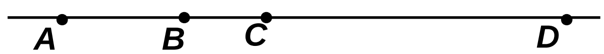
Note: This practice test is only used to help determine what areas
We should spend our time teaching.

1. A square pyramid has a volume of 108 cubic inches and the ratio of length of its altitude to the perimeter of its base is 3 : 8. A plane parallel to its base divides the pyramid into two solids one of which is a smaller pyramid whose slant height is $\sqrt{10}$. Compute the volume of the smaller pyramid.
2. A right triangle has a hypotenuse of length 65. If the length of the long leg is increased by 4 and the length of the short leg is decreased by 8, the length of the hypotenuse is unchanged. What is the perimeter of the original right triangle?
3. *A*, *B*, *C* and *D* are 4 collinear points ordered on a line as indicated in the diagram below. If $AD = 203$, $\frac{AC}{BD} = \frac{4}{9}$ and $\frac{AB}{CD} = \frac{10}{27}$, compute *BC*.



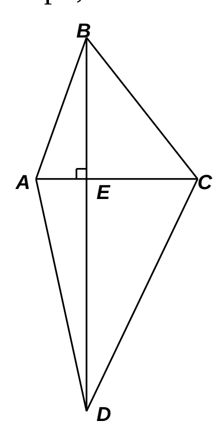
4. If $\frac{a+b}{c} = 2$ and $\frac{a+c}{b} = 3$, compute $\frac{b+c}{a}$.
5. *A* and *B* are distinct two-digit positive integers with digits reversed. *A* and *B* are both prime, with $A < B$.

   Let *p* denote the number of ordered pairs (*A*, *B*).
   Let *C* = the <u>minimum</u> value of $|A - B|$ and
   D = the <u>maximum</u> value of $|A - B|$.

   <u>*How many*</u> integers *x* are there in the range $p \cdot C < x < p \cdot D$?
6. Let the binary operation $(*)$ be defined as follows:

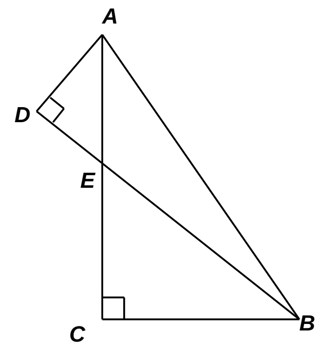
$$a * b = \begin{cases} a + ab, & \text{when } b \text{ is a proper fraction} \\ b - ab, & \text{when } b \text{ is an improper faction} \end{cases}$$

   Compute $\left(6 * \frac{2}{3}\right) * \left(\frac{3}{4} * \frac{3}{2}\right)$.
7. If $\sqrt{-40-9i} = A + Bi$, compute $\left(\frac{A}{B}\right)^2$.
8. A train travels 150 miles in *w* hours. If the rate of the train were increased by *x* mph, the train would arrive at its destination in 2 less hours. Find *x* in terms of *w*.
9. Given: quadrilateral *ABCD* with perpendicular diagonals and
   $AB = 13$, $BC = 15$, $BD = 52$, $AC = 14$
   To the nearest integer, what is the perimeter of ΔADE?



10. The polynomial $x^{24} - x^{8} - 256x^{16} + 256$ can be written as the product of *N* binomial factors of the form $(x^a \pm b)$, where *a* and *b* are positive integers. Determine the <u>maximum</u> value of *N*.

11. In ΔABC, $m\angle C = m\angle D = 90°$, $AB = 4$,
$m\angle BAC = 30°$ and $BC = EC$.
Find BD in simplified radical form.



12. $\overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$ and $\overleftrightarrow{AB}$ is a transversal intersecting $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ in points M and N respectively.
P is a point between the parallel lines such that
$$m\angle NMP = 3m\angle PMD$$
$$m\angle MNP = 4m\angle PNF$$
If $m\angle AMD = (7x-40)°$ and $m\angle MNF = (5x)°$,
find $m\angle P$.

13. In ΔABC, $m\angle A = 30°$, $a = 10$, $b = 15$ and $\angle B$ is as large as possible.
Determine the <u>exact</u> value of $\sin C$.

