Chapter 1

Structural Integration Management

1.1 simple example

Now we need to build the code block shown in the Figure 1.1 to connect all the relevant functions. In our code, we can also write the following blocks:

1.1.1 Problem Formulation: 1D Beam with deteriorating stiffness

We model a 1D fixed-fixed beam under a point load at mid-span with a predefined geometry, Loading condition, boundary condition, material properties are summarized in the following Table 1.1:

Table 1.1: Configuration of Steel Truss Bridge Structure

Geometry Property	length L Cross Section A	$1m$ $1.0m^2$
Machanical Property	Youngs modulus E	$E(t=0) = 210e9Pa$
Material Property	Density ρ	$7800kg/m^3$
Loading	Mid point load F	10kN
Boundary Condition	$u_L = 0$, $u_R = 0$	

To model as a POMDP model, we will think through the following components of the model

- States space $\mathbb{S}$: is a continuous spaces $\mathbb{R}^N$ containing the Youngs modulus for all elements $\mathbf{E} = [E_1, E_2, \cdots, E_{n_{elements}}]$. In other words, we will assume that the only changing parameters over time is the mechanical property Youngs modulus. The mass (usually depends on density and geometry) is constant if we assume that the density and geometry do not change over time.

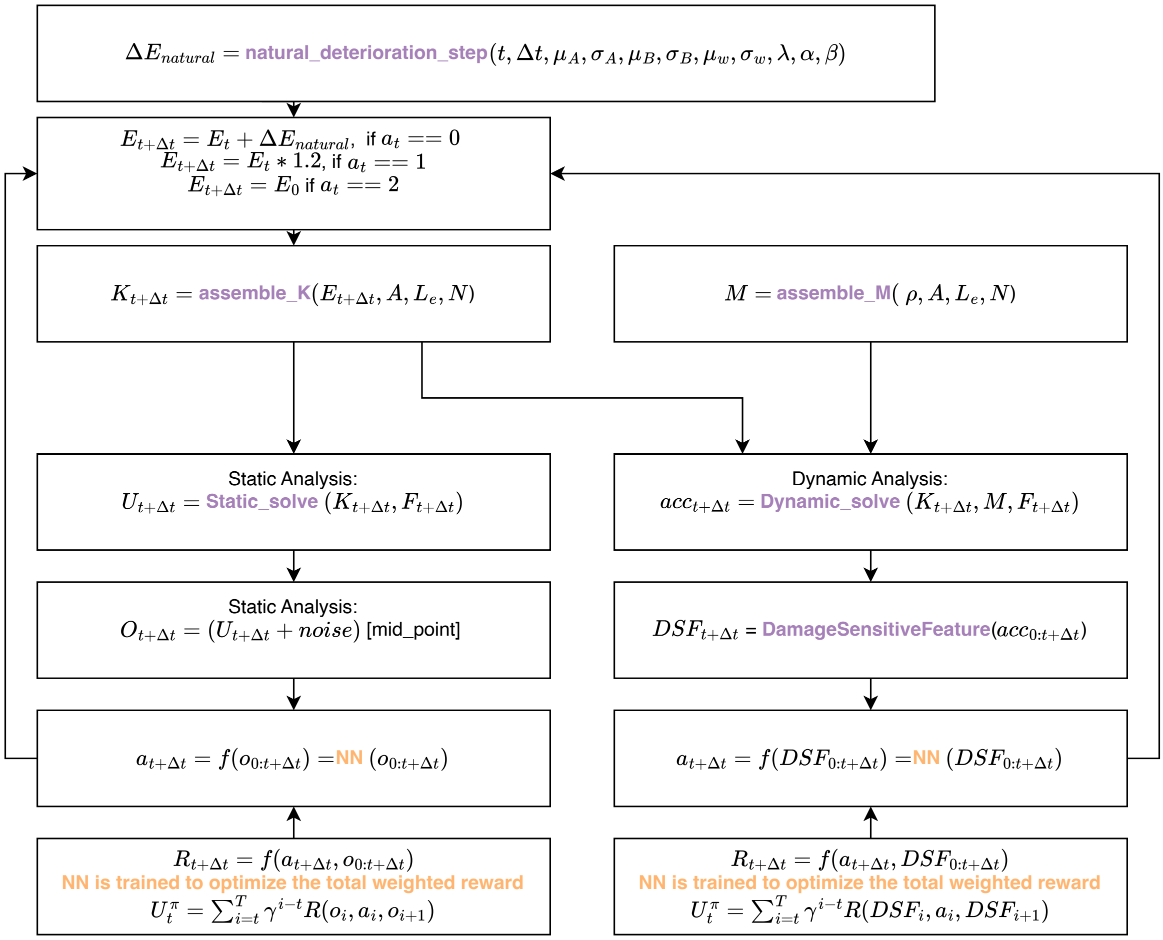

Figure 1.1: Flowchart diagram

- Action space $\mathbb{A}$: is a discrete space contains three elements $\mathfrak{a}_0, \mathfrak{a}_1, \mathfrak{a}_2$

  The state-dependent sequence of actions is defined as policy π. There could be two types of policies, the deterministic policy $\pi(\mathfrak{a}|\mathfrak{s}) : \mathbb{S} \to \mathbb{A}$ (mapping from the state space to the action space) and the stochastic policy $\pi(\mathfrak{a}|\mathfrak{s}) : \mathbb{S} \times \mathbb{A} \to \mathbb{R} \in [0, 1]$(mapping from the state space to the probability of actions). Depending on whether the action space are continuous or discrete we can have the conditional probability density function (PDF) and the conditional probability mass function (PMF).

