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#1(a)

Information content for each picture,

$$I(S_k) = \log_2\left(\frac{1}{P_k}\right),$$

given $P_k = \frac{1}{8}$, $I(S_k) = \log_2 \frac{1}{1/8} = 3$ b/element

$= \cancel{3\ b/sampl}$ #

#1(b)

Information transmission rate, $R = r\,H(X)$ [b/s]

$$H(X) = \sum_{i=1}^{8} P_k \log_2\left(\frac{1}{P_k}\right)$$

$$= \sum_{i=1}^{8} \frac{1}{8} \log_2\left(\frac{1}{1/8}\right)$$

$= 3$ b/element

$r = (211\,000)(30) = 6330(10^3)$ elements/s

$R = r\,H(X)$

$= 6330(10^3)(\cancel{4}\ 3)$

$= 18990(10^3)$

$= 18.99$ Mb/s #

1(c)

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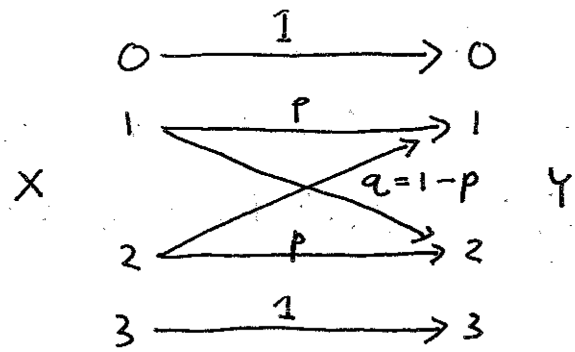
#2

Calculate the capacity for discrete channel below, assume $r_s = 1$ symbol/s.



$P(X=0) = P$

$P(X=1) = Q$

$P(X=2) = Q$

$P(X=3) = P$

Channel capacity, $C = \max\left[H(X) - H(X|Y)\right] r_s$

Given $r_s = 1$ symbol/s, $C = \max\left[H(X) - H(X|Y)\right]$

$$= \max\left[I(X;Y)\right]$$

$$H(X) = -\sum_{i=0}^{3} P(x=i)\log_2 P(x=i)$$

$$= -P\log_2 P - Q\log_2 Q - Q\log_2 Q - P\log_2 P$$

$$= -2P\log_2 P - 2Q\log_2 Q$$

$$H(X|Y) = \sum_{i=0}^{3} P(x=i)\, H(Y|x=i)$$

$$= P\log_2 1 + Q(-p\log_2 p - q\log_2 q) + Q(-p\log_2 p - q\log_2 q) + P\log_2 1$$

$$= 2Q(-p\log_2 p - q\log_2 q)$$

$$= -2Q(p\log_2 p + q\log_2 q)$$

Let $\alpha = -(p \log_2 p + q \log_2 q)$

then $H(X|Y) = 2\alpha Q$

$$I(X;Y) = H(X) - H(X|Y)$$
$$= -2P \log_2 P - 2Q \log_2 Q - 2\alpha Q$$

We know that $2P + 2Q = 1$

Maximize $I(X;Y)$ with respect to P, with $Q = \frac{1}{2} - p$

$$I(X;Y) = -2P \log_2 P - 2(\tfrac{1}{2} - P)\log_2(\tfrac{1}{2} - P) - 2\alpha(\tfrac{1}{2} - P)$$

$$\frac{dI(X;Y)}{dP} = 0$$

$$2\left[-\log_2 e - \log_2 P + \log_2 e + \log_2(\tfrac{1}{2} - P) + \alpha\right] = 0$$

$$-\log_2 P + \log_2(\tfrac{1}{2} - P) + \alpha = 0$$
$$\log P - \log(\tfrac{1}{2} - P) + \alpha = 0$$
$$-\log P + \log(\tfrac{1}{2} - P) = \alpha = 0$$

Let $\beta = 2^\alpha$, $\log \frac{\beta}{P}(\frac{1}{2} - P) = 0$

$$P = \frac{\beta}{2(1+\beta)} = \frac{2^\alpha}{2(1+2^\alpha)}$$

$$Q = \frac{1}{2(1+2^\alpha)}$$

The channel capacity is

$$-2(P \log P + Q \log Q + \alpha Q) r_s = \log \frac{2(\beta+1)}{\beta} \text{ bits/sec}$$

