Effects of Labor Mobility on Firm and Country Productivity

Bas Machielsen & Walter Verwer

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1 Introduction

In the three latest decades, the European Union has expanded by granting membership to a large number of countries. At the same time, it has also witnessed an enormous increase in intra-European migration, particularly after the entering into force of the Schengen agreements. The European Union is also set to expand in the future by absorbing some of the Balkan countries. More recently, it is likely that the European countries have experienced *decreases* in intra-European labor mobility, induced by the Covid-19 crisis and accompanying mobility restrictions ranging from requiring negative Covid-19 tests to closing borders.

Similarly, the United States has seen large surges in migration in the last decades, with push-backs back and forth, the latest push-back famously enacted by former president Donald Trump. Elsewhere, there is talk of the African Union implementing a Schengen-like system between various African countries.

All of these trends point to increased cooperation and convergence of national labor markets to supra-national alternatives. To analyze this, we develop a simple general equilibrium model (a special case of the Melitz model), in which we compare situations of separate versus unified labor markets, and contrast the export position and productivity levels of the separate countries with the export position of the unified country.

We show that improving labor mobility likely has welfare-improving consequences, which accrue particularly to high-productivity firms, whereas there might also be important distributional effects, including adverse effects on the wages of (some) local workers. Additionally, we gather data and show that the predictions of the model find support in the data.

2 Model setup

We propose a general equilibrium framework akin to Melitz. We have a representative consumer in each country with CES-preferences indexed by σ. We have a continuum of varieties, and a continuum of producers each producing one variety. Our setting is almost identical to Melitz (2003), in the sense that we consider a CES-consumer and two symmetric countries, but differs in the fact that we analyze two situations with different production function for the producers. In Melitz (2003),

3 Equilibrium without Labor Mobility

In the disunited equilibrium, the Netherlands and Poland can export to the United States (but not to each other), but they cannot use each others labor inputs. Hence, their production function is constrained by $l_F = 0$. Using this assumption, and solving the producers problem yields the following equilibrium price conditions for each good i:

$$p_i = \frac{\sigma}{\sigma - 1} \frac{w_H}{\phi_i} \frac{1}{\alpha} \frac{I \cdot P^{\sigma-1} p_i}{\phi_i}^{\frac{1-\alpha}{\alpha}} \tag{1}$$

This is an implicit equation for p_i. For n varieties, we have $n+1$ unknowns. Normalizing w_H then yields a unique solution for the price vector p_i and consequently, firm profits as a function of p_i.

Then, we consider equilibrium exports (of Home and Foreign to a Third Country).

Assumption: (maybe) We suppose that $w_H > w_F$ and $\phi_{H,i} > \phi_{F,i}$ for all i.

4 Equilibrium with labor mobility

In the equilibrium with international labor mobility, Home can use two sources of labor, one from Home, and one from External, to trade with Foreign. We have defined the production function of firms as a Cobb-Douglas production function with output elasticity α. That is, firms' production is according to:

$$q(\phi) = \phi\left(l_H^{\alpha} \cdot (1 + l_E)^{1-\alpha}\right), \tag{2}$$

where we now allow for positive values of l_E. In this way we allow a firm to choose to hire External labor inputs, which leads to productivity gains through the functional form we imposed. The profit function for firms producing domestically is defined as follows:

$$\pi(\phi) = q(\phi) \cdot p(\phi) - w_H \cdot l_H - w_E \cdot l_E - f \tag{3}$$

Solving the above optimization problem, given consumers' optimal demand, we obtain the following equilibrium price for good i.

$$p_i(\phi_i) = \left(\frac{\sigma}{\sigma - 1}\right)\left[\frac{w_H}{\phi}\left(\frac{(1-\alpha)}{\alpha}\frac{w_H}{w_E}\right)^{1-\alpha} + \frac{w_E}{\phi}\left(\frac{(1-\alpha)}{\alpha}\frac{w_H}{w_E}\right)^{\alpha}\right] \tag{4}$$