- Observation space $\mathbb{O}$: is a continous observation space containing the observation of the mid-point displacement observations

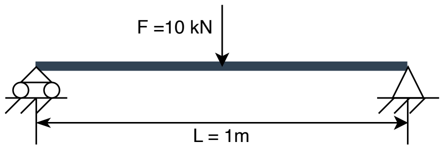

Figure 1.2: Geometrical Property of Steel Truss Bridge Structure

- Transition probability $\mathbf{T}$: $s_{t+\Delta t} \leftarrow \mathbf{T}(s_t, a_t)$ The natural degradation process is modelled as the basic deterioration and repairs are applied element-wise
  - a_0: do nothing $\rightarrow$ natural deterioration. We could define a deterioration level as $D(t) \stackrel{\text{def}}{=\!=} E(t=0) - E(t)$ to indicate the deterioration extent from the beginning to the time t. The deterioration of the infrastructures is usually modelled by the mixed Levy process including the gradual deterioration (aging process) and the sudden deterioration (jump process).

$$E(t) = Gradual(t) + Jump(t) = G(t) + J(t) \tag{1.1}$$

    The gradual process $G(t)$ focuses on the material aging, wearing, corrosion of the environment etc. The Jump process $J(t)$ focuses on those infrastures in the overloading areas, earthquake zone etc. Below is the mathematical model we choose to model the deterioration process. For the gradual deterioration, we could use a simple rate function or a Gamma process. For the jump process, we could use a Compound Poisson Process shown below.

    * gradual deterioration (aging process) modelled as a simple rate function [**ellingwood2005risk**]:

$$G(t) = At^B e^{w(t)} \tag{1.2}$$

      where A is the random variable modelling the deterioration rate, B is the random variable modelling the nonlinearity effect in terms of a lower law in time and $w(t)$ models the gaussian stochastic process noise. Realization plot of an aging process is shown in Figure 1.3 The changes of the deterioration can be approximated by the derivative $\frac{dD}{dt}\Delta t$ if we treate $w(t)$ as a constant number w_k

$$\Delta G(t) = G(t) - G(t + \Delta t) \approx ABt^{B-1}e^{w_k}\Delta t \tag{1.3}$$

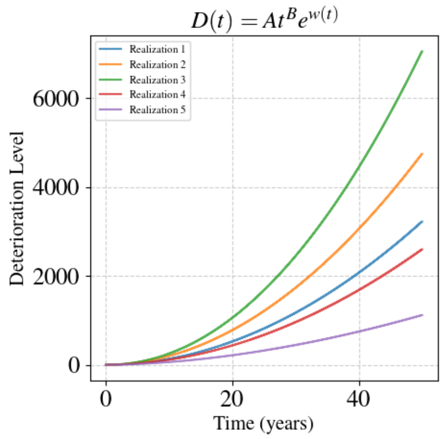

Figure 1.3: Gradual deterioration realization plot modeled by a simple rate function

