Beyond Static Assumptions: the Predictive Justified Perspective Model for Epistemic Planning

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Abstract

Epistemic Planning (EP) is an important research area dedicated to reasoning about the knowledge and beliefs of agents in multi-agent cooperative or adversarial settings. The Justified Perspective (JP) model is the state-of-the-art approach to solving EP problems with efficiency and expressiveness. However, all existing EP methods inherit the static environment assumption from classical planning. This limitation hinders the application of EP in fields such as robotics with multi-agent settings, where the environment contains changing variables. In this paper, we propose an extension of the JP model, namely, the *Predictive Justified Perspective* (PJP) model, to remove this assumption. Instead of assuming that beliefs remain unchanged since the last observation, the PJP model uses all past observations to form predictions about the changing variables. The definition of the prediction function with examples is provided, and it is demonstrated that it can work with arbitrary nesting. We then implemented the PJP model in several well-known domains and compared it with the JP model in the experiments. The results indicated that the PJP model performs exceptionally well across various domains, demonstrating its potential in improving EP applications in robotics.

1 Introduction

Epistemic Planning (EP) is a popular research field that reasons about agents' higher-order knowledge and beliefs. With the capability of modelling others' knowledge and belief, it can be potentially applied in the *Multi-Agent System* (MAS) or *Human-Agent Interaction* (HAI) scenarios. There are some exiting works extend EP into MAS and HAI, such as: using contingent epistemic planning to handle multiagent implicit coordination by converting to *Full Observable Non-Deterministic* (FOND) problem (Engesser et al. 2017; Engesser and Miller 2020), or applying epistemic reasoning system on a humanoid robot to perform false-belief tasks (Dissing and Bolander 2020).

However, all existing approaches in EP inherit the assumption from classical planning that the environment does not change unless the agent causes the change. This "static environment" assumption is reasonable in AI planning but not in many other fields. The necessity to model continu-

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ous changing variable in the environment generates the gap between EP and its application, such as robotics.

The state-of-the-art EP is usually solved by three main approaches. *Dynamic Epistemic Logic* (DEL) based approach was firstly proposed by Bolander and Andersen (2011), and it maintains a Kripke structure (Fagin et al. 1995) using an event-based model which requires explicit action effects to specify modal logic changes. Pre-compilation strategy is another approach that transforms the epistemic problem into a more manageable form, such as a classical planning problem (Kominis and Geffner 2015; Muise et al. 2015, 2022; Cooper et al. 2019). The main challenge of this approach is the high cost of pre-compiling steps when the epistemic formulae depth increase.

Both DEL and pre-compilation methods have the bounded epistemic formulae depth, as they require predefining the depth when defining the problem. To address these challenges, a novel state-based approach, namely *Planning with Perspectives* (PWP) (Hu, Miller, and Lipovetzky 2022), was proposed. By leveraging the external functions and lazy evaluation, PWP can offload the epistemic formula reasoning from the planner, which improve both efficiency and expressiveness. However, the PWP approach only handles knowledge (not belief). A recent continuation study introduced *Justified Perspectives* (JP) model to handle the belief as an enhancement to the PWP method (Hu, Miller, and Lipovetzky 2023). Both PWP and JP approaches are state-based (action-model-free), which makes them more suitable to be embedded on other applications.

However, the JP model is developed based on the intuition that individuals maintain unchanged beliefs in the absence of contradictory evidence. Mathematically, this updating process of belief values can be seen as a *Zero-Order Hold* (ZOH) model. It means JP model cannot handle continuously changing variables, which may lead to erroneous beliefs and faulty reasoning.

We use a simplified Grapevine example (Muise et al. 2022) to explain our idea.

Example 1. There are two agents in the room, Agent a and Agent b. Agent a holds a secret changing value as which is represented by a first-order polynomial (as = x + 2, where x is the state index). It is common knowledge that as is represented by a first-order polynomial, but the coefficients are unknown to others. Agents can share their own secret and

follows by an action stop until they can share again. The task is for Agent b to form a correct belief about as when Agent a stops sharing.

To determine the coefficients, it requires at least two observations, a valid plan would be:

Plan 1 share(a), stop, share(a), stop

In the existing JP model, Agent b believes as's value as [3,3,3,5,5] from state s_0 to s_4 , because b sees as at state s_1 and state s_3 , while the true value of as should be [2,3,4,5,6]. Thus, the belief of b is incorrect.

These faulty reasoning in practical applications may lead to low efficiency or even cause system failure, such as collision of robots. Thus, this work aims to propose a new model to empower the existing JP model to deal with the changing environment. In this paper, we introduce the planning languages and the JP model, as well as define the problem mathematically in Section 2. Following this, the new model is proposed in Section 3. In Section 4 and 5, we demonstrated the implementation in three distinct domains and compare to the JP model on the correctness, soundness and optimality.

