# Effects of Labor Mobility on Firm and Country Productivity

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## 1 Introduction

In the three latest decades, the European Union has expanded by granting membership to a large number of countries. At the same time, it has also witnessed an enormous increase in intra-European labor mobility, particularly after the entering into force of the Schengen agreement. [1] The European Union is also set to expand in the future by absorbing some of the Balkan countries. More recently, it is likely that the European countries have experienced *decreases* in intra-European labor mobility, induced by the Covid-19 crisis and accompanying mobility restrictions ranging from requiring negative Covid-19 tests to closing borders.[2] Similarly, the United States have seen large surges in migration in the last decades, but also push-backs, the latest famously enacted by former president Donald Trump. [3] Elsewhere, there is talk of the African Union implementing a Schengen-like system between various African countries.[4]

All of these trends point to increased cooperation and convergence of national labor markets to supranational alternatives, but recent times have shown that this is fragile. To analyze the effect of labor mobility, we develop a simple general equilibrium model (a special case of the Melitz model), in which we compare situations of separate versus unified labor markets, and contrast the export position and productivity levels of a country being able to use only domestic labor, with the same country being able to use both domestic and external labor.

## 2 Model setup

We propose a general equilibrium framework akin to Melitz (2003). We have a representative consumer in each country with CES-preferences indexed by $\sigma$. We have a continuum of varieties, and a continuum of producers each producing one variety. Our setting is almost identical to Melitz (2003), in the sense that we consider a CES-consumer and two symmetric countries, but differs in the fact that we analyze two situations, one in which we impose no international labor mobility, and the second in which we do not. We consider a Cobb-Douglas production function of the following type:

$$q(\phi) = \phi \cdot l_H^{\alpha} \cdot (1 + l_E)^{1-\alpha}. \tag{1}$$

In the above, $l_H$ refers to labor input from the home country, and $l_E$ is external labor input. The external labor input can be seen as labor that comes from some exogenous source, that can capture international labor mobility. We abstract from labor market-related general equilibrium effects, because the focus lies on the availability of labor and the consequences for producers.

[1] European Commission, 2014, accessed on June 18, 2021

[2] Deutsche Welle, 2021, last accessed on June 25, 2021

[3] BBC, 2020, last accessed June 25, 2021

[4] The Independent, 2016, last accessed on June 25, 2021

## 2.1 Equilibrium without Labor Mobility

In order to analyze a situation without international labor mobility, we take equation 1, and impose $l_E = 0$. Hence, the production function under no international labor mobility takes the following form:

$$q(\phi) = \phi \cdot l_H^{\alpha}. \tag{2}$$

In this situation it can be observed that firms can only hire domestic workers. Using the standard Melitz framework, and the modified production function, we obtain the equilibrium conditions postulated below.

$$\tilde{\phi} = \left( \int_{\phi^*}^{\bar{\phi}} \mu(\phi^*)\phi^{\sigma-1} d\phi \right)^{\frac{1}{\sigma-1}} \tag{3}$$

$$\tilde{\phi_X} = \left( \int_{\phi_X^*}^{\bar{\phi}} \mu(\phi_X^*)\phi^{\sigma-1} d\phi \right)^{\frac{1}{\sigma-1}} \tag{4}$$

$$(1 - G(\phi^*)) \left( \left( \frac{\tilde{\phi}}{\phi^*} \right)^{\partial} - 1 \right) f + (1 - G(\phi_X^*)) \left( \left( \frac{\tilde{\phi}_X}{\phi_X^*} \right)^{\partial} - 1 \right) f_X = \theta \cdot f_e \tag{5}$$

$$\frac{\phi_X^*}{\phi^*} = \left( \frac{f_X}{f} \right)^{\frac{1}{\partial(1-\sigma)}} \cdot \tau^{-\frac{1}{\partial}} \tag{6}$$

In the above, we have defined $\partial = \frac{(1-\sigma)}{\sigma(\alpha-1)-\alpha}$. The derivation of the equilibrium conditions can be found in the appendix.