* gradual deterioration modelled as a gamma process

$$G(t) \sim \Gamma(\alpha, \beta), \tag{1.4}$$

where α, β denotes the aging rate. Gamma Process is a time continuous, parameter space continuous, pure jump Levy process. Its path is not continuous, but combined by small jumps, which is a very suitable model for real deteriorations where the seemly-continuous wearing process is actually combined by a series of discrete broken of chemical bonds in microscope.
The increment of Gamma process also follows a Gamma process $\Delta G(t) = G(t + \Delta t) - G(t) \sim \Gamma(\alpha\Delta t, \beta)$.
Gamma process has independent increments, and the increment over any interval has a Gamma distribution.

* sudden deterioration: The sudden deterioration can be modelled as a homogeneous compound Poisson Process(CPP) [**van2009survey**, **sanchez2016reliability**]

$$J(t) = \sum_{i=1}^{N(t)} J_i \sim CPP(t; \lambda, F_J(j)) \tag{1.5}$$

where the number of jumps in the time interval t $N(\Delta t) \sim \mathrm{PoissonProcess}(\lambda)$ with λ denoting the frequency of sudden disaster; the amplitude of each jump $J_i \sim F_J(j)$ are independent and identically distributed random variables following a given distribution $F_J(j)$ e.g. a Gamma

distribution or Lognormal(μ, σ^2) with μ and σ representing the extend of the damage from each disaster. Realization plot of a CPP process is shown in Figure 1.4
Compound Poisson Process has also independent and stationary increments, so The change of the deterioration during the time interval Δt is also a CPP process:

$$\Delta J(t) = J(t + \Delta t) - J(t) = \sum_{i=N(t)+1}^{N(t+\Delta t)} J_i \sim CPP(t; \lambda\Delta t, F_J(j)) \quad (1.6)$$

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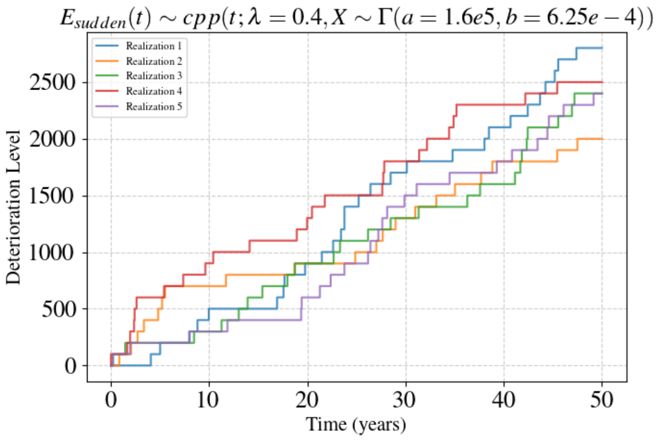

Figure 1.4: sudden deterioration realization plot modelled by a CPP process

In summary the deterioraion model parameters are defined in the Table 1.2

- a_1: minor repair $\rightarrow$ $E(t + \Delta t) = E(t) + (E(t = 0) - E(t)) \cdot \alpha_{repair}$
- a_2: full replacement $\rightarrow$ $E(t + \Delta t) = E(t = 0)$

where $\Delta t = T/N$, T is the total time span e.g. 50 years and N is the total number of decision steps.