2 Preliminary

2.1 Planning Extension

In AI planning, the *Planning Domain Definition Language* (PDDL) serves as a foundational framework and is widely used to describe planning problems and domains (Haslum et al. 2019). With the increasing demand, PDDL 2.1 has enhanced the modeling of numerical resources, enabling a better expression of the continuous numerical effects of ongoing actions (Fox and Long 2003).

Fox and Long (2006) proposed PDDL+, aimed at meeting the planning needs of hybrid systems. PDDL+ models complex dynamic changes by introducing continuous processes. In PDDL+, continuous processes represent the continuous changes of state variables. The process starts when its preconditions are met and ends when the preconditions are no longer satisfied. So, the duration of a process is not fixed but varies dynamically based on the system's state.

To further enhance expressiveness, Geffner (2000) introduced Functional STRIPS (F-STRIPS), which allows usage of external function in planning. This idea was elaborated upon by Francès and Geffner (2015), and has since been applied in many planning variation fields, such as, epistemic planning (Hu, Miller, and Lipovetzky 2022, 2023) and generalized planning (Lei, Lipovetzky, and Ehinger 2024). This extension allows the definition of complex state variables and calculations through external functions, thereby improving its expressive capabilities.

2.2 Justified Perspective Model

The JP model (Hu, Miller, and Lipovetzky 2023) is build on the foundation of the PWP model by incorporating the belief operator B_i which is used to capture intuition (Goldman 1979): unless they see evidence to the contrary, agents believe that what they have seen before is true. Specifically, when the agent infers unobservable entities, if there is no evidence suggesting that these entities are no longer valid, they

generate a justified belief by retrieving information from their memory. The JP model aims to address some of the limitations of the PWP model and further enhance the modeling capability for multi-agent beliefs.

A JP **Signature** is defined as a tuple:

$$\Sigma = (Agt, V, D_{v_1}, \dots, D_{v_k}, \mathbb{R}),$$

where Agt represents a finite set of agent identifiers containing m agents. The set V is a finite set of variables such that $Agt \subseteq V$. For each variable $v_i \in V$, D_{v_i} denotes a potentially infinite domain of constant symbols. Additionally, $\mathbb R$ is a finite set of predicate symbols. The domains can be either discrete or continuous, and the overall set of values is given by $D = \bigcup_{v \in V} D_v$.

The language $L(\Sigma)$ of JP model is defined by the grammar:

$$\alpha ::= r(\vec{t}) \mid \neg \alpha \mid \alpha \wedge \alpha \mid S_i v \mid S_i \alpha \mid K_i \alpha,$$
$$\varphi ::= \alpha \mid B_i \varphi.$$

where $r(\vec{t})$ represents a predicate symbol applied to terms \vec{t} , $\vec{t} \subset V$ and $r \in \mathbb{R}$.

We denote **R** as a set of all predicates $r(\vec{t})$. With above signature and language, the JP model is proposed as follows.

Definition 1 (JP Model). The JP **model** M is defined as:

$$M = (Agt, V, D_{v_1}, \dots, D_{v_k}, \pi, O_1, \dots, O_m),$$

where Agt denotes the set of m agents. The set V represents the variables, while D_{v_1},\ldots,D_{v_k} are the domains associated with the variables v_1,\ldots,v_k , respectively. A state is represented as a set of assignments that matches the variable $v\in V$ and its domain D_v , and $\mathrm{dom}(s)$ is used as the set of variables in state s. A global state is a complete assignment, while the local state might be a partial assignment, t is an interpretation function, t : t is true, t is true in t is an interpretation function function function symbols that establish the observation relationships among different elements (Definition 2).

They denoted the S as the state space, \vec{s} as a sequence of state from any plan, both $\vec{s}[t]$ and s_t as the state at timestamp t in given sequence \vec{s} , and \vec{S} as the sequence space.

Definition 2 (Observation Function). An observation function for Agent $i, O_i : S \to S$, is a function that takes a state and returns a subset of that state, representing the part of the state visible to Agent i.

The following properties must hold for a observation function O_i for all $i \in Agt$ and $s \in S$:

- 1. $O_i(s) \subseteq s$
- 2. $O_i(s) = O_i(O_i(s))$
- 3. If $s \subseteq s'$, then $O_i(s) \subseteq O_i(s')$

The **Retrieval function** is introduced to retrieve the value of a specific variable in the last observed timestamp.

