# Where Common Knowledge Cannot Be Formed, Common Belief Can – Planning with Multi-Agent Belief Using Group Justified Perspectives

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#### **Abstract**

Epistemic planning is the sub-field of AI planning that focuses on changing knowledge and belief. It is important in both multi-agent domains where agents need to have knowledge/belief regarding the environment, but also the beliefs of other agents, including nested beliefs. When modeling knowledge in multi-agent settings, many models face an exponential growth challenge in terms of nested depth. A contemporary method, known as Planning with Perspectives (PWP), addresses these challenges through the use of perspectives and set operations for knowledge. The JP model defines that an agent's belief is justified if and only if the agent has seen evidence that this belief was true in the past and has not seen evidence to suggest that this has changed. The current paper extends the JP model to handle group belief, including distributed belief and common belief. We call this the Group Justified Perspective (GJP) model. Using experimental problems crafted by adapting well-known benchmarks to a group setting, we show the efficiency and expressiveness of our GJP model at handling planning problems that cannot be handled by other epistemic planning tools.

## 1 Introduction and Motivation

Epistemic planning is a sophisticated branch of automated planning that integrates elements from both classical planning and epistemic logic. It allows the agents to reason about not only the physical world but also other agents' knowledge and beliefs. It is suitable in solving multi-agent cooperative or adversarial tasks.

There are two traditional research directions to solving epistemic planning problems: explicitly maintain all epistemic relations, such as Kripke frames (Kominis and Geffner 2015; Bolander and Andersen 2011; Bolander 2014; Cooper et al. 2021); or require an expensive pre-compilation step to convert an epistemic planning problem into a classical planning problem (Muise et al. 2022). Research from both directions faces exponential growth in terms of the epistemic formulae depth.

Recently, Hu, Miller, and Lipovetzky (2022) proposed a lazy state-based approach called *Planning with Perspectives* (PWP) that uses F-STRIPS (Geffner 2000) to reason about agent's seeing relation and knowledge. Their intuition is to use **perspective functions** to model the part of a state that each agent can see, and evaluate epistemic formulae from this. In short, an agent knows a proposition if it can see

the variables involved in the proposition, and that proposition is true. They allow perspective functions to be implemented in F-STRIPS external functions, which means new logics can be created; for example, they model proper epistemic knowledge bases (Lakemeyer and Lespérance 2012) and Big Brother logic (Gasquet, Goranko, and Schwarzentruber 2014) in continuous domains, with impressive computational results. Hu, Miller, and Lipovetzky (2023) extended their model to model belief as well as knowledge, permitting e.g. conflicting belief between agents. However, their model could only reason about single agent nested belief, not group belief operators such as common belief.

In this paper, we extend their work to model uniform belief, distributed belief and common belief. We follow the intuition that when people reason about something they cannot see, they generate justified beliefs by retrieving the information they have seen in the past (Goldman 1979).

However, applying this intuition of 'belief is past knowledge' naïvely to group belief is neither complete nor consistent. It is possible that some agents in a group see value changes that affect their own knowledge and belief while the group's belief stays the same. In addition, it is possible to form a common belief about a proposition even if there was no prior common knowledge about this previously. For example, consider agent a looking in a box and seeing a coin with heads, and then agent b looking into the box a minute after agent a and seeing it is heads. At no point did they see the coin at the same time, so they cannot form common knowledge that the coin is heads (it may have changed in the minute in between). However, they can form a common belief that it is heads because they each saw heads and have no evidence to suggest the value has changed.

We illustrate this idea with an extended (from the *coin* example from Hu, Miller, and Lipovetzky (2023)) false-belief example.

**Example 1.** There are two agents a and b, and there is a number  $n \in \mathbb{N}$  inside a box. The number can only be seen by the agents when they are peeking into the box. The agents know whether the others are peeking into the box. The actions that agents can do are: peek and return. They cannot peek into the box at the same time, so they need to return to allow the other agent to peek. There are two hidden actions add and subtract, and their effects, are only visible to the agents who are peeking into the box. Initially, both agents a

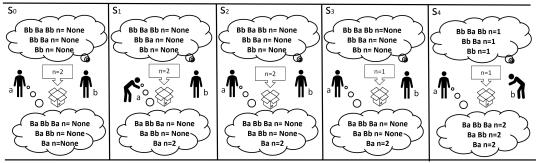


Figure 1: Plan 1.1, where **n=None** means agent does not know the value of **n**.

and b are not peeking, and the value of n is 2. The task is to generate a plan such that: the common belief between a and b is n < 3.

As shown in Figure 1, a valid plan to achieve both group beliefs above would be:

#### **Plan 1.1.** peek(a), return(a), subtract, peek(b)

At the end of the plan, agent a does not know if n=2 – an agent only knows something if they can see it. Hu, Miller, and Lipovetzky (2023) model individual belief using *justified belief* – justified belief can be derived from an agent's "memory" (Goldman 1979) – an agent believes something if they knew it earlier and have no evidence to suggest it has changed.

In the above, agents a and b do not peek into the box at the same time. So, at no point neither the statement "agent a knows that agent b knows n < 3"  $(K_a K_b n < 3)$ nor  $K_bK_an < 3$  hold. Further, the common knowledge  $CK_{\{a,b\}}n < 3$  does not hold. However, we assert that the common belief  $CB_{\{a,b\}}n < 3$  should hold if agents have memory. Since agent a sees n = 2 after step 1 and agent b sees n = 1 after step 4, both  $B_a n = 2$  and  $B_b n = 1$ hold, which implies both  $B_a n < 3$  and  $B_b n < 3$ . In addition, since agent a sees agent b peeking into the box after step 4 and  $B_a n = 2$ ,  $B_a B_b n = 2$  should hold. Similarly,  $B_b B_a n = 1$  should hold. Therefore, we have both  $B_aB_bn < 3$  and  $B_bB_an < 3$ . Given that a and b both saw that each other peeked in the box, and saw that each saw that each peeked into the box, etc, both a and b believe each other believes n < 3 with infinite depth. From the definition by Fagin et al. (2003), this constitutes common belief.

In this paper, we propose group perspective functions to reason about uniform belief, distributed belief and common belief. We discuss an implementation that extends an existing epistemic planning tool, and report experiments on key domains in epistemic planning Our results show that we can efficiently and expressively solve interesting problems with group belief, even with a basic blind search algorithm.

#### 2 Background

## 2.1 Epistemic planning

In epistemic planning, the most popular approach, Kripke structures, model belief and knowledge logic using the possible worlds. The core idea is to use a set of accessibility relations  $\mathcal{R}_i$  to represent whether agent i can distinguish between two states (possible worlds). An agent knows (believes) a formula  $\phi$  if it is true in every world that the agent considers possible. The difference between knowledge and belief lies in the properties of  $\mathcal{R}_i$ . To reason about knowledge, the agent's accessibility relations need to be Reflexive and Transitive and Euclidean, while the Reflexivity (Axiom T) is lost in belief. Semantically speaking, if  $K_i\varphi$  (agent i knows  $\varphi$  is true) holds (Axiom T), then  $\varphi$  holds; while if  $B_i \varphi$  (agent i believes  $\varphi$ ), it is not necessarily the case that  $\varphi$  holds. In short: agents can have incorrect beliefs, but not incorrect knowledge. For group beliefs, there are mainly three types: uniform beliefs, also known as shared beliefs; distributed beliefs; and common beliefs.

