

# A dynamic model of the type-2 inositol trisphosphate receptor

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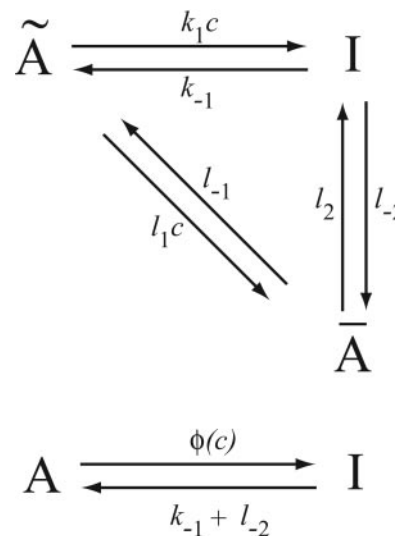
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The dynamic properties of the inositol (1,4,5)-trisphosphate (IP<sub>3</sub>) receptor are crucial for the control of intracellular Ca<sup>2+</sup>, including the generation of Ca<sup>2+</sup> oscillations and waves. However, many models of this receptor do not agree with recent experimental data on the dynamic responses of the receptor. We construct a model of the IP<sub>3</sub> receptor and fit the model to dynamic and steady-state experimental data from type-2 IP<sub>3</sub> receptors. Our results indicate that, (i) Ca<sup>2+</sup> binds to the receptor using saturating, not mass-action, kinetics; (ii) Ca<sup>2+</sup> decreases the rate of IP<sub>3</sub> binding while simultaneously *increasing* the steady-state sensitivity of the receptor to IP<sub>3</sub>; (iii) the rate of Ca<sup>2+</sup>-induced receptor activation increases with Ca<sup>2+</sup> and is faster than Ca<sup>2+</sup>-induced receptor inactivation; and (iv) IP<sub>3</sub> receptors are sequentially activated and inactivated by Ca<sup>2+</sup> even when IP<sub>3</sub> is bound. Our results emphasize that measurement of steady-state properties alone is insufficient to characterize the functional properties of the receptor.

Oscillations and waves in the concentration of free intracellular calcium (Ca<sup>2+</sup>) are seen in many cell types and are known to be an important intra- and intercellular signaling system. It is thus of interest to determine the mechanisms underlying such complex dynamic behavior. One of the most important of these mechanisms is the inositol trisphosphate receptor (IPR), which also functions as a Ca<sup>2+</sup> channel. There is now a great deal of experimental evidence that in many cell types, oscillations and waves of Ca<sup>2+</sup> are mediated in major part by the release through the IPR of Ca<sup>2+</sup> from the endoplasmic reticulum, and that it is the modulation of the IPR by Ca<sup>2+</sup> and by inositol (1,4,5)-trisphosphate (IP<sub>3</sub>) that causes such complex dynamic behavior (1–7).

Models of the IPR thus play a central role in models of Ca<sup>2+</sup> oscillations and waves, and a number of such models have been constructed (8–15). Most of them assume that the IPR can be modulated by the binding of Ca<sup>2+</sup> and IP<sub>3</sub>, and that Ca<sup>2+</sup> plays a dual role, both activating and inactivating the IPR. However, recent experimental data are forcing us to reevaluate older models. One major problem with almost all older models of the IPR is that they assume that Ca<sup>2+</sup> binds to the IPR with mass action kinetics, in which case the rate of IPR activation and inactivation increases approximately linearly with Ca<sup>2+</sup> concentration ([Ca<sup>2+</sup>]). However, experiments show that as [Ca<sup>2+</sup>] varies over five orders of magnitude, the rate of IPR inactivation varies over approximately only a single order of magnitude (16–18). This discrepancy can be avoided if we assume that each transition between receptor states is modulated by Ca<sup>2+</sup> but is more complex than just simple Ca<sup>2+</sup> binding by mass action kinetics.

Consider, for example, the reaction scheme shown in Fig. 1. If we assume that  $\tilde{A}$  and  $\bar{A}$  are in instantaneous equilibrium, we have  $c\tilde{A} = L_1\bar{A}$ , where  $L_1 = l_{-1}/l_1$ , and  $c$  denotes [Ca<sup>2+</sup>]. Hence, letting  $A = \tilde{A} + \bar{A}$ , we have  $dA/dt = (k_{-1} + l_{-2})I - \phi(c)A$ , where  $\phi(c) = c(k_1L_1 + l_2)/c + L_1$ . Thus, this scheme is a simple way in which saturating binding kinetics can be incorporated into a model. It is similar to the Michaelis–Menten model of an enzyme-catalyzed reaction, in which a saturating



**Fig. 1.** Reaction diagram of a Ca<sup>2+</sup>-dependent transition from state A to state I ( $c$  is Ca<sup>2+</sup> concentration). If  $\tilde{A}$  and  $\bar{A}$  are in equilibrium, and if  $A = \tilde{A} + \bar{A}$ , then the upper reaction scheme is equivalent to the lower.

reaction rate is obtained by assuming the existence of an intermediate complex. In this introductory model, the state  $\bar{A}$  plays a role similar to that of the enzyme complex. By assuming that the original state  $\tilde{A}$  (analogous to the substrate) is in fast equilibrium with state  $\bar{A}$ , we attain saturating kinetics of the Michaelis–Menten type.

