Chapter 1

Bayesian Filtering

This chapter is a summary of Bayesian Filtering based on my Master thesis understanding and the manuscript from [**chen2003bayesian**].

1.1 State-space formulations

Now let consider the following generic stochastic filtering problem in a dynamic state-space form written in continuous time domain.

$$\dot{\mathbf{s}}_t = \mathsf{f}(t, \mathbf{s}_t, \mathbf{a}_t, \mathbf{d}_t) \tag{1.1a}$$
$$\mathbf{o}_t = \mathsf{g}(t, \mathbf{s}_t, \mathbf{a}_t, \mathbf{v}_t) \tag{1.1b}$$

where equation Eq. 1.1a is called state eqation and Eq. 1.1b is the measurement equation (or observation equation). where

- $\mathbf{s}_t \in \mathbb{S}$ represents the state vector.
- $\mathbf{a}_t \in \mathbb{A}$ represents the system input vector as driving force (action vector) in a controlled environment.
- $\mathbf{o}_t \in \mathbb{O}$ represents the observation vector.
- $\mathsf{f}(\cdot) : \mathbb{R}^{N_\mathbb{S}} \to \mathbb{R}^{N_\mathbb{S}}$
- $\mathsf{g}(\cdot) : \mathbb{R}^{N_\mathbb{S}} \to \mathbb{R}^{N_\mathbb{O}}$
- $\mathbf{d}_t$ is the process (dynamical) noise
- $\mathbf{v}_t$ is the measurement (observation) noise

In practice more common form is the discrete-time equations, in the discrete time space, $t \in (0, T)$ is discretized to $n \in \{1, 2, \cdots, N\}$

- $\mathbf{s}_t, t \in (0, T)$ becomes $\mathbf{s}_n, n \in \{1, \cdots, N\}$
- $\mathbf{o}_t, t \in (0, T)$ becomes $\mathbf{o}_n, n \in \{1, \cdots, N\}$
- white noise $\mathbf{d}_t, t \in (0, T)$ becomes $\mathbf{d}_n, n \in \{1, \cdots, N\}$
- white noise $\mathbf{v}_t, t \in (0, T)$ becomes $\mathbf{v}_n, n \in \{1, \cdots, N\}$

$$\mathbf{s}_{n+1} = f(\mathbf{s}_n, \mathbf{d}_n) \tag{1.2a}$$
$$\mathbf{o}_n = g(\mathbf{s}_n, \mathbf{v}_n) \tag{1.2b}$$

Eq. 1.2a characterizes the state transition probability $p(\mathbf{s}_{n+1}|\mathbf{s}_n)$, whereas the observation equation Eq. 1.2b describes the observation probability $p(\mathbf{o}_n|\mathbf{s}_n)$.

The Eq. 1.2 can be further simplified under the linear Quadratic Gaussian assumption as:

$$\mathbf{s}_{n+1} = \mathbf{F}_{n+1,n}\mathbf{s}_n + \mathbf{d}_n \tag{1.3a}$$
$$\mathbf{o}_n = \mathbf{G}_n\mathbf{s}_n + \mathbf{v}_n \tag{1.3b}$$

For this simplified discrete dynamic state space governing equation with Linear Gaussian Quadratic assumption, Kalman filter could be implemented to get the analytical solution.

1.2 Sequential Monte Carlo Simulation

Some Items explanations

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