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#1 (a)

For a full-scale sinusoidal modulating signal with amplitude A, show that

$$(SNR)_o = \left(\frac{S}{N_q}\right)_o = \frac{3}{2}L^2 \quad \text{or}$$

$$\left(\frac{S}{N_q}\right)_{o\,dB} = 1.76 + 20\log 2^n = 1.76 + 6.02n \text{ dB}$$

The step size is $\Delta = \frac{2A}{L}$, where L is the number of quantizing level

The average quantizing noise power, $N_q = \langle q_e^2 \rangle = \frac{\Delta^2}{12}$

$$\neq \cancel{\frac{(2A)^2}{12}} \qquad = \frac{(2A/L)^2}{12}$$

$$\neq \cancel{\frac{A^2}{12}} \qquad = \frac{4A^2}{12L^2} = \frac{A^2}{3L^2}$$

$$(SNR)_o = \left(\frac{S}{N_q}\right)_o = \frac{A^2/2}{A^2/3L^2} = \frac{3}{2}L^2 \;\#$$

Expressing in decibels

$$\left(\frac{S}{N_q}\right)_{o\,db} = 10\log\left(\frac{S}{N_q}\right)_o$$
$$= 10\log\frac{3}{2}L^2$$
$$= 10\log\frac{3}{2} + 10\log L^2$$
$$= 1.76 + 20\log L \quad [dB]$$

In binary system, $L = 2^n$ where n is the number of binary digits

Thus, $$\left(\frac{S}{N_q}\right)_{o\,db} = 1.76 + 20\log 2^n$$
$$= 1.76 + n(20)(\log 2)$$
$$= 1.76 + 6.02n \quad [dB] \;\#$$

$2\pi f t = f_w$
$2\pi f_m = 5000$
$f = 2\pi(5000)$
$5000 = f$
$2(2500)\pi t =$

#1 (b)

(i) The output signal-to-quantizing-noise ratio for a full-scale sinusoid is given by equation

$$\left(\frac{S}{N_q}\right)_{OdB} = 1.76 + 20 \log 2^n$$

$$= 1.76 + 6.02n \quad [dB]$$

Given $n = 16$,

$$\left(\frac{S}{N_q}\right)_{OdB} = 1.76 + 6.02 \times 16 = 98.08 \text{ dB}$$ #

(ii) The input bitrate is

$$2 \times 44.1 \text{ kb/s} \times 16 = 2(44.1)(10^3)(16) \text{ b/s}$$
$$= 1.4112 (10^6) \text{ b/s}$$
$$= 1.4112 \text{ Mb/s}$$ #

With additional 100% overhead, the output bitrate is

$$2 \times 1.4112 \text{ Mb/s} = 2.8224 \text{ Mb/s}$$ #

(iii) The number of bits recorded on the CD, worth an hour is

$$(60 \times 60 \text{ s})(2.8224 \text{ Mb/s}) = 3.6(10^3)(2.8224)(10^6) \text{ b}$$
$$= 10.16 (10^9) \text{ b}$$
$$= 10.16 \text{ Gb}$$ #

(iv) The number of bits to store the dictionary is,

$$(1500)(2)(100)(8)(6)(7) \text{ b} = 100.8 (10^6) \text{ b}$$
$$= 100.8 \text{ Mb}$$

With additional 100% overhead, the number of comparable books that can be stored is,

$$\frac{10.16 \text{ Gb}}{2 \times 100.8 \text{ Mb}} = \frac{(10.16)(10^9)}{2(100.8)(10^6)}$$
$$= 50.4$$ #

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#2

Minimum sampling rate based on the given signal

$$m(t) = 10 \cos 2000\pi t \cos 800\pi t$$

Using trigonometric identities,

$$\cos(A)\cos(B) = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$$

Thus, $m(t) = \frac{10}{2}\left[\cos(2000\pi t + 800\pi t) + \cos(2000\pi t - 800\pi t)\right]$

$$= 5(\cos 2800\pi t + \cos 1200\pi t)$$

$m(t)$ is band-limited with $f_m = 1400$ Hz.