Uniform belief, denoted  $EB_G\varphi$ , is straightforward — it means that everyone in group G believes proposition  $\varphi$ . There are a number of approaches to model uniform belief (Iliev 2013; French et al. 2013).

**Distributed belief**, denoted  $DB_G\varphi$ , combines the beliefs of all agents in group G. It is, effectively, the pooled beliefs of group G if the agents were to "communicate" everything they believe to each other. Any model has to consider the pooled beliefs from each agent and the pooled beliefs from the group that are not held by any of its individual agents, but are held by the group. For example, if agent a believes x=1 (and nothing else) and agent b believes y=1 (and nothing else), distributively, the group  $\{a,b\}$  believes that x=y, even though no individual agent believes this. Distributed belief is challenging because agents can have conflicting beliefs: if agent a believes x=1 and agent b believes x=1, what should the distributed belief be?

There are two main approaches to model distributed belief: (1) belief merging (Konieczny 2000; Konieczny and Pérez 2002; Everaere, Konieczny, and Marquis 2015); and (2) merging the agents' epistemic accessibility relations (Halpern and Moses 1992; Wáng and Ågotnes 2013; Roelofsen 2007; Ågotnes and Wáng 2017; Solaki 2020; Li 2021; Goubault et al. 2023; van der Hoek, van Linder, and Meyer 1999). Typically, merging conflicting beliefs is

<sup>&</sup>lt;sup>1</sup>Note: we do not have any existing approach to compare to.

solved using some form of ordering over agents or propositions, meaning that some agents (propositions) receive priority over others. In this paper, we give two definitions of distributed belief: one that accepts inconsistent distributed belief; and one in which conflicting beliefs are removed entirely, leading to a modal operator that obeys the axiom of consistency (axiom D). In what is the most closely related work to ours, Herzig et al. (2020) combine the two approaches of belief merging and the merging of accessibility relations to define a logic for modeling explicit and implicit distributed beliefs. Explicit distributed belief is obtained from each agent's individual belief base; while implicit belief is derived from the group's collective belief base. In addition, they also introduce customised belief combination operators to model consistent distributed beliefs.

Common belief, denoted  $CB_G\varphi$ , is defined as: all agents in G believe  $\varphi$ , all agents in G believe that all agents in G believe  $\varphi$ , all agents in G believe ..., up to an infinite depth of nesting. The existing work (Meggle 2003; Schwarzentruber 2011; Bonanno 1996; Heifetz 1999; Bonanno and Nehring 2000) reasons for belief on belief bases or possible worlds. Hu, Miller, and Lipovetzky (2022) form the common knowledge of a group by finding the fixed point intersection of all perspectives from all agents in the group, showing that this fixed point always exists within a finite bound. However, this approach cannot handle justified beliefs.

## 2.2 Planning with Perspectives

Hu, Miller, and Lipovetzky (2022) propose a perspective model to lazily evaluate epistemic (knowledge) formulae with external functions, called *Planning with Perspectives* (PWP). They adapt the seeing operator  $S_i$  from Cooper et al. (2016) for individual agent i following the intuition that "seeing is knowing"; formally,  $K_i\varphi \leftrightarrow \varphi \land S_i\varphi$ . That is, agent i knows  $\varphi$  iff  $\varphi$  is true and it sees the value of  $\varphi$ .

They define a state as a set of variable assignments x = e, and reason about agents' epistemic formulae based on what the agents can observe from their local state. The key is to define a **perspective function** for each agent i that takes a state and returns a subset of that state, which represents the part of the state that is observable to agent i. The perspective function  $O_i: \mathcal{S} \to \mathcal{S}$  is a function<sup>2</sup>, where  $\mathcal{S}$  is the set of all (partial and complete) states. The following properties must hold on a perspective function  $O_i$ :

- (1)  $O_i(s) \subseteq s$
- $(2) O_i(s) = O_i(O_i(s))$
- (3) If  $s \subseteq s'$ , then  $O_i(s) \subseteq O_i(s')$

These properties ensure that: (1) everything that an agent sees is part of the current state; (2) if an agent observes part of the world, then it observes that it observes that part of the world (it sees what it sees); and (3) the observation function is monotonic. Consider the example in Figure 1. At the final state  $s_4$ , the global state is  $\{peeking_a \rightarrow$ 

 $false, peeking_b \rightarrow true, n \rightarrow 1$ , while the observations of agent a are  $O_a(s_4) = \{peeking_a \rightarrow false, peeking_b \rightarrow true\}$ .

In addition, they also introduce group perspective functions to model group knowledge. They use set operators for the distributed perspective, which forms distributed knowledge, and a fix point for common perspective, which forms the common knowledge.

Their intuition is to use perspective function  $O_i$  to define semantics for seeing relation  $S_i$ , and using the equivalence relation (Cooper et al. 2016),  $S_i\varphi \wedge \varphi \equiv K_i\varphi$ , to define semantics for knowledge  $K_i$ . They define a sound and complete semantics (called the *complete semantics*) for both single and group operators based on the perspective function  $O_i$ . They also define a *ternary semantics* for higher computational efficiency. They show this ternary semantics is complete for a fragement of the logic, the so-called *logically separable formula*, which excludes propositions such as some tautologies and contradictions.

They provide some general perspective functions and show how perspective functions can be customised for specific domains. This provides a level of expressiveness not possible in declarative planning languages. Over several benchmarks, PWP solves problems faster than the state-of-the-art approach (Muise et al. 2022), and adds flexibility in modelling by using external functions.

#### 2.3 Justified Perspective Model

Hu, Miller, and Lipovetzky (2023) extend the PWP approach with the Justified Perspective (JP) model, which reasons about belief as well as knowledge. They introduce the belief operator  $B_i$  and reason about an agent's belief by generating agent's *justified perspective*. Their intuition is that agents believe something if they saw it in the past and have no evidence to suggest it has changed. Compared to their previous perspective model in the PWP approach, which was purely based on the observation of the current state, JP model forms perspectives also based on previous observations. However, JP model only models single agent's nested belief under multi-agent settings, not group belief. We define group belief in the current paper.

The signature, language and model are the same as ours (defined in Section 3 .1), except does not have those group operators. Their model uses the following notation:  $dom^3(s) \subseteq V$ , where V from the signature, a sequence of states is denoted by  $\vec{s}$ , the set of all possible state sequences is denoted by  $\vec{s}$ , and a state in a sequence  $\vec{s}$  at timestamp t is denoted by  $s_t$  or s[t].

The **retrieval function** R identifies the value of the variable v with respect to timestamp ts, which is the latest time an agent saw v in the given sequence  $\vec{s}$ . The formal definition they gave is as follows:

**Definition 1** (Retrieval Function (Hu, Miller, and Lipovetzky 2023)). Given a sequence of states  $\vec{s}$ , a timestamp ts and a variable v, the retrieval function,  $R: \vec{S} \times \mathbf{N} \times V \to D$ , is

<sup>&</sup>lt;sup>2</sup>Note they use f to represent the perspective function, while we are instead consistent with Hu, Miller, and Lipovetzky (2023) by using O to differentiate a PWP perspective function from a justified perspective function (see Section 2 .3).