#

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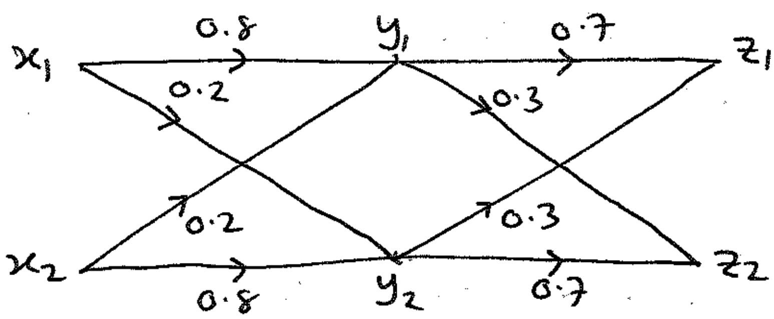
#3



(a) Find the channel matrix of the resultant channel

Channel 1, $P[Y/X] = \begin{bmatrix} P(y_1/x_1) & P(y_2/x_1) \\ P(y_1/x_2) & P(y_2/x_2) \end{bmatrix}$

$= \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}$ #

Channel 2, $P[Z/Y] = \begin{bmatrix} P(z_1/y_1) & P(z_2/y_1) \\ P(z_1/y_2) & P(z_2/y_2) \end{bmatrix}$

$= \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}$ #

$$P[Z/X] = P[Y/X] \cdot P[Z/Y]$$

$$= \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix} \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}$$

$$= \begin{bmatrix} 0.62 & 0.38 \\ 0.38 & 0.62 \end{bmatrix}$$ #

(b) Find $P(z_1)$ & $P(z_2)$ if $P(x_1) = 0.6$ & $P(x_2) = 0.4$.

$$P[z|x] = \begin{bmatrix} 0.62 & 0.38 \\ 0.38 & 0.62 \end{bmatrix} = \begin{bmatrix} P(z_1/x_1) & P(z_2/x_1) \\ P(z_1/x_2) & P(z_2/x_2) \end{bmatrix}$$

$$\begin{aligned} P(z_1) &= (0.62)(0.6) + (0.38)(0.4) \\ &= 0.372 + 0.152 \\ &= 0.524 \ \# \end{aligned}$$

$$\begin{aligned} P(z_2) &= (0.38)(0.6) + (0.62)(0.4) \\ &= 0.228 + 0.248 \\ &= 0.476 \ \# \end{aligned}$$

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#4 A binary code for error control has the parameters $(n, k, d) = (39, 18, 10)$. Determine the error detection capability of this code.

Error detection capability $= d - 1 = 10 - 1 = 9$ #

#5 A binary code is defined by the generator matrix. Determine the minimum distance of this code.

$$G = \begin{bmatrix} 11010110 \\ 01101011 \end{bmatrix}$$

The minimum distance is equal to the smallest non-zero weight among the code words.

Non-zero codewords	Hamming weight
11010110	5
01101011	5
10111101	6

$\therefore$ The minimum distance $= 5$ #

6

G is the generator matrix of a linear code. This code is extended by adding an overall parity check bit to each codeword so that the Hamming weight of each resulting code is even.

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

(a) Determine the size of the extended code.

The size of current code is $(6,3)$.

The size of extended code is $(7,3)$. ~~$(n+1, n+1-k)$~~ ~~which is~~ ~~$(7,3)$~~.

$c_e = [c \mid x]$ where x is 0 if the weight of c is even, x is 1 if the weight of c is odd

(b) ~~If G is~~ If $G = [I_k \mid P]$, then $H = [-P^T \mid I_{n-k}]$

parity check matrix $H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

Parity check matrix of the extended code, ~~H_E~~

$$H_e = \left[\begin{array}{c|c} H & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} \\ \hline 1 \ldots & 1 \end{array}\right]$$

$$= \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$ #

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#6 (c) Develop the codewords and determine the minimum distance of the extended code.