From the price equation we can derive the equilibrium conditions. The method of deriving this is similar as we did in section 3. The equilibrium conditions for the endogenous variables $\tilde{\phi}$, ϕ^*, $\tilde{\phi}_x$ and ϕ_x^*, in the case of international labor mobility can be summarized by the following equations:

$$\tilde{\phi} = \left(\int_{\phi^*}^{\bar{\phi}} \phi^{\sigma-1}\mu(\phi)d\phi\right)^{\frac{1}{\sigma-1}} \tag{5}$$

$$\tilde{\phi_x} = \left(\int_{\phi_x^*}^{\bar{\phi}} \phi^{\sigma-1}\mu(\phi)d\phi\right)^{\frac{1}{\sigma-1}} \tag{6}$$

$$(1 - G(\phi^*))\left(\frac{1}{\gamma(\phi^*, \tilde{\phi})} - 1\right) f + (1 - G(\phi_x^*))\left(\frac{1}{\gamma(\phi_x^*, \tilde{\phi}_x)} - 1\right) f_x = \theta f_e \tag{7}$$

$$\gamma(\phi_x^*, \phi^*) = \tau^{\sigma-1}\frac{f_x}{f} \tag{8}$$

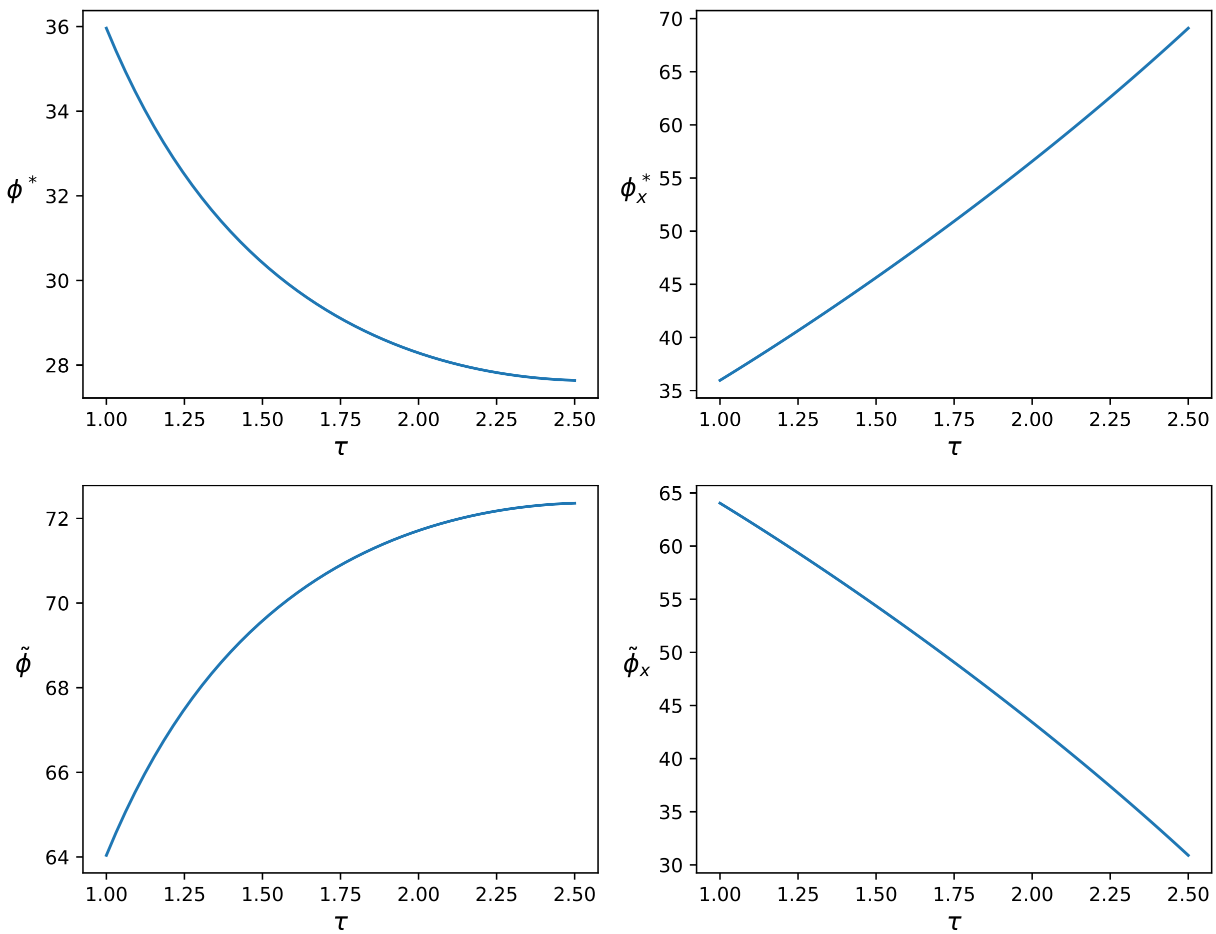

Figure 1: Labor mobility model numerical simulation. Parameters: **FILL IN RIGHT PARAMETERS + VALUES!**

5 Numerical Illustration

5.1 Without International Labor Mobility

5.2 With International Labor Mobility

6 Data

7 Analysis

Appendices

A.1 Full specification of model without labor mobility

A.1.1 Autarchy

There is a representative consumer with CES-utility over a continuum of goods:

$$U = \left(\int_{\omega \in \Omega} q(\omega)^{\rho} d\omega \right)^{\frac{1}{\rho}} \tag{9}$$

Households minimize expenditure per unit utility $U = 1$, which yields the Hicksian demand at $U = 1$ for each variety i:

$$q_i = \left(\int_{\omega \in \Omega} p(\omega)^{\frac{\rho}{\rho-1}} \right)^{-\frac{1}{\rho}} \cdot p_i^{\frac{1}{\rho-1}} \tag{10}$$

The expenditures P per unit utility are then $q \cdot p$, which amounts to:

$$P = \left(\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}} \tag{11}$$