Definition 3 (Retrieval Function). The retrieval function ${\cal R}$ is formally defined as:

$$R(\vec{s}, ts, v) = \begin{cases} s_{ts}(v) & \text{if } v \in \text{dom}(s_{ts}) \\ s_{\max(\text{lts})}(v) & \text{else if lts} \neq \{\} \\ s_{\min(\text{rts})}(v) & \text{else if rts} \neq \{\} \\ \text{None} & \text{otherwise} \end{cases}$$

where

$$lts = \{j \mid v \in s_j \land j < ts\}, rts = \{j \mid v \in s_j \land 0 \le ts < j \le |\vec{s}|\}.$$

The retrieval function R is a critical mechanism in the JP model for determining the value of a variable v at a given timestamp ts within a sequence of states \vec{s} . It assesses the visibility of v by checking its presence in the state at ts, then searches backward to the most recent previous state, then forward to the earliest subsequent state where v is seen. If v is not found within the sequence, R returns None. This function ensures the accurate reflection of an agent's beliefs based on the most recent observations.

A **justified perspective function** represents how an agent views the sequence of states within a plan.

Definition 4 (Justified Perspective Function). A perspective function for Agent $i, f_i : \vec{S} \to \vec{S}$, is defined as:

$$f_i([\mathbf{s}_0,\ldots,\mathbf{s}_n])=[\mathbf{s}'_0,\ldots,\mathbf{s}'_n],$$

where for all $t \in [0, n]$ and all $v \in \text{dom}(s_t)$:

$$\begin{split} s_t' &= \{v = e \,|\, l_t = \max(\text{ats}(v))\},\\ \text{ats}(v) &= \{j \,|\, v \in \text{dom}(O_i(s_j)) \land j \leq t\} \cup \{-1\},\\ e &= R([s_0, \dots, s_t], l_t, v). \end{split}$$

The justified perspective function enables agents to form reasonable beliefs based on available evidence and observations in complex environments. This function typically utilizes timestamps to track the observations of agents, allowing them to establish rational beliefs in the current state. By nesting perspective functions, agents can create intricate belief structures, where their beliefs not only rely on their own observations but also on the observations and beliefs of other agents.

Then, a ternary semantics is proposed for JP, which employs three truth values: 0 (false), 1 (true), and $\frac{1}{2}$ (unknown). This ternary semantics aims to enhance efficiency by avoiding the need to iterate over all global states S_G .

Definition 5 (Ternary semantics). A function T is defined, omitting the model M for readability:

(a)
$$T[\vec{s}, r(\vec{t})] = 1$$
 if $\pi(s_n, r(\vec{t})) = true$;
 0 else if $\pi(s_n, r(\vec{t})) = false$;
 $\frac{1}{2}$ otherwise

- (b) $T[\vec{s}, \phi \wedge \psi] = \min(T[\vec{s}, \phi], T[\vec{s}, \psi])$
- (c) $T[\vec{s}, \neg \varphi] = 1 T[\vec{s}, \varphi]$
- (d) $T[\vec{s}, S_i v] = \frac{1}{2} \text{ if } i \notin s_n \text{ or } v \notin s_n;$

0 else if
$$v \notin O_i(s_n)$$
;
1 otherwise

(e)
$$T[\vec{s}, S_i \varphi] = \frac{1}{2} \text{ if } T[\vec{s}, \varphi] = \frac{1}{2} \text{ or } i \notin \text{dom}(s_n)$$

 $0 \text{ else if } T[O_i(s_n), \varphi] = \frac{1}{2}$
 1 otherwise

(f)
$$T[\vec{s}, K_i \varphi] = T[\vec{s}, \varphi \wedge S_i \varphi]$$

(g)
$$T[\vec{s}, B_i \varphi] = T[f_i(\vec{s}), \varphi]$$

where s_n is the final state in sequence \vec{s} ; that is, $s_n = \vec{s}(|\vec{s}|)$.

By generating justified belief and applying the ternary semantics, JP model addresses the limitation of PWP, which can handle knowledge but not belief (g).

3 Predictive Justified Perspective Model

Now, we formally propose the *Predictive Justified Perspective* (PJP) Model to model continuous changing environment. The PJP model uses the same signature, language and semantics as the JP model, as shown in Section 2.2.

To model the change of the dynamic variables, our model use process¹-like variables, namely *processual* variables. The set of processual variables is denoted as V_p , and defined as follows.

Definition 6 (Processual Variables). Given \mathcal{T} is a set that includes all processual variables types, V_p is defined as:

$$V_p = (V, \mathcal{T}, type, coef),$$

where

$$type: V \to \mathcal{T},$$

 $coef: V \to \mathbb{N}^*.$

Processual variables are defined to describe the changing environment. For each variable $v \in V_p$, the type and coefficients are defined as type(v) and coef(v) to indicate the changing rules. The type static is considered as a base case, because it is only changed by agent's action effects. Thus, $coef(v) = \{\}$ for all type(v) = static. Static processual variables are the same as variables, $v \in V$, in the JP model as introduced in Definition 1.