A 1 is appended at the end of the codewords to produce even parity. Thus, the minimum distance of the extended code is $m+1 = 3+1 = 4$ #

The generator matrix of the extended code is

$$G_e = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

	code word
$[0\ 0\ 0]G \Rightarrow$	0000000
$[0\ 0\ 1]G \Rightarrow$	001 0111
$[0\ 1\ 0]G \Rightarrow$	010 1011
$[0\ 1\ 1]G \Rightarrow$	011 1100
$[1\ 0\ 0]G \Rightarrow$	100 1101
$[1\ 0\ 1]G \Rightarrow$	101 1010
$[1\ 1\ 0]G \Rightarrow$	110 0110
$[1\ 1\ 1]G \Rightarrow$	111 0001

(d) What are the error detection and correction capabilities of the new code?

The new code is able to detect and correct a single error, and is able to detect up to 3 errors but not correct any.

(e) Find the coding gain of the extended code?

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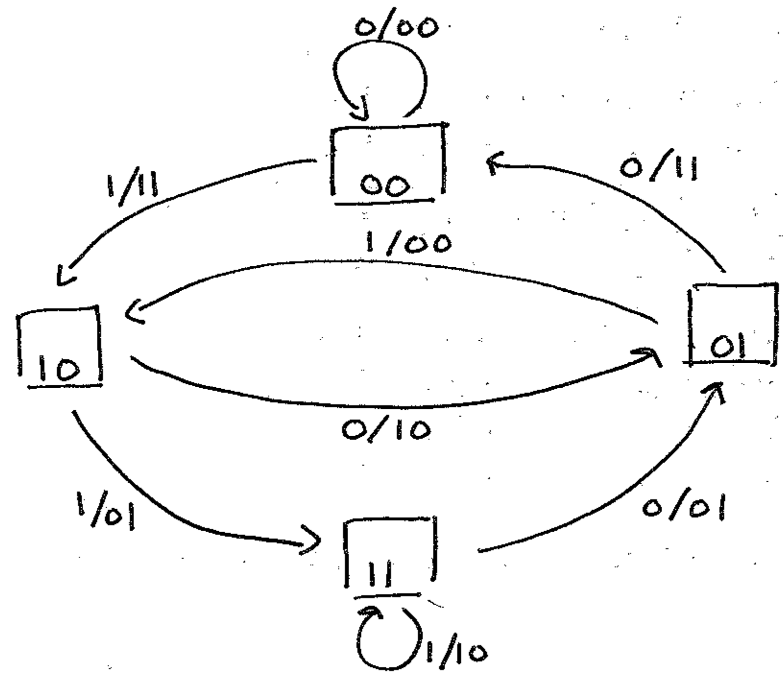
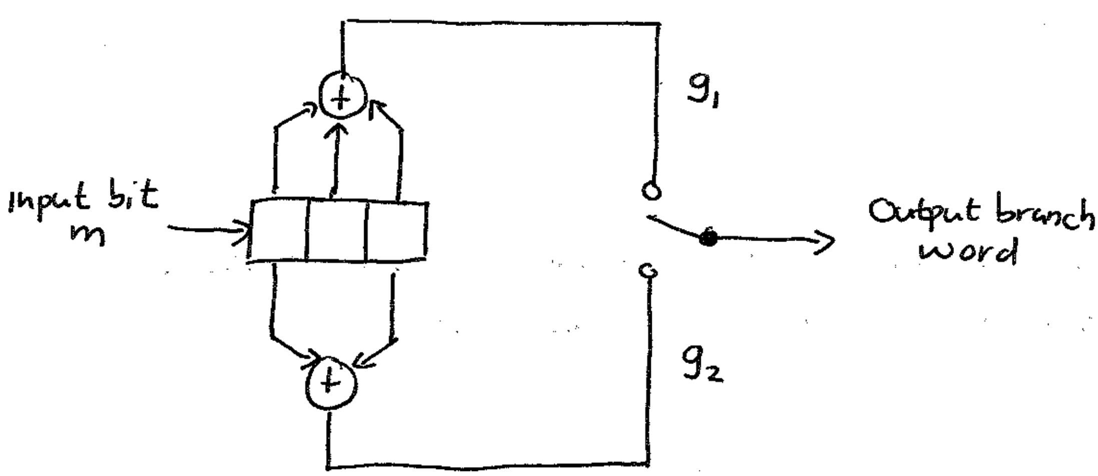
#7 A ½ rate convolutional code with constraint length of 3 is described by the generator polynomial below and the state diagram is depicted in Figure P5.