 $<sup>^{3}</sup>$ Function dom(s) returns all variables appeared in the input state s.

defined as:

$$R(\vec{s}, ts, v) = \begin{cases} s_{ts}(v) & \text{if } v \in \text{dom}(s_{ts}) \\ s_{max(lts)}(v) & \text{else if } lts \neq \{\} \\ s_{min(rts)}(v) & \text{else if } rts \neq \{\} \\ None & otherwise \end{cases}$$
e: 
$$lts = \{j \mid v \in s_j \land j < ts\}$$

 $\begin{array}{l} lts = \{j \mid v \in s_j \land j < ts\} \\ rts = \{j \mid v \in s_j \land 0 \leq ts < j \leq |\vec{s}|\} \end{array}$ where:

Their intuition is: if v is in the state of timestamp ts from  $\vec{s}$ , then the value is found; else if v is in the previous states in the given justified perspective, then they assume the v stays unchanged since max(lts), which is the most recent time v in the given perspective before ts; else if v has not been seen in the given justified perspective before, then they assume the v stays unchanged to min(rts), which is the closest time v's value revealed in the givin perspective after ts; otherwise, v is None, as the given perspective does not contain v at all.

A **justified perspective** is a function:  $f_i : \vec{S} \to \vec{S}$ , that: the input  $\vec{s}$  represents the sequence of states of a plan from a particular perspective, which could be an agent's perspective or the global perspective; and, the output is a sequence of (local) states that i believes, which is i's justified perspective. It contains i's observation of the input (global or local) sequence, as well as i's memory, which both can be generated by the Retrieval Function R. They give the definition of the justified perspective function as follows:

**Definition 2** (Justified Perspective Function (Hu, Miller, and Lipovetzky 2023)). A justified perspective function for agent  $i, f_i : \vec{S} \to \vec{S}$ , is defined as follows:

$$f_{i}([s_{0},...,s_{n}]) = [s'_{0},...,s'_{n}]$$
 where for all  $t \in [0,n]$  and all  $v \in \text{dom}(s_{t})$ :
$$s'_{t} = \{v = e \mid \text{lt} = \max(ats(v))\},\ ats(v) = \{j \mid v \in \text{dom}(O_{i}(s_{j})) \land j \leq t\} \cup \{-1\},\ e = R([s_{0},...,s_{t}],\text{lt},v)$$

This defines that  $f_i$  takes a sequence of states (from parent perspective, could be global), and returns the sequence of perspectives that agent i believes. The function ats(v)defines the set of timestamps at which variable v was visible to agent i, lt is the most recent (last) timestamp that agent isaw variable v, e was identified as the value i should believe v is at time lt in the current parent perspective  $\vec{s}$ , and the set of assignments v = e forms  $s'_t$ .

**Semantics** are also provided by them in the format of complete semantics and ternary semantics. Here, we give their ternary semantics<sup>4</sup> as follows:

**Definition 3** (Ternary semantics (Hu, Miller, and Lipovetzky 2023)). A function T is defined, omitting the model M for readability:

(a) 
$$T[\vec{s}, r(\vec{t})] = 1 \text{ if } \pi(s_n, r(\vec{t})) = true;$$
  
 $0 \text{ else if } \pi(s_n, r(\vec{t})) = false;$   
 $\frac{1}{2} \text{ otherwise}$ 

(b) 
$$T[\vec{s}, \phi \wedge \psi] = \min(T[\vec{s}, \phi], T[\vec{s}, \psi])$$

(c) 
$$T[\vec{s}, \neg \varphi] = 1 - T[\vec{s}, \varphi]$$

(d) 
$$T[\vec{s}, S_i v] = \frac{1}{2} \text{ if } i \notin \text{dom}(s_n) \text{ or } v \notin \text{dom}(s_n);$$
  
 $0 \text{ else if } v \notin \text{dom}(f_i(s_n));$ 

(e) 
$$T[\vec{s}, S_i \varphi] = \frac{1}{2} \text{ if } T[\vec{s}, \varphi] = \frac{1}{2} \text{ or } i \notin \text{dom}(s_n)$$
  
 $0 \text{ else if } T[f_i(s_n), \varphi] = \frac{1}{2}$ 

(f) 
$$T[\vec{s}, K_i \varphi] = T[\vec{s}, \varphi \wedge S_i \varphi]$$

(g) 
$$T[\vec{s}, B_i \varphi] = T[f_i(\vec{s}), \varphi]$$

where: predicate  $r \in \mathbb{R}$  is from the Signature  $\Sigma$  and evaluation function  $\pi$  is from the Model M (Section 3.1);  $s_n$ is the final state in sequence  $\vec{s}$  (  $s_n = \vec{s}[|\vec{s}|]$ );  $0, \frac{1}{2}$  and 1 represent False, Unknown and True respectively.

Given the observation functions for each agent, Hu, Miller, and Lipovetzky implemented their own planner following F-STRIPS (Geffner 2000). Their experimental results show that it is state-of-the-art in most domains, except the ones with large branching factors, such as Grapevine (Muise et al. 2022), due to the naïve blind search algorithm used.

## **Group Justified Perspective Model**

In this section, we formally propose our group justified perspective (GJP) model by adding group operations for uniform belief, distributed belief, and common belief to the JP model (Hu, Miller, and Lipovetzky 2023), inheriting the existing group modal operators from PWP approach (Hu, Miller, and Lipovetzky 2022).

## 3.1 Preliminaries

A signature is a tuple  $\Sigma = (Agt, V, D_{v_1}, \dots D_{v_n}, \mathbb{R})$ , in which Agt is a finite set of n agent identifiers, V is a finite set of variables such that  $Agt \subseteq V$  (agent identifiers can be used as variables),  $D_{v_i}$  is a possibly infinite domain of constant symbols, one for each variable  $v_i \in V$ , and  $\mathbb{R}$  is a finite set of predicate symbols. Domains can be discrete or continuous, and the set of all values is defined as  $D = \bigcup_{v \in V} D_v$ .

**Definition 4** (Language). Language  $\mathcal{L}(\Sigma)$  is defined by the grammar:

$$\alpha ::= r(\vec{t}) \mid \neg \alpha \mid \alpha \land \alpha \mid S_i v \mid S_i \alpha \mid K_i \alpha$$

$$\alpha ::= ES_G \alpha \mid DS_G \alpha \mid CS_G \alpha \mid EK_G \alpha \mid DK_G \alpha \mid CK_G \alpha$$

$$\varphi ::= \alpha \mid B_i \varphi \mid EB_G \varphi \mid DB_G \varphi \mid CB_G \varphi$$

where  $r \in \mathbb{R}$ ,  $\vec{t} \subseteq V$  are the terms of r,  $r(\vec{t})$  are predicates and R is the set of all predicates. The group seeing operators, ES, DS and CS, and knowledge operators, EK, DKand CK are from the PWP model (Hu, Miller, and Lipovetzky 2022), while the  $B_i$  operator is from the JP model (Hu, Miller, and Lipovetzky 2023). In this paper, we add the operators  $EB_G\varphi$ ,  $DB_G\varphi$  and  $CB_G\varphi$  to represent that agents from group G jointly, distributedly and commonly believe  $\varphi$ respectively.

