Field Projection Theory (FPT)

A higher-dimensional field framework unifying mass, charge, interaction, and spacetime via projection

1 Abstract

Field Projection Theory (FPT) posits that all observable properties in four-dimensional spacetime—such as mass, charge, matter, and force—are emergent effects of a single, higher-dimensional continuous field. These effects arise via projection and compactification mechanisms applied over a product manifold $\mathcal{M} = \mathbb{R}^{1,3} \times \mathcal{C}^n$, where $\mathcal{C}^n$ is a compact internal space. The theory is constructed using principles of symmetry, the variational method, and standard quantum field theoretic structures.

2 Geometric Structure

We assume the physical universe is a $(4+n)$-dimensional manifold:

- **Spacetime:** $x^\mu \in \mathbb{R}^{1,3}$ (3 spatial + 1 time dimension)
- **Internal dimensions:** $y^n \in \mathcal{C}^n$ (compactified space, such as T^n or S^n)
- **Total manifold:** $\mathcal{M} = \mathbb{R}^{1,3} \times \mathcal{C}^n$

3 Fundamental Field

A single scalar or tensor field $\chi(x^\mu, y^n)$ is defined over $\mathcal{M}$. This field is considered ontologically fundamental. All known particles and interactions are emergent properties derived from the behavior, modes, and symmetries of χ.

4 Lagrangian and Variational Basis

The dynamics of χ are determined by the action:

$$S[\chi] = \int_{\mathcal{M}} \left[\frac{1}{2} G^{MN} (D_M \chi)^* (D_N \chi) - V(\chi) \right] \sqrt{-G}\, d^4x\, d^ny$$

Where:

- G^{MN}: Metric over $\mathcal{M}$ (includes gravity)
- $D_M = \partial_M + igA_M$: Gauge-covariant derivative
- $V(\chi)$: Potential function, chosen for vacuum stability and symmetry breaking

  Applying the Euler-Lagrange equation:

$$\frac{\delta S}{\delta \chi} = 0 \Rightarrow \Box_{(4+n)} \chi - \frac{\partial V}{\partial \chi^*} = 0$$

5 Mode Decomposition and Projection Derivation

We decompose the field χ using eigenfunctions $f_k(y^n)$ of the Laplacian on the compact space:

$$\Delta_{\mathcal{C}^n} f_k(y^n) = -\lambda_k f_k(y^n)$$

Then expand:

$$\chi(x^\mu, y^n) = \sum_k \phi_k(x^\mu) f_k(y^n)$$

Substitute into the action and integrate over y^n:

$$S[\phi_k] = \int d^4x \sum_k \left[\frac{1}{2} \eta^{\mu\nu} \partial_\mu \phi_k^* \partial_\nu \phi_k - \frac{1}{2} \lambda_k |\phi_k|^2 - V(\phi_k) \right]$$

Thus each ϕ_k behaves as a 4D field with effective mass $m_k^2 = \lambda_k$.

6 Projection to Observable Physics

Observable 4D fields $\phi(x^\mu)$ are obtained via projection over internal space:

$$\phi(x^\mu) = \int_{\mathcal{C}^n} \chi(x^\mu, y^n) f(y^n)\, d^ny$$

Choosing $f(y^n)$ as $f_k(y^n)$ selects $\phi_k(x^\mu)$.

7 Emergence of Physical Phenomena

Observable properties emerge from χ as follows:

- **Mass:** Effective mass arises via:

$$m^2 \sim \int_{\mathcal{C}^n} (\partial_{y^n}\chi)^2 \, d^n y$$

- **Charge:** From conserved Noether charges under internal symmetries (U(1), SU(N), etc.).
- **Forces:** Gauge fields $A_M(x^\mu, y^n)$ yield familiar forces after compactification.
- **Matter Fields:** Fermionic matter arises from spinor fields $\psi(x^\mu, y^n)$ via KK decomposition.
- **Gravity:** 4D Einstein gravity arises from the higher-dimensional metric G_{MN}.
- **Stability:** Localized solitonic/topological solutions explain stable particle-like behavior.

8 Quantum Structure

Quantization is performed through the path integral:

$$Z = \int \mathcal{D}\chi \, e^{iS[\chi]}$$

This governs all quantum behavior and correlation functions of projected fields.

9 Theoretical Merits

FPT satisfies key criteria for unification:

- **Symmetry-grounded**
- **Action-derived**
- **Geometrically driven**
- **Unifying matter, force, and spacetime**
- **Extendable (SUSY, curvature, strings)**

Conclusion

Field Projection Theory (FPT) provides a mathematically grounded approach to unifying physical interactions and structure. Observable reality in 4D spacetime may be no more than the projected behavior of a master field existing in higher-dimensional space. This model suggests a complete, elegant theory of emergence, potentially resolving gaps left by particle-based frameworks.