

Algorithm 4: Intersect

Comp175: Introduction to Computer Graphics Spring 2016

Due: Monday March 28th at 11:59pm

Your Names: Walton Lee

Skylar Tom

Your CS Logins: wlee10

stom01

$$p_{film} = \left(-1 + \frac{2x}{x_{max}}, -1 + \frac{2y}{y_{max}}, -1 \right)$$

1 Instructions

Complete this assignment only with your teammate. You may use a calculator or computer algebra system. All your answers should be given in simplest form. When a numerical answer is required, provide a reduced fraction (i.e. $1/3$) or at least three decimal places (i.e. 0.333). Show all work; write your answers on this sheet. This algorithm handout is worth 3% of your final grade for the class.

2 Generating rays

For this assignment, you need to shoot a ray from the eye point through the center of each pixel.

[1 point] Given a pixel with screen-space coordinates x and y , and the width and height of the screen x_{max} and y_{max} , what is the corresponding point, p_{film} , on the normalized film plane (note that this means that the film plane is at the distance of 1 from the eye point)? Assume that this is taking place after all of the perspective viewing transformations have been applied except for the unhinging transformation. Remember that a pixel y -value of 0 corresponds to the top of the screen.

[1 point] In **Camera**, you transformed a point from world space to screen space by using the normalizing transformation. In **Intersect** and **Ray**, you need to transform p_{film} on the normalized film plane into an untransformed world-space point, p_{world} , by performing the viewing transformation. Using only the components of the viewing transformation (and/or their inverses) write the equation for $M_{film-to-world}$, the composite transformation matrix that transforms p_{film} to p_{world} , such that $p_{world} = M_{film-to-world} * p_{film}$.

$$M_{film-to-world} = (T_{vw}^{-1} R_{wzxy}^{-1} S_{xy}^{-1})$$

$$\begin{bmatrix} 1 & 0 & 0 & c_x \\ 0 & 1 & 0 & c_y \\ 0 & 0 & 1 & c_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_x u_y u_z 0 \\ v_x v_y v_z 0 \\ w_x w_y w_z 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tan(\frac{\theta_x}{2}) f & 0 & 0 & 0 \\ 0 & \tan(\frac{\theta_y}{2}) f & 0 & 0 \\ 0 & 0 & f & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

[1 point] Given your eye-point p_{eye} and the world-space point on the normalized film plane p_{world} give the equation for the world-space ray you want to shoot into the scene. Specify your ray in the format $p + t\vec{d}$, where p is a

point and \vec{d} is a normalized vector.

$$r(t) = P_{eye} + t(P_{world} - P_{eye})$$

3 Plane-Ray Intersection

Write out the plane-ray intersect equation in terms of t . Use the definition of a ray used in question 1, i.e. $p + t\vec{d}$. To get you started you might want to define an intersection point as $(x, y, z) = (p_x + \vec{d}_x t, p_y + \vec{d}_y t, p_z + \vec{d}_z t)$, where p is the eyepoint, and \vec{d} is the direction of the ray we are shooting.

For this exercise, show how you would compute the intersection for two sides of the cube, namely the planes where $x=0.5$ and $x=-0.5$.

[1 points] Plane $x = 0.5$

$$p_x + \vec{d}_x t = 0.5$$

$$t = \frac{0.5 - p_x}{\vec{d}_x} \quad \vec{d}_x \neq 0$$

intersection point: if $t > 0$
 $(p_x + \vec{d}_x t, p_y + \vec{d}_y t, p_z + \vec{d}_z t)$

[1 points] Plane $x = -0.5$

$$p_x + \vec{d}_x t = -0.5$$

$$t = \frac{-0.5 - p_x}{\vec{d}_x} \quad \vec{d}_x \neq 0$$

same as above

4 Cone-Ray Intersection

Special Instructions: For this problem, show all your work and circle, box, or bold your final answers.

Write out both of the cone-ray intersect equations in terms of t . There are two equations: one for the body of the cone, and one for the bottom cap. For your cone, use the same dimensions that you did in **Shapes**. Use the definition of a ray used in question 1, i.e. $p + t\vec{d}$. To get you started you might want to define an intersection point as $(x, y, z) = (p_x + \vec{d}_x t, p_y + \vec{d}_y t, p_z + \vec{d}_z t)$, where p is the eyepoint, and \vec{d} is the direction of the ray we are shooting. Looking over the the derivation of the implicit equations for the cylinder in the Raytracing lecture might prove to be useful.

Recall that the equation of a circle on the 2D XZ coordinate plane is $x^2 + z^2 = r^2$. Think of our canonical unit cone as an infinite number of "differential" circles in the XZ plane stacked on top of one another in the Y direction; the

bottommost circle has a radius of $1/2$ and the topmost circle has a radius of 0 . Then the equation of the unit cone is $x_2 + z_2 = k(y)^2$, where k linearly interpolates the radius of the differential circle from $1/2$ at the base to 0 at the top (see the lecture notes for a slightly more intuitive equation for the cone).

The intersection points you compute are possible intersection points and need to be examined further (such as the $-0.5 \leq y \leq 0.5$ restriction for the body of the cylinder in the lecture notes). However for this problem you are NOT required to list these restrictions.

Note that in your program you will need to find intersection points by finding a value for t . If you do not find an explicit formula for t (ie. $t = \text{some value(s)}$) for both the cone and the cap then you will have a very hard time writing the program.

Finally, the equations you write should not use vectors but should be functions of the individual components of the vectors. By reducing your equations after deriving them, you eliminate unnecessary computation and thereby optimize your code before you even write it!

5 How to Submit

Hand in a PDF version of your solutions using the following command:

`provide comp175 a4-alg`

[3 points] Cone Body

Solve for

$$(p_x + d_x t)^2 + (p_z + d_z t)^2 = \left(\frac{1}{2} \left(\frac{1}{2} - (y + d_y t) \right) \right)^2$$

Quadratic formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 4d_x^2 + d_y^2 + 4d_z^2$$

$$b = 8p_x d_x + 2p_y d_y - 8p_z d_z$$

$$c = (-16x^2 + 4y^2 - 16z^2 + 1)$$

[2 points] Cone Cap

Quadratic Formula for

$$At^2 + Bt + C = 0$$

where

$$A = d_x^2 + d_z^2$$

$$B = 2d_x p_x + 2d_z p_z$$

$$C = p_x^2 + p_z^2 - \frac{1}{4}$$