

Numerical Analysis HW2

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1 Problem1

选取精度为 0.00001, 得到 $p_2 = -0.8654740331$ 。

若采取 $p_0 = 0$, 根据 $p_n = p_{n-1} - \frac{f(x)}{f'(x)}$, $f'(p_0) = f'(0) = 0$, 故不能作为分母, 因此不能使用牛顿方法

2 Problem2

2.1 (i)

$f(x) = b - \frac{1}{x}$, $f'(x) = \frac{1}{x^2}$, 可得

$$x_{n+1} = 2x_n - bx_n^2$$

$$\epsilon_k = \frac{\frac{1}{b} - x_k}{\frac{1}{b}} = 1 - bx_k$$

$$\epsilon_{k+1} = \frac{\frac{1}{b} - x_{k+1}}{\frac{1}{b}} = \frac{\frac{1}{b} - 2x_k + bx_k^2}{\frac{1}{b}} = (1 - bx_k)^2$$
$$\epsilon_{k+1} = \epsilon_k^2$$

2.2 (ii)

对任意 $x_n \in (0, \frac{2}{b})$, $x_n + 1 = x_n(2 - bx_n) > 0$ (数学归纳法), 所以存在 $x_1 \in (0, \frac{1}{b})$ 与 $k \in (0, 1)$, 使得

$$g(x) = 2x - bx^2$$

$$g'(x) = 2 - 2bx_1 = 2(1 - bx_1) < k$$

根据收敛性定理, x 收敛于 $\frac{1}{b}$

3 Problem 3

代码附在 code 文件夹中

3.1 a.

$$x_2 = [0.50016669 \ 0.25080364 \ -0.51738743]$$

3.2 b.

$$x_2 = [4.35087719 \ 18.49122807 \ -19.84210526]$$

4 Problem 4

代码附在 code 文件夹中

4.1 a.

$$[1.04122820973906, 1.06586158726298, 0.928075247526470] \text{ 此时, } g(x)=0.0361977296438597$$

4.2 b.

$$[0.510630019323346, 1.01246048957044, -0.468346515234894], \text{ 此时, } g(x)=0.0487719856095837$$

过程中发现精确解为 $[0.5, 1, -0.5]$

5 Problem 5

5.1 a.

关注四个偏导函数,

$$\frac{\partial g_1}{\partial x_1} = \frac{x_1}{5} \leq 0.3$$

$$\frac{\partial g_1}{\partial x_2} = \frac{x_2}{5} \leq 0.3$$

$$\frac{\partial g_2}{\partial x_1} = \frac{x_2^2 + 1}{10} \leq 0.325$$

$$\frac{\partial g_2}{\partial x_2} = \frac{x_1 x_2}{5} \leq 0.45$$

取 $K = 0.91$, 则

$$\left| \frac{\partial g_i}{\partial x_i} \right| \leq \frac{K}{n}$$

5.2 b.

$$x^{(0)} = (0, 1)^t$$

$$x^{(1)} = (0.9, 0.8)^t$$

$$x^{(2)} = (1.045 \ 0.9046)^t$$