi_m + \alpha_2\Phi_m^2 + \alpha_3\Phi_m^3$$

$$A_4 = \beta_0 + \beta_1\Phi_m + \beta_2\Phi_m^2 + \beta_3\Phi_m^3$$

Φ_m: receiver geomagnetic latitude



$$\left.\begin{matrix} \alpha_0, \alpha_1, \alpha_2, \alpha_3 \\ \beta_0, \beta_1, \beta_2, \beta_3 \end{matrix}\right\}$$

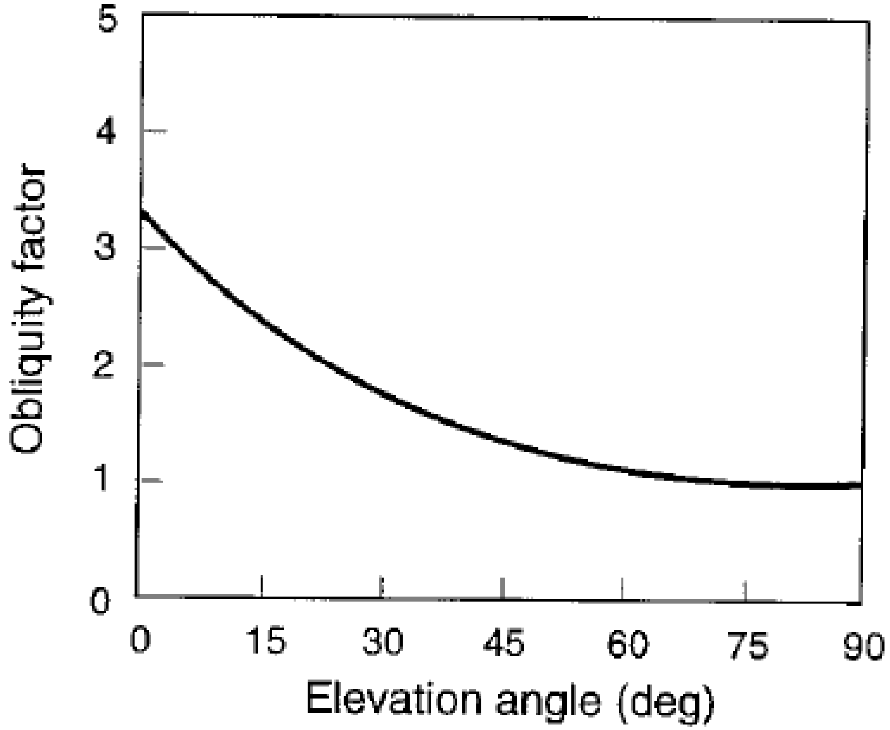
- Broadcasted in the navigation message
- Updated once every 6 days
- Based on ~370 sensors measurements

Obliquity Factor Approximation

$$OF_I = \frac{1}{\cos\zeta'} = \frac{1}{\sqrt{1-\left(\frac{R_E \sin\zeta}{R_E + h_I}\right)^2}}$$

$$\downarrow$$

$$OF_I \approx 1.0 + 16.0 \times (0.53 - el)^3$$



Broadcast model corrects up to 50% ionosphere error

Dual Frequency Receiver Estimation of Absolute TEC

Recall code measurement model: $\rho = r + I_\rho + T + c\left(\delta t_u - \delta t^s\right) + \varepsilon_\rho$

L1: $\rho_1 = r + I_{\rho 1} + T + c\left(\delta t_u - \delta t^s\right) + \varepsilon_{\rho 1} = I_{\rho 1} + \rho_{1IF} \approx I_{\rho 1} + \boxed{\rho_{IF}}$ $\qquad I_{\rho 1} = \dfrac{40.3TEC}{f_1^2}$

L2: $\rho_2 = r + I_{\rho 2} + T + c\left(\delta t_u - \delta t^s\right) + \varepsilon_{\rho 2} = I_{\rho 2} + \rho_{\rho 2IF} \approx I_{\rho 2} + \boxed{\rho_{IF}}$ $\qquad I_{L2} = \dfrac{40.3TEC}{f_{L2}^2}$

ρ_{IF}: ionosphere-free range

We can compute TEC based on ρ_1 and ρ_2:

$$\rho_1 - \rho_2 = I_{\rho 1} - I_{\rho 2} = 40.3TEC\left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right) \Rightarrow TEC = \frac{f_1^2 f_2^2}{40.3\left(f_1^2 - f_2^2\right)}\left(\rho_2 - \rho_1\right)$$