Minimum sampling rate, $f_s = 2f_m$

$$= 2800 \text{ Hz}$$ #

$2\pi f t = 2800\pi t$

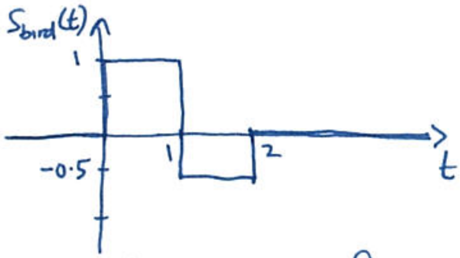
$f = \frac{2800\pi t}{2\pi t}$

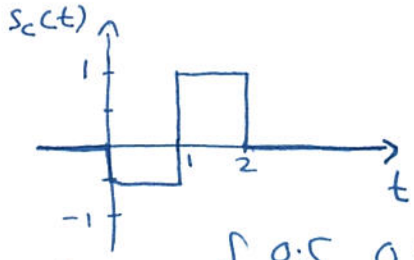
$= 1400$ Hz

3 (a)

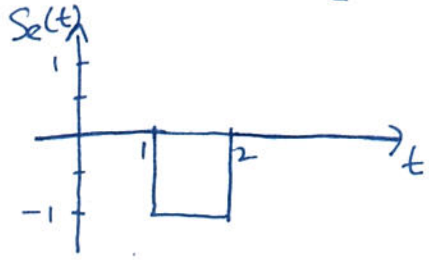
The orthogonal basis functions for

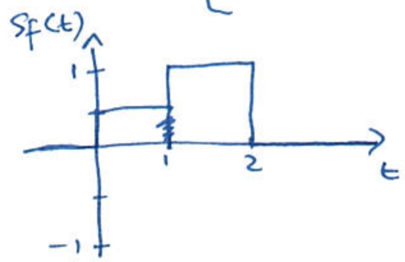
$$S_{bird}(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ -0.5 & 1 \leq t \leq 2 \end{cases} \qquad S_{commercial\ airlines} = \begin{cases} -0.5 & 0 \leq t \leq 1 \\ 1 & 1 \leq t \leq 2 \end{cases}$$



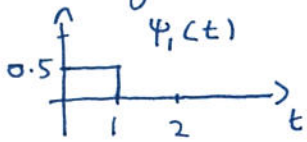


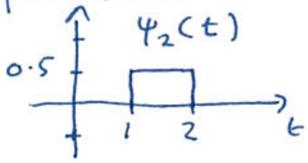
$$S_{enemy\ jet\ fighter} = \begin{cases} 0 & 0 \leq t \leq 1 \\ -1 & 1 \leq t \leq 2 \end{cases} \qquad S_{frendly\ jet\ fighter} = \begin{cases} 0.5 & 0 \leq t \leq 1 \\ 1 & 1 \leq t \leq 2 \end{cases}$$





By inspection, the signals can be expressed in terms of the following two basis functions





$$S_{bird}(t) = 2\varphi_1(t) - \varphi_2(t)$$

$$S_{enemy}(t) = -2\varphi_2(t)$$

$$S_{commercial}(t) = -\varphi_1(t) + 2\varphi_2(t)$$

$$S_{friendly}(t) = \varphi_1(t) + 2\varphi_2(t)$$

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#3(b)

$\langle S_{bird}(t), S_{enemy\ jet\ fighter}(t) \rangle$

$= \int_0^2 S_{bird}(t)\, S_{enemy\ jet\ fighter}(t)\, dt$

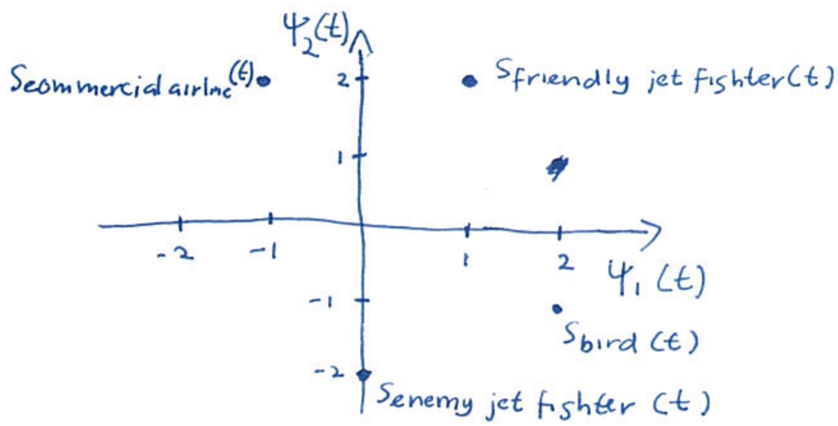
$= \int_0^1 (1)(0)\, dt + \int_1^2 (0.5)(-1)\, dt$

$= -0.5 \neq 0$

$\therefore$ $S_{bird}(t)$ and $S_{enemy\ jet\ fighter}(t)$ are not orthogonal.