## An IPR Model

A diagram of the IPR model is given in Fig. 2. Although it appears to contain a multiplicity of states, there are specific reasons for each one. The background structure is simple. Ignoring the various tildes, hats, and primes, we see that a receptor,  $R$ , can bind Ca<sup>2+</sup> and inactivate to state  $I_1$ , or it can bind IP<sub>3</sub> and open to state  $O$ . State  $O$  can then shut (state  $S$ ) or bind Ca<sup>2+</sup> and activate to state  $A$ . State  $A$  can then bind Ca<sup>2+</sup> and inactivate to state  $I_2$ . This structure is clearer in Fig. 3.

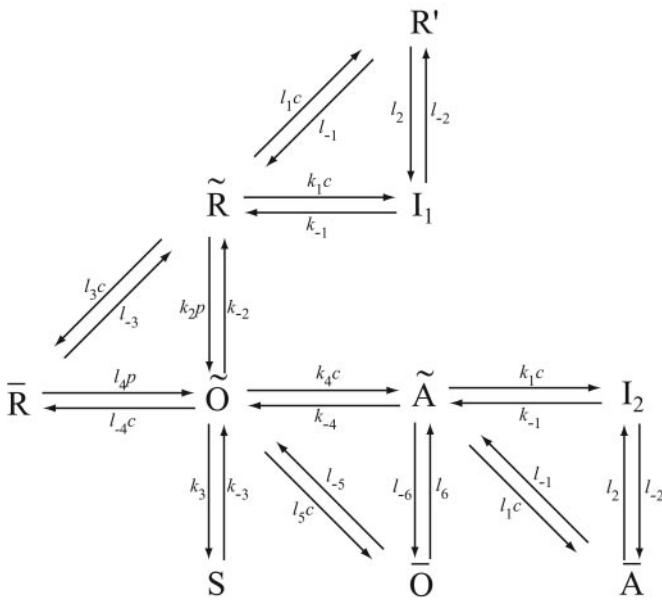
States  $\tilde{R}$ ,  $\tilde{O}$ ,  $\tilde{A}$ , and  $R'$  are used to give Ca<sup>2+</sup>-dependent transitions that have saturable kinetics, as in the simple example of Fig. 1. These states will ultimately disappear, leaving behind only functions of  $c$ . Note that the inactivated states  $I_1$  and  $I_2$  both have Ca<sup>2+</sup> bound to the same site, but  $I_2$

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Abbreviations: IP<sub>3</sub>, inositol (1,4,5)-trisphosphate; IPR, IP<sub>3</sub> receptor.

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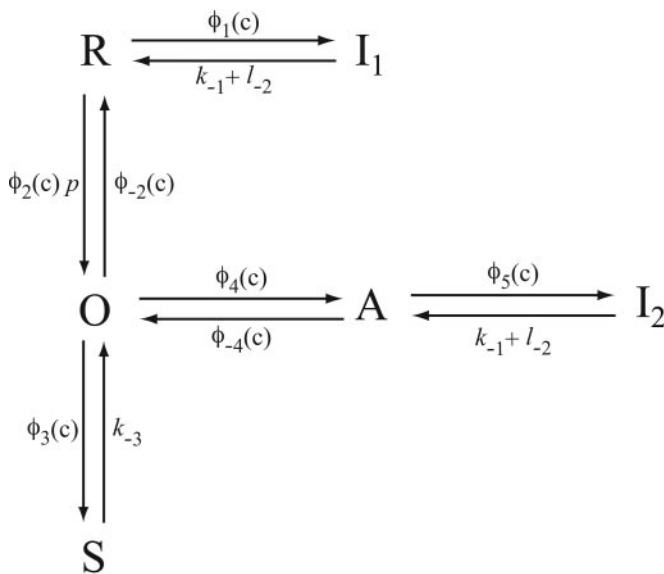
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**Fig. 2.** The full IPR model. R, receptor; O, open; A, activated; S, shut; I, inactivated.  $c$  is  $[Ca^{2+}]$ ;  $p$  is  $[IP_3]$ .

also has  $IP_3$  and one other  $Ca^{2+}$  ion bound. For simplicity, we assume that the rate of  $Ca^{2+}$  binding to the inactivating site is independent of whether  $IP_3$  is bound, or whether the receptor has been activated by  $Ca^{2+}$  (but see ref. 19 and later discussion). The  $\bar{R}$ ,  $\bar{R}$ ,  $\bar{O}$  triangle models  $Ca^{2+}$ -dependent binding of  $IP_3$ ;  $Ca^{2+}$  modulates the interconversion of the receptor between two states, each of which can bind  $IP_3$  with different kinetics. In type-3 IPR,  $Ca^{2+}$  enhances  $IP_3$  binding but inhibits  $IP_3$  binding in type-1 IPR (20–22). The reaction scheme shown here incorporates either possibility, depending on the parameter values. As we shall see, our model predicts that  $Ca^{2+}$  increases the rate of  $IP_3$  binding to type-2 IPR.