<sup>&</sup>lt;sup>4</sup>Because the ternary semantics is used in the implementation for the experiments, while the complete Boolean semantics are provided in the appendix.

This grammar prevents agents from seeing or knowing a belief, such as  $K_i B_G \varphi$ . Although those formulae can be modelled, they are equivalent to the existing operators. If an agent (group) knows that another agent (group) believes a proposition, this can only hold if the agent (group) knows that the other agent (group) knows that proposition. For example, if  $K_i B_j \varphi$  holds, then for i to know that j believes  $\varphi$ , it must be able to currently see that j believes  $\varphi$ , which means that i sees that j sees that  $\varphi$  holds, therefore  $K_iK_j\varphi$ must hold. That is,  $K_iB_j\varphi$  is equivalent to  $K_iK_j\varphi$ .

A **model** is defined as  $M = (\Sigma, \pi, O_1, \dots, O_n)$ . A state  $s: V \to D$  is an assignment from variables to values; e.g. [x = 1, y = 2]. A global state is a complete assignment for all variables in V, while a local state is a partial assignment (some variables may not be assigned). The expression s(v) denotes the value of variable v in state s. The set of all models is denoted  $\mathcal{M}$ .  $\pi$  is an interpretation function  $\pi: \mathcal{S} \times \mathbf{R} \to \{true, false\}$  that determines whether the atomic term  $r(\vec{t})$  is true in s. r is undefined if any of its arguments  $t_i$  is a variable  $v \in V$  that is not assigned a value in a local state s, i.e.  $v \notin dom(s)$ . In addition, we denote  $\tau$  to represent the value of a variable is None (not seen).

**Functions:** Observation function,  $O_i$ , and Justified Perspective Function,  $f_i$ , are the same as in Section 2.2 and

In addition, we propose the following theorem<sup>5</sup> on f:

**Theorem 3.1.** For any agent  $i \in A$  and perspective  $\vec{s} \in \vec{S}$ :  $f_i(\vec{s}) = f_i(f_i(\vec{s}))$ 

Semantics: We inherit the ternary semantics from PWP (Hu, Miller, and Lipovetzky 2022) and JP model (Hu, Miller, and Lipovetzky 2023). The items (a)-(g) follows Definition 3. The item (h)-(m) are the group semantics item (f)-(j) from (Hu, Miller, and Lipovetzky 2022), which can be found in Appendix, since they are not relevant to the contribution of this paper.

#### 3.2 Semantics for Group Belief

In this section, we define group justified perspective function for uniform belief, distributed belief, and common belief and add ternary semantics for them.

**Uniform Belief** is straightforward. Since a uniform belief of  $\varphi$  is that everyone in the group believes  $\varphi$ , the uniform justified perspective function is just a set union of everyone's individual justified perspectives.

**Definition 5.** (Uniform Justified Perspectives)  $ef_G(\vec{s}) = \bigcup_{i \in G} f_i(\vec{s})$ 

$$ef_G(\vec{s}) = \bigcup_{i \in G} f_i(\vec{s})$$

**Definition 6** (Ternary Semantics for Uniform Belief). Omitting the model M for readability, uniform belief  $EB_G$  for group G is defined:

(m) 
$$T[\vec{s}, EB_G\varphi] = \min(\{T[\vec{g}, \varphi] \mid \forall \vec{g} \in ef_G(\vec{s})\})$$

The ternary value of  $T[\vec{s}, EB_G\varphi]$  depends on the agent that holds the most conservative beliefs of  $\varphi$ .

Distributed Belief is more challenging than distributed knowledge. The Knowledge Axiom  $T: K_i \varphi \Rightarrow \varphi$ , which states that knowledge must be true, does not hold for belief. This means that agents can hold incorrect beliefs. If we simply take the distributed union of the perspectives for all agents  $i \in G$ , as it is done in PWP, we could obtain conflicting beliefs, so the implicit distributed belief would be inconsistent. To ensure consistency, we form the group distributed justified perspective instead of just uniting each agent's justified perspective. Intuitively, agents follow their own observations and "listen" to agents that have seen variables more recently. The distributed perspective function  $df^*$  is defined as follows.

**Definition 7** (Distributed Justified Perspectives). The distributed justified perspective function for a group of agent G is defined as follows:

$$df_G([s_0,\ldots,s_n]) = [s_0',\ldots,s_n']$$
 where for all  $t \in [0,n]$  and all  $v \in \text{dom}(s_t)$ : 
$$s_t' = \{v = e \mid \text{lt} = \max(ats(v)) \land e \neq \text{None}\},$$
 
$$ats(v) = \{j \mid v \in \text{dom}(O_G(s_j)) \land j \leq t\} \cup \{-1\},$$
 
$$e = R([s_0,\ldots,s_t],\text{lt},v)$$
 
$$O_G(s) = \bigcup_{i \in G} O_i(s)$$

In this definition, the group distributed justified perspective follows everyone's observation, and uses the Retrieval function R (in Definition 1) to identify the value of the variables which are or were not seen by any agent from the group. Intuitively, given any agent i in the group, the value from i's observation in timestamp t,  $O_i(s_t)$ , which leads to knowledge, must be true (Axiom T) in  $s_t$ . While, the value of an unseen variable is determined by anyone in the group that saw it last. To be specific, the last timestamp that group sees variable v, lt = max(ats(v)), is determined by the group observation (formed by union), and then, value e is retrieved by identifying the closest value that is consistent with it. So, this definition mimics the definition of belief from Definition 2, except that the value of a variable v in a state  $s'_t$  is taken by the agent(s) that have the most recent view of it.

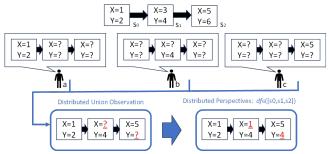


Figure 2: State sequence  $\vec{s}$  and  $df_G(\vec{s})$  in Example 2

**Example 2.** Let the set of variables be  $V = \{x, y\}$ , domains be  $D_x = D_y = \{1, 2\}$ , and a state<sup>6</sup> sequence be  $\vec{s} = [s_0, s_1, s_2]$ , where:  $s_0 = 1-2$ ,  $s_1 = 3-4$  and  $s_2 = 5-6$ .

<sup>&</sup>lt;sup>5</sup>The proof Theorem 3 .1 can be found in appendix.

<sup>&</sup>lt;sup>6</sup>We use the shorthand m-n to represent the state [x = m, y = n]. We use  $\tau$  to represent the 'value' of an unseen variable.

Assume a sees x and y in  $s_0$ , while b sees y in  $s_1$  and c sees x in  $s_2$ . So,  $O_a(\vec{s}) = [1-2,\tau-\tau,\tau-\tau]$ ,  $O_b(\vec{s}) = [\tau-\tau,\tau-4,\tau-\tau]$  and  $O_c(\vec{s}) = [\tau-\tau,\tau-\tau,5-\tau]$ . This is visualised in Figure 2.