Definition 7 (Model). The PJP model M is defined as:

$$M = (Agt, V_p, D_{v_1}, \dots, D_{v_k}, \pi, O_1, \dots, O_m, PR),$$

where PR is a set of predictive retrieve functions pr_x (Definition 8); V_p is the processual variables (Definition 6); and the rest of the elements are adopted from the JP model (refer to Definition 1).

The PJP model extends the definition of the JP model by changing V to V_p and adding PR to introduce the dynamics. This is done by using the *Predictive Retrieval Function* and *Predictive Justified Perspective Function*, as defined in Definition 8 and 9.

A high-level definition is given for any predictive retrieval function, $pr_x \in PR$, to handle the variables represented by different changing types.

Definition 8 (Predictive Retrieval Function). A predictive retrieval function $pr_x : \{O_0, \dots, O_m\} \times \vec{S} \times V \times N \to D$

¹The idea of the process is from PDDL+ (Fox and Long 2006).

takes the input of an agent's observation function, a state sequence, a variable and a timestamp (natural number), and outputs the predicted value of that variable at a given timestamp, where \boldsymbol{x} is the changing type of that variable.

The prediction function pr_x estimates the value of the given variable v at timestamp t by deriving v's changing pattern based on the given state sequence \vec{s} and agent's observation function O_i . The base case of the pr_x is the type **static**, which is domain independent. While all the other pr_x for domain dependent types from \mathcal{T} are needed to be defined by the modeller. Here, we give the definition of the static predictive retrieval function pr_{static} , and show a popular pr_x example (Definition 8.2) in Example 2.

Definition 8.1 (Static Predictive Retrieval Function). The static predictive retrieval function $pr_{static} \in PR$ can be defined as:

where
$$pr_{static}(O_i, \vec{s}, v, t) = e$$
 where
$$e = \vec{s}_{t'}(v)$$

$$t' = \begin{cases} max(\operatorname{lts}) & \text{if } \operatorname{lts} \neq \{\} \\ min(\operatorname{rts}) & \text{if } \operatorname{rts} \neq \{\} \\ None & \text{otherwise} \end{cases}$$

$$\operatorname{lts} = \{j \mid j \in \operatorname{ats}(v) \land 0 \leq j \leq t\}$$

$$\operatorname{rts} = \{j \mid j \in \operatorname{ats}(v) \land j > t\}$$

$$\operatorname{ats}(v) = \{j \mid v \in \operatorname{dom}(O_i(s_j))\}.$$

Function $pr_{static}(O_i, \vec{s}, v, t)$ returns Agent i's deduced value of v at timestamp t given state sequence \vec{s} and changing type as static. Static changing type indicates the value of the variable should stay unchanged unless the agent sees otherwise. Set $\operatorname{ats}(v)$ is the set of timestamps at which v has been observed by $i.\ lts = \{\}$ represents i has never seen v before or at v, while v = v after v and our intuition follows Retrieval Function v (Definition 3) from JP model.

Definition 9 (Predictive Justified Perspective Function). The Predictive Justified Perspective (PJP) function f_i is formally defined as:

$$f_i(\vec{s}) = [v \to e \mid e = pr_x(O_i, \vec{s}, v, t) \mid t \in \{0, \dots, n\}]$$

where

$$\begin{split} x = \begin{cases} static & \text{if } |\mathsf{ats}(v)| < |coef(v)| \\ type(v) & \text{if } |\mathsf{ats}(v)| \ge |coef(v)| \end{cases} \\ \mathsf{ats}(v) = \{j \, | \, v \in \mathsf{dom}(O_i(s_j))\} \end{split}$$

The PJP Function f_i generates a sequence of local states for Agent i. Each variable v is assigned with a deduced value (by pr_x) at the timestamp t. The pr_x used for each variable v is determined by the type of v and its coefficient size, as well as the number of observations of v from i. If the type is static, the value of v is the same as it returned by the retrieval function from JP model in justified perspective function (Definition 4). In addition, the PJP function assumes the type of pr_x is static, if the number of observations is not sufficient (v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v | v |

To demonstrate the effectiveness of the PJP model, we provide an example using the original Grapevine domain (Hu, Miller, and Lipovetzky 2023), which allows lying compared to Example 1.

Example 2. There are 3 agents, a, b and c and two rooms r1 and r2. The actions include move, share and lie. Similar to Example 1, Agent a has a secret as which is represented by a first-order polynomial (as = x + 2). Since a can lie about as, the shared value of as, denoting as sas, may differ from the actual as. For simplicity, in this example, the deceptive value of as is assumed as -1. Initially, all agents are in r1. The task is for Agent a to deceive a and a to make each of them has different belief about sas.