14. Find the ordered pair (x, y) so that $9x43y5$ is the smallest number divisible by 33.

15. Three vertices of <u>parallelogram</u> $PQRS$ are $P(2, 1)$ $Q(6,11)$ and $S(12, 9)$.|
Determine the equation of $\overleftrightarrow{PR}$, in $ax + by + c = 0$ form, where a, b and c are integers, $a > 0$ and $\text{GCF}(a, b, c) = 1$.

16. Solve for x over the reals. $$\frac{2^x - 2^{-x}}{2} = -1.875$$

17. A <u>round trip</u> between A and B consists of a trip from A to B at a constant rate of R_1 and a return trip from B to A at a constant rate of R_2.
The average speed (R) on a round trip varies <u>directly</u> as the distance between A and B and <u>inversely</u> as the sum of the elapsed times traveling back and forth.

$R = 48$ when $(R_1, R_2) = (40, 60)$ and $D = 30$.
Compute R when $(R_1, R_2) = (8, 12)$ and $D = 2008$.

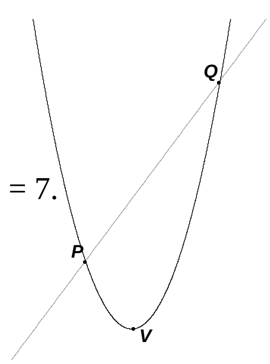
18. The interior angles of regular polygon P measure $x°$.
The interior angles of regular polygon Q measure $\left(x + \frac{1}{2}\right)^{\circ}$.
If Q has 8 more sides than P, compute the number of diagonals in Q.

19. The points $P(6, 5)$, $Q(11, 7)$ and R lie on a parabola whose vertex is at $V(2, 1)$.
The axis of symmetry is parallel to one of the coordinate axes.
The <u>focus</u> of the parabola lies on $\overline{QR}$.
Compute the (x, y) coordinates of the point R.

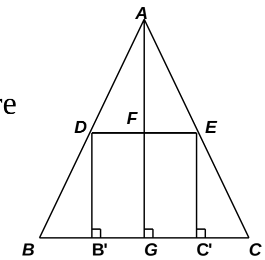
20. Solve for x. $$\frac{4}{5 - \frac{3+x}{3}} = \frac{16}{4 + \frac{8}{3 - \frac{6}{x}}}$$

21. Solve for θ, where $0° < \theta < 360°$: $$\frac{\sin\theta}{\sqrt{3} + \sqrt{3}\cos\theta} = -1$$

22. The line $2x - y + 7 = 0$ intersects $y = Ax^2 + Bx + C$ at $x = -2$ and $x = 7$.
The low point V has coordinates $(1, -3)$
Compute the value of C.



23. Given: ΔABC is isosceles, $\overline{DE} \parallel \overline{BC}$, $\dfrac{FG}{AG} = \dfrac{2}{3}$ and $DEC'B'$ is a square



Express $area(\Delta AFD) : area(DEC'B') : area(DECB)$
as a simplified ratio.

24. On the interstate Mario traveling 100 mph passed a state trooper with a radar gun parked beside the road. The trooper immediately decided to give chase. 48 seconds after Mario passed the state trooper's parked car, the trooper had gone 1/6 of a mile and had reached his top speed of 121 mph, which he maintained until he overtook Mario. How long after Mario passed the trooper was he apprehended? Express your answer in minutes and seconds, accurate to the nearest second.

25. Let $f(x) = Ax^3 + Bx^2 + Cx + D$.

If $f(-a) = -f(a)$, $f(-3) + 2f(3) + f(5) = 3$ and one zero of $Ax^3 + Bx^2 + Cx + D = 0$ is 4, determine the ordered quadruple (A, B, C, D).

