

NA HW4.

Problem 1 a.

$$L_{n,k}(x) = \prod \frac{x-x_i}{x_k-x_i}$$

$$\left\{ \begin{array}{l} L_{2,0}(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \quad | \quad f(x_0)=1 \\ L_{2,1}(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} \quad | \quad f(x_1)=e^{0.6} \cos 0.9 \\ L_{2,2}(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} \quad | \quad f(x_2)=e^{1.2} \cos 1.8 \end{array} \right.$$

$$P(x) = \sum_{k=0}^{\infty} f(x_k) \cdot L_{n,k}(x) = \underbrace{-11.72x^2 + 3.81x + 1}_{(code is in the file)}$$

$$\sum = \left| \frac{f^{(3)}(\xi(x))}{3!} (x-x_0)(x-x_1)(x-x_2) \right|$$

$$f' = 2e^{2x} \cos 3x + e^{2x} (-3 \sin 3x)$$

$$f'' = e^{2x} (-9 \sin 3x - 46 \cos 3x)$$

$$f''' = e^{2x} (-9 \sin 3x - 46 \cos 3x) \Rightarrow g(\xi) = f'''(\xi) = e^{2\xi} (-9 \sin 3\xi - 46 \cos 3\xi)$$

$$g'(\xi) = e^{2\xi} (120 \sin 3\xi - 119 \cos 3\xi) \rightarrow \xi = \frac{1}{3} \arctan \frac{120}{119}$$

$$\sum = \max_{\xi \in [0, 0.6]} \left\{ \left| \frac{f'''(\xi)}{6} \right| \right\} \max_{x \in [0, 0.6]} \left\{ |(x-x_0)(x-x_1)(x-x_2)| \right\} = 0.113$$

b. 13.3: $P(x) = -0.13x^2 + 0.897x + 0.885$

$$\sum = 0.01423$$

Problem 2.

$$P_3(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

$$\begin{cases} a_0 = 0 \\ a_0 + \frac{1}{2}a_1 + \frac{1}{4}a_2 + \frac{1}{8}a_3 = y \\ a_0 + a_1 + a_2 + a_3 = 3 \\ a_0 + 2a_1 + 4a_2 + 8a_3 = 2 \\ a_3 = 6 \end{cases} \quad \begin{cases} a_1 = 1 \\ a_2 = -20 \\ a_3 = 6 \end{cases} \Rightarrow y = \frac{17}{4}$$

Problem 3.

$$P_{2,3} = \frac{1}{x_3 - x_2} [(x - x_2) P_3 - (x - x_3) P_2] \Big|_{x=0.4} = 2 \cdot 4$$

$$\text{cp: } 4 \cdot (-0.1 \cdot 8 + 0.35 P_2) = 2 \cdot 4 \Rightarrow P_2 = 4$$

$$\begin{matrix} x_0 = 0 \\ x_1 = 1 \end{matrix}$$

$$P_{0,1} = \frac{1}{x_1 - x_0} [(x - x_0) P_1 - (x - x_1) P_0] = x P_1 - (x-1) P_0 = 2x + 1$$

$$x_2 = 2$$

$$P_{0,2} = \frac{1}{x_2 - x_0} [(x - x_0) P_2 - (x - x_2) P_0] = \frac{1}{2} [x P_2 - (x-2) P_0] = x + 1$$

$$x_3 = 3$$

$$\Rightarrow \begin{cases} (P_1 - P_0)x + P_0 = 2x + 1 \\ (P_2 - P_0)x + 2P_0 = 2x + 2 \end{cases} \Rightarrow \begin{cases} P_0 = 1 \\ P_1 = P_2 = 3 \end{cases}$$

$$\Rightarrow P_{1,2} = \frac{1}{x_2 - x_1} [(x - x_1) P_2 - (x - x_2) P_1] = 3[(x-1) - (x-2)] = 3$$

$$\Rightarrow P_{0,1,2} = \frac{1}{x_3 - x_0} [(x - x_0) P_{1,2} - (x - x_3) P_{0,1}] = \frac{1}{2} [x \cdot 3 - (x-2)(2x+1)]$$

$$= \frac{1}{2} [3x - (2x^2 - 3x - 2)] = \frac{1}{2} (x^2 + 5x + 2) = -x^2 + 3x + 1$$

$$\Rightarrow P_{0,1,2,3}(x) = \frac{1}{x_3 - x_0} [(x - x_0) P_{1,2,3} - (x - x_3) P_{0,1,2}] \Big|_{x=2.5}$$

$$= \frac{1}{3} [2 \cdot 5 \cdot 3 + 0.5 \cdot \frac{9}{4}] = 2.875$$

Problem 4

$$\begin{array}{ll}
 x_0=0 & f(x_0)=1 \\
 x_1=0.4 & f(x_1)=3 \\
 x_2=0.7 & f(x_2)=6
 \end{array}
 \quad
 \begin{array}{ll}
 f(x_0, x_1)=5 \\
 f(x_1, x_2)=10
 \end{array}
 \quad
 \begin{array}{l}
 f(x_0, x_1, x_2)=\frac{50}{7}
 \end{array}$$