Plan 3 A valid plan is as follows: lie(a), stop, share(a), stop, move(c, r2), lie(a), stop.

At the beginning, all agents see the effects of the actions: lie(a), stop, share(a), stop. Thus, they should obtain the same belief. However, after action move(c, r2), c no longer in r_1 , and won't be able to "see" the following sas value. Agent b would obtain a new belief after a lies again. Agent a believes that b's belief changed while c's belief doesn't as c left the room.

In both Example 1 and 2, the changing variable is in the first-order polynomial. Here, we provide our definition of the predictive retrieval function for the first-order polynomial.

Definition 8.2 (First-order Polynomial Predictive Retrieval Function). The predictive retrieval function for the first-order polynomial, $pr_{1st-poly}(O_i, \vec{s}, v, t)$, can be defined as:

$$pr_{1st_poly}(O_i, \vec{s}, v, t) = \frac{(t - t_1)(e_1 - e_2)}{t_1 - t_2} + e_1,$$

where

$$\begin{cases} \text{if rts} = \{\}, & t_2 = max(\text{lts}), t_1 = max(\text{lts} \setminus \{t_2\}) \\ \text{if lts} = \{\}, & t_1 = min(\text{rts}), t_2 = min(\text{rts} \setminus \{t_1\}) \\ \text{otherwise}, & t_1 = max(\text{lts}), t_2 = min(\text{rts}) \end{cases} \\ \text{ats}(v) = \{j \mid v \in \text{dom}(O_i(s_j))\} \\ \text{rts} = \{j \mid j \in \text{ats}(v) \land j > t\} \\ \text{lts} = \{j \mid j \in \text{ats}(v) \land j \leq t\} \\ e_1 = s_{t_1}(v), e_2 = s_{t_2}(v) \end{cases}$$

The predictive retrieval function for first-order polynomial identifies the most recent two timestamps that Agent i observes v, denoted as t_1 and t_2 ($t_1 \neq t_2$), such that: $t_1 < t_2 \leq t$, if timestamp t is at or after the agent's latest observation of v; $t < t_1 < t_2$, if timestamp t is before the agent's first observation of v; $t_1 \leq t < t_2$, otherwise.

Regard to Example 2, the shared secret value sas is [-,-1,-,5,-,-,-1,-]. Agent b and c observe sas as, [-,-1,-,5,-,-,-1,-] and [-,-1,-,5,-,-,-] respectively. The value of sas in b's predictive justified perspective, for example in timestamp 4, should be $f_b(\vec{s})[4](sas) = pr_{1st_poly}(O_b, \vec{s}, sas, 4) = 3$. This is calculated by identifying t_1 and t_2 (3 and 6), and using sas's values (5 and -1) to get its value in timestamp 4.

Since both a and b stay in r_1 the whole time, $f_b(\vec{s})(sas) = f_b(\vec{s})(sas) = f_b(f_a(\vec{s}))(sas)$. Following

Definition 9, $f_a(\vec{s})(sas)$ is [-4, -1, 2, 5, 3, 1, -1, -3]. After applying f_c on a's predictive justified perspective, we have $f_c(f_a(\vec{s}))(sas) = [-4, -1, 2, 5, 8, 11, 14, 17]$.

4 Implementation

The PJP model, similar to the JP model, is implemented using similar language as PDDL+ by introducing the external function idea from F-STRIPS. The prediction Process is integrated into the external function to handle belief update during the planning process.

4.1 PDDL+ encoding

Following PDDL2.1, the signature of V is encoded as functions, while, the \mathcal{T} , type and coef are defined as rules in our encoding.

The type static (line 2) has no coefficient, while the type first-order polynomial has two coefficients. The second square bracket represent agent's initial knowledge of the coefficients, which is none in the given example.

Similar to the encoding in JP, the epistemic formulae appear in the action precondition and goal as external functions, while they could also be included in the action effects. Two types of external function, @ep and @jp are provided. External function @ep is for evaluating an normal epistemic formula, for example, $\neg B_b(shared_valueas) = 6$, as shown below:

```
1 (= (@ep ("- b [b]") (= (shared_value as)
6)) ep.true)
```

While, the external function, @jp, represents agent's perspective, which in this work is powered by PJP function. It takes a state variable as its second argument, and return the value of that variable in the querying epistemic perspective. A PDDL example of action *sharing_others_secret* is provided below:

The epistemic formula precondition (Line 6), indicates that the agent has a belief (not None) of this value. This can also be done by using @ep function and unknown from the ternary semantics (Definition 5). The epistemic formula can be in action effects as well. Although Agent ?a's belief of (shared_value ?s) is not part of the global state, its value is deduced by the external function @jp and assigned to (shared_value ?s) in the global state. This could not be done by any other encoding in other epistemic planning approaches.