## 2.2 Equilibrium with labor mobility

Now for the situation in which we allow for international labor mobility, we allow for $l_E \geq 0$. In this way we allow a firm to choose to hire external labor inputs, which leads to productivity gains through the functional form we imposed. The equilibrium conditions for the endogenous variables $\tilde{\phi}$, $\tilde{\phi}_x$, $\phi^*$, and $\phi_x^*$, in the case of international labor mobility can be summarized by the following equations:

$$\tilde{\phi} = \left( \int_{\phi^*}^{\bar{\phi}} \phi^{\sigma-1} \mu(\phi) d\phi \right)^{\frac{1}{\sigma-1}} \tag{7}$$

$$\tilde{\phi}_X = \left( \int_{\phi_X^*}^{\bar{\phi}} \phi^{\sigma-1} \mu(\phi) d\phi \right)^{\frac{1}{\sigma-1}} \tag{8}$$

$$(1 - G(\phi^*)) \left( \frac{r(\tilde{\phi})}{r(\phi^*)} - 1 \right) f + (1 - G(\phi_X^*)) \left( \frac{r(\tilde{\phi}_X)}{r(\phi_X^*)} - 1 \right) f_X = \theta f_e \tag{9}$$

$$\frac{r(\phi_X^*)}{r(\phi^*)} = \frac{f_X}{f} \tau^{\sigma-1} \tag{10}$$

Again, the derivation of the equilibrium conditions can be found in the appendix.

## 3 Analysis

In order to analyze the effect of increased international labor mobility, we will compare the two situations with numerical solutions based on changes in $\tau$ [5]. We do this since our model is most sensitive to changes in $\tau$. The range of $\tau$ chosen is around 1.3, which is based on the work of Felbermayr et al. (2013) for U.S. data.

Looking at figure 1, the first hypothesis generated by our model is that $\Delta\tilde{\phi} = \tilde{\phi}_{LM} - \tilde{\phi}_{WLM} > 0$. The interpretation of this finding is that the average productivity of a firm that is able to hire external workers is higher than the average productivity of a firm that is unable to do so. Relating this finding to for example the Schengen agreement, as a consequence of the policy we expect an increase in the average productivity of firms.

The second hypothesis that follows from our model is that an increase in international labor mobility leads to an increase in the average productivity of exporting firms for plausible values of $\tau$. If $\tau$ becomes excessively large, then we observe a decrease in average productivity of exporting firms. That is, there is a negative relationship between $\tau$ and $\Delta\tilde{\phi}_X$.

The mechanism leading to an increase in the threshold (and thus average) productivity parameters, both for domestic and exporting firms, is the following: compared to a non-labor mobility situation, all firms can now pick a better input mix, but firms with a higher $\phi$ have a higher productivity gain than firms with a lower $\phi$, which can be seen by taking the first derivative of the production function with respect to the inputs. This means that a unit of (home or external) labor is *more* productive for firms with a higher $\phi$. In equilibrium, with the same demand, that means that prices will drop for each variety, and that the firm at the threshold for exporting, and also at the threshold for producing rather than exiting, will decide not to export (respectively, produce) anymore.

A corollary of this mechanism is our third hypothesis: since the gains in productivity from the availability of external labor are asymmetrically distributed, that is, skewed towards firms with a higher $\phi$, we establish that an increase in international labor mobility leads to a decrease in the share of exporting firms. We established that both thresholds increase, but since the gains in productivity are greater for high-$\phi$ firms, the firms at the lower-end of the productivity distribution face more firms that stand to benefit more from labor mobility, than firms at the upper-end of the productivity distribution. Hence, these firms face more competition than firms at the upper-end of the productivity distribution, hence, the lower (i.e. domestic) threshold will increase more. Furthermore, this relationship is amplified by an increase in $\tau$.

