```
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 2
 3
        % The file may not be re-distributed without explicit authorization
        % from Krishna Sankar.
  5
        % Checked for proper operation with Octave Version 3.0.0
  6
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 8
        % Version
                                     : 1.0
                                     : 24 January 2010
 9
        % Date
10
        11
12
        % Script for computing the BER for BPSK modulation in 3 tap ISI
13
        st channel. Minimum Mean Square Error (MMSE) equalization with 7 tap
14
        % and the BER computed (and is compared with Zero Forcing equalization)
15
16
17
        N = 10<sup>6</sup>; % number of bits or symbols
        Eb_N0_dB = [0:15]; % multiple Eb/N0 values
18
19
        K = 3;
20
        mH = 3; nH = 2^mH-1; kH = nH-mH; % Hamming (7,4)
21
22
        ref = [0 \ 0 \ ; \ 0 \ 1; \ 1 \ 0 \ ; \ 1 \ 1 \ ];
23
24
25
        ipLUT = [0]
                                 0
                                         0
                                                0;...
26
                                 0
                                                0; . . .
27
                          1
                                 1
                                        0
                                                0; . . .
                          0
                                        1
                                 0
                                                1];
28
29
30
        for ii = 1:length(Eb N0 dB)
31
              % Transmitter
32
33
              ip = rand(1,N)>0.5; % generating 0,1 with equal probability
34
              s = 2*ip-1; % BPSK modulation 0 -> -1; 1 -> 0
35
36
             % Channel model, multipath channel
37
              nTap = 3;
             ht = [0.2 \ 0.9 \ 0.3];
38
             L = length(ht);
39
40
41
              chanOut = conv(s,ht);
42
             n = 1/sqrt(2)*[randn(1,N+length(ht)-1) + j*randn(1,N+length(ht)-1)]; % white gaussian noise,
        0dB variance
43
44
              % Noise addition
              y = chan0ut + 10^(-Eb_N0_dB(ii)/20)*n; % additive white gaussian noise
45
46
47
              % Channel coding - block code
48
              ip bc = encode(ip,nH,kH,'hamming/binary'); % Hamming coding
49
              ip_bc = reshape(ip_bc,1,size(ip_bc));
              s_bc = 2*ip_bc-1; \% BPSK modulation 0 -> -1; 1 -> 0
50
51
              chanOut_bc = conv(s_bc,ht);
52
              n_bc = 1/sqrt(2)*[randn(1,size(ip_bc,2)+length(ht)-1) + j*randn(1,size(ip_bc,2)+length
        (ht)-1)]; % white gaussian noise, 0dB variance
             y_bc = chanOut_bc + 10^(-Eb_NO_dB(ii)/20)*n_bc; % additive white gaussian noise
53
54
              % Channel coding - convolutional coding, rate - 1/2, generator polynomial - [7,5] octal
55
56
              ip_cc1 = mod(conv(ip,[1 1 1 ]),2);
57
              ip_cc2 = mod(conv(ip,[1 0 1 ]),2);
              ip\_cc = [ip\_cc1; ip\_cc2];
58
             ip\_cc = ip\_cc(:).
