Mathematical Derivation Framework for Field Projection Theory (FPT)

June 28, 2025

Introduction

This document provides the mathematical origin and justification for the core equations used in the Field Projection Theory (FPT). FPT assumes that all observable phenomena (mass, charge, spacetime) emerge from a continuous field χ defined on a higher-dimensional manifold $\mathcal{M} = \mathbb{R}^{1,3} \times \mathcal{C}^n$. Below we explain how the standard equations were derived from known physics.

1 1. Higher-Dimensional Field Definition

Let $\chi(x^\mu, y^n)$ be a scalar or tensor field on $\mathcal{M}$, where:

- x^μ are coordinates on 4D spacetime: $\mathbb{R}^{1,3}$
- y^n are coordinates on a compact n-dimensional internal manifold $\mathcal{C}^n$

2 2. Action and Lagrangian from Variational Principle

The field's dynamics are governed by a higher-dimensional action:

$$S[\chi] = \int_{\mathcal{M}} \left[\frac{1}{2} G^{MN} (D_M \chi)^* (D_N \chi) - V(\chi) \right] \sqrt{-G}\, d^4x\, d^n y \tag{1}$$

This follows from general relativistic field theory in curved space, where G^{MN} is the higher-dimensional metric, and D_M is the gauge-covariant derivative.

3 3. Mode Expansion over Internal Manifold

The field χ is decomposed using a basis of eigenfunctions on the compact space:

$$\chi(x^\mu, y^n) = \sum_k \phi_k(x^\mu) f_k(y^n) \tag{2}$$

Here $f_k(y^n)$ satisfy:

$$\Delta_{\mathcal{C}^n} f_k(y^n) = -\lambda_k f_k(y^n) \tag{3}$$

This is the standard spectral decomposition from Kaluza-Klein theory and harmonic analysis.

4 4. Projection to 4D

By integrating over $\mathcal{C}^n$, we project χ to 4D:

$$\phi_k(x^\mu) = \int_{\mathcal{C}^n} \chi(x^\mu, y^n) f_k^*(y^n)\, d^n y \tag{4}$$

This yields observable fields $\phi_k(x^\mu)$ with mass:

$$m_k^2 = \lambda_k \tag{5}$$

The masses arise as eigenvalues of the Laplacian on $\mathcal{C}^n$.

5 5. Effective 4D Action

Plugging the decomposition into the action and integrating gives:

$$S_{\text{eff}}[\phi_k] = \int d^4x \sum_k \left[\frac{1}{2} \eta^{\mu\nu} \partial_\mu \phi_k^* \partial_\nu \phi_k - \frac{1}{2} \lambda_k |\phi_k|^2 - V(\phi_k) \right] \tag{6}$$

This is the effective low-energy action in 4D.

Conclusion

All key equations in FPT arise from well-established methods in theoretical physics: variational principles, spectral theory, and compactification. No arbitrary assumptions are made beyond the dimensional structure of $\mathcal{M}$. This document forms the foundational mathematical justification for FPT.