Our fourth and final hypothesis is that the increase in average productivity is smaller for exporting firms than for domestically producing firms. This follows from our previous stated result that the gains from increased labor mobility accrue more to the firms with a higher $\phi$. Hence, we expect to observe that the threshold for exporting increases less than the threshold for producing domestically. Therefore, this implies that the average productivity increases more for domestically producing firms that for exporting firms.

[5] The code for all our simulations is available on https://github.com/walterverwer/melitz_simulations

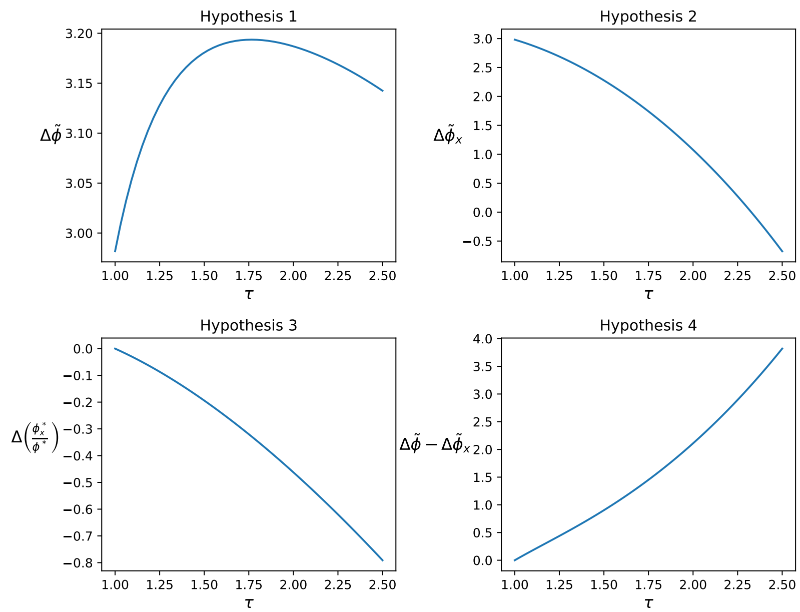


**Figure 1:** Graph containing the hypotheses. We use $\Delta$ to indicate the situation with labor mobility minus without labor mobility. Parameters: $\phi \sim U(0, 100), f = 1, f_x = 1, f_e = 2, \theta = 0.5, \sigma = 2, \alpha = 0.8, \tau \in [1, 2.5], w_H = 2$, and $w_E = 1$.

## 4 Conclusion

This essay attempted to build on the Melitz (2003) framework to analyze the consequences for two symmetric economies of increased labor mobility, modeled as the opportunity to utilize external labor when trading with another country in monopolistic competition. Comparing a free-trade equilibrium without labor mobility to a framework with labor mobility, this model yields several testable hypotheses: we find that the average productivity of both exporting and non-exporting firms increases, the share of exporting firms decreases, and the increase in average productivity is higher for non-exporting firms than for exporting firms.

Finally, further theoretical research can be conducted by focusing more explicitly on the role of labor markets and explicit migration-decisions in this process. In our model, labor is exogenous and disconnected from the income of the representative consumer. Secondly, labor markets are implicitly supposed to be perfectly competitive. The endogenization of labor would lead to consumer income depending on supplied labor, which would complicate the standard Melitz set-up. A more accurate model of competition on the labor market and a model involving the decision to migrate could further increase realism and make the gains to the availability of external labor contingent on decision-making by external laborers.