So, we can see from Example 2 that forming distributed belief is about finding the observation from each agent and deducting the value of group unseen variables, following the same intuition as JP Model (In Section 2 .3).

**Definition 8** (Ternary Semantics for Distributed Belief). The distributed ternary semantics are defined using function T, omitting the model M for readability:

(n) 
$$T[\vec{s}, DB_G\varphi] = T[df_G(\vec{s}), \varphi]$$

This semantics guarantee the group distributed justified belief is consistent. That is done by only merging agents' observation into the group distributed observation, which is consistent with the global state, and deducing the value of unseen variables from it. This definition is particularly nice as many existing definitions of distributed belief require us to define preference relations over e.g. agents or states, to resolve conflicts; see e.g. (Liau 2003). In our definition, the preference relation is implicit, and prefers more recent observations over older observations.

**Common belief** is the infinite nesting of belief. Our definition avoids having to calculate the infinite regression by calculating the fixed point of the group's perspectives.

**Definition 9** (Common Justified Perspectives). Given a set of perspectives (that is, a set of sequences of states)  $\vec{S}$ , the common justified perspective is defined as:

$$cf_G(\vec{s}) = \begin{cases} \bigcup_{\vec{s} \in \vec{S}} ef_G(\vec{s}) & \text{if } \bigcup_{\vec{s} \in \vec{S}} ef_G(\vec{s}) = \vec{s} \\ cf_G(\bigcup_{\vec{s} \in \vec{S}} ef_G(\vec{s})) & \text{otherwise.} \end{cases}$$

The function applies a set union on the uniform perspectives of the group for each input perspective. Then, the common perspective function repeatedly calls itself by using the output of one iteration as the input of the next iteration, until the input set and output set are the same, which means a convergence of the common perspectives. Semantically speaking, each iteration adds one level deeper nested perspectives of everyone's uniform belief for evaluation on whether everyone in the group believes.

**Definition 10** (Ternary Semantics for Common Belief). The group ternary semantics are defined using function T, omitting the model M for readability:

(o) 
$$T[\vec{s}, CB_G\varphi] = \min(\{T[\vec{g}, \varphi] \mid \forall \vec{g} \in cf_G(\{\vec{s}\})\})$$

The common justified perspectives function  $cf_G$  contains the fixed point of all agent's perspectives, their perspectives about others' perspectives, and so on to the infinite depth. Although the depth is infinite, the definition of  $cf_G$  converges in finite iterations<sup>7</sup>:

**Theorem 3.2.** Given a sequence of states of length n, the upper bound for the number of iteration for  $cf_G(\vec{S})$  converges is  $2^{|V|\times n}$ .

Although in the worst case scenario, the maximum number of iterations would be  $|cf_G(\{\vec{s}\})|$ , practically, in our ex-

periments, we find that it converges after a few iterations (see Section 4).

An example for group justified perspective functions is provided using the same problem in Example 1<sup>8</sup> as follows:

**Example 3.** For example with Plan 1 .1, let  $G = \{a,b\}$  We have  $ef_G(\vec{s})$  is  $\{[\tau,2,2,2,2],[\tau,\tau,\tau,\tau,1]\}$ . Then, since the current  $ef_G(\vec{s})$  is not equal to  $\{\vec{s}\}$ , we nestedly apply  $ef_G(\vec{s})$  to generate  $ef_G(\{\vec{s}\})$ . From proof of Theorem 3 .1, we have  $f_a(f_a(\vec{s})) = f_a(\vec{s})$  and  $f_b(f_b(\vec{s})) = f_b(\vec{s})$ .  $f_b(f_a(s))$  is  $[\tau,\tau,\tau,\tau,2]$  and  $f_a(f_b(s))$  is  $[\tau,\tau,\tau,\tau,1]$ . So that, the current  $ef_G(\vec{s})$  is a set that contains the following perspectives: 1:  $[\tau,2,2,2,2]$ ; 2:  $[\tau,\tau,\tau,\tau,1]$ ; 3:  $[\tau,\tau,\tau,\tau,2]$ ; 4:  $[\tau,\tau,\tau,\tau,1]$ .

Since this is also not equal to  $ef_G(\vec{s})$ , we again apply  $ef_G$  on each perspective. Item (1) and (2) result in the same set, as the previous step, while both  $f_a$  and  $f_b$  on item (3) result in item (3) itself and both  $f_a$  and  $f_b$  on item (4) result in item (4) itself. Now, we have that  $ef_G(\{\vec{s}\})$  has converged.

For ternary semantics, following Example 3, we have  $ef_G(\vec{s}) = \{[\tau,2,2,2,2], [\tau,\tau,\tau,\tau,1]\}$ . The ternary representation  $T[\vec{s},EB_Gn<3]$  is evaluated as the minimum value in  $\{T[[\tau,2,2,2,2],n<3],T[[\tau,\tau,\tau,\tau,1],n<3]\}$ , which is  $\{1,1\}$  due to  $\{\pi(2,<3),\pi(1,<3)\}$ .

For the  $T[\vec{s}, CB_Gn < 3]$ , the (converged) common group justified perspectives is  $\{[\tau, 2, 2, 2, 2], [\tau, \tau, \tau, \tau, 1], [\tau, \tau, \tau, \tau, 2], [\tau, \tau, \tau, \tau, 1]\}$ , which is evaluated as  $\{1, 1, 1, 1\}$ . Therefore,  $T[\vec{s}, CB_Gn < 3]$  is 1.

## 4 Experiments

Since there are no planning benchmarks for group belief, we select three domains (Number, Grapevine and Big Brother Logic) from existing work (Hu, Miller, and Lipovetzky 2022, 2023) and add several challenging problem instances (7 for each domain) that use group belief, including instances with inconsistent or nested beliefs.

#### 4.1 Implementation

The source code of the planner, the domain, the problem and external function files, as well as experimental results, are downloadable from: **omitted to anonymity**. We extend the F-STRIPS planner from (Hu, Miller, and Lipovetzky 2023). To demonstrate the efficiency of our model instead of the particular search algorithms, we use the BrFS (breadth-first search) search algorithm with duplicate removal. The experiments are run on a Linux machine (Ubuntu 20.04) with 8 CPUs (Intel i7-10510U 1.80GHz) with 16GB RAM. The external functions, implemented in Python, evaluate the belief formulae (either in action preconditions or goals) as search nodes are generated. We implement the group justified perspective model and its corresponding ternary semantics.

#### 4.2 Domains

**Number** is an adapted domain from the origin coin domain (Hu, Miller, and Lipovetzky 2023). The problem settings, as described in Example 1, are of agents taking turns to peek into a box containing a changeable number.

<sup>&</sup>lt;sup>7</sup>The proof of Theorem 3 .2 can be found in appendix.

<sup>&</sup>lt;sup>8</sup>For simplicity, we only show the value of the number n instead of the agent's local state. The variables  $peeking_a$  and  $peeking_b$ ) are visible to all agents at all times.