$g_1 = [1\ 1\ 1]$, $g_2 = [1\ 0\ 1]$



(a) Draw the encoder corresponding to this code.



(b) Find the output of the encoder if the input message is 1011.

$$
\begin{array}{l}
u_1[0] = (1+0+0) = 1 \\
u_2[0] = (1 \quad +0) = 1 \\
\hline
u_1[1] = (0+1+0) = 1 \\
u_2[1] = (0 \quad +0) = 0 \\
\hline
u_1[2] = (1+0+1) = 0 \\
u_2[2] = (1 \quad +1) = 0 \\
\hline
u_1[3] = (1+1+0) = 0 \\
u_2[3] = (1 \quad +0) = 1 \\
\hline
u_1[4] = (0+1+1) = 0 \\
u_2[4] = (0 \quad +1) = 1 \\
\hline
u_1[5] = (0+0+1) = 1 \\
u_2[5] = (0 \quad +1) = 1 \\
\hline
u_1[6] = (0+0+0) = 0 \\
u_2[6] = (0 \quad +0) = 0
\end{array}
$$

The output of the encoder is 11 10 00 01 01 11 ✱

(c) Find the free distance of this code.

(d) What is the error detection and correction capability provided by this encoder?

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#7(e) Consider that this encoder is used over a binary symmetric channel (BSC). Assume that the initial encoder state is the ØØ state. At the output of the BSC, the sequence Z = [1 1 0 0 0 0 1 0 1 1 rest all "0"] is received. Find the maximum likehood path through the trellis diagram, and determine the first 5 decoded information bits. (If a tie occurs between any two merged path, choose the upper branch entering the particular state).

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#8 Consider the (7,4) Hamming code defined by the generator polynomial

$$g(X) = 1 + X + X^3$$

(a) Proof that this generator is a valid generator polynomial for this code.

~~n=4~~ $n = 7, k = 4, m = n - k = 7 - 4 = 3$

By definition, $g(x)$ is a factor of $1 + x^n \Rightarrow 1 + x^7$

The polynomial $1 + x^7$ has the following factor:

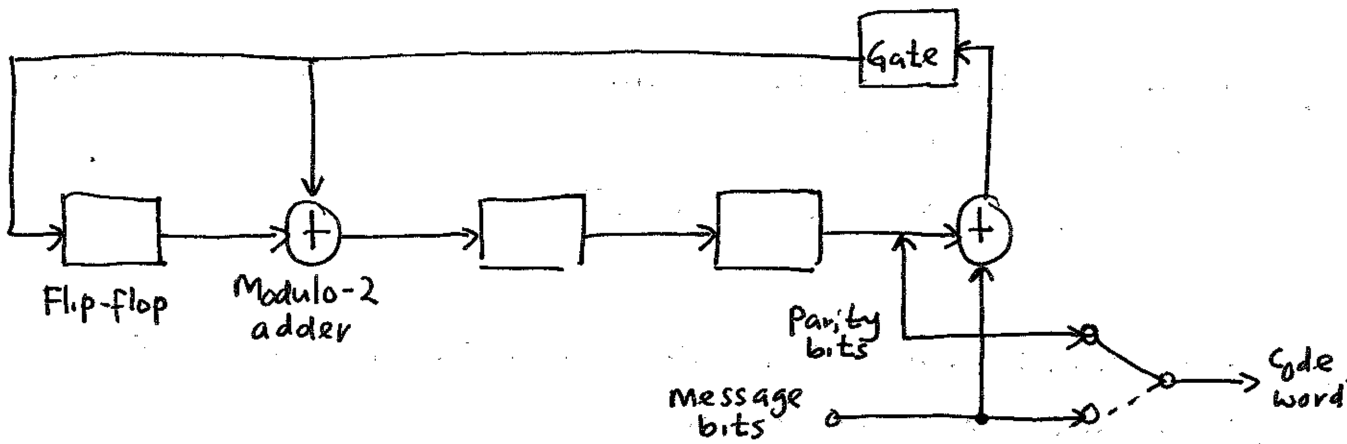
$$1 + x^7 = (1 + x)(1 + x + x^3)(1 + x^2 + x^3)$$