## References

Felbermayr, G., Jung, B., & Larch, M. (2013). Optimal tariffs, retaliation, and the welfare loss from tariff wars in the melitz model. *Journal of International Economics*, *89*(1), 13–25. https://doi.org/10.1016/j.jinteco.2012.06.001

Melitz, M. J. (2003). The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity. *Econometrica*, *71*(6), 1695–1725. https://ideas.repec.org/a/ecm/emetrp/v71y2003i6p1695-1725.html

# Appendices

## A.1 Full specification of model without labor mobility

### A.1.1 Autarchy

There is a representative consumer with CES-utility over a continuum of goods:

$$U = \left( \int_{\omega \in \Omega} q(\omega)^{\rho} d\omega \right)^{\frac{1}{\rho}} \tag{11}$$

Households minimize expenditure per unit utility $U = 1$, which yields the Hicksian demand at $U = 1$ for each variety $i$:

$$q_i = \left( \int_{\omega \in \Omega} p(\omega)^{\frac{\rho}{\rho-1}} \right)^{-\frac{1}{\rho}} \cdot p_i^{\frac{1}{\rho-1}} \tag{12}$$

The expenditures $P$ per unit utility are then $q \cdot p$, which amounts to:

$$P = \left( \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}} \tag{13}$$

where $\sigma = \frac{1}{1-\rho}$. Since Hicksian demand is homogeneous in $U$, and in CES utility functions, $I = U \cdot P$, we can reconstruct the Walrasian demand for each variety $i$ as:

$$q_i = I \cdot P^{\sigma-1} \cdot p_i^{-\sigma} \tag{14}$$

There is a continuum of producers, each producing their own variety with production function $q_i = \phi_i \cdot l_H^{\alpha} \cdot (1 + l_E)^{1-\alpha}$. In this first setting, $l_E = 0$, so that $q_i = \phi_i \cdot l_H^{\alpha}$. Producers pick the labor input $l_H$ such that their costs are minimized. Their profit functions are then equal to:

$$\pi_i(q_i) = q_i \cdot p_i - w_N \cdot \left( \frac{q_i}{\phi_i} \right)^{\frac{1}{\alpha}} - f \tag{15}$$

Differentiating this w.r.t. $q_i$, the first order condition for profit maximization yields:

$$q_i^{-\frac{1}{\sigma}} I^{\frac{1}{\sigma}} P^{\frac{\sigma-1}{\sigma}} = \frac{w_H}{\alpha} \cdot \left( \frac{q_i}{\phi_i} \right)^{\frac{1-\alpha}{\alpha}} \cdot \frac{1}{\phi_i} \tag{16}$$

Then inserting the mapping from $q_i$ to $p_i$ from the demand side (equation 14), and solving this equation for $p_i$, gives us the equilibrium price for each variety:

$$p_i = I^{\frac{1}{\sigma}} P^{\frac{\sigma-1}{\sigma}} Z_i^{-\frac{1}{\sigma}} \tag{17}$$

where:

$$Z_i = \left[ \left( \frac{w_H}{\alpha} \right) \phi_i^{-\frac{1}{\alpha}} P^{\frac{1-\sigma}{\sigma}} I^{-\frac{1}{\sigma}} \right]^{\frac{-\alpha}{\sigma(\alpha-1)-\alpha}} \tag{18}$$

Taking this definition of $p_i$ and multiplying it with the definition of $q_i$ in equation 14 yields an expression for the revenues:

$$r(\phi_i) = IP^{\sigma-1} \left( I^{\frac{1}{\sigma}} P^{\frac{\sigma-1}{\sigma}} Z_i \right)^{1-\sigma} = I^{\frac{1}{\sigma}} P^{\frac{(\sigma-1)}{\sigma}} Z^{1-\sigma} \tag{19}$$

The ratio of revenues of producers of two varieties $i$ and $j$ is then:

$$\frac{r(\phi_i)}{r(\phi_j)} = \left(\frac{Z_i}{Z_j}\right)^{1-\sigma} = \left[\left(\frac{\phi_i}{\phi_j}\right)^{\frac{1}{\sigma(\alpha-1)-\alpha}}\right]^{1-\sigma} = \left(\frac{\phi_i}{\phi_j}\right)^{\delta} \tag{20}$$

where $\delta = \frac{(1-\sigma)}{\sigma(\alpha-1)-\alpha}$. We then use this relationship to define the ratio of revenues between the threshold firm and the average firm: $r(\phi^*) = r(\tilde{\phi}) \cdot \left(\frac{\phi^*}{\tilde{\phi}}\right)^{\delta}$. Inserting this in the zero-profit cutoff condition, as in Melitz (2003) yields:

$$r(\tilde{\phi}) = \sigma \cdot f \cdot \left(\frac{\tilde{\phi}}{\phi^*}\right)^{\delta} \tag{21}$$