4.2 PR

Five processual variable types, which are first-order polynomial, second-order polynomial, power function, first-order modulus, and static, are implemented in this work. For different types, different number of observations are needed to deduce its coefficients and predict its value. Without knowing any coefficients, the first-order polynomial requires two observations (Definition 8.2), while if one of the coefficients

PDDL Example 1: sharing_others_secret

```
1
    (:action sharing_others_secret
            :parameters (?a - agent, ?s -
                secret)
3
            :precondition (
                 (= (own ?a ?s) 0)
4
5
                 (= (sharing) 0)
6
                 (!= (@jp ("b [?a]") (
                    shared_value ?s)) jp.none
7
8
            :effect (
9
                 (assign (shared_loc ?s) (
                    agent_loc ?a))
10
                 (assign (shared_value ?s) (
                    @jp ("b [?a]") (
                     shared_value ?s)))
11
                 (assign (sharing) 1)
12
            )
13
```

are known by the agents as the initial belief, only one observation is sufficient.

The expressiveness of our model is not only reflected by the example types provided, but also by PJP model's ability (Definition 8) to incorporate other prediction functions for other processual variable types. In addition to traditional mathematical models, the PJP model has its potential to be implemented with a variety of approaches, such as linear regression, support vector machines and neural networks.

5 Experiments

Three experiments are conducted on distinct problem domains: Big Brother Logic, Number, and Grapevine. The latest JP model serves as the control group to evaluate the performance of our model.

The experimental platform comprises a laptop equipped with a 12th Generation Intel $^{\otimes}$ Core $^{\text{TM}}$ i7-12700H processor (2.30 GHz) and 16 GB of RAM, running the Windows 11 operating system. The timeout is set to 300 seconds and memory out is set to 8GB.

To control the influence from the searching algorithm, the vanilla version of the *Breadth-First Search (BFS)* is used for both approaches to focus on the demonstration of the model's capability.

The outcome metrics included: the solvability (|Solvable|), the plan length (|Plan|), the number of expanded nodes (|Exp|), the number of generated nodes (|Gen|), the execution time (TIME), the average call time (T_c) and the goal conditions (Goal).

5.1 Big Brother Logic (BBL)

In the BBL problem domain, which was proposed by Gasquet, Goranko, and Schwarzentruber (2014), there are several stationary cameras positioned in a two-dimensional space free of obstacles, where the cameras do not obstruct each other's line of sight. Each camera can take two actions: **clockwise_rotation** and **counterclockwise_rotation**. Using

	PJP Model						JP Model						Goal
	Solvable	Plan	Exp	Gen	TIME(s)	$T_c(s)$	Solvable	Plan	Exp	Gen	TIME(s)	$T_c(s)$	
B1	True	4	15	31	0.0	1.548	True*	8	303	607	2.2	3.410	$B_a B_b v = t$
B2	True	4	15	31	0.0	1.113	True*	11	2434	4869	31.1	6.198	$B_a B_b v = t \wedge B_a B_c v = t$
В3	True	5	31	63	0.1	1.281	Timeout	_	43656	87313	_	3.360	$B_a B_b B_c v = t$
B4	True	5	62	125	0.3	1.950	Timeout	_	34172	68345	_	4.316	$B_a B_b v = t \wedge B_a B_c v = t \wedge B_a B_d v = t$
В5	True	5	62	125	0.2	1.938	Timeout	-	37032	74065	-	3.951	$B_a B_b B_c B_d v = t$
N1	False	_	93	93	0.0	0.088	False	_	93	93	0.0	0.0	$B_a v = 5$
N2	True	3	5	9	0.0	0.116	True	3	5	9	0.0	0.0	$B_a v = 7$
N3	True	4	9	13	0.0	0.0	False	_	93	93	0.0	0.022	$B_a v = 9$
N4	True	5	14	22	0.0	0.236	False	_	93	93	0.0	0.135	$B_b B_a v = 11 \wedge B_a B_b v = 11$
N5	False	_	93	93	0.0	0.206	False	_	93	93	0.0	0.143	$B_b B_a v = 13 \wedge B_a B_b v = 13$
N6	True	7	30	46	0.0	0.274	False	_	93	93	0.0	0.184	$B_b B_a v = 15 \wedge B_a B_b v = 15$
N7	True	8	48	67	0.0	0.333	False	_	93	93	0.0	0.169	$B_b B_a v = 17 \wedge B_a B_b v = 17$
N8	True	6	21	29	0.0	0.072	False	_	29	29	0.0	0.0	$B_a v = 37$
N9	True	2	3	5	0.0	0.0	False	-	13	13	0.0	0.0	$B_a v = 9$
G1	True	1	1	10	0.0	0.369	True	1	1	10	0.0	0.0	$tas = 3 \wedge B_b sas = 1$
G2	True	4	201	759	0.5	0.541	False	_	32866	32866	19.5	0.261	$tas = 6 \wedge B_b sas = 7$
G3	True	1	7	37	0.0	0.423	True	1	7	37	0.0	0.152	$tas = 3 \wedge B_b sas = 3$
G4	True	2	37	133	0.1	0.593	True	2	37	133	0.0	0.116	$tas = 4 \wedge B_b sas = 3 \wedge B_a B_b sas = 3$
G5	True	4	543	1987	1.6	0.568	False	_	32866	32866	36.4	0.337	$tas = 6 \wedge B_a B_b sas = 6 \wedge not sharing$
G6	True	7	23688	32344	59.5	1.172	False	_	32866	32866	24.8	0.375	$B_c sas = 9 \wedge B_a B_c sas = None$
G7	True	7	25935	32344	58.0	1.101	False	-	32866	32866	27.9	0.411	$B_c sas = 9 \land \neg B_a B_c sas = 9$