59
              s\_cc = 2*ip\_cc-1; % BPSK modulation 0 -> -1; 1 -> 0
60
61
              chanOut_cc = conv(s_cc,ht);
62
              n_cc = 1/sqrt(2)*[randn(1,size(ip_cc,2)+length(ht)-1) + j*randn(1,size(ip_cc,2)+length(ht)-1) + j*randn(1,size(ip_cc,2)+leng
        (ht)-1)]; % white gaussian noise, 0dB variance
             y_cc = chan0ut_cc + 10^{-Eb_N0_dB(ii)/20}*n_cc; % additive white gaussian noise
63
64
65
              % zero forcing equalization
             hM = toeplitz([ht([2:end]) zeros(1,2*K+1-L+1)], [ ht([2:-1:1]) zeros(1,2*K+1-L+1) ]);
66
67
              d = zeros(1,2*K+1);
             d(K+1) = 1;
68
69
              c_zf = [inv(hM)*d.'].';
```

```
70
              yFilt_zf = conv(y,c_zf);
              yFilt_zf = yFilt_zf(K+2:end);
 71
              yFilt_zf = conv(yFilt_zf,ones(1,1)); % convolution
 72
 73
              ySamp zf = yFilt zf(1:1:N); % sampling at time T
 74
 75
              % mmse equalization
 76
              hAutoCorr = conv(ht,fliplr(ht));
              hM = toeplitz([hAutoCorr([3:end]) zeros(1,2*K+1-L)], [ hAutoCorr([3:end]) zeros(1,2*K+1-L) ]);
 77
 78
              hM = hM + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*10^{-6} + 1/2*
 79
              d = zeros(1,2*K+1);
              d([-1:1]+K+1) = fliplr(ht);
 80
 81
              c_{mmse} = [inv(hM)*d.'].';
 82
              yFilt_mmse = conv(y,c_mmse);
 83
              yFilt_mmse = yFilt_mmse(K+2:end);
 84
              yFilt_mmse = conv(yFilt_mmse,ones(1,1)); % convolution
 85
              ySamp_mmse = yFilt_mmse(1:1:N); % sampling at time T
 86
 87
              % zero forcing equalization - block code
 88
              hM = toeplitz([ht([2:end]) zeros(1,2*K+1-L+1)], [ ht([2:-1:1]) zeros(1,2*K+1-L+1) ]);
 89
              d = zeros(1,2*K+1);
              d(K+1) = 1;
 90
              c_zf = [inv(hM)*d.'].';
 91
 92
              yFilt_zf_bc = conv(y_bc,c_zf);
              yFilt_zf_bc = yFilt_zf_bc(K+2:end);
 93
 94
              yFilt_zf_bc = conv(yFilt_zf_bc,ones(1,1)); % convolution
 95
              ySamp_zf_bc = yFilt_zf_bc(1:1:size(ip_bc,2)); % sampling at time T
 96
 97
              % zero forcing equalization - convolutional code
 98
              hM = toeplitz([ht([2:end]) zeros(1,2*K+1-L+1)], [ht([2:-1:1]) zeros(1,2*K+1-L+1)]);
 99
              d = zeros(1,2*K+1);
              d(K+1) = 1;
100
              c_zf = [inv(hM)*d.'].';
101
102
              yFilt zf cc = conv(y cc,c zf);
103
              yFilt zf cc = yFilt zf cc(K+2:end);
104
              yFilt_zf_cc = conv(yFilt_zf_cc,ones(1,1)); % convolution
105
              ySamp_zf_cc = yFilt_zf_cc(1:1:size(ip_cc,2)); % sampling at time T
106
107
              % mmse equalization - block code
108
              hAutoCorr = conv(ht,fliplr(ht));
109
              hM = toeplitz([hAutoCorr([3:end]) zeros(1,2*K+1-L)], [ hAutoCorr([3:end]) zeros(1,2*K+1-L) ]);
              hM = hM + 1/2*10^{-Eb_N0_dB(ii)/10}*eye(2*K+1);
110
111
              d = zeros(1,2*K+1);
112
              d([-1:1]+K+1) = fliplr(ht);
113
              c_{mmse} = [inv(hM)*d.'].';
114
              yFilt_mmse_bc = conv(y_bc,c_mmse);
115
              yFilt_mmse_bc = yFilt_mmse_bc(K+2:end);
116
              yFilt_mmse_bc = conv(yFilt_mmse_bc,ones(1,1)); % convolution
117
              ySamp_mmse_bc = yFilt_mmse_bc(1:1:size(ip_bc,2)); % sampling at time T
118
119
              % mmse equalization - convolutional code
120
              hAutoCorr = conv(ht,fliplr(ht));