where $\sigma = \frac{1}{1-\rho}$. Since Hicksian demand is homogeneous in U, and in CES utility functions, $I = U \cdot P$, we can reconstruct the Walrasian demand for each variety i as:

$$q_i = I \cdot P^{\sigma-1} \cdot p_i^{-\sigma} \tag{12}$$

There is a continuum of producers, each producing their own variety with production function $q_i = \phi_i \cdot l_H^{\alpha} \cdot (1 + l_E)^{1-\alpha}$. In this first setting, $l_E = 0$, so that $q_i = \phi_i \cdot l_H^{\alpha}$. Producers pick the labor input l_H such that their costs are minimized. Their profit functions are then equal to:

$$\pi_i(q_i) = q_i \cdot p_i - w_N \cdot \left(\frac{q_i}{\phi_i} \right)^{\frac{1}{\alpha}} - f \tag{13}$$

And the first order condition for profit maximization yields:

$$q_i^{-\frac{1}{\sigma}} I^{\frac{1}{\sigma}} P^{\frac{\sigma-1}{\sigma}} = \frac{w_H}{\alpha} \cdot \left(\frac{q_i}{\phi_i} \right)^{\frac{1-\alpha}{\alpha}} \cdot \frac{1}{\phi_i} \tag{14}$$

Solving this equation for q_i, then inserting the mapping from q_i to p_i from the demand side (equation 12) gives us the equilibrium price for each variety:

$$p_i = I^{\frac{1}{\sigma}} P^{\frac{\sigma-1}{\sigma}} Z_i^{-\frac{1}{\sigma}} \tag{15}$$

where:

$$Z_i = \left[\left(\frac{w_H}{\alpha}\right)\phi_i^{-\frac{1}{\alpha}} P^{\frac{1-\sigma}{\sigma}} I^{-\frac{1}{\sigma}}\right]^{\frac{-\alpha}{\sigma(\alpha-1)-\alpha}} \tag{16}$$

Taking this definition of p_i and multiplying it with the definition of q_i in equation 12 yields an expression for the revenues:

$$r(\phi_i) = IP^{\sigma-1}\left(I^{\frac{1}{\sigma}} P^{\frac{\sigma-1}{\sigma}} Z_i\right)^{1-\sigma} = I^{\frac{1}{\sigma}} P^{\frac{(\sigma-1)}{\sigma}} Z^{1-\sigma} \tag{17}$$