So there are two possible generator polynomials

~~for~~ $g_1(x) = 1 + x + x^3$, and

$g_2(x) = 1 + x^2 + x^3$

(b) Draw the block diagram for this encoder.



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(c) The code word 0111001 is sent over a noisy channel, producing the received word 0101001 that has a single error. Determine the syndrome polynomial $s(X)$ for this received word, and show that it is identical to the error polynomial $e(X)$. Draw the decoder block diagram.

Received word, $\cancel{R(X}$ $r(X) = q(X)g(X) + s(X)$

where $q(X)$ is the quotient, $s(X)$ is the remainder aka the syndrome polynomial.

Given $r(X) = X + X^3 + X^6$ and

$c(X) = X + X^2 + X^3 + X^6$

$$\begin{array}{r|l} & X^3 + X \\ \hline X^3 + X + 1 & X^6 \quad\quad + X^3 \quad + X \\ & X^6 \quad + X^4 + X^3 \\ \hline & \quad\quad X^4 \\ & \quad\quad X^4 \quad + X^2 + X \\ \hline & \quad\quad\quad\quad X^2 \end{array}$$

$r(x) = (x + x^3)(1 + x + x^3) + x^2$

$\therefore$ $\cancel{s(X) \leq}$

$s(x) = X^2$

$e(X) = r(X) + c(X) = X^2 = s(X)$ ⋕

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SKLAR P6.26

(a) Consider a data sequence encoded with a (127,64) BCH code and then modulated using coherent 16-ary PSK. If the received E_b/N_0 is 10 dB, find the MPSK probability of symbol error, the probability of code-bit error (assuming that a Gray code is used for symbol-to-bit assignment), and the probability of information-bit error.

MPSK probability of symbol error, $P_E = 2Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\frac{\pi}{M}\right)$

$$\frac{E_s}{N_0} = \frac{E_c}{N_0}\log_2 M \quad , \text{ where } M=16 \text{ , and}$$

$$= \frac{E_b}{N_0} \times \frac{f_b}{B} \times \log_2 M$$

$$= 10 \times \frac{64}{127} \times 4$$

$$= 20.157$$

$$= 13.04 \text{ dB}$$

$$\frac{E_c}{N_0} = \cancel{10 \times \frac{64}{127}} = 10\log_{10}\left(\frac{E_b}{N_0}\right) + 10\log_{10}\left(\frac{f_b}{B}\right)$$

$$= 10\log_{10}10 + 10\log_{10}\left(\frac{64}{127}\right)$$

$$= 10 - 2.9762 = 7.024 \text{ dB}$$

$$P_E = (2Q)(6.35)(0.195) = 2Q(1.24)$$

$$= 2(0.1075) = 0.215$$

Probability of code-bit error, $P_b = P_E/k = 0.215/4 = 0.054$

Probability of information-bit error is expressed by the following approximation, and (127,64) BCH can correct up to $t=10$ error patterns.

$$P_B \cong \frac{1}{n}\sum_{j=t+1}^{n} j\binom{n}{j}p^j(1-p)^{n-j} \quad , \; p = 0.054$$

$$\cong \frac{1}{127}\Big[11\binom{127}{11}(0.054^{11})(1-0.054)^{116} + 12\binom{127}{12}(0.054^{12})(1-0.054)^{115} + 13\binom{127}{13}(0.054^{13})(1-0.054)^{114} + \dots\Big]$$

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SKLAR P6.26

(b) For the same probability of information-bit error found in part (a), determine the value of E_b/N_0 required if the modulation in part (a) is changed to coherent orthogonal 16-ary FSK. Explain the difference.

