Agent with generative model $P(s_t, o_t | a_{t-1}, s_{t-1})$

$q(s_t|O_t) = \mathbf{b}_t = Posterior(\mathbb{S})$ at time step t, $b_t(E_t) = P(E_t|O_t, a_{t-1}, \mathbf{b}_{t-1})$

Belief Update Rule, this is theory, in practice we may use the Sequential Monte Carlo SImulation to get posterior like the MCMC or Importance Sampling
$b_{t+\Delta t}(E_{t+\Delta t}) = P(E_{t+\Delta t}|O_{t+\Delta t}, a_t, \mathbf{b}_t) = \frac{P(O_{t+\Delta t}|E_{t+\Delta t},a_t)P(E_{t+\Delta t}|a_t,\mathbf{b}_t)}{P(O_{t+\Delta t}|a_t,\mathbf{b}_t)}$

Transition Model $\mathbf{T}_{belief} = P(\mathbf{b}_t|\mathbf{b}_{t-1}, a_{t-1})$
$P(\mathbf{b}_{t+\Delta t}|\mathbf{b}_t, a_t) = \int_{O_{t+\Delta t}\in\mathbb{O}} P(\mathbf{b}_{t+\Delta t}|\mathbf{b}_t, a_t, O_{t+\Delta t})dO_{t+\Delta t}$

Observation Model
$P(O_{t+\Delta t}|\mathbf{b}_t, a_t) = \int_{E_{t+\Delta t}\in\mathbb{S}} P(O_{t+\Delta t}|E_{t+\Delta t}, a_t) \int_{E_t\in\mathbb{S}} P(E_{t+\Delta t}|E_t, a_t) b_t(E_t) dE_t dE_{t+\Delta t}$

Surprise: A measure of how bad a model fits the observations it tries to explain
$S_t = -\ln(Evidence) = -\ln(P(O_{t+\Delta t}|a_t, \mathbf{b}_t))$

Bayesian Suprise: quantifies the difference between a prior and posterior probability of the state
$BayesianSurprise = D_{KL}[P_{prior}||P_{post}] = \mathbb{E}_{P_{prior}}[\ln P_{prior} - \ln P_{post}]$

Perceptual Learning is to minimize the Variational Free Energy to infer $q(s_t|o_t)$
Variational Free Energy:
$F_t = -\mathbb{E}_{q(s_t|o_t)}[\ln(p(o_t|s_t)) + D_{KL}[q(s_t|o_t), p(s_t|s_{t-1}, a_{t-1})]$

Looking into future time steps, the agent has a desired distribution $\tilde{P}(o_{t+1})$ over future observations o_τ, for $\tau \in \{t, t+1, \cdots, T\}$
Expected Free Energy:
$G_{AIF}(o_t) = \mathbb{E}_{q(s_{t:T}, o_{t+1:T}, a_{t:T-1}|o_t)}[\ln \frac{q(s_{t+1:T}, a_{t:T-1}|s_t)}{P(s_{t+1:T}, o_{t+1:T}|s_t, a_{t:T-1})}]$

Biased generative model: given the prior preference $\tilde{P}(o_{t+1})$, the generative model is approximated by a biased generative model $\tilde{P}(s_{t+1:T}, o_{t+1:T}|s_t, a_{t:T-1})$
$\tilde{P}(s_{t+1:T}, o_{t+1:T}|s_t, a_{t:T-1}) = \prod_{\tau=t}^{T-1} \tilde{P}(o_{\tau+1}|s_\tau, a_\tau)q(s_{\tau+1}|o_{\tau+1})$

Then the expected Free Energy becomes:
$G_{AIF}(o_t) = \mathbb{E}_{q(s_{t:T}, o_{t+1:T}, a_{t:T-1}|o_t)}[\ln \frac{q(s_{t+1:T}, a_{t:T-1}|s_t)}{\prod_{\tau=t}^{T-1} \tilde{P}(o_{\tau+1}|s_\tau, a_\tau)q(s_{\tau+1}|o_{\tau+1})}]$

Action is selected according to a policy
$a_{t+\Delta t} \sim \pi(a|\mathbf{b}_{t+\Delta t})$
The policy is supposed to minimize the expected free energy
$\pi = argmax_{a_{t:T-1}} G^{\pi}_{AIF}(o_t)$

World with generative process

$\Delta E_{natural} =$ **natural_deterioration_step**$(t, \Delta t, \mu_A, \sigma_A, \mu_B, \sigma_B, \mu_w, \sigma_w, \lambda, \alpha, \beta)$

$E_{t+\Delta t} = E_t + \Delta E_{natural}$, if $a_t == 0$
$E_{t+\Delta t} = E_t * 1.2$, if $a_t == 1$
$E_{t+\Delta t} = E_0$ if $a_t == 2$

$K_{t+\Delta t} =$ **assemble_K**$(E_{t+\Delta t}, A, L_e, N)$

$M =$ **assemble_M**(ρ, A, L_e, N)

Static Analysis:
$U_{t+\Delta t} =$ **Static_solve** $(K_{t+\Delta t}, F_{t+\Delta t})$
$O_{t+\Delta t} = (U_{t+\Delta t} + noise)$ [mid_point]

Dynamic Analysis:
$acc_{t+\Delta t} =$ **Dynamic_solve** $(K_{t+\Delta t}, M, F_{t+\Delta t})$
$DSF_{t+\Delta t}$ = **DamageSensitiveFeature**$(acc_{0:t+\Delta t})$

All kinds of observations