ID	Exp	Gen	Common Max Avg		External   calls   Avg Time(ms)		Total   Time(s)	p	Goals
			IVIUX	7 TV S	Catto	Avg Time(ms)	Time(s)		
N0	39	140	0	0	207	0.085	0.048	4	$EB_G n < 2$
N1	7	25	0	0	33	0.095	0.006	2	$DB_G n < 2$
N2	39	140	4	2.199	207	0.255	0.075	4	$CB_G n < 2$
N3	120	435	4	2.461	668	0.414	0.354	6	$EB_Gn < 2 \land \neg CB_Gn < 2$
N4	347	1273	5	2.716	2041	0.798	1.906	8	$EB_GEB_Gn < 2 \land \neg CB_Gn < 2$
N5	31	111	3	2.134	161	0.307	0.068	4	$\neg EB_G n = 1 \land \neg EB_G n = 2 \land CB_G n < 2$
N6	50	177	3	1.649	257	0.301	0.107	4	$B_aCB_Gn=2\wedge B_bCB_Gn=1$
G0	5	35	4	3.028	41	3.17	0.149	1	$CB_Gsct_a = t$
G1	66	450	4	3.195	691	5.94	4.542	4	$EB_Gsct_a = t \wedge CB_Gsct_a = t \rightarrow \frac{1}{2}$
G2	240	1828	5	3.496	3282	9.984	35.764	6	$EB_GEB_Gsct_a = t \wedge CB_Gsct_a = t \rightarrow \frac{1}{2}$
G3	103	913	4	3.018	1138	4.882	6.14	3	$B_b C B_G s c t_a = f \wedge C B_{\{a,c,d\}} s c t_a = t$
G4	328	2959	4	2.664	3792	10.717	42.776	4	$CB_{\{b,c\}}CB_Gsct_a = f \wedge CB_{\{a,d\}}sct_a = t$
G5	66	450	4	3.195	691	6.469	4.89	4	$DB_GEB_Gsct_a = t \wedge CB_Gsct_a = t \rightarrow \frac{1}{2}$
G6	70	455	0	0	734	1.326	1.396	4	$DB_GEB_Gsct_a = t \wedge B_aEB_Gsct_a = t \rightarrow \frac{1}{2}$
BBL0	2	8	4	3.111	11	9.387	0.107	1	$CB_Go_2 = 2$
BBL1	5	19	4	3.143	26	10.617	0.279	2	$EB_Go_2 = 2 \wedge CB_Go_2 = 2 \rightarrow \frac{1}{2}$
BBL2	177	708	4	3.492	1011	21.724	22.072	5	$CB_Go_1 = 1$
BBL3	189	756	4	3.485	1067	42.853	45.841	5	$CB_G(o_1 = 1 \land o_2 = 2)$
BBL4	1595	6380	0	0	10295	0.486	6.05	9	$EB_G(o_1 = 1 \land o_2 = 2 \land o_3 = 3)$
BBL5	1595	6380	0	0	10295	0.39	5.007	9	$EB_Go_1 < o_2$
BBL6	2	8	0	0	11	0.663	0.009	1	$DB_{G}o_{1} < o_{2}$

Table 1: Result for three domains (N0-N6, G0-G6, BBL0-BBL6 are instances for Number, Grapevine and BBL respectively). G represents the group of agents  $-\{a,b\}$  for Number and BBL; and,  $\{a,b,c,d\}$  for Grapevine. 'Exp" and "Gen" are the number of nodes expanded and generated during search, "Max" and "Avg" under "Common" as the maximum and average level of nesting required to compute  $cf_G$ , |calls| and "Avg Time(ms)" under "External" as the number and average time of external function calls, and |p| as plan length. Since we implement the ternary semantics, we denote the ternary evaluation result  $T[\vec{s},\varphi]$  equal to  $0,\frac{1}{2}$  and 1 as  $\neg\varphi$ ,  $\varphi \to \frac{1}{2}$  and  $\varphi$  respectively.

**Grapevine** is a benchmark domain in epistemic planning (Muise et al. 2022). In two adjacent rooms, 4 agents, in the same room, each has their own secret. All agents can move between two rooms and *share* or *lie* about a secret  $sct_i$ , if either the secret is their own secret, or they have heard the secret from someone. That is, they need to believe the secret  $(B_i sct_i)$  before they can share or lie about it.

**Big Brother Logic (BBL)** (Gasquet, Goranko, and Schwarzentruber 2014) is a domain that stationary cameras that can turn and observe with a certain angular range in a 2-dimension plane. For simplicity, we limit the camera's angle of turning to a set  $\{0^{\circ}, \pm 45^{\circ}, \pm 90^{\circ}, \pm 135^{\circ}, 180^{\circ}\}$  and the field is in a  $5 \times 5$  grid. Initially, camera a and b locates at (3,3) and (1,1) with direction of  $-135^{\circ}$  and  $90^{\circ}$ , while  $o_1$ ,  $o_2$  and  $o_3$  locates at (0,0), (2,2) and (3,3), with value 1, 2 and 3 respectively.

#### 4.3 Results

The results can be found in Table 1. All group beliefs, except common belief, can be evaluated easily, which is indicated by the external function computation time (N1, N2, G6 and BBL4-6). It takes longer to evaluate common belief because of the level of nesting in finding converged common perspectives; however, the number iterations of  $cf_G$  to find the fixed-point is around 2.2-3.5 – much less than the worst case identified in Theorem 3 .2. Typically, the higher the level of nesting in finding cf, the longer it takes to compute. It is worth noting that the nesting of the common be-

lief did not significantly increase external function computational unit time per call, which can be found in G4.

#### 5 Conclusion and Future work

In this paper, we define an extension to the Justified Perspective model (Hu, Miller, and Lipovetzky 2023) to handle group beliefs; implement its ternary semantics as a model-free planning tool; and demonstrate its expressiveness and efficiency on new featured domains. The results show that our approach can effectively handle multi-agent epistemic planning problems with group beliefs and do so efficiently, even with a simple prototype F-STRIPS planner implementing a Breadth First Search.

For future work, we will implement efficient search algorithms for our planner. Potential search algorithms may need to create novel non-Markovian search algorithms, as our GJP model works with state sequences. In addition, although there is a growing body of research in epistemic planning, there are still no uniform standards on either epistemic planning language or benchmark domains. It would be valuable to revisit existing approaches and model our benchmarks in those. Besides, current epistemic planning approaches often rely on classical planning assumptions. Future research could broaden the field's applicability by relaxing these assumptions, particularly in dynamic environments and in human-agent interaction domains with formal human belief model.

#### A Proof for Theorem 3.1

**Theorem 3.1.** For any agent  $i \in A$  and perspective  $\vec{s} \in \vec{S}$ :  $f_i(\vec{s}) = f_i(f_i(\vec{s}))$ 

*Proof.* Let  $s(v) = \tau$  be  $v \notin s$  in the following content.

The base case is a sequence with one element  $[s_0]$ . In this case, the above lemma holds trivially, because:  $f_i([s_0]) = [O_i(s_0)]$ ;  $f_i(f_i([s_0])) = f_i([O_i(s_0)]) = [O_i(O_i(s_0))]$ ; and,  $O_i(s_0) = O_i(O_i(s_0))$ .