              hM = toeplitz([hAutoCorr([3:end]) zeros(1,2*K+1-L)], [ hAutoCorr([3:end]) zeros(1,2*K+1-L) ]);
121
122
              hM = hM + 1/2*10^{-Eb_N0_dB(ii)/10}*eye(2*K+1);
123
              d = zeros(1,2*K+1);
124
              d([-1:1]+K+1) = fliplr(ht);
125
              c mmse = [inv(hM)*d.'].';
126
              yFilt_mmse_cc = conv(y_cc,c_mmse);
              yFilt_mmse_cc = yFilt_mmse_cc(K+2:end);
127
128
              yFilt_mmse_cc = conv(yFilt_mmse_cc,ones(1,1)); % convolution
129
              ySamp_mmse_cc = yFilt_mmse_cc(1:1:size(ip_cc,2)); % sampling at time T
130
              % receiver - hard decision decoding
131
132
              ipHat_zf = real(ySamp_zf)>0;
133
              ipHat_zf_bc = real(ySamp_zf_bc)>0;
              ipHat_zf_bc = decode(ipHat_zf_bc,nH,kH,'hamming/binary');
134
135
              ipHat_zf_bc = reshape(ipHat_zf_bc,1,N);
              ipHat_mmse = real(ySamp_mmse)>0;
136
137
              ipHat_mmse_bc = real(ySamp_mmse_bc)>0;
              ipHat_mmse_bc = decode(ipHat_mmse_bc,nH,kH,'hamming/binary');
138
139
              ipHat_mmse_bc = reshape(ipHat_mmse_bc,1,N);
140
              ipHat zf cc = real(ySamp zf cc)>0;
141
              ipHat_mmse_cc = real(ySamp_mmse_cc)>0;
```

```
142
        for kk = 1:2
143
144
        % Viterbi decoding
145
        pathMetric = zeros(4,1); % path metric
146
        if (kk == 1)
        survivorPath_v_zf = zeros(4,length(ySamp_zf_cc)/2); % survivor path
147
148
        length_y = length(ySamp_zf_cc)
149
        survivorPath v mmse = zeros(4,length(ySamp mmse cc)/2); % survivor path
150
151
        length_y = length(ySamp_mmse_cc)
152
        endif
153
        for iii = 1:length_y/2
154
155
           if (kk == 1)
156
           r = ipHat_zf_cc(2*iii-1:2*iii); % taking 2 coded bits
157
           else
           r = ipHat_mmse_cc(2*iii-1:2*iii); % taking 2 coded bits
158
159
           endif
160
           st computing the Hamming distance between ip coded sequence with [00;01;10;11]
161
           rv = kron(ones(4,1),r);
162
163
           hammingDist = sum(xor(rv,ref),2);
164
           if (iii == 1) || (iii == 2)
165
166
               % branch metric and path metric for state 0
              bm1 = pathMetric(1,1) + hammingDist(1);
167
168
               pathMetric_n(1,1) = bm1;
               survivorPath(1,1) = 1;
169
170
171
               % branch metric and path metric for state 1
              bm1 = pathMetric(3,1) + hammingDist(3);
172
              pathMetric n(2,1) = bm1;
173
174
               survivorPath(2,1) = 3;
175
176
              % branch metric and path metric for state 2
               bm1 = pathMetric(1,1) + hammingDist(4);
177
178
               pathMetric_n(3,1) = bm1;
179
               survivorPath(3,1) = 1;
180
               % branch metric and path metric for state 3
181
182
               bm1 = pathMetric(3,1) + hammingDist(2);
183
               pathMetric_n(4,1) = bm1;
               survivorPath(4,1) = 3;
184
185
186
           else
               % branch metric and path metric for state 0
187
              bm1 = pathMetric(1,1) + hammingDist(1);
188
189
               bm2 = pathMetric(2,1) + hammingDist(4);
               [pathMetric_n(1,1) idx] = min([bm1,bm2]);
190
191
               survivorPath(1,1) = idx;
192
               % branch metric and path metric for state 1
193
              bm1 = pathMetric(3,1) + hammingDist(3);
194
              bm2 = pathMetric(4,1) + hammingDist(2);
