USA
AMC 12/AHSME
2011

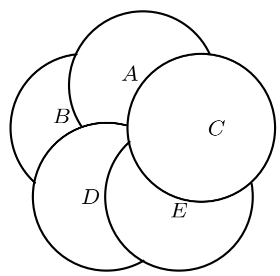
A

1. A cell phone plan costs \$20 dollars each month, plus 5 cents per text message sent, plus 10 cents for each minute used over 30 hours. In January Michelle sent 100 text messages and talked for 30.5 hours. How much did she have to pay?

(A) \$24.00 **(B)** \$24.50 **(C)** \$25.50 **(D)** \$28.00 **(E)** \$30.00

2. There are 5 coins placed flat on a table according to the figure. What is the order of the coins from top to bottom?

(A) (C, A, E, D, B) **(B)** (C, A, D, E, B) **(C)** (C, D, E, A, B) **(D)** (C, E, A, D, B) **(E)** $(C, E, D, A,$



3. A small bottle of shampoo can hold 35 milliliters of shampoo, whereas a large bottle can hold 500 milliliters of shampoo. Jasmine wants to buy the minimum number of small bottles necessary to completely fill a large bottle. How many bottles must she buy?

(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

4. At an elementary school, the students in third grade, fourth grade, and fifth grade run an average of 12, 15, and 10 minutes per day, respectively. There are twice as many third graders as fourth graders, and twice as many fourth graders as fifth graders. What is the average number of minutes run per day by these students?

(A) 12 **(B)** $\frac{37}{3}$ **(C)** $\frac{88}{7}$ **(D)** 13 **(E)** 14

5. Last summer 30

(A) 20 **(B)** 30 **(C)** 40 **(D)** 50 **(E)** 60

6 The players on a basketball team made some three-point shots, some two-point shots, and some one-point free throws. They scored as many points with two-point shots as with three-point shots. Their number of successful free throws was one more than their number of successful two-point shots. The team's total score was 61 points. How many free throws did they make?

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

7 A majority of the 30 students in Ms. Demeanor's class bought pencils at the school bookstore. Each of these students bought the same number of pencils, and this number was greater than 1. The cost of a pencil in cents was greater than the number of pencils each student bought, and the total cost of all the pencils was \$17.71. What was the cost of a pencil in cents?

(A) 7 **(B)** 11 **(C)** 17 **(D)** 23 **(E)** 77

8 In the eight-term sequence A, B, C, D, E, F, G, H, the value of C is 5 and the sum of any three consecutive terms is 30. What is $A + H$?

(A) 17 **(B)** 18 **(C)** 25 **(D)** 26 **(E)** 43

9 At a twins and triplets convention, there were 9 sets of twins and 6 sets of triplets, all from different families. Each twin shook hands with all the twins except his/her sibling and with half the triplets. Each triplet shook hands with all the triplets except his/her siblings and half the twins. How many handshakes took place?

(A) 324 **(B)** 441 **(C)** 630 **(D)** 648 **(E)** 882

10 A pair of standard 6-sided fair dice is rolled once. The sum of the numbers rolled determines the diameter of a circle. What is the probability that the numerical value of the area of the circle is less than the numerical value of the circle's circumference?

(A) $\frac{1}{36}$ **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{5}{18}$

11 Circles A, B, and C each have radius 1. Circles A and B share one point of tangency. Circle C has a point of tangency with the midpoint of $\overline{AB}$. What is the area inside circle C but outside circle A and circle B?

(A) $3 - \frac{\pi}{2}$ **(B)** $\frac{\pi}{2}$ **(C)** 2 **(D)** $\frac{3\pi}{4}$ **(E)** $1 + \frac{\pi}{2}$

12 A power boat and a raft both left dock A on a river and headed downstream. The raft drifted at the speed of the river current. The power boat maintained a constant speed with respect to the river. The power boat reached dock B downriver, then immediately turned and traveled back upriver. It eventually met the raft on the river 9 hours after leaving dock A. How many hours did it take the power boat to go from A to B?

(A) 3 **(B)** 3.5 **(C)** 4 **(D)** 4.5 **(E)** 5

$\boxed{13}$ Triangle ABC has side-lengths $AB = 12$, $BC = 24$, and $AC = 18$. The line through the incenter of $\triangle ABC$ parallel to $\overline{BC}$ intersects $\overline{AB}$ at M and $\overline{AC}$ at N. What is the perimeter of $\triangle AMN$?

(A) 27 **(B)** 30 **(C)** 33 **(D)** 36 **(E)** 42

$\boxed{14}$ Suppose a and b are single-digit positive integers chosen independently and at random. What is the probability that the point (a, b) lies above the parabola $y = ax^2 - bx$?