$$\frac{P_B(M)}{P_E(M)} = \frac{M/2}{M-1}$$

~~PE~~ = Probability of symbol error, $P_E = \frac{M-1}{M/2} P_b$

$$= \frac{15}{8} \times 0.054$$

$$= 0.10125$$

$$P_E(M) \leq (M-1)\, Q\left(\sqrt{\frac{E_s}{N_0}}\right) = 0.10125$$

$$Q\left(\sqrt{\frac{E_s}{N_0}}\right) = \frac{0.10125}{16} = 0.0063$$

From Q $\sqrt{\frac{E_s}{N_0}} = 2.50$

$$\frac{E_s}{N_0} = 6.25$$

$$\frac{E_c}{N_0} = \frac{1}{k}\left(\frac{E_s}{N_0}\right) = \frac{1}{4}(6.25) = 1.5625$$

$$\frac{E_b}{N_0} = \frac{127}{64}\left(\frac{E_c}{N_0}\right) = \frac{127}{64}(1.5625) = 3.1 = 4.91\text{ dB}$$ #

The E_b/N_0 for 16-ary FSK is less than E_b/N_0 for 16-ary PSK for the given error performance.

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SKLAR P8.9 For each of the following conditions, design an interleaver for a communication system operating over a bursty noise channel at a transmission rate of 19,200 code symbols/s.

(a) A contiguous noise burst typically lasts for 250 ms. The system code consists of a (127,36) BCH code with $d_{min} = 31$. The end-to-end delay is not to exceed 5 s.

~~Maximum number of~~
Error correcting capability, $t_{max} = \dfrac{d_{min} - 1}{2} = \dfrac{31-1}{2} = 15$

Number of symbol error in a noise burst of 250 ms

$= (0.25\text{ s})(19200 \text{ symbols/s})$

$= 4800$ symbols ~~error~~

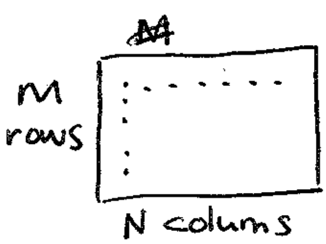
$= bN$

A code block of 127 bits can correct 15 errors.

Let $b = 15$, $bN = 4800$

$15N = 4800$

$N = 4800/15 = 320$ #



$M - b = 127$

$M - 15 = 127$

$M = 127 + 15 = 142$ #

Use a block interleaver of 142×320.

The interleaver end-to-end delay is approximately 2MN.

End-to-end delay $\cong 2MN$

$\cong \dfrac{2(142)(320)\text{ symbol}}{19200 \text{ symbol/s}}$

$= 4.73\text{ s} < 5\text{ s}$ #

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SKLAR P8.9

(b) A contiguous noise burst typically lasts for 20 ms. The system code ~~contain~~ consists of a rate ½ convolutional code with a feedback decoding algorithm that corrects an average of 3 symbols in a sequence of 21 symbols. The end-to-end delay is not to exceed 160 ms.

Number of symbol error in a noise burst of 20 ms

$= (0.02\,s)(19200 \text{ symbol}/s)$

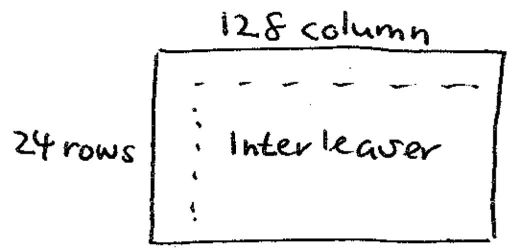
$= 384$ symbols.

$bN = 384$ number of errors can be corrected.

Let $b = 3$, $N = 384/3 = 128$ //

$M - b = 21$ ← sequence of symbols

$M = 21 + 3 = 24$ //



$$\text{End-to-end delay} \triangleq 2MN = \frac{2(24)(128) \text{ symbols}}{19200 \text{ symbols}/s}$$

$$= 0.32\,s$$

$$= 320 \text{ ms} > 160 \text{ ms}.$$

Use convolutional interleaver of size 24×128 with half of the delay (320 ms), which is 160 ms.