MET1413 ADVANCED DIGITAL COMMUNICATION
2013/14 semester II
Homework 4
Due: 17th April 2014

1. A TV picture contains 211,000 picture elements. Suppose that the each element is considered digital signals with 8 different level of brightness. Assume that each brightness has equal probability:
a. What is the information content for each picture?
b. Determine the information transmission rate if 30 separate pictures are sent per seconds.
c. Assume that the Malay language consists of about 50000 equiporbable words (a highly unrealistic assumption). What is the information content of a typical 1000 word message?

2. Calculate the capacity for discrete channel shown in Figure below, Assume r_s=1 symbol/s.

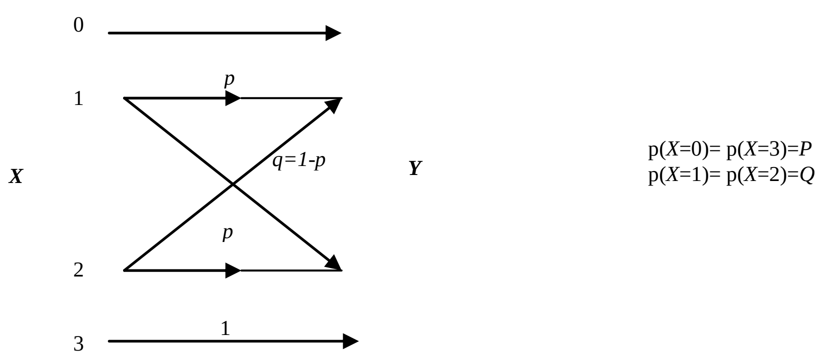

Figure P2

3. Figure below shows an up-link and down-link satellite communication channel model.
   a. Find the channel matrix of the resultant channel.
   b. Find $P(z_1)$ and $P(z_2)$ if $P(x_1)$=0.6 and $P(x_2)$=0.4.

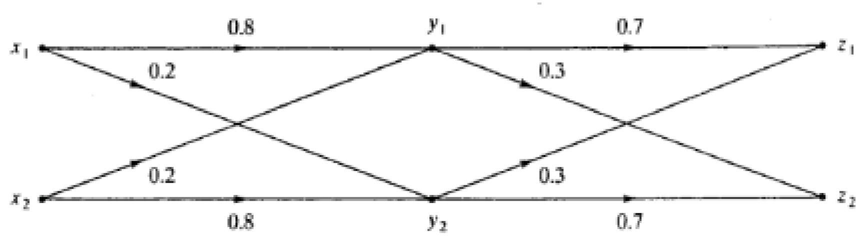

Figure P3

4. A binary code for error control has the parameters (n, k, d)=(39, 18, 10). Determine the error detection capability of this code.

5. A binary code is defined by the generator matrix. Determine the minimum distance of this code.

$$G = \begin{bmatrix} 11010110 \\ 01101011 \end{bmatrix}$$

6. **G** is the generator matrix of a linear code. This code is extended by adding an overall parity check bit to each codeword so that the Hamming weight of each resulting code is even.

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

   a. Determine the size of the extended code.
   b. Find the parity check matrix of the extended code.
   c. Develop the codewords and determine the minimum distance of the extended code?
   d. What are the error detection and correction capabilities of the new code?
   e. Find the coding gain of the extended code?

7. A ½ rate convolutional code with constraint length of 3 is described by the generator polynomial below and the state diagram is depicted in Figure P5.

$$\mathbf{g}_1 = [1 \quad 1 \quad 1]; \qquad \mathbf{g}_2 = [1 \quad 0 \quad 1]$$

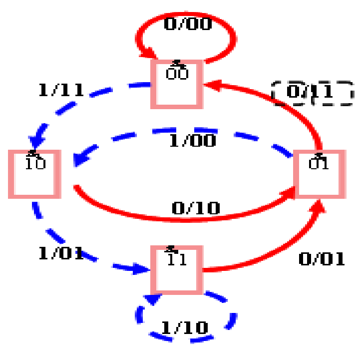

Figure P5

   a. Draw the encoder corresponding to this code.
   b. Find the output of the encoder if the input message is 1011.
   c. Find the free distance of this code.
   d. What is the error detection and correction capability provided by this encoder?
   e. Consider that this encoder is used over a binary symmetric channel (BSC). Assume that the initial encoder state is the 00 state. At the output of the BSC, the sequence Z = (1 1 0 0 0 0 1 0 1 1 rest all "0") is received. Find the maximum likelihood path through the trellis diagram, and determine the first 5 decoded information bits. (If a tie occurs between any two merged paths, choose the upper branch entering the particular state).
   f. Identify any channel bits in Z that were inverted by the channel during transmission.

8. Consider the (7, 4) Hamming code defined by the generator polynomial

$$g(X) = 1 + X + X^3$$

a. Proof that this generator is a valid generator polynomial for this code.
b. Draw the block diagram for this encoder.
c. The code word 0111001 is sent over a noisy channel, producing the received word 0101001 that has a single error. Determine the syndrome polynomial $s(X)$ for this received word, and show that it is identical to the error polynomial $e(X)$. Draw the decoder block diagram.

Do problems from Sklar's: P6.26, P8.9