(A) $\frac{11}{81}$ **(B)** $\frac{13}{81}$ **(C)** $\frac{5}{27}$ **(D)** $\frac{17}{81}$ **(E)** $\frac{19}{81}$

$\boxed{15}$ The circular base of a hemisphere of radius 2 rests on the base of a square pyramid of height 6. The hemisphere is tangent to the other four faces of the pyramid. What is the edge-length of the base of the pyramid?

(A) $3\sqrt{2}$ **(B)** $\frac{13}{3}$ **(C)** $4\sqrt{2}$ **(D)** 6 **(E)** $\frac{13}{2}$

$\boxed{16}$ Each vertex of convex pentagon $ABCDE$ is to be assigned a color. There are 6 colors to choose from, and the ends of each diagonal must have different colors. How many different colorings are possible?

(A) 2520 **(B)** 2880 **(C)** 3120 **(D)** 3250 **(E)** 3750

$\boxed{17}$ Circles with radii $1, 2$, and 3 are mutually externally tangent. What is the area of the triangle determined by the points of tangency?

(A) $\frac{3}{5}$ **(B)** $\frac{4}{5}$ **(C)** 1 **(D)** $\frac{6}{5}$ **(E)** $\frac{4}{3}$

$\boxed{18}$ Suppose that $|x + y| + |x - y| = 2$. What is the maximum possible value of $x^2 - 6x + y^2$?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

$\boxed{19}$ At a competition with N players, the number of players given elite status is equal to

$$2^{1+\lfloor \log_2 (N-1) \rfloor} - N.$$

Suppose that 19 players are given elite status. What is the sum of the two smallest possible values of N?

(A) 38 **(B)** 90 **(C)** 154 **(D)** 406 **(E)** 1024

$\boxed{20}$ Let $f(x) = ax^2 + bx + c$, where a, b, and c are integers. Suppose that $f(1) = 0$, $50 < f(7) < 60$, $70 < f(8) < 80$, and $5000k < f(100) < 5000(k + 1)$ for some integer k. What is k?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

$\boxed{21}$ Let $f_1(x) = \sqrt{1 - x}$, and for integers $n \geq 2$, let $f_n(x) = f_{n-1}(\sqrt{n^2 - x})$. If N is the largest value of n for which the domain of f_n is nonempty, the domain of f_N is c. What is $N + c$?

(A) -226 **(B)** -144 **(C)** -20 **(D)** 20 **(E)** 144

$\boxed{22}$ Let R be a square region and $n \geq 4$ an integer. A point X in the interior of R is called *n-ray partitional* if there are n rays emanating from X that divide R into n triangles of equal area. How many points are 100-ray partitional but not 60-ray partitional?

(A) 1500 **(B)** 1560 **(C)** 2320 **(D)** 2480 **(E)** 2500

$\boxed{23}$ Let $f(z) = \frac{z+a}{z+b}$ and $g(z) = f(f(z))$, where a and b are complex numbers. Suppose that $|a| = 1$ and $g(g(z)) = z$ for all z for which $g(g(z))$ is defined. What is the difference between the largest and smallest possible values of $|b|$?

(A) 0 **(B)** $\sqrt{2} - 1$ **(C)** $\sqrt{3} - 1$ **(D)** 1 **(E)** 2

$\boxed{24}$ Consider all quadrilaterals $ABCD$ such that $AB = 14$, $BC = 9$, $CD = 7$, $DA = 12$. What is the radius of the largest possible circle that fits inside or on the boundary of such a quadrilateral?

(A) $\sqrt{15}$ **(B)** $\sqrt{21}$ **(C)** $2\sqrt{6}$ **(D)** 5 **(E)** $2\sqrt{7}$

$\boxed{25}$ Triangle ABC has $\angle BAC = 60^\circ$, $\angle CBA \leq 90^\circ$, $BC = 1$, and $AC \geq AB$. Let H, I, and O be the orthocenter, incenter, and circumcenter of $\triangle ABC$, respectively. Assume that the area of the pentagon $BCOIH$ is the maximum possible. What is $\angle CBA$?

(A) 60° **(B)** 72° **(C)** 75° **(D)** 80° **(E)** 90°

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B

1 What is
$$\frac{2+4+6}{1+3+5} - \frac{1+3+5}{2+4+6}?$$

(A) -1 **(B)** $\frac{5}{36}$ **(C)** $\frac{7}{12}$ **(D)** $\frac{147}{60}$ **(E)** $\frac{43}{3}$

2 Josanna's test scores to date are 90, 80, 70, 60, and 85. Her goal is to raise her test average at least 3 points with her next test. What is the minimum test score she would need to accomplish this goal?