The ratio of revenues of producers of two varieties i and j is then:

$$\frac{r(\phi_i)}{r(\phi_j)} = \left(\frac{Z_i}{Z_j}\right)^{1-\sigma} = \left[\left(\frac{\phi_i}{\phi_j}\right)^{\frac{1}{\sigma(\alpha-1)-\alpha}}\right]^{1-\sigma} = \left(\frac{\phi_i}{\phi_j}\right)^{\delta} \tag{18}$$

where $\delta = \frac{(1-\sigma)}{\sigma(\alpha-1)-\alpha}$. We then use this relationship to define the ratio of revenues between the threshold firm and the average firm: $r(\phi^*) = r(\tilde{\phi}) \cdot \left(\frac{\phi^*}{\tilde{\phi}}\right)^{\delta}$. Inserting this in the zero-profit cutoff condition, as in Melitz (2003) yields:

$$r(\tilde{\phi}) = \sigma \cdot f \cdot \left(\frac{\tilde{\phi}}{\phi^*}\right)^{\delta} \tag{19}$$

Substituting this in the free-entry condition gives:

$$(1 - G(\phi^*)) \cdot \left[\left(\frac{\tilde{\phi}}{\phi^*}\right)^{\delta} - 1\right] \cdot f = \theta \cdot f_e \tag{20}$$

Together with:

$$\tilde{\phi} = \left(\int_{\phi^*}^{\bar{\phi}} \mu(\phi^*)\phi^{\sigma-1} d\phi\right)^{\frac{1}{\sigma-1}} \tag{21}$$

[1]

equations 20 and 21 form the equilibrium average and threshold productivity parameters.

[1] $\mu(\phi^*)$ represents the conditional productivity distribution, conditional on the random variable (productivity) ϕ being greater than or equal to ϕ^*.

A.1.2 Free trade

Using parallel definitions to equation 13, the definition of q_i in terms of p_i, and the definition of iceberg costs, we can write the profit of a firm producing a particular variety in the foreign market as follows:

$$\pi_x(\phi_i) = \frac{I_F P_F^{\sigma-1}(p(\phi_i)\cdot\tau)^{1-\sigma}}{\sigma} - f_X \tag{22}$$

from which we can also deduce the zero-cutoff condition for exporting, such that the fraction as a function of ϕ_X^* on the RHS of equation 22 equals f_X.

The relationship between ϕ_X^* and ϕ^* is obtained by dividing the expressions for f_X and f. Solving for the ratio $\frac{\phi_X^*}{\phi^*}$, we obtain:

$$\frac{\phi_X^*}{\phi^*} = \left(\frac{f_X}{f}\right)^{\frac{1}{\delta(1-\sigma)}} \cdot \tau^{-\frac{1}{\delta}} \tag{23}$$

where δ is defined as before.

The free-entry condition in a free-trade environment is now:

$$(1-G(\phi^*))\left(\frac{r(\tilde{\phi})}{\sigma} - f\right) + (1-G(\phi_X^*))\left(\frac{r(\tilde{\phi_X})}{\sigma} - f_x\right) = \theta \cdot f_e \tag{24}$$

After substituting in the zero-cutoff profit conditions for the domestic market and for the foreign market, equation 24 simplifies to:

$$(1-G(\phi^*))\left(f\cdot\left(\frac{\tilde{\phi}}{\phi^*}\right)^{\delta} - f\right) + (1-G(\phi_X^*))\left(f_X\cdot\left(\frac{\tilde{\phi_X}}{\phi_X^*}\right)^{\delta} - f_X\right) = \theta \cdot f_e \tag{25}$$

Together, equations 25, 23, 21 and $\tilde{\phi_X} = \left(\int_{\phi_X^*}^{\bar{\phi}} \mu(\phi_X^*)\phi^{\sigma-1}d\phi\right)^{\frac{1}{\sigma-1}}$ form the equilibrium conditions that determine the domestic and export threshold and average productivity parameters.

A.2 Full specification of model with labor mobility

A.2.1 Autarchy

The demand-side is identical to the demand side with a singe labor input, as in equation 12.