- Observation model $\mathbf{O}(o_{t+\Delta t} | s_{t+\Delta t}, a_t)$:

Table 1.2: Prior distribution of deterioration model parameters

Parameter	Distribution	Mean	cv
A	Lognormal	$1.94 \cdot 10^{-4}$	0.4
B	Normal	2.0	0.1
ω_k	Normal	-0.005	0.1
D_i	Lognormal	3.75	0.25
$N(t)$	Poisson	$0.04 \cdot t$	$0.04 \cdot t$

- Static analysis: Observation from the static analysis is the midpoint displacement. It is continuous observation. The displacement vector is calculated via

$$u_{t+\Delta t} = StaticSolver(K(E(t+\Delta t)), F_{t+\Delta t}). \tag{1.7}$$

We could generate the synthetic observations by adding the noise

$$o_{t+\Delta t} = f(u_{t+\Delta t} + \mathbf{N}(0, \sigma^2)). \tag{1.8}$$

- Dynamic analysis: We could generate the synthetic observation from dynamic analysis is the acceleration time series data.

$$acc_{t+\Delta t} = dynamicSolver(K(E(t+\Delta t)), F_{t+\Delta t}). \tag{1.9}$$

We could use the Vibration-based SHM method to extract the damage sensitive feature from the acceleration time serires data.

$$DSF_{t+\Delta t} = DamageSensitiveFeatureExtraction(acc_{0:t+\Delta t}) \tag{1.10}$$

- Reward Model $\mathbf{r}(s, a)$:

Accumulated Discount Reward for the whole episode T is defined as the weighted sum of reward at each time step:

$$R = R(s_0, a_0, \cdots, s_T.a_T) = \sum_{i=0}^{T} \gamma^{i-t} R(s_i, a_i) \tag{1.11}$$

where the discount factor $\gamma \in [0, 1]$. It weights more on the current reward than the future reward. When $\gamma = 0$: only the current reward matters; when $\gamma = 1$: rewards in all steps equally matter.

The total reward is also composed of three parts: $R = R_{insp} + R_{disp} + R_{repair} + R_{replace} + R_{failure}$

Table 1.3: Cost Definition in the beam monitoring process

Cost due to displacement	$R_{disp} = -\sum_{i=1}^{T} k_i u_i^2$ or simpler $-\beta\|u_i\|$	$k_i = 10$ for $i = 0, \cdots, T$ $\beta = 50$
Cost due to repair	$R_{repair} = -c_{repair} \cdot n_{repair}$	$c_{repair} = 500$ n_{repair} is the total repair times
Cost due to replace	$R_{replace} = -c_{replace} \cdot n_{replace}$	$c_{replace} = 2000$ $n_{replace}$ is the total replace times
Cost due to inspection	$R_{insp} = -c_{insp} \cdot n_{inspection}$	$c_{insp} = 200$ n_{insp} is the total inspection times
Cost due to failure	$R_{failure} = -c_{failure}$	$c_{failure} = 10000$ is the equivalent in During time T , the structure failure

1.2 Proposed Framework

This section presents the proposed framework for addressing the research problem. The overall architecture is depicted in ??.