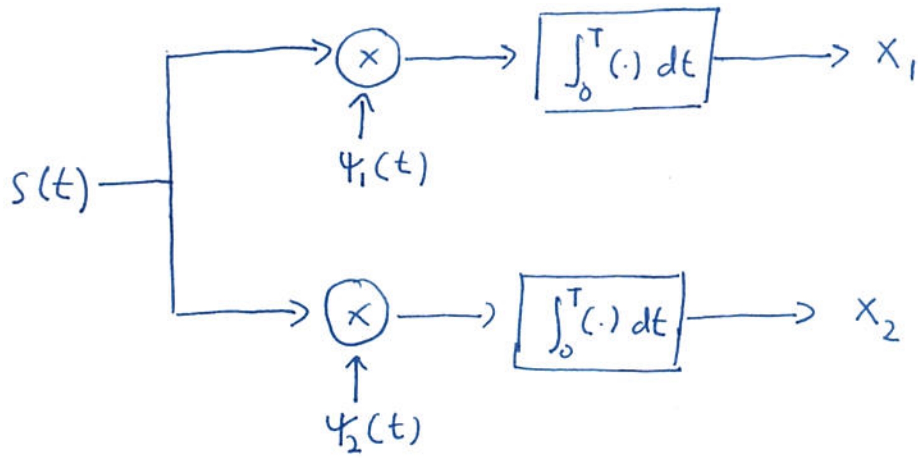
#3(c)

Signal space representation of the signals,



#3 (d)

Sketch the optimum correlator receiver of the system,



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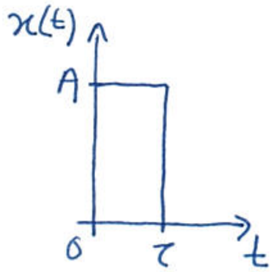
#4(a)

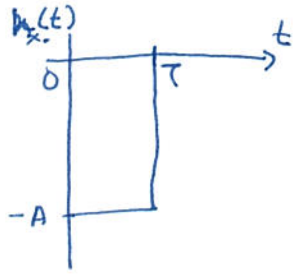
Sketch the causal matched filter impulse responses.

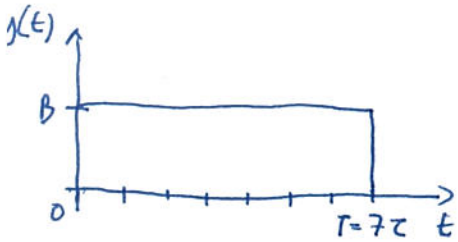
=> time-reversed and delayed version of the input signal



$$x(t) = \begin{cases} A & \text{for } 0 \le t \le \tau \\ 0 & \text{for the rest} \end{cases}$$

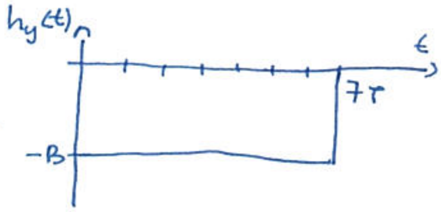
$$h_x(t) = \begin{cases} -A & \text{for } 0 \le t \le \tau \\ 0 & \text{for the rest} \end{cases}$$





$$y(t) = \begin{cases} B & \text{for } 0 \le t \le 7\tau \\ 0 & \text{for the rest} \end{cases}$$

$$h_y(t) = \begin{cases} -B & \text{for } 0 \le t \le 7\tau \\ 0 & \text{for the rest} \end{cases}$$



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#5

The maximum quantizing error

$(q_e)_{max} = \frac{\Delta}{2}$, where Δ is the step size

and the step size, $\Delta = \frac{2m_p}{L}$ where m_p is the peak amplitude. L is the number of quantizing level

$$(q_e)_{max} = \frac{\Delta}{2} = \frac{(2m_p/L)}{2} = \frac{m_p}{L}$$

$$\frac{m_p}{L} \leq 0.01 m_p$$

$$L \geqslant \frac{m_p}{0.01 m_p} \Rightarrow L \geqslant 100$$

Assume $L = 128 = 2^7$ ∴ number of bits required is 7.

The Nyquist frequency, $f_s = 2f_m = 2(2000) = 4000$ Hz

25% above f_s, $f_s = 1.25(4000) = 5000$ Hz

8 message signals, $8(5000) = 40\,000$ Hz

Encoded by 7-bit, the bit rate is $\frac{1}{T_b} = 7(40000)$

$= 280\,000$ b/s

The minimum transmission bandwidth required is,

$$f_B = \frac{1+\alpha}{2T} \text{ [Hz]}, \quad \alpha = 0.2, \quad \frac{1}{T} = 280000$$

$$= \frac{1+0.2}{2}(280000)$$

$$= 168\,000 = 168(10^3)$$

$$= 168 \text{ kHz}$$ #

#6

Let $H(\omega)$ is the frequency response of a linear filter.