Substituting this in the free-entry condition gives:

$$(1 - G(\phi^*)) \cdot \left[\left(\frac{\tilde{\phi}}{\phi^*}\right)^{\delta} - 1\right] \cdot f = \theta \cdot f_e \tag{22}$$

Together with:

$$\tilde{\phi} = \left(\int_{\phi^*}^{\tilde{\phi}} \mu(\phi^*)\phi^{\sigma-1} d\phi\right)^{\frac{1}{\sigma-1}} \tag{23}$$

[6]

equations 22 and 23 form the equilibrium average and threshold productivity parameters.

### A.1.2 Free trade

Using parallel definitions to equation 15, the definition of $q_i$ in terms of $p_i$, and the definition of iceberg costs, we can write the profit of a firm producing a particular variety in the foreign market as follows:

$$\pi_x(\phi_i) = \frac{I_F P_F^{\sigma-1}(p(\phi_i) \cdot \tau)^{1-\sigma}}{\sigma} - f_X \tag{24}$$

from which we can also deduce the zero-cutoff condition for exporting, such that the fraction as a function of $\phi_X^*$ on the RHS of equation 24 equals $f_X$.

The relationship between $\phi_X^*$ and $\phi^*$ is obtained by dividing the expressions for $f_X$ and $f$. Solving for the ratio $\frac{\phi_X^*}{\phi^*}$, we obtain:

$$\frac{\phi_X^*}{\phi^*} = \left(\frac{f_X}{f}\right)^{\frac{1}{\delta(1-\sigma)}} \cdot \tau^{-\frac{1}{\delta}} \tag{25}$$

where $\delta$ is defined as before.

The free-entry condition in a free-trade environment is now:

$$(1 - G(\phi^*))\left(\frac{r(\tilde{\phi})}{\sigma} - f\right) + (1 - G(\phi_X^*))\left(\frac{r(\tilde{\phi_X})}{\sigma} - f_x\right) = \theta \cdot f_e \tag{26}$$

After substituting in the zero-cutoff profit conditions for the domestic market and for the foreign market, equation 26 simplifies to:

[6] $\mu(\phi^*)$ represents the conditional productivity distribution, conditional on the random variable (productivity) $\phi$ being greater than or equal to $\phi^*$.

$$(1-G(\phi^*))\left(f\cdot\left(\frac{\tilde{\phi}}{\phi^*}\right)^{\delta}-f\right)+(1-G(\phi_X^*))\left(f_X\cdot\left(\frac{\tilde{\phi_X}}{\phi_X^*}\right)^{\delta}-f_X\right)=\theta\cdot f_e \tag{27}$$

Together, equations 27, 25, 23 and $\tilde{\phi_X}=\left(\int_{\phi_X^*}^{\tilde{\phi}}\mu(\phi_X^*)\phi^{\sigma-1}d\phi\right)^{\frac{1}{\sigma-1}}$ form the equilibrium conditions that determine the domestic and export threshold and average productivity parameters.

## A.2 Full specification of model with labor mobility

### A.2.1 Autarchy

The demand-side is identical to the demand side with a singe labor input, as in equation 14.

