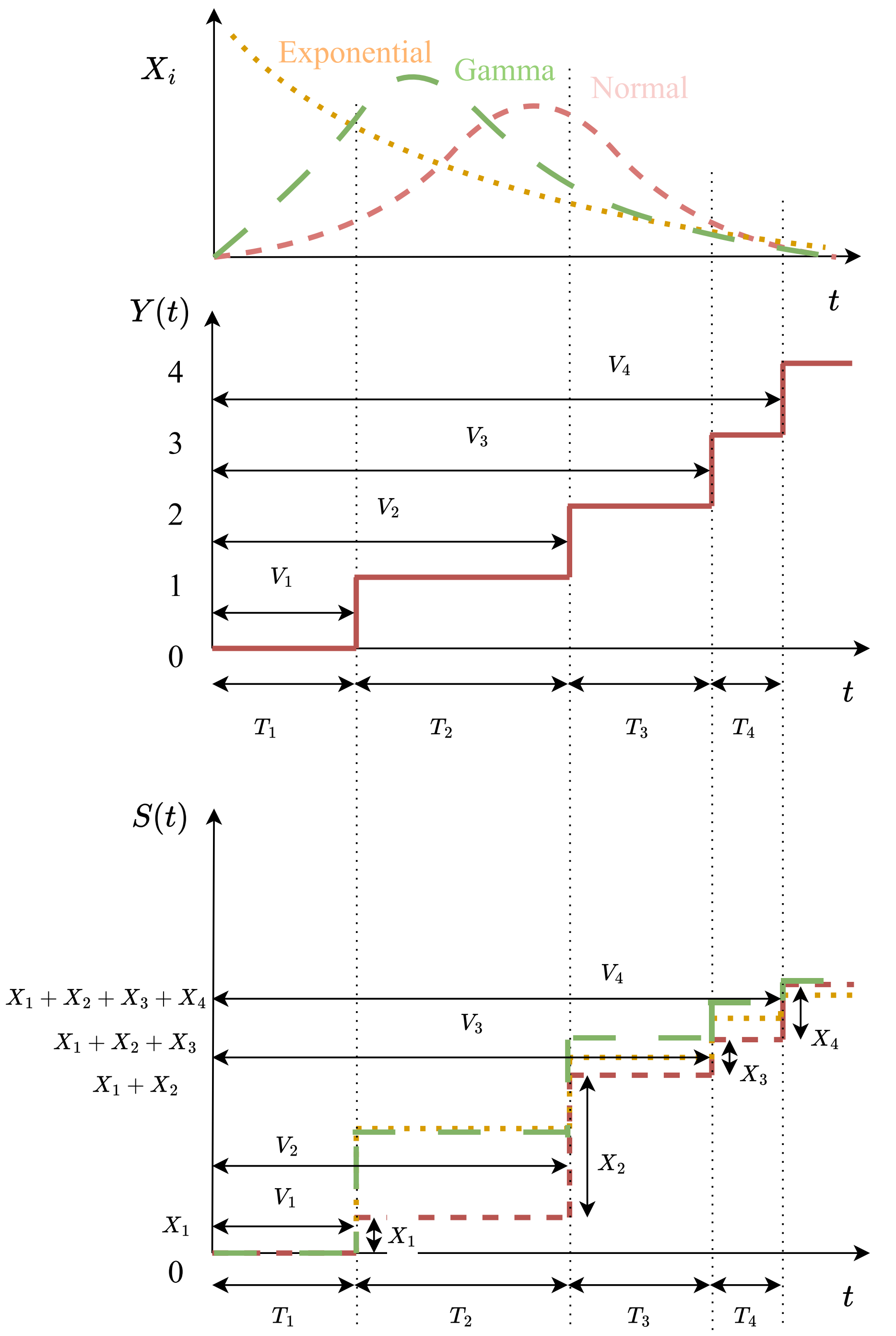
Non-stationary Compound Poisson Process $(\lambda(t), f_{X_i})$

Parameters are continuous, states are discrete



The magnitude of i^{th} event $X_i \in (0, +\infty)$ could follow any specified distribution for RV X_i with mean μ_X and standard deviation σ_X e.g. continuous distribution: normal, gamma or exponential; discrete distribution:

The total number of successes until time t

$$Y(t) \in \{0, 1, 2, \cdots\} \sim poisson(\nu), \nu = \int_0^t \lambda(\tau)d\tau, \lambda > 0,$$

$$\lim_{y\to\infty, p\to 0, yp\to\lambda t} binomial(y, p) = poisson(\lambda t)$$

PMF $p_Y(y) = \frac{\nu^y}{y!}e^{-\nu} = \frac{(\lambda t)^y}{y!}e^{-\lambda t}$

CDF $F_Y(y) = 1 - p(1-p)^y$

$$\mu_Y = \nu = \int_0^t \lambda(\tau)d\tau, \sigma_Y^2 = \nu = \int_0^t \lambda(\tau)d\tau$$

The time or distance between $(i-1)^{th}$ and i^{th} success:

$$T_i \in (0, +\infty) \sim exp(\lambda), \lambda > 0$$

PDF $f_{T_i}(t) = \lambda e^{-\lambda t}$

CDF $F_{T_i}(t) = 1 - e^{-\lambda t}$

$$\mu_{T_i} = \frac{1}{\lambda}, \sigma_{T_i} = \frac{1}{\lambda}$$

The time until obtaining the k^{th} success:

$$V_k \in (0, +\infty) \sim Gamma(\lambda, k), \lambda > 0, k > 0$$

PDF $f_{V_k}(v) = \frac{\lambda^k v^{k-1} e^{-\lambda v}}{\Gamma(k)}$

CDF $F_{V_k}(v) = \frac{\gamma(k, \lambda v)}{\Gamma(k)}$

$$\mu_{V_k} = \frac{k}{\lambda}, \sigma_{V_k} = \frac{\sqrt{k}}{\lambda}$$

The accumulated total amount until time t

$$S(t) = \sum_{i=1}^{Y(t)} X_i \in (0, +\infty) \sim CompoundPoisson(\nu), \nu = \lambda t, \lambda > 0, t > 0$$

$$\mathbb{E}[S(t)] = \mathbb{E}[\mathbb{E}[S(t)|Y(t)]] = \mathbb{E}[Y(t)\mathbb{E}[X]] = \mu_X \int_0^t \lambda(\tau)d\tau$$

$$\mathbb{D}[S(t)] = \mathbb{E}[\mathbb{D}[S(t)|Y(t)]] + \mathbb{D}[\mathbb{E}[S(t)|Y(t)]] = \mathbb{E}[\sigma_X^2 Y(t)] + \mathbb{D}[\mu_X Y(t)] = (\mu_X^2 + \sigma_X^2) \int^t \lambda(\tau)d\tau$$