We can also compute ionosphere-free range based on ρ_1 and ρ_2: $\rho_{IF} = \dfrac{f_1^2\rho_1 - f_2^2\rho_2}{f_1^2 - f_2^2}$

Dual Frequency RX Estimation of Relative TEC

Recall carrier measurement model: $\phi = \frac{1}{\lambda}\left(r - I_\rho + T\right) + \frac{c}{\lambda}\left(\delta t_u - \delta t^s\right) + N + \varepsilon_\phi$

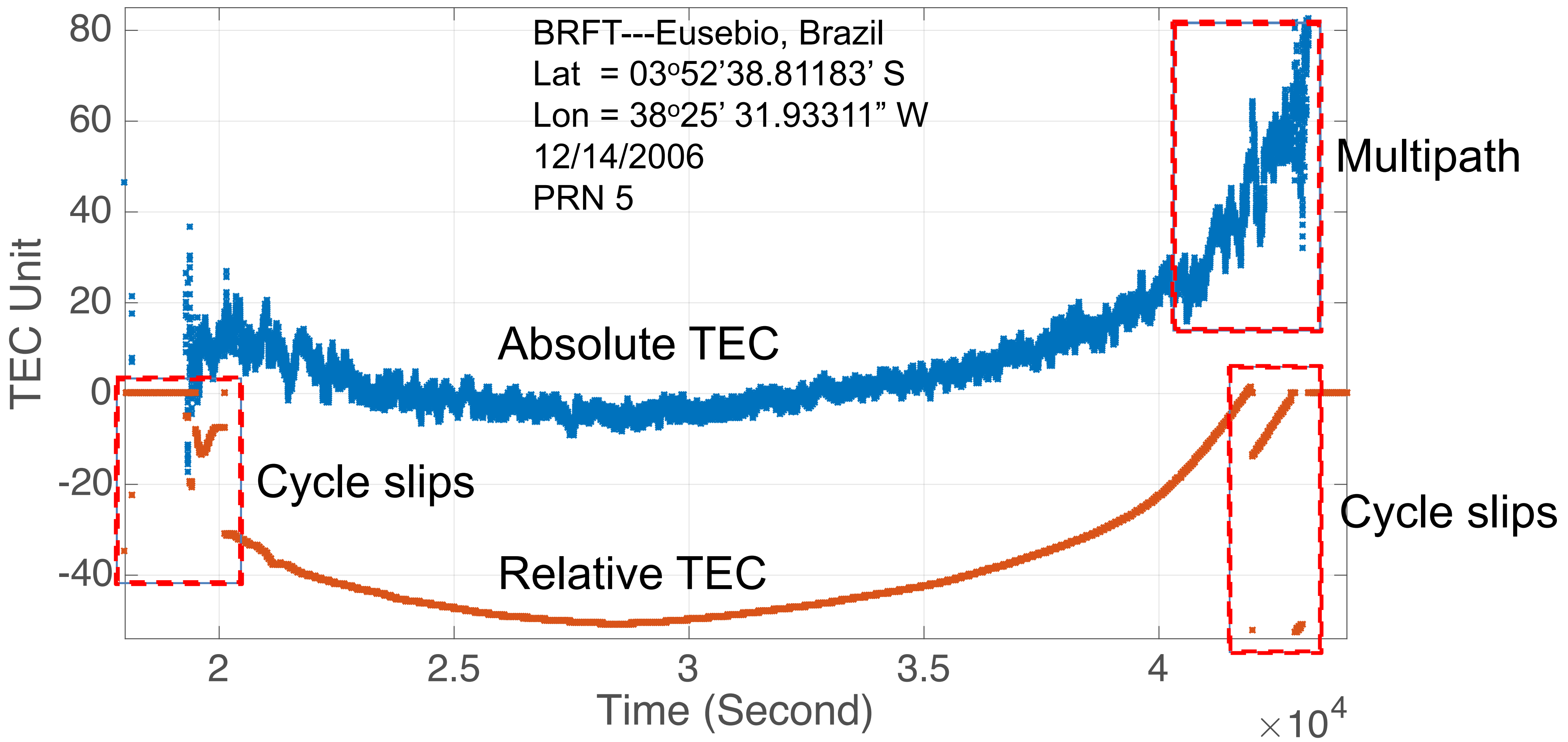
L1: $\phi_1 = -I_{\rho 1} + \phi_{IF} + N_1$