Table 1: Experimental results (BBL: B1-B5, Number: N1-N9, Grapevine: G1-G7)

the same simplification method as Hu, Miller, and Lipovetzky (2023), the rotation angles are set to be enumerated from the set $0^{\circ}, \pm 45^{\circ}, \pm 90^{\circ}, \pm 135^{\circ}, 180^{\circ}$, with the rotation angle increment being 45° . In addition, the field of view of the camera assume to covers angles strictly between 0 and 90 degrees, excluding the endpoints.

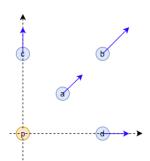


Figure 1: Initial State of BBL

Initially (Figure 1), four rotatable cameras, namely a,b,c and d, positioned at (1,1),(2,2),(0,2) and (2,0), and faced at $45^{\circ},45^{\circ},90^{\circ}$ and 0° , respectively, with a stationary object p at (0,0). To demonstrate dynamic changing variables, all cameras continuously rotate clockwise with a constant angular velocity, except **only camera** a can take rotation actions. Thus, the directions of b,c,d are represented by first-order modulus with one known coefficient $(y=x+c_2\mod 8)$, where c_2 is the initial direction of each agent.

The results of BBL demonstrated that the PJP model solves all problems with an optimal solution, while the original JP model could provide a sub-optimal plan², or even an invalid plan (marked as "*" in **B1**, **B2**). As the level of nest-

ing increases, the original JP model was unable to find a plan within 300 seconds time limit.

To be specific in **B1**, the optimal plan for a would be turn 4 times. While, the plan found by JP model is that a turns counterclockwise and clockwise until b looks at desired direction (-135°) , and then turns 4 times to see p. However, in the mean time, b keeps turning which results the belief deduced by a is false. That is, the problem was solvable in the mindset of JP model, but the plan JP model found was not sound.

5.2 Number

The number problem domain is inspired by the coin problem introduced by Hu, Miller, and Lipovetzky (2023). There are two agents, and instead of a coin, a number is placed in the box. Agents can take the actions **peek** and **return**. The number can only be observed by the agent who is peeking into the box. Agents can see each other and the actions they take, indicating whether the other agent is peeking into the box. However, unlike the coin problem, agents must return to its original state after peeking, thereby unable to keep watching the number changes.

The agents are denoted as a and b, and the number are denoted as v. Three different processual variable types for v were tested for different problem instances. Specifically, those were first-order polynomial (v=2x+1) for instances N1 to N7, second-order polynomial ($v=x^2+1$) for N8 and power ($v=3^x$) for N9, respectively.

Instances N1 to N3 showed the belief states of a single agent. Neither model found a solution in N1: the PJP model could not identify the coefficient of v with only one observation, while the JP model could only obtain values when the peeking action was performed. While the PJP model could find solutions in N2 and N3, but the JP model can only find a solution in N2.

N4 to N7 showed higher-order belief states. The JP model could not find any solutions. On the other hand, for the PJP

²The JP model returned invalid optimal plan instead of valid sub-optimal plan, as it is powered by BFS.

model, in N4, one agent learned the correct rule while the other was unable to predict correctly after returning, due to they only peeked once. In N6, both agents learned the rule, while one agent was peeking, and in N7, both agents were in a non-peeking state and still gained correct belief about each other on v.

In **N8** and **N9**, similar to **N3**, the PJP model needs different numbers (3, 1, 2, respectively) of the observation to deduce the pattern when the rule is different.