26. The 258th natural number that is not divisible by either 3 or 7 is k.
Compute the sum of the prime factors of k.

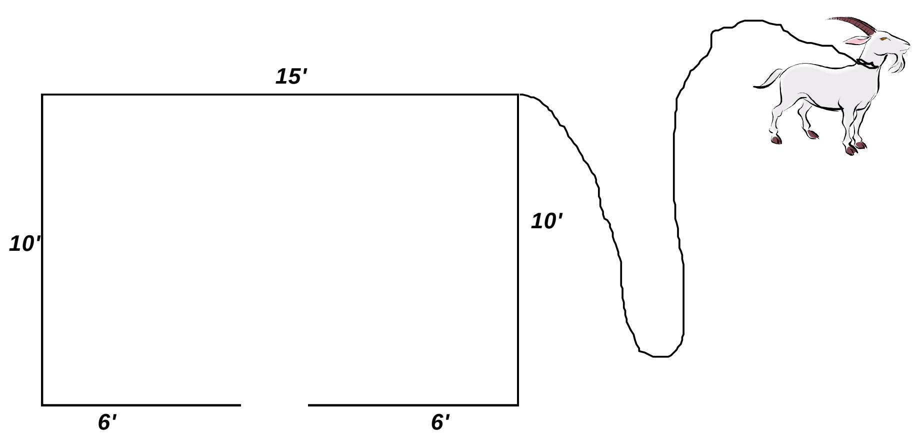
27. Given: $\begin{cases} y = \sin^3 t \\ x = \cos^3 t \end{cases}$, where $0 \le t \le 2\pi$.

Compute all real values of y for which $x = \dfrac{64}{125}$.

28. Tom and his sister Sherry are two of the oldest living tortoises. You've probably seen them in the Slowski's TV ad Comcast vs. FIOS. Presently, Tom is 18 years older than his sister. Four score and seven years ago**, their ages were two-digit numbers with the digits reversed and the sum of their ages was 110. How old is Tom now?

** a score is 20 years (from Abraham Lincoln's Gettysburg Address)

29. A goat is tied with a 20' rope to a 15' x 10' shed as shown. The shed has an open 3' doorway. In terms of π, compute the total area where the goat can roam.



30. AB, $3AB$, $18A$ form an increasing geometric progression.
A^3, $A + B + 1$, B form a decreasing arithmetic progression.
If A, B, C and D are real numbers and $(A+Bi)^3 = C + Di$, where $i = \sqrt{-1}$,
compute $\dfrac{C}{D}$.

31. Connecting the points $A(3, 7)$, $B(-1, 2)$, $C(3x, -x)$ and $D(10, 1)$ you have an outline of the deck in my backyard. It's a nondescript convex quadrilateral and finding its area is baffling my builders. If $x > 0$ and A and C are opposite vertices, compute the coordinates of C so the area of my deck is 52 square units.

Note: The area of a triangle with vertices at (x_1, y_1), (x_2, y_2) and (x_3, y_3) can be computed
as $\dfrac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$.

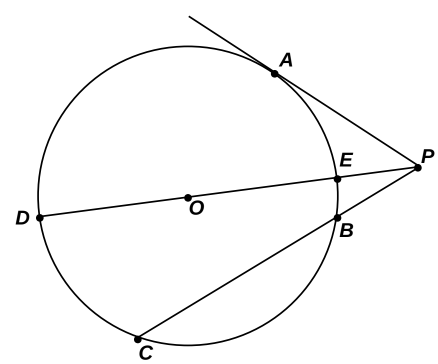
32. Given: $\sqrt{\dfrac{9}{16} - \dfrac{9}{25}} = \dfrac{3}{4} - \dfrac{3}{x}$ for some integer value of x.
Compute $\sqrt{\dfrac{2^x \cdot 4^{3x+4}}{8^{2x+2}}}$.

33. Given: $\begin{cases} \sin A = \dfrac{k}{\sqrt{41}} \\ \cos A = \dfrac{k+1}{\sqrt{41}} \end{cases}$ and $k > 0$. Compute $\sec\left(A - \dfrac{5\pi}{2}\right) \cdot \cos(A - 5\pi)$.

34. Compute the two rational roots of the following quadratic:
$$4A(48A+1) = 5(16 - 201A).$$

35. Given: $\overrightarrow{PA}$ is tangent to circle O,
$PA = 14$,
$CB = 3PB$ and $EP = 2\sqrt{PB}$

Compute the area of circle O.



36. The 5th term in the expansion of $\left(\dfrac{1}{2}x^2 + Ax^{-1}\right)^7$ is $\dfrac{70}{81}x^T$.
Compute all possible ordered pairs (A, T).
Note: The 1st term in the expansion is $\dfrac{x^{14}}{128}$.