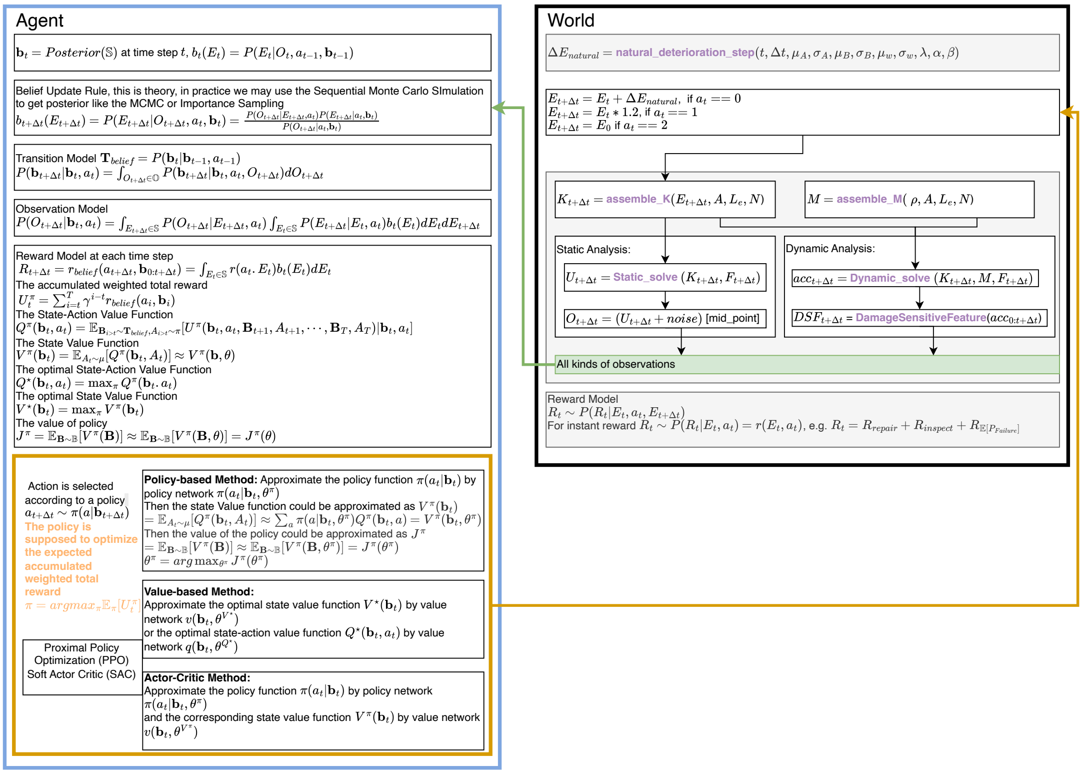

Figure 1.5: Flowchart Belief MDP diagram

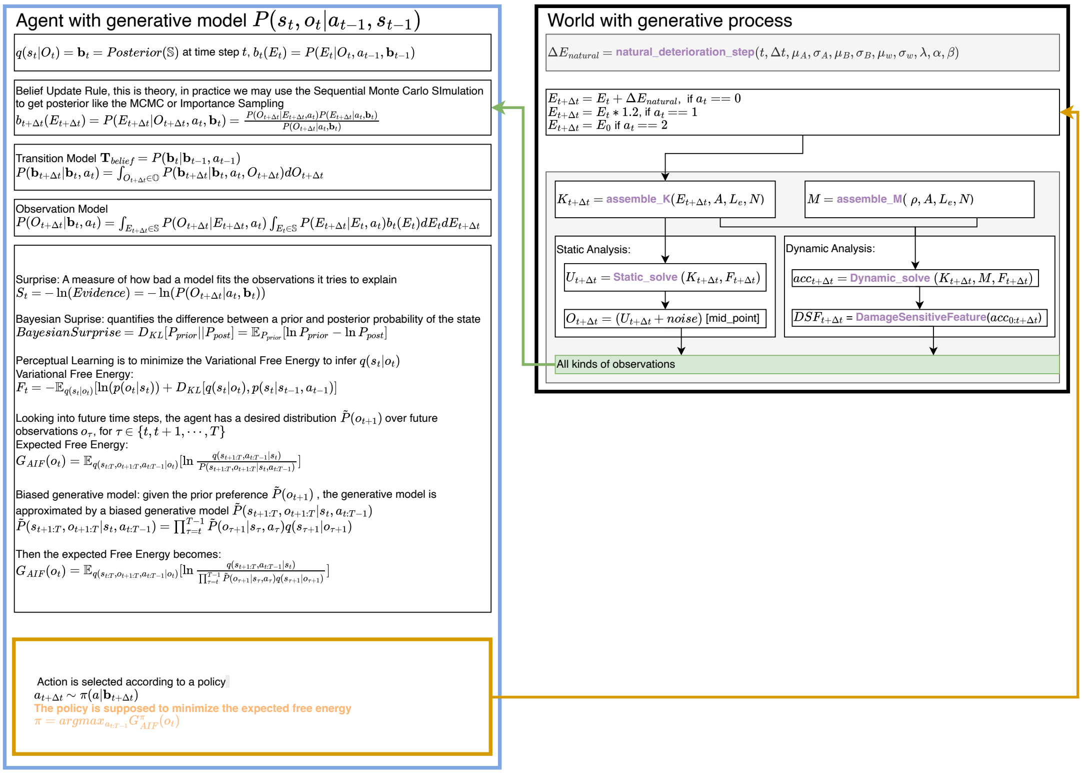

Figure 1.6: Flowchart Active Inference diagram