195
               [pathMetric_n(2,1) idx] = min([bm1,bm2]);
196
               survivorPath(2,1) = idx+2;
197
198
199
              % branch metric and path metric for state 2
200
               bm1 = pathMetric(1,1) + hammingDist(4);
               bm2 = pathMetric(2,1) + hammingDist(1);
201
               [pathMetric_n(3,1) idx] = min([bm1,bm2]);
202
              survivorPath(3,1) = idx;
203
204
205
              % branch metric and path metric for state 3
206
              bm1 = pathMetric(3,1) + hammingDist(2);
207
               bm2 = pathMetric(4,1) + hammingDist(3);
208
               [pathMetric_n(4,1) idx] = min([bm1,bm2]);
209
               survivorPath(4,1) = idx+2;
210
211
           end
212
213
           pathMetric = pathMetric_n;
```

```
214
            if (kk == 1)
            survivorPath_v_zf(:,iii) = survivorPath;
215
216
            survivorPath v mmse(:,iii) = survivorPath;
217
218
            endif
219
220
221
         end
222
223
         % trace back unit - ZF
224
         currState = 1;
225
         ipHat_zf_cc = zeros(1,length(ySamp_zf_cc)/2);
226
         for jj = length(ySamp_zf_cc)/2:-1:1
227
            prevState = survivorPath_v_zf(currState,jj);
228
            ipHat_zf_cc(jj) = ipLUT(currState,prevState);
229
            currState = prevState;
230
         end
231
         % trace back unit - MMSE
232
233
         currState = 1;
234
         ipHat_mmse_cc = zeros(1,length(ySamp_mmse_cc)/2);
         for jj = length(ySamp_mmse_cc)/2:-1:1
    prevState = survivorPath_v_mmse(currState,jj);
235
236
            ipHat_mmse_cc(jj) = ipLUT(currState,prevState);
237
238
            currState = prevState;
239
240
241
         % counting the errors
242
         nErr_zf(1,ii) = size(find([ip- ipHat_zf]),2);
         nErr_zf_bc(1,ii) = size(find([ip- ipHat_zf_bc]),2);
nErr_zf_cc(1,ii) = size(find([ip- ipHat_zf_cc(1:N)]),2);
243
244
245
         nErr_mmse(1,ii) = size(find([ip- ipHat_mmse]),2);
         nErr mmse bc(1,ii) = size(find([ip- ipHat mmse bc]),2);
246
247
         nErr mmse cc(1,ii) = size(find([ip-ipHat mmse <math>cc(1:N)]),2);
248
249
     end
250
      simBer_zf = nErr_zf/N; % simulated ber
251
      simBer_zf_bc = nErr_zf_bc/N; % simulated ber
252
253
      simBer_zf_cc = nErr_zf_cc/N; % simulated ber
254
      simBer mmse = nErr mmse/N; % simulated ber
255
      simBer_mmse_bc = nErr_mmse_bc/N; % simulated ber
      simBer_mmse_cc = nErr_mmse_cc/N; % simulated ber
256
257
      theoryBer = 0.5*erfc(sqrt(10.^(Eb_N0_dB/10))); % theoretical ber
258
259
     % plot
260
     close all
261
      figure
      semilogy(Eb N0 dB,simBer zf(1,:),'b+-','Linewidth',2);
262
263
      hold on
      semilogy(Eb_N0_dB,simBer_mmse(1,:),'ro-','Linewidth',2);
264
265
      hold on
      semilogy(Eb_N0_dB,simBer_zf_bc(1,:),'g*-','Linewidth',2);
266
      hold on
267
268
      semilogy(Eb_N0_dB,simBer_mmse_bc(1,:),'bx-','Linewidth',2);
269
      hold on
270
      semilogy(Eb_N0_dB,simBer_zf_cc(1,:),'ms-','Linewidth',2);
271
      hold on
272
      semilogy(Eb_N0_dB,simBer_mmse_cc(1,:),'cd-','Linewidth',2);
      axis([0 14 10^-5 0.5])
273
274
      grid on
      legend('sim-zf', 'sim-mmse', 'sim-zf-hamming', 'sim-mmse-hamming', 'sim-zf-convolutional', 'sim-
275
      mmse-convolutional');
276
      xlabel('Eb/No, dB');
      ylabel('Bit Error Rate');
277
278
      title('Bit error probability curve for BPSK in ISI with ZF & MMSE equalizer');
279
      print("q4.jpg");
```