On the supply-side, we again have a continuum of producers, who can now produce according to:

$$q(\phi) = \phi\left(l_H^{\alpha}\cdot(1+l_E)^{1-\alpha}\right), \tag{26}$$

Input efficiency and profit maximization leads to the following equilibrium price for variety i:

$$p_i(\phi_i) = \left(\frac{\sigma}{\sigma-1}\right)\left[\frac{w_H}{\phi}\left(\frac{(1-\alpha)}{\alpha}\frac{w_H}{w_E}\right)^{1-\alpha} + \frac{w_E}{\phi}\left(\frac{(1-\alpha)}{\alpha}\frac{w_H}{w_E}\right)^{\alpha}\right] \tag{27}$$

The revenues for a firm producing variety i is then equal to:

$$\begin{aligned} r(\phi_i) &= I \cdot P^{\sigma-1} \cdot p(\phi_1)^{1-\sigma} \\ &= I \cdot P^{\sigma-1} \cdot \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \cdot \left[\left(\frac{w_H}{\phi_i}\left[\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{w_H}{w_E}\right)\right]^{1-\alpha}\right) + \left(\frac{w_E}{\phi_i}\left[\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{w_H}{w_E}\right)\right]^{\alpha}\right)\right]^{1-\sigma} \end{aligned} \tag{28}$$

The ratio of revenues between firm i and firm j is then:

$$\frac{r(\phi_i)}{r(\phi_j)} = \frac{\left[\left(\frac{w_H}{\phi_i}\left[\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{w_H}{w_E}\right)\right]^{1-\alpha}\right) + \left(\frac{w_E}{\phi_i}\left[\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{w_H}{w_E}\right)\right]^{\alpha}\right)\right]^{1-\sigma}}{\left[\left(\frac{w_H}{\phi_j}\left[\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{w_H}{w_E}\right)\right]^{1-\alpha}\right) + \left(\frac{w_E}{\phi_j}\left[\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{w_H}{w_E}\right)\right]^{\alpha}\right)\right]^{1-\sigma}} \hat{=} \gamma_{i,j} \tag{29}$$

The canceling out of various terms also means that $\gamma = \left(\frac{p(\phi_i)}{p(\phi_j)}\right)^{1-\sigma}$. Similar to equation 19, we can use the ratio of revenues to formulate the revenues of the average firm as a function of the revenues of the threshold firm:

$$r(\tilde{\phi}) = \frac{r(\phi^*)}{\gamma_{\phi^*,\tilde{\phi}}} \tag{30}$$

Rewriting this as a function of $r(\phi^*)$, and then substituting this in the zero-profit condition, we obtain that:

$$r(\tilde{\phi}) = \frac{f \cdot \sigma}{\gamma_{\phi^*,\tilde{\phi}}} \tag{31}$$

Finally, substituting this in the free-entry condition, we find that:

$$(1 - G(\phi^*))\left(\frac{1}{\gamma_{\phi^*,\tilde{\phi}}} - 1\right) \cdot f = \theta \cdot f_e \tag{32}$$

Together with 21, these are the autarchic equilibrium conditions.

A.2.2 Free trade

The free trade discussion closely parallels the free trade discussion without labor mobility. In this case, the relationship between ϕ_X^* and ϕ^* in 23 is characterized by a more complicated expression, which we will not express analytically:

$$\tau^{\sigma-1}\frac{f_X}{f} = \gamma_{\phi^*,\tilde{\phi}} \tag{33}$$

After using expression 31 and its counterpart for exporting firms, and substituting these in the free entry condition, we obtain the following:

$$(1-G(\phi^*))\left(\frac{1}{\gamma_{\phi^*,\tilde{\phi}}}-1\right)\cdot f + (1-G(\phi_X^*))\cdot\left(\frac{1}{\gamma_{\phi_X^*,\tilde{\phi}_X}}-1\right)\cdot f_X = \theta\cdot f_e \tag{34}$$

Then, equations 33, 21, $\tilde{\phi_X} = \left(\int_{\phi_X^*}^{\bar{\phi}} \mu(\phi)\phi^{\sigma-1}d\phi\right)^{\frac{1}{\sigma-1}}$, and equation 34 form the equilibrium conditions that determine the threshold and average productivity parameters for exporting and non-exporting firms when they can also use external labor.