For a sequence for more than one element,  $f_i(\vec{s})[t]$  depends on the  $[O_i(s_0), \ldots, O_i(s_t)]$  and  $s_0, \ldots, s_t$ . Let v be any variable in V.

Any variable  $v \in V$  can be classified into one of the following conditions:

```
1. v \in O_i(s_t)
2. v \notin O \land v \in O_i(f_i(\vec{s})[t])
3. v \notin O \land v \notin O_i(f_i(\vec{s})[t])
```

For Condition (1), we have  $f_i(\vec{s})[t](v) = O_i(s_t)(v) = s_t(v)$  and  $f_i(f_i(\vec{s}))[t](v) = O_i(f_i(\vec{s})[t])(v) = O_i(O_i(s_t))(v) = O_i(s_t)(v) = s_t(v)$ , since: $v \in O_i(s_t)$ ;  $O_i(s_t) = O_i(O_i(s_t))$ ; and,  $O_i(s_t) \subseteq f_i(\vec{s})[t] \to O_i(O_i(s_t)) \subseteq O_i(f_i(\vec{s})[t])$ . Thus,  $f_i(\vec{s})[t](v) = f_i(f_i(\vec{s}))[t](v)$ .

For Condition (2), we have  $f_i(f_i(\vec{s}))[t](v) = O_i(f_i(\vec{s})[t])(v) = f_i(\vec{s})[t](v)$ . For Condition (3), whether  $f_i(\vec{s})[t](v) = f_i(f_i(\vec{s}))[t](v)$  holds depends on whether  $f_i(\vec{s})[t-1](v) = f_i(f_i(\vec{s}))[t-1](v)$ . Then, by recursively apply the above reasoning, we can show if the v matches Condition (3) for every timestamp from t, t-1 to 0, it matches with our base case above.

Overall, 
$$f_i(\vec{s}) = f_i(f_i(\vec{s}))$$
 holds.

### **B** Proof for Theorem 3.2

**Theorem 3.2.** Given a sequence of states of length n, the upper bound for the number of iterations for  $cf_G(\vec{S})$  converges is  $2^{|V| \times n}$ .

*Proof.* Since for each variable in the last state of a justified perspective  $\vec{w}$ , its value is either visible (same as its in the last state of the global perspective), or not visible (same as its in the second-last state from  $\vec{w}$ ), the number of possible states in each index of a justified perspectives is  $2^{|V|}$ . So, the number of possible perspectives given a global state sequence  $\vec{s}$  with length of n is  $2^{|V| \times n}$ . In calculating  $cf_G$ , either the base case holds (that is, combining the perspective of the group for all  $\vec{s} \in \vec{S}$  does not change the common perspective), so it terminates and adds no new perspectives; or the recursive step holds. In this case, the input of the cf function is set that contains perspectives from each agent in the format of  $\vec{S} = \{f_i(\vec{s}), f_i(\vec{s}'), \dots \mid \forall i \in G\}$ . Then, we apply  $f_i$  for each agent j in the group G on each perspective from  $\vec{S}$  as  $\vec{S}' = \bigcup_{\vec{s} \in \vec{S}} ef_G(\vec{s})$ . For each  $f_i(\vec{s})$  from  $\vec{S}$ , we have  $f_i(f_i(\vec{s}))$ in  $\vec{S}'$  for each agent j in group G. With Theorem , we have  $f_i(f_i(\vec{s})) = f_i(\vec{s})$  when i = j. Therefore, we have  $\vec{S} \subseteq \vec{S}'$ . At worst, we add one new sequence each iteration, meaning that  $cf_G(\vec{S})$  converges by at most  $2^{|V| \times n}$  iterations.

### **C** Semantics

The language for this full semantics is defined in Definition 4. In order to provide semantics for CS, we need to introduce the perspective (observation) function cO from Hu, Miller, and Lipovetzky (2022)'s paper.

**Definition 11** (Common Observation Function (Hu, Miller, and Lipovetzky 2022)). Given a group of agents G and the current state s, the common observation of the group can be defined as:

fined as: 
$$cO(G,s) = \begin{cases} s & \text{if } s = \bigcap_{i \in G} O_i(s) \\ cO(G,\bigcap_{i \in G} O_i(s)) & \text{otherwise.} \end{cases}$$

The variables that are not visible to any agent in the group G are filtered out until the remaining set becomes a fixed point set. That is, every variable in the set is commonly seen by the group G.

## **C.1** Ternary Semantics

Here we provide full ternary semantics. Similar to the complete semantics: item a-g (Hu, Miller, and Lipovetzky 2023) are for individual modal operators using justified perspective function; item h-l (Hu, Miller, and Lipovetzky 2022) are for group seeing and knowledge operators using group perspective functions; item m-o are for group belief operators defined in Section 3.2.

**Definition 12** (Ternary Semantics). The ternary semantics are defined using function T, omitting the model M for readability:

(a) 
$$T[\vec{s}, r(\vec{t})] = 1$$
 if  $\pi(\vec{s}[n], r(\vec{t})) = true$ ;  $0$  if  $\pi(\vec{s}[n], r(\vec{t})) = false$ ;  $\frac{1}{2}$  otherwise  
(b)  $T[\vec{s}, \phi \land \psi] = \min(T[\vec{s}, \phi], T[\vec{s}, \psi])$   
(c)  $T[\vec{s}, \neg \varphi] = 1 - T[\vec{s}, \varphi]$   
(d)  $T[\vec{s}, S_i v] = \frac{1}{2}$  if  $i \notin \vec{s}[n]$  or  $v \notin \vec{s}[n]$   
 $0$  if  $v \notin O_i(\vec{s}[n])$   
 $1$  otherwise  
(e)  $T[\vec{s}, S_i \varphi] = \frac{1}{2}$  if  $T[\vec{s}, \varphi] = \frac{1}{2}$  or  $i \notin \vec{s}[n]$ ;  $0$  if  $T[\vec{O}_i(\vec{s}), \varphi] = \frac{1}{2}$ ;  $1$  otherwise  
(f)  $T[\vec{s}, K_i \varphi] = T[\vec{s}, \varphi \land S_i \varphi]$   
(g)  $T[\vec{s}, B_i \varphi] = T[\vec{s}, \varphi \land S_i \varphi]$   
(g)  $T[\vec{s}, B_i \varphi] = T[f_i(\vec{s}), \varphi]$   
(h)  $T[\vec{s}, ES_G \alpha] = \min(\{T[\vec{s}, S_i \alpha] \mid i \in G\})$   
(i)  $T[\vec{s}, DS_G v] = \frac{1}{2}$  if  $v \notin \vec{s}[n]$  or  $\forall i \in G, i \notin \vec{s}[n]$ ;  $0$  if  $v \notin \bigcup_{i \in G} O_i(\vec{s}[n])$ ;  $1$  otherwise  
(j)  $T[\vec{s}, DS_G \varphi] = \frac{1}{2}$  if  $T[\vec{s}, \varphi] = T[\vec{s}, \neg \varphi] = \frac{1}{2}$  or  $\forall i \in G, i \notin \vec{s}[n]$   
 $0$  if  $T[s', \varphi] = T[s', \neg \varphi] = \frac{1}{2}$  where  $s' = \bigcup_{i \in G} O_i(\vec{s}[n])$ ;  $1$  otherwise  
(k)  $T[\vec{s}, CS_G v] = \frac{1}{2}$  if  $v \notin \vec{s}[n]$  or  $\exists i \in G, i \notin \vec{s}[n]$ ;