L2: $\phi_2 = -I_{\rho 2} + \phi_{IF} + N_2$

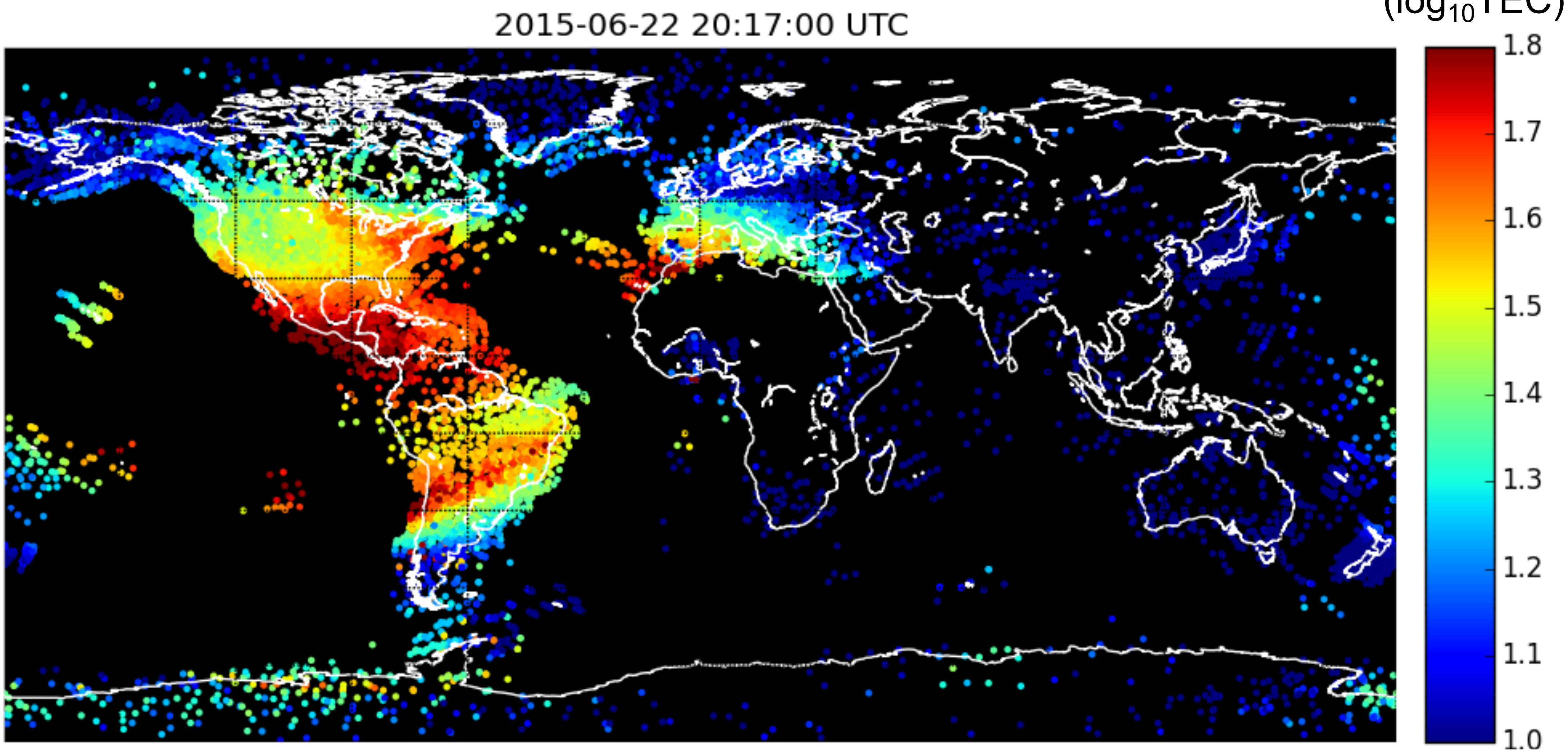
$$TEC = \frac{f_1^2 f_2^2}{40.3\left(f_1^2 - f_2^2\right)}\left(\lambda_1\left(\phi_1 - N_1\right) - \lambda_2\left(\phi_2 - N_2\right)\right)$$

Relative TEC: $\Delta TEC = \frac{f_1^2 f_2^2}{40.3\left(f_1^2 - f_2^2\right)}\left(\lambda_1\phi_1 - \lambda_2\phi_2\right)$

Example Dual Frequency Ionosphere TEC Measurements



Global Vertical TEC Map



Courtesy of Drs. Anthea Coster and Juha Vierinen, Haystack Observatory

Coverage and Homework

- Lecture 18: P157-169
- Lecture 19: Ch. 6
- Project 7: Compute TEC using dual frequency GPS measurements downloaded from IGS station of your choice. Due Date: Monday, 12/4 midnight.