On the supply-side, we again have a continuum of producers, who can now produce according to:

$$q(\phi)=\phi\left(l_H^{\alpha}\cdot(1+l_E)^{1-\alpha}\right), \tag{28}$$

Input efficiency and profit maximization leads to the following equilibrium price for variety $i$:

$$p_i(\phi_i)=\left(\frac{\sigma}{\sigma-1}\right)\left[\frac{w_H}{\phi}\left(\frac{(1-\alpha)}{\alpha}\frac{w_H}{w_E}\right)^{1-\alpha}+\frac{w_E}{\phi}\left(\frac{(1-\alpha)}{\alpha}\frac{w_H}{w_E}\right)^{\alpha}\right] \tag{29}$$

The revenues for a firm producing variety $i$ is then equal to:

$$r(\phi_i)=I\cdot P^{\sigma-1}\cdot p(\phi_i)^{1-\sigma} \tag{30}$$

$$=I\cdot P^{\sigma-1}\cdot\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\cdot\left[\left(\frac{w_H}{\phi_i}\left[\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{w_H}{w_E}\right)\right]^{\alpha-1}\right)+\left(\frac{w_E}{\phi_i}\left[\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{w_H}{w_E}\right)\right]^{\alpha}\right)\right]^{1-\sigma} \tag{31}$$

The ratio of revenues between firm $i$ and firm $j$ is then:

$$\frac{r(\phi_i)}{r(\phi_j)}=\frac{\left[\left(\frac{w_H}{\phi_i}\left[\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{w_H}{w_E}\right)\right]^{\alpha-1}\right)+\left(\frac{w_E}{\phi_i}\left[\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{w_H}{w_E}\right)\right]^{\alpha}\right)\right]^{1-\sigma}}{\left[\left(\frac{w_H}{\phi_j}\left[\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{w_H}{w_E}\right)\right]^{\alpha-1}\right)+\left(\frac{w_E}{\phi_j}\left[\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{w_H}{w_E}\right)\right]^{\alpha}\right)\right]^{1-\sigma}}\coloneqq\gamma(\phi_i,\phi_j) \tag{32}$$

The canceling out of various terms also means that $\gamma(\phi_i,\phi_j)=\left(\frac{p(\phi_i)}{p(\phi_j)}\right)^{1-\sigma}$. Similar to equation 21, we can use the ratio of revenues to formulate the revenues of the average firm as a function of the revenues of the threshold firm:

$$r(\tilde{\phi})=\frac{r(\phi^*)}{\gamma(\phi^*,\tilde{\phi})} \tag{33}$$

Rewriting this as a function of $r(\phi^*)$, and then substituting this in the zero-profit condition, we obtain that:

$$r(\tilde{\phi})=\frac{f\cdot\sigma}{\gamma(\phi^*,\tilde{\phi})} \tag{34}$$

Finally, substituting this in the free-entry condition, we find that:

$$(1-G(\phi^*))\left(\frac{1}{\gamma(\phi^*,\tilde{\phi})}-1\right)\cdot f=\theta\cdot f_e \tag{35}$$

Together with 23, these are the autarchic equilibrium conditions.

### A.2.2 Free trade

The free trade discussion closely parallels the free trade discussion without labor mobility. In this case, the relationship between $\phi_X^*$ and $\phi^*$ in equation 25 is characterized by a more complicated expression, which we will not express analytically:

$$\tau^{\sigma-1}\frac{f_X}{f} = \gamma(\phi^*, \tilde{\phi}) \tag{36}$$

After using expression 34 and its counterpart for exporting firms, and substituting these in the free entry condition, we obtain the following:

$$(1 - G(\phi^*))\left(\frac{1}{\gamma(\phi^*, \tilde{\phi})} - 1\right) \cdot f + (1 - G(\phi_X^*)) \cdot \left(\frac{1}{\gamma(\phi_X^*, \tilde{\phi}_X)} - 1\right) \cdot f_X = \theta \cdot f_e \tag{37}$$

Then, equations 36, 23, $\tilde{\phi_X} = \left(\int_{\phi_X^*}^{\tilde{\phi}} \mu(\phi_X^*)\phi^{\sigma-1} d\phi\right)^{\frac{1}{\sigma-1}}$, and equation 37 form the equilibrium conditions that determine the threshold and average productivity parameters for exporting and non-exporting firms when they can also use external labor.