$$0 \quad \text{if } v \notin cO(G, \vec{s}[n]);$$
 
$$1 \quad \text{otherwise}$$
 
$$(1) \quad T[\vec{s}, CS_G\varphi] = \frac{1}{2} \quad \text{if } T[s, \varphi] = T[s, \neg \varphi] = \frac{1}{2} \quad \text{or } \exists i \in G, \ i \notin s;$$
 
$$0 \quad \text{if } T[\overrightarrow{cO}(G, \vec{s}), \varphi] = \frac{1}{2}, \quad \text{and } T[\overrightarrow{cO}(G, \vec{s}), \neg \varphi] = \frac{1}{2} \quad \text{1} \quad \text{otherwise}$$
 
$$(m) \quad T[\vec{s}, EB_G\varphi] = \quad \min(\{T[\vec{g}, \varphi] \mid \forall \vec{g} \in ef_G(\vec{s})\})$$
 
$$(n) \quad T[\vec{s}, DB_G\varphi] = \quad T[df_G(\vec{s}), \varphi]$$
 
$$(o) \quad T[\vec{s}, CB_G\varphi] = \quad \min(\{T[\vec{g}, \varphi] \mid \forall \vec{g} \in ef_G(\{\vec{s}\})\})$$

where:  $\alpha$  is a variable v or a formula  $\varphi$ ;  $\vec{s}[n]$  is the final state in sequence  $\vec{s}$ ; that is,  $\vec{s}[n] = \vec{s}[|\vec{s}|]$ ;  $\vec{O}_i(\vec{s}) = [O_i(\vec{s}[0]), \dots, O_i(\vec{s}[n])]; \text{ and, } \overrightarrow{cO}(G, \vec{s}) =$  $[cO(G, \vec{s}[0]), \ldots, cO(G, \vec{s}[n])].$ 

## **C.2** Complete Semantics

Here, we provide the full complete semantics, which includes: semantics for the individual modal logic operators using justified perspective functions (item a-g) (Hu, Miller, and Lipovetzky 2023); semantics for the group seeing and knowledge operators using group perspective functions (item h-1) (Hu, Miller, and Lipovetzky 2022); And, the semantics for group belief operators is similar as we provided in Section 3.2.

**Definition 13** (Complete semantics). The complete semantics for justified perspective are defined as:

cs for justified perspective are defined as:

(a) 
$$(M, \vec{s}) \models r(\vec{t})$$
 iff  $\pi(\vec{s}[n], r(\vec{t})) = true$ 

(b)  $(M, \vec{s}) \models \phi \land \psi$  iff  $(M, \vec{s}) \models \phi$  and  $(M, \vec{s}) \models \psi$ 

(c)  $(M, \vec{s}) \models \neg \varphi$  iff  $(M, \vec{s}) \not\models \varphi$ 

(d)  $(M, \vec{s}) \models S_i v$  iff  $v \in O_i(\vec{s}[n])$ 

(e)  $(M, \vec{s}) \models S_i \varphi$  iff  $\forall \vec{g} \in \vec{S}_{tc}, (M, \vec{g}[\vec{O}_i(\vec{s})]) \models \varphi$ 

or  $\forall \vec{g} \in \vec{S}_{tc}, (M, \vec{g}[\vec{O}_i(\vec{s})]) \models \neg \varphi$ 

(f)  $(M, \vec{s}) \models K_i \varphi$  iff  $(M, \vec{s}) \models \varphi \land S_i \varphi$ 

(g)  $(M, \vec{s}) \models B_i \varphi$  iff  $\forall \vec{g} \in \vec{S}_{tc}, (M, \vec{g}[f_i(\vec{s})]) \models \varphi$ 

(h)  $(M, \vec{s}) \models ES_G \alpha$  iff  $\forall \vec{t} \in G, (M, \vec{s}) \models S_i \alpha$ 

(i)  $(M, \vec{s}) \models DS_G \psi$  iff  $\forall \vec{t} \in G, (M, \vec{g}(\vec{d}(\vec{s}))) \models \varphi$ 

or  $\forall \vec{g} \in \vec{S}_{tc}, (M, \vec{g}(\vec{d}(\vec{s}))) \models \varphi$ 

(k)  $(M, \vec{s}) \models CS_G \psi$  iff  $v \in cO(G, \vec{s}[n])$ 

(l)  $(M, \vec{s}) \models CS_G \varphi$  iff  $\forall \vec{g} \in \vec{S}_{tc}, (M, \vec{g}(\vec{d}(\vec{s}))) \models \varphi$ 
 $(M, \vec{g}(\vec{cO}(G, \vec{s}))) \models \varphi$ 

$$(M, \vec{g} \langle \overrightarrow{cO}(G, \vec{s}) \rangle) \vDash \neg \varphi$$

$$(m) \ (M, \vec{s}) \vDash EB_G \varphi \ \text{iff} \ \forall \vec{w} \in ef_i(\vec{s}), \\ \forall \vec{g} \in \vec{S}_{tc}, (M, \vec{g}[\vec{w}]) \vDash \varphi$$

$$(n) \ (M, \vec{s}) \vDash DB_G \varphi \ \text{iff} \ \forall \vec{g} \in \vec{S}_{tc}, (M, \vec{g}[df_G(\vec{s})]) \vDash \varphi$$

or  $\forall \vec{g} \in \vec{S}_{tc}$ ,

(ii) 
$$(M, \vec{s}) \models DB_G \varphi$$
 iii  $\forall g \in S_{tc}, (M, g[u_{i}G(\vec{s})]) \models G(M, \vec{s}) \models CB_G \varphi$  iff  $\forall \vec{w} \in cf_G(\{\vec{s}\}), \forall \vec{q} \in \vec{S}_{tc}, (M, \vec{g}[\vec{w}]) \models \varphi$ 

where:  $\alpha$  is a variable v or a formula $\varphi$ ;  $\vec{s}[n]$  is the final state in sequence  $\vec{s}$ ; that is,  $\vec{s}[n] = \vec{s}[|\vec{s}|]; \ \vec{g}(\vec{s}) = \vec{g}[0]\langle \vec{s}[0]\rangle, \dots, \vec{g}[n]\langle \vec{s}[n]\rangle];$  $[O_i(\vec{s}[0]),\ldots,O_i(\vec{s}[n])]; \quad \vec{d}(\vec{s})$  $\vec{O}_i(\vec{s})$  $[\bigcup_{i \in G} O_i(\vec{s}[0]), \ldots, \bigcup_{i \in G} O_i(\vec{s}[n])]$  and,  $\overrightarrow{cO}(G, \vec{s})$  $[cO(G, \vec{s}[0]), \ldots, cO(G, \vec{s}[n])].$ 

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