(A) 80 **(B)** 82 **(C)** 85 **(D)** 90 **(E)** 95

3 LeRoy and Bernardo went on a week-long trip together and agreed to share the costs equally. Over the week, each of them paid for various joint expenses such as gasoline and car rental. At the end of the trip it turned out that LeRoy had paid A dollars and Bernardo had paid B dollars, where $A < B$. How many dollars must LeRoy give to Bernardo so that they share the costs equally?

(A) $\frac{A+B}{2}$ **(B)** $\frac{A-B}{2}$ **(C)** $\frac{B-A}{2}$ **(D)** $B-A$ **(E)** $A+B$

4 In multiplying two positive integers a and b, Ron reversed the digits of the two-digit number a. His errorneous product was 161. What is the correct value of the product of a and b?

(A) 116 **(B)** 161 **(C)** 204 **(D)** 214 **(E)** 224

5 Let N be the second smallest positive integer that is divisible by every positive integer less than 7. What is the sum of the digits of N?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 9

6 Two tangents to a circle are drawn from a point A. The points of contact B and C divide the circle into arcs with lengths in the ratio $2:3$. What is the degree measure of $\angle BAC$?

(A) 24 **(B)** 30 **(C)** 36 **(D)** 48 **(E)** 60

7 Let x and y be two-digit positive integers with mean 60. What is the maximum value of the ratio $\frac{x}{y}$?

(A) 3 **(B)** $\frac{33}{7}$ **(C)** $\frac{39}{7}$ **(D)** 9 **(E)** $\frac{99}{10}$

8 Keiko walks once around a track at exactly the same constant speed every day. The sides of the track are straight, and the ends are semicircles. The track has width 6 meters, and

it takes her 36 seconds longer to walk around the outside edge of the track than around the inside edge. What is Keiko's speed in meters per second?

(A) $\frac{\pi}{3}$ **(B)** $\frac{2\pi}{3}$ **(C)** π **(D)** $\frac{4\pi}{3}$ **(E)** $\frac{5\pi}{3}$

$\boxed{9}$ Two real numbers are selected independently at random from the interval [-20, 10]. What is the probability that the product of those numbers is greater than zero?

(A) $\frac{1}{9}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{4}{9}$ **(D)** $\frac{5}{9}$ **(E)** $\frac{2}{3}$

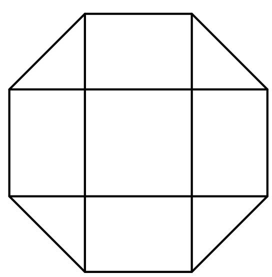
$\boxed{10}$ Rectangle $ABCD$ has $AB = 6$ and $BC = 3$. Point M is chosen on side AB so that $\angle AMD = \angle CMD$. What is the degree measure of $\angle AMD$?

(A) 15 **(B)** 30 **(C)** 45 **(D)** 60 **(E)** 75

$\boxed{11}$ A frog located at (x, y), with both x and y integers, makes successive jumps of length 5 and always lands on points with integer coordinates. Suppose that the frog starts at $(0, 0)$ and ends at $(1, 0)$. What is the smallest possible number of jumps the frog makes?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

$\boxed{12}$ A dart board is a regular octagon divided into regions as shown. Suppose that a dart thrown at the board is equally likely to land anywhere on the board. What is probability that the dart lands within the center square?



(A) $\frac{\sqrt{2}-1}{2}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{2-\sqrt{2}}{2}$ **(D)** $\frac{\sqrt{2}}{4}$ **(E)** $2 - \sqrt{2}$

$\boxed{13}$ Brian writes down four integers $w > x > y > z$ whose sum is 44. The pairwise positive differences of these numbers are $1, 3, 4, 5, 6,$ and 9. What is the sum of the possible values for w?

(A) 16 **(B)** 31 **(C)** 48 **(D)** 62 **(E)** 93

$\boxed{14}$ A segment through the focus F of a parabola with vertex V is perpendicular to $\overline{FV}$ and intersects the parabola in points A and B. What is $\cos(\angle AVB)$?

(A) $-\frac{3\sqrt{5}}{7}$ **(B)** $-\frac{2\sqrt{5}}{5}$ **(C)** $-\frac{4}{5}$ **(D)** $-\frac{3}{5}$ **(E)** $-\frac{1}{2}$

$\boxed{15}$ How many positive two-digit integers are factors of $2^{24}-1$?

(A) 4 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 14

$\boxed{16}$ Rhombus $ABCD$ has side length 2 and $\angle B = 120^\circ$. Region R consists of all points inside the rhombus that are closer to vertex B than any of the other three vertices. What is the area of R?

