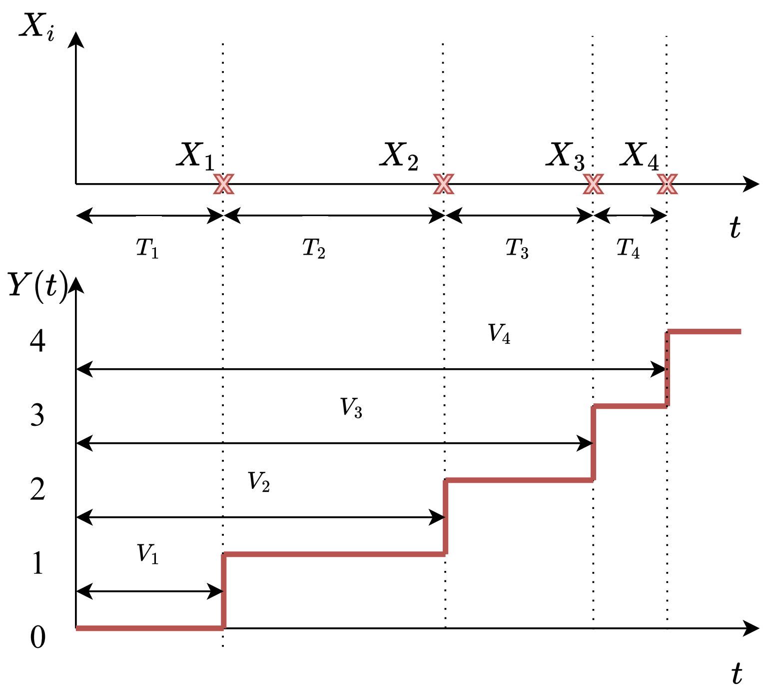
Poisson Process $(t; \lambda)$

Parameters are continuous, states are discrete



The occurence of an event at given time t

$$X_i(t) \in \{0,1\} \sim bernoulli(p = \frac{\lambda t}{n} = \lambda \Delta t)$$

PMF $p_{X_i}(x) = (\lambda \Delta t)^x (1 - \lambda \Delta t)^{(1-x)}$
where $x \in \{0,1\}$

$$\mu_{X_i} = p, \sigma_{X_i} = \sqrt{np(1-p)}$$

The total number of occurences until time t

$$Y(t) \in \{0,1,2,\cdots\} \sim poisson(\nu), \nu = \gamma t, \gamma > 0, t > 0$$

$$\lim_{n\to\infty, p\to 0, np\to\lambda t} binomial(n,p) = poisson(\lambda t)$$

PMF $p_Y(y) = \frac{\nu^x}{x!} e^{-\nu} = \frac{(\lambda t)^x}{x!} e^{-\lambda t}$

CDF $F_Y(y) = 1 - p(1-p)^n$

$$\mu_Y = p, \sigma_Y = \sqrt{np(1-p)}$$

The time or distance interval between $(i-1)^{th}$ and i^{th} occurence:

$$T_i \in (0, +\infty) \sim exp(\lambda), \lambda > 0$$

PDF $f_{T_i}(t) = \lambda e^{-\lambda t}$

CDF $F_{T_i}(t) = 1 - e^{-\lambda t}$

$$\mu_{T_i} = \frac{1}{\lambda}, \sigma_{T_i} = \frac{1}{\lambda}$$

The time until obtaining the k^{th} occurence:

$$V_k \in (0, +\infty) \sim Gamma(\lambda, k), \lambda > 0, k > 0$$

PDF $f_{V_k}(v) = \frac{\lambda^k v^{k-1} e^{-\lambda v}}{\Gamma(k)}$

CDF $F_{V_k}(v) = \frac{\gamma(k, \lambda v)}{\Gamma(k)}$

$$\mu_{V_k} = \frac{k}{\lambda}, \sigma_{V_k} = \frac{\sqrt{k}}{\lambda}$$