The output signal at time $t = T$ is

$$a_i(T) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)\, S(\omega)\, e^{j\omega T}\, d\omega \qquad \text{—— (1)}$$

where $S(W)$ is the Fourier transform of the input signal

Let $S_n(\omega)$ is the power spectrum of the input noise

The output noise power is

$$N_0 = E[n_0^2(t)]$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} S_n(\omega)\, |H(\omega)|^2\, d\omega \qquad \text{—— (2)}$$

Combine (1) and (2), the output SNR is

$$\left(\frac{S}{N}\right)_0 = \frac{\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)\, S(\omega)\, e^{j\omega T}\, d\omega\right]^2}{\frac{1}{2\pi}\int_{-\infty}^{\infty} S_n(\omega)\, |H(\omega)|^2\, d\omega}$$

$$= \frac{\frac{1}{2\pi}\left|\int_{-\infty}^{\infty} H(\omega)\, S(\omega)\, e^{j\omega T}\, d\omega\right|^2}{\int_{-\infty}^{\infty} S_n(\omega)\, |H(\omega)|^2\, d\omega} \qquad \text{—— (3)}$$

Using Schwarz's inequality, which is

$$\left|\int_{-\infty}^{\infty} f_1(x) f_2(x)\, dx\right|^2 \leq \int_{-\infty}^{\infty} |f_1(x)|^2\, dx \int_{-\infty}^{\infty} |f_2(x)|^2\, dx$$

the equality holds if $f_1(x) = k f_2^*(x)$

Set $f_1(\omega) = \sqrt{S_n(\omega)}\, H(\omega)$ & $f_2(\omega) = \dfrac{S(\omega)\, e^{j\omega T}}{\sqrt{S_n(\omega)}}$

$$\left|\int_{-\infty}^{\infty} H(\omega)\, S(\omega)\, e^{j\omega T} d\omega\right|^2 \leq \int_{-\infty}^{\infty} S_n(\omega)\, |H(\omega)|^2\, d\omega \int_{-\infty}^{\infty} \frac{|S(\omega)|^2}{S_n(\omega)}\, d\omega \qquad \text{—— (4)}$$

The equality holds if

$$\sqrt{S_n(\omega)}\, H(\omega) = k\, \frac{S^*(\omega)\, e^{j\omega T}}{\sqrt{S_n(\omega)}}$$

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#6 cont.

Substitute (4) into (3),

$$\left(\frac{S}{N}\right)_0 \leq \frac{\frac{1}{2\pi}\left[\int_{-\infty}^{\infty} S_n(\omega)|H(\omega)|^2 d\omega \int_{-\infty}^{\infty} \frac{|S(\omega)|^2}{S_n(\omega)} d\omega\right]}{\int_{-\infty}^{\infty} S_n(\omega)|H(\omega)|^2 d\omega}$$

$$\leq \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{|S(\omega)|^2}{S_n(\omega)} d\omega \quad \text{——} \ (5)$$

$(SNR)_0$ is maximum when the equality holds, which is when

$$\sqrt{S_n(\omega)}\, H(\omega) = k\, \frac{S^*(\omega)\, e^{j\omega T}}{\sqrt{S_n(\omega)}}$$

$$H(\omega) = k\, \frac{S^*(\omega)\, e^{j\omega T}}{S_n(\omega)}$$

#

#7

Using equation

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \leq 10^{-4}$$

$$= Q(x) \quad \text{where} \quad x = \sqrt{\frac{2E_b}{N_0}}$$

From Q-function table, $Q(x) = 0.0001$

$$x = 3.72$$

$$\sqrt{\frac{2E_b}{N_0}} = 3.72$$

$\frac{2E_b}{N_0} = 3.72^2$ and ~~$\frac{N_0}{2}$ is g~~ given $\frac{N_0}{2} = 10^{-5}$

$$E_b = 3.72^2 (10^{-5})$$

Thus, the maximum bit rate that can be sent with the given bit error probability is

$$R = \frac{1}{T} = \frac{1}{3.72^2(10^{-5})} = 7226.27 \text{ bits/s}$$

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8(a)

Evaluate the average energy

$$s_0(t) = 0$$

$$s_1(t) = s_2(t) = s_3(t) = A \cos\left(\omega t + \frac{2\pi}{3}\right)$$

$$\text{Average energy, } E_{avg} = \frac{1}{4}\left[\int s_0^2\,dt + \int s_1^2\,dt + \int s_2^2\,dt + \int s_3^2\,dt\right]$$

$$= \frac{1}{4}\left[0 + \frac{A^2T}{2} + \frac{A^2T}{2} + \frac{A^2T}{2}\right]$$

$$= \frac{3A^2T}{8} \#$$

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