## A.3 Numerical Illustrations

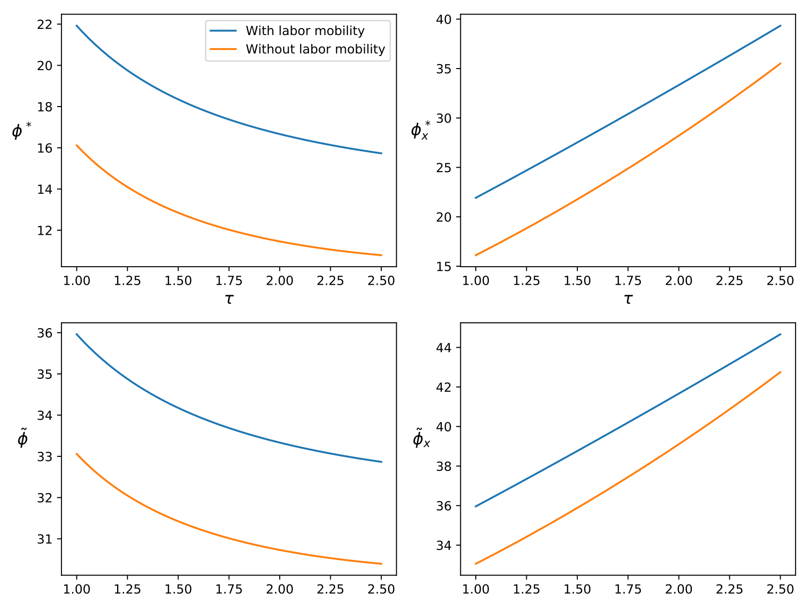


**Figure 2:** The effect of a change in $\tau$ for the model with and without international labor mobility. Parameters: $f = 1, f_x = 1, f_e = 2, \theta = 0.5, \sigma = 2, \alpha = 0.8, \tau \in [1, 2.5], w_H = 2$, and $w_E = 1$.

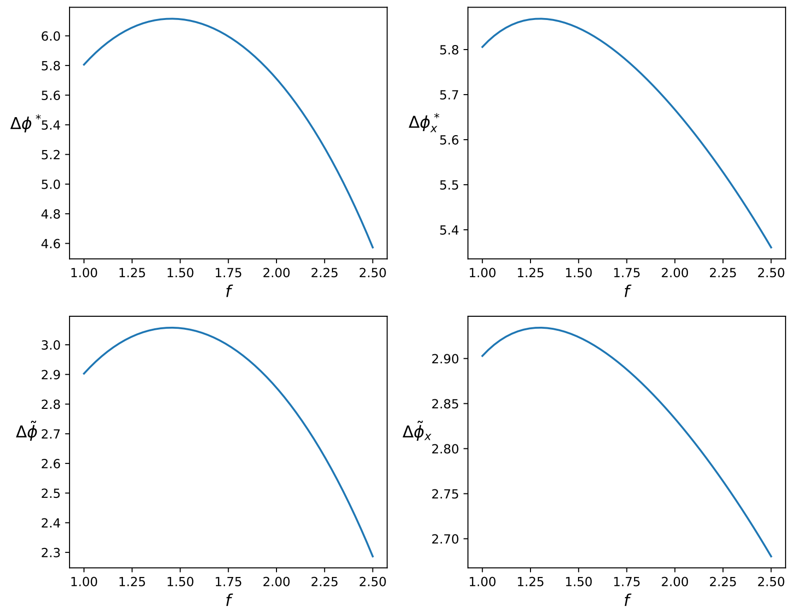


**Figure 3:** Comparing the effect of a change in $f$ on the difference between the models with and without international labor mobility. Parameters: $f \in [1, 2.5]$, $f_x = 1$, $f_e = 2$, $\theta = 0.5$, $\sigma = 2$, $\alpha = 0.8$, $\tau = 1$, $w_H = 2$, and $w_E = 1$.