(A) $\frac{\sqrt{3}}{3}$ **(B)** $\frac{\sqrt{3}}{2}$ **(C)** $\frac{2\sqrt{3}}{3}$ **(D)** $1+\frac{\sqrt{3}}{3}$ **(E)** 2

$\boxed{17}$ Let $f(x) = 10^{10x}, g(x) = \log_{10}\left(\frac{x}{10}\right), h_1(x) = g(f(x))$, and $h_n(x) = h_1(h_{n-1}(x))$ for integers $n \geq 2$. What is the sum of the digits of $h_{2011}(1)$?

(A) $16,081$ **(B)** $16,089$ **(C)** $18,089$ **(D)** $18,098$ **(E)** $18,099$

$\boxed{18}$ A pyramid has a square base with sides of length 1 and has lateral faces that are equilateral triangles. A cube is placed within the pyramid so that one face is on the base of the pyramid and its opposite face has all its edges on the lateral faces of the pyramid. What is the volume of this cube?

(A) $5\sqrt{2}-7$ **(B)** $7-4\sqrt{3}$ **(C)** $\frac{2\sqrt{2}}{27}$ **(D)** $\frac{\sqrt{2}}{9}$ **(E)** $\frac{\sqrt{3}}{9}$

$\boxed{19}$ A lattice point in an xy-coordinate system is any point (x, y) where both x and y are integers. The graph of $y = mx + 2$ passes through no lattice point with $0 < x \leq 100$ for all m such that $\frac{1}{2} < m < a$. What is the maximum possible value of a?

(A) $\frac{51}{101}$ **(B)** $\frac{50}{99}$ **(C)** $\frac{51}{100}$ **(D)** $\frac{52}{101}$ **(E)** $\frac{13}{25}$

$\boxed{20}$ Triangle ABC has $AB = 13$, $BC = 14$, and $AC = 15$. The points D, E, and F are the midpoints of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ respectively. Let $X \neq E$ be the intersection of the circumcircles of $\triangle BDE$ and $\triangle CEF$. What is $XA + XB + XC$?

(A) 24 **(B)** $14\sqrt{3}$ **(C)** $\frac{195}{8}$ **(D)** $\frac{129\sqrt{7}}{14}$ **(E)** $\frac{69\sqrt{2}}{4}$

$\boxed{21}$ The arithmetic mean of two distinct positive integers x and y is a two-digit integer. The geometric mean of x and y is obtained by reversing the digits of the arithmetic mean. What is $|x-y|$?

(A) 24 **(B)** 48 **(C)** 54 **(D)** 66 **(E)** 70

$\boxed{22}$ Let T_1 be a triangle with sides $2011, 2012$, and 2013. For $n \geq 1$, if $T_n =$ and D, E, and F are the points of tangency of the incircle of to the sides AB, BC and AC, respectively, then

T_{n+1} is a triangle with side lengths AD, BE, and CF, if it exists. What is the perimeter of the last triangle in the sequence (T_n)?

(A) $\frac{1509}{8}$ **(B)** $\frac{1509}{32}$ **(C)** $\frac{1509}{64}$ **(D)** $\frac{1509}{128}$ **(E)** $\frac{1509}{256}$

$\boxed{23}$ A bug travels in the coordinate plane, moving only along the lines that are parallel to the x-axis or y-axis. Let $A = (-3, 2)$ and $B = (3, -2)$. Consider all possible paths of the bug from A to B of length at most 20. How many points with integer coordinates lie on at least one of these paths?

(A) 161 **(B)** 185 **(C)** 195 **(D)** 227 **(E)** 255

$\boxed{24}$ Let $P(z) = z^8 + (4\sqrt{3} + 6)z^4 - (4\sqrt{3} + 7)$. What is the minimum perimeter among all the 8-sided polygons in the complex plane whose vertices are precisely the zeros of $P(z)$?

(A) $4\sqrt{3} + 4$ **(B)** $8\sqrt{2}$ **(C)** $3\sqrt{2} + 3\sqrt{6}$ **(D)** $4\sqrt{2} + 4\sqrt{3}$ **(E)** $4\sqrt{3} + 6$

$\boxed{25}$ For every m and k integers with k odd, denote by $[\frac{m}{k}]$ the integer closest to $\frac{m}{k}$. For every odd integer k, let $P(k)$ be the probability that

$$[\frac{n}{k}] + [\frac{100-n}{k}] = [\frac{100}{k}]$$

for an integer n randomly chosen from the interval $1 \leq n \leq 99!$. What is the minimum possible value of $P(k)$ over the odd integers K in the interval $1 \leq k \leq 99$?

(A) $\frac{1}{2}$ **(B)** $\frac{50}{99}$ **(C)** $\frac{44}{87}$ **(D)** $\frac{34}{67}$ **(E)** $\frac{7}{13}$