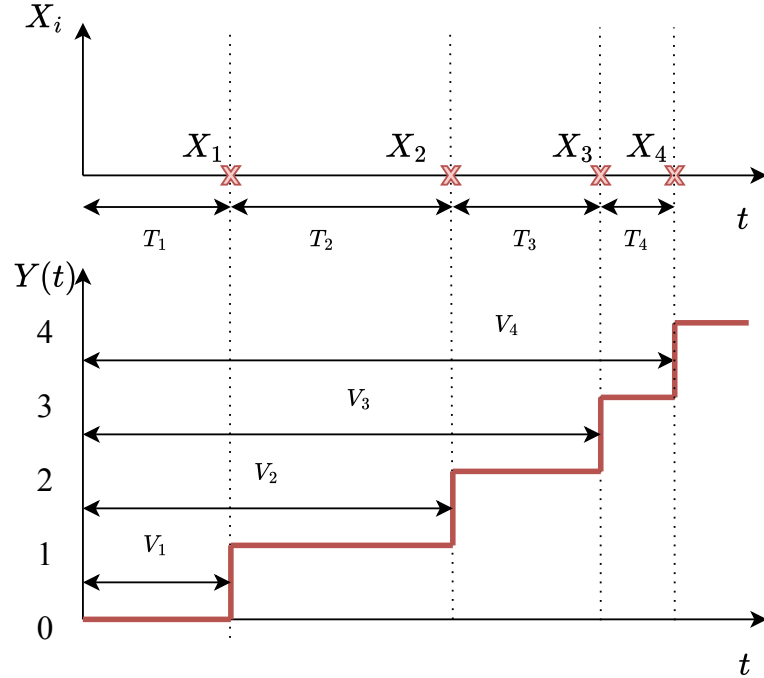


Poisson Process $(t; \lambda)$

Parameters are continuous, states are discrete



The occurrence of an event at given time t

$$X_i(t) \in \{0, 1\} \sim \text{bernoulli}(p = \frac{\lambda t}{n} = \lambda \Delta t)$$

The total number of occurrences until time t

$$Y(t) \in \{0, 1, 2, \dots\} \sim \text{poisson}(\nu), \nu = \gamma t, \gamma > 0, t > 0$$

$$\lim_{n \rightarrow \infty, p \rightarrow 0, np \rightarrow \lambda t} \text{binomial}(n, p) = \text{poisson}(\lambda t)$$

The time or distance interval between $(i - 1)^{th}$ and i^{th} occurrence:

$$T_i \in (0, +\infty) \sim \text{exp}(\lambda), \lambda > 0$$

The time until obtaining the k^{th} occurrence:

$$V_k \in (0, +\infty) \sim \text{Gamma}(\lambda, k), \lambda > 0, k > 0$$

$$\text{PMF } p_{X_i}(x) = (\lambda \Delta t)^x (1 - \lambda \Delta t)^{(1-x)}$$

where $x \in \{0, 1\}$

$$\mu_{X_i} = p, \sigma_{X_i} = \sqrt{np(1-p)}$$

$$\text{PMF } p_Y(y) = \frac{\nu^y}{y!} e^{-\nu} = \frac{(\lambda t)^y}{y!} e^{-\lambda t}$$

$$\text{CDF } F_Y(y) = 1 - p(1-p)^n$$

$$\mu_Y = \nu = \lambda t, \sigma_Y = \nu = \lambda t$$

$$\text{PDF } f_{T_i}(t) = \lambda e^{-\lambda t}$$

$$\text{CDF } F_{T_i}(t) = 1 - e^{-\lambda t}$$

$$\mu_{T_i} = \frac{1}{\lambda}, \sigma_{T_i} = \frac{1}{\lambda}$$

$$\text{PDF } f_{V_k}(v) = \frac{\lambda^k v^{k-1} e^{-\lambda v}}{\Gamma(k)}$$

$$\text{CDF } F_{V_k}(v) = \frac{\gamma(k, \lambda v)}{\Gamma(k)}$$

$$\mu_{V_k} = \frac{k}{\lambda}, \sigma_{V_k} = \frac{\sqrt{k}}{\lambda}$$