Problem 5

$$\begin{cases} a_0 + a_1x + a_2x^2 + a_3x^3 & x \in [0,1] \\ S(x) = b_0 + b_1(x-1) + b_2(x-1)^2 + b_3(x-1)^3 & x \in [1,2] \end{cases}$$

$$f(0)=0, f(1)=1, f(2)=2$$

$$\begin{cases} a_0=0 \\ a_0+a_1+a_2+a_3=1 \\ b_0=1 \\ b_0+b_1+b_2+b_3=2 \end{cases}$$

12k, -.- = 3x^2 + 3x^3

$$\begin{cases} a_0+2a_1+3a_2+b_3=b_1 \\ 2a_2+6a_3=b_2 \\ a_2=0 \\ b_2+6b_3=0 \end{cases}$$

$$\Rightarrow \begin{cases} a_0=1 & b_0=1 \\ a_1=1 & b_1=1 \\ a_2=0 & b_2=0 \\ a_3=0 & b_3=0 \end{cases}$$

$$\Rightarrow S(x) = \begin{cases} x & x \in [0,1] \\ x & x \in [1,2] \end{cases} \Rightarrow S(x)=x \quad x \in [1,2]$$

$$\begin{cases} a_0=0 \\ a_0+a_1+a_2+a_3=1 \\ b_0=1 \\ b_0+b_1+b_2+b_3=2 \end{cases}$$

$$\begin{cases} a_0+2a_1+3a_2+b_3=b_1 \\ 2a_2+6a_3=b_2 \\ a_1=1 \\ b_1+b_2+b_3=1 \end{cases}$$

$$\Rightarrow \begin{cases} a_0=0 & b_0=1 \\ a_1=1 & b_1=1 \\ a_2=0 & b_2=0 \\ a_3=0 & b_3=0 \end{cases}$$

$$\Rightarrow f(x)=x \quad x \in [0,2]$$

Problem 6

a. $h=0.1 \quad x_0=1.1$

$$f'(1.1) = \frac{1}{h} \left[-\frac{3}{2}f(1.1) + 2f(1.2) - \frac{1}{2}f(1.3) \right] = 17.76971$$

$$f'(1.2) = \frac{1}{h} \left[-\frac{1}{2}f(1.1) + \frac{1}{2}f(1.3) \right] = 22.19364$$

$$f'(1.3) = \frac{1}{h} \left[-\frac{1}{2}f(1.2) + \frac{1}{2}f(1.4) \right] = 27.10375$$

$$f'(1.4) = \frac{1}{h} \left[\frac{1}{2}f(1.2) - 2f(1.3) + \frac{3}{2}f(1.5) \right] = 32.51085$$

b. 13] 38:

$$f'(8.1) = 3.09205$$

$$f'(8.5) = 3.139975$$

$$f'(8.3) = 3.11615$$

$$f'(8.7) = 3.163525$$

Problem 7

$$M = N(h) + k_1 h^2 + k_2 h^4 + k_3 h^6 + \dots$$

$$M = N\left(\frac{h}{3}\right) + \frac{1}{9}k_1 h^2 + \frac{k_2}{81} h^4 + k_3 \left(\frac{h}{3}\right)^6 + \dots$$

$$M = N\left(\frac{h}{9}\right) + \frac{1}{81}k_1 h^2 + \frac{k_2}{9^4} h^4 + k_3 \left(\frac{h}{9}\right)^6 + \dots$$

$$\begin{cases} a_1 + \frac{1}{9}a_2 + \frac{1}{81}a_3 = 0 & \textcircled{1} \\ a_2 + \frac{1}{3^4}a_2 + \frac{1}{9^4}a_3 = 0 & \textcircled{2} \end{cases}$$

$$\textcircled{1} - \textcircled{2} \Rightarrow \frac{1}{9}(1 - \frac{1}{9})a_2 + \frac{1}{81}(1 - \frac{1}{81})a_3 = 0$$

$$\text{If } a_3 = 81 \cdot a_2 = -10 \quad a_1 = \frac{1}{9}$$

$$\Rightarrow M = \left[\frac{1}{9}N(h) - 10N\left(\frac{h}{3}\right) + 81N\left(\frac{h}{9}\right) \right] + O(h^6)$$

Problem 8

$$1. E(a_0, a_1) = \sum_{i=1}^4 [y_i - (a_1 x_i + a_0)]^2$$

$$\frac{\partial E}{\partial a_0} = 2 \sum_{i=1}^4 (y_i - (a_1 x_i + a_0))(-1) = -2 (\sum y_i - a_1 \sum x_i - 4a_0) = 0$$

$$\Rightarrow 48 - 11a_1 - 4a_0 = 0 \quad ①$$

$$\frac{\partial E}{\partial a_1} = 2 \sum_{i=1}^4 [y_i - (a_1 x_i + a_0)](-x_i) = 0$$

$$\Rightarrow \sum x_i y_i - a_1 \sum x_i^2 - a_0 \sum x_i = 0$$

$$\Rightarrow 172 - a_1 \cdot 45 - a_0 \cdot 11 = 0 \quad ②$$

$$\Rightarrow a_1 = \frac{160}{59} \quad a_0 = \frac{-768}{59} \Rightarrow y = \frac{160x + 268}{59}$$

$$2. y_1' = \frac{268}{59} \quad E_1 = 24.3\%$$

$$y_2' = \frac{588}{59} \Rightarrow E_2 = 24.6\%$$

$$y_3' = \frac{908}{59} \quad E_3 = 9.92\%$$

$$y_4' = \frac{1068}{59} \quad E_4 = 9.49\%$$