5.3 Grapevine

Grapevine, which is a benchmark problem in EP (Muise et al. 2022), describes the scenario that a few agents in two adjacent rooms, and they can choose to share secrets or move, while they can share their true secrets or lies, as well as share what they believe about the others' secret. In this work, as shown in Example 1 and 2, the secrets became a number instead of a binary.

The actions that each agent can take are: move_left, move_right, share_own_secret, lie_own_secret, share_other_secret and stop.

There are a total of three agents, a, b, c, all initially located in room r_1 . Since one secret value was sufficient for the experiments, only the true secret value as and its shared value sas were discussed. Same as the previous examples, as was represented by a first-order polynomial, as = x + 2, while the false value (lying value) in sas was always 1 for simplicity.

Instances **G1** and **G2** showed the scenarios involving lying. Both the JP model and the PJP model could obtain the false sas when lying (**G1**). But in **G2**, the JP model could not find a plan as the agent can only lie the value of sas as "1" rather than "7". But for PJP model, with the capability of prediction, the agent could make other agents generate a false belief, by manipulating the sas at different states. It makes the value of sas from others' belief not necessarily be an observed value from state history. For example, the plan for the PJP model in **G2** was $['lying_own_secret\ a\ sas', 'stop\ sas']$.

G3 to G5 showed the scenarios involving sharing secret values. Both the JP model and the Predictive JP model could obtain the correct secret values when sharing secrets (G3). Before learning the secret value variation rules, both models had similar belief update methods (G4). However in G5, only the PJP model could obtain the correct secret values when no one is sharing secrets (notsharing).

G6 and **G7** showed the complex scenarios of agents sharing other agents' secret values. In both goals, $B_c sas = 9$, which indicates c believes the true value of as, while a: has no belief of c on sas in **G6** and has incorrect belief of c on sas in **G7**. The plan for **G6** was that c leave r_1 first, and b heard from a sharing sas twice (meaning b knows the true pattern of as), then, b move to r_2 to share the value of sas (what b believes) to c, while a does not know that. The plan for **G7** was similar, except c left the room after hearing sas from a once. Thus, a believes c believe sas was still the value shared by a for the first time.

6 Related work and Discussion

Due to the lack of prediction capability in the previous research, the epistemic planning had limited studies in the highly-dynamic system.

A series of pioneer studies (Bramblett, Gao, and Bezzo 2023; Bramblett and Bezzo 2023) were conducted to introduce the concept of the epistemic planning into the multirobot coverage problem, in which robot can move around dynamically. To deal with the robots' movement when disconnecting, a series of sorted targeted positions for each agent has been generated in the initialisation phase as a common knowledge. In this case, the robot could use this consensus when disconnecting by assuming the others will follow their sorted targeted position list. They demonstrated that the epistemic logic with certain level of reasoning capability on the unobservable state could be applied in a highly-dynamic MAS problem. However, comparing to the PJP model, these works had limited nesting depth (2) and could not reason the changing pattern of the other agents state.

The PJP model introduce processual variables v_p to model dynamic environments. By forming agent's predictive justified perspectives using pr_x and f_i , the agents are able to reason about unseen changing variables with meaningful prediction. The PJP model adopted the strength of JP model, including arbitrary nesting and action-model free, which makes it suitable for further applications.

One of the most popular AI-driven application fields is robotics, specifically *Multi-Robot System* (MRS) or *Human-Robot Interaction* (HRI). In MRS, the challenge is not only the robot dynamics, but also non-deterministic components, for example, the results from the simulation and experiment showed significant differences due to the noise in a multirobot coverage study (Li et al. 2024). The PJP model showed good potential to be implemented with filters (e.g. linear regression) to reject the noise. In physical HRI application, the PJP model could potentially be used to predict human behaviour based on the measurements and generate the appropriate plan for robots to cooperatively complete the task with human.

7 Conclusion & Future Work

In conclusion, to fill the gap between the dynamic environments and the "static environment" assumption in existing EP studies, we proposed the PJP model by introducing the processual variables V_p and predictive retrieval functions PR in to the model. Thus, the modeller is able to make agents generate beliefs with reasonable predictions of changing variables and potentially be applied in various applications. Our approach retains the advantages of the JP model being action-model-free and capable of arbitrary nesting beliefs.

The current PJP model has limited error correction capability and would be easily affected by incorrect observations, leading to inaccurate learned rules. Incorporating confidence levels in predicted values could potentially reduce the impact of the outliers and enhance the robustness of the model. Moreover, the proposed model requires the pre-definition of processual variable types and corresponding mathematical

models, constraining its applications where the consensus is impossible to be reached. A potential solution is to introduce the learning-based methods to eliminate the predefined rules, improving the model's adaptability. Finally, all the changing variables in this model are assumed to be independent. In the future work, to relax this assumption, the Jacobi Method could be used to approximate and find the values of those dependent variables.

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