Numerical Analysis HW3

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1 Problem1

1.1 a.

$X - \widetilde{X} = (0.2, 0.5, -0.4)^T$,so $||X - \widetilde{X}||_\infty = 0.5$
$AX - b = (0, -0.3, -0.2)^T$,so$||AX - b||_\infty = 0.3$

1.2 b.

$X - \widetilde{X} = (0.33, 0.9, -0.8)^T$,so $||X - \widetilde{X}||_\infty = 0.9$
$AX - b = (0.27, -0.26, 0.21)^T$,so$||AX - b||_\infty = 0.27$

2 Problem2

$$||A||_2^2 = \sqrt{\rho(A^T A)} = \sqrt{\rho(A^2)}$$
$$\rho(A^2) = max|\lambda_A^2| = (max|\lambda_A|)^2 = \rho^2(A)$$
$$||A||_2^2 = \sqrt{\rho^2(A)} = \rho(A)$$

3 Problem3

codes are in the file.

3.1 a.

Result:$x_1 = 10.005851 \quad x_2 = 1.005093$

3.2 b.

Result:$x_1 = 0.000000 \quad x_2 = 10.000000 \quad x_3 = 0.142857$

4 Problem4

4.1 a.

$$x_1 = (1.25, -1.33333333, 0.2)^T$$

$$x_2 = (1.63333333, -0.98333333, 0.23333333)^T$$

$$x_3 = (1.55416667, -0.86666667, -0.06)^t$$

4.2 b.

$$x_1 = (-2, 2, 0)^T$$

$$x_2 = (-1, 1, -1)^T$$

$$x_3 = (-1.75, 1.75, -0.5)^T$$

5 Problem5

5.1 a.

GS:

$$x_7 = (0.03515081, -0.23682839, 0.65786182)^T$$

Jacobi:

$$x_{10} = (0.03507839, -0.23692617, 0.65780145)^T$$

5.2 b.

GS:

$$x_4 = (0.9957475, 0.95787375, 0.79157475)^T$$

Jacobi:

$$x_6 = (0.995725, 0.957775, 0.79145)^T$$

6 Problem6

反证法：如果不线性无关，那么必定存在 x_1, x_2, 使得

$$k_1 x_1 + k_2 x_2 = 0$$

$$Ak_1 x_1 + Ak_2 x_2 = 0$$

$$\rho_1 k_1 x_1 + \rho_2 k_2 x_2 = 0$$

$$\rho_1 k_1 x_1 - \rho_2 k_1 x_1 = 0$$

得到 $\rho_1 = \rho_2$, 与题意矛盾，所以假设不成立。

7 Problem7

反证法：假设 A 不可逆，那么 $|A| = 0$, 对于 AX=0，存在非零解 $X = x_1, x_2...x_n$，对第 i 行，有

$$\sum_{j=1}^{n} a_{ij}x_j = 0$$

取 $x_k = maxx_1, x_2...x_n$, 则对第 k 行，有

$$\sum_{j\neq k}^{n} a_{kj}x_j = -a_{kk}x_k$$

又 A 为对角严格占优矩阵,

$$|\sum_{j\neq k}^{n} a_{kj}x_j| \leq |x_k||\sum_{j\neq k}^{n} a_{kj}| < |a_{kk}||x_k|$$

矛盾！所以假设不成立