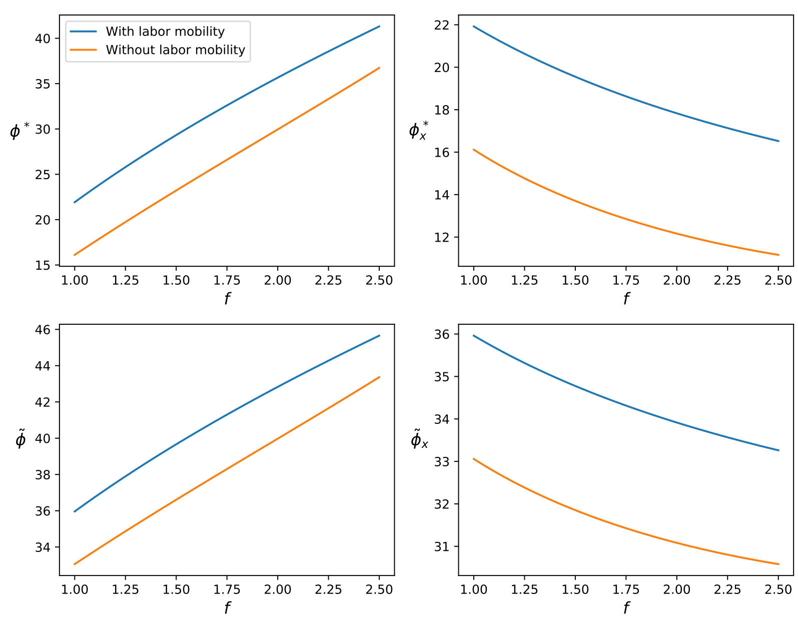


**Figure 4:** The effect of a change in $f$ for the model with and without international labor mobility. Parameters: $f \in [1, 2.5]$, $f_x = 1$, $f_e = 2$, $\theta = 0.5$, $\sigma = 2$, $\alpha = 0.8$, $\tau = 1$, $w_H = 2$, and $w_E = 1$.

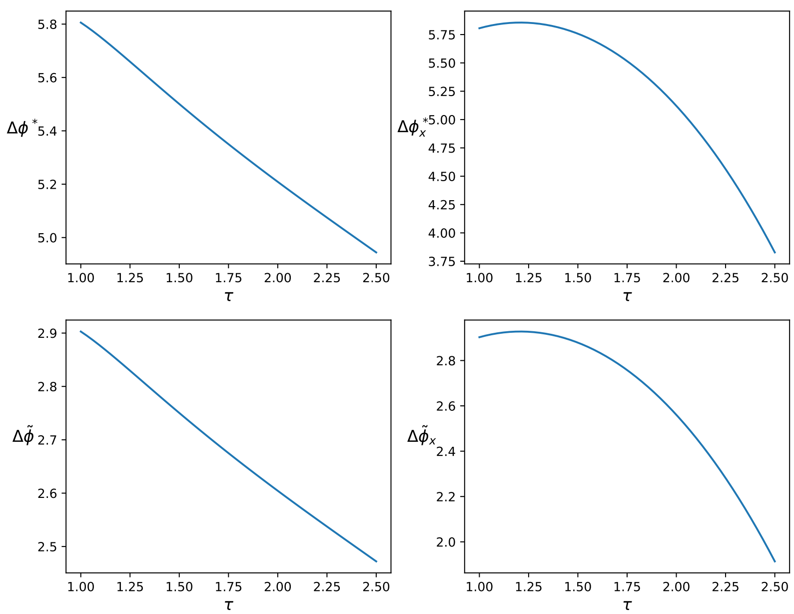


**Figure 5:** Comparing the effect of a change in $\tau$ on the difference between the models with and without international labor mobility. Parameters: Parameters: $f = 1, f_x = 1, f_e = 2, \theta = 0.5, \sigma = 2, \alpha = 0.8, \tau \in [1, 2.5], w_H = 2$, and $w_E = 1$.