To derive the model equations, we first define  $K_i = k_{-i}/k_i$  and  $L_i = l_{-i}/l_i$  for every appropriate integer  $i$ . We also let  $c$  and



**Fig. 3.** Simplified diagram of the IPR model. Given the fast equilibria described in the text, this diagram is equivalent to that in Fig. 2.

$p$  denote  $[Ca^{2+}]$  and  $[IP_3]$ , respectively. Then, assuming that the transitions  $\bar{R} \rightleftharpoons \bar{R}$ ,  $\bar{O} \rightleftharpoons \bar{O}$ ,  $\bar{A} \rightleftharpoons \bar{A}$  and  $\bar{R} \rightleftharpoons \bar{R}'$  are fast and in instantaneous equilibrium, we get  $c\bar{R} = L_3\bar{R}$ ,  $c\bar{R} = L_1R'$ ,  $c\bar{O} = L_5\bar{O}$ , and  $c\bar{A} = L_1\bar{A}$ . We now define the new variables  $R = \bar{R} + \bar{R} + R'$ ,  $O = \bar{O} + \bar{O}$ ,  $A = \bar{A} + \bar{A}$ . Then

$$\frac{dR}{dt} = \phi_{-2}O - \phi_2pR + (k_{-1} + l_{-2})I_1 - \phi_1R,$$

$$\frac{dO}{dt} = \phi_2pR - (\phi_{-2} + \phi_4 + \phi_3)O + \phi_{-4}A + k_{-3}S,$$

$$\frac{dA}{dt} = \phi_4O - \phi_{-4}A - \phi_5A + (k_{-1} + l_{-2})I_2,$$

$$\frac{dI_1}{dt} = \phi_1R - (k_{-1} + l_{-2})I_1,$$

$$\frac{dI_2}{dt} = \phi_5A - (k_{-1} + l_{-2})I_2,$$

where

$$\phi_1(c) = \frac{(k_1L_1 + l_2)c}{L_1 + c(1 + L_1/L_3)},$$

$$\phi_2(c) = \frac{k_2L_3 + l_4c}{L_3 + c(1 + L_3/L_1)},$$

$$\phi_{-2}(c) = \frac{k_{-2} + l_{-4}c}{1 + c/L_5},$$

$$\phi_3(c) = \frac{k_3L_5}{L_5 + c},$$

$$\phi_4(c) = \frac{(k_4L_5 + l_6)c}{L_5 + c},$$

$$\phi_{-4}(c) = \frac{L_1(k_{-4} + l_{-6})}{L_1 + c},$$

$$\phi_5(c) = \frac{(k_1L_1 + l_2)c}{L_1 + c},$$

and where  $R + O + A + S + I_1 + I_2 = 1$ . Thus, given the fast equilibria above, Fig. 2 is equivalent to Fig. 3.

The model assumes that the binding of  $IP_3$  and  $Ca^{2+}$  is sequential, not independent. So, for instance,  $Ca^{2+}$  can bind to the activating site only after  $IP_3$  has bound. Other models have made similar assumptions (10–12, 14) but without the correct dynamic behavior. Experimental data (23) indicate that the binding of  $IP_3$  and  $Ca^{2+}$  are not independent events, with  $Ca^{2+}$  being unable to bind to the activating site until  $IP_3$  has first bound.

There are three inactivated states;  $I_1$ ,  $I_2$ , and  $S$ . States  $I_1$  and  $I_2$  have a  $Ca^{2+}$  bound to the same inactivating site, but  $I_2$  also has  $IP_3$  bound and  $Ca^{2+}$  bound to the activating site. State  $I_1$  is necessary so that the receptor can inactivate in the absence of  $IP_3$  (16). State  $I_2$  is necessary so that an activated receptor may be inactivated by  $Ca^{2+}$ . State  $S$  is necessary so that an open receptor can inactivate in the absence of  $Ca^{2+}$  (18, 23).

We assume the  $IP_3$  receptor consists of four independent and identical subunits and allows  $Ca^{2+}$  current when all four subunits are in state  $O$ , or all four are in state  $A$ , or some

intermediate combination (for instance, when three are in state O, and one is in state A). Furthermore, we shall assume that the more subunits there are in state A, the greater the open probability of the receptor. With these assumptions, the open probability of the receptor is most conveniently written as  $(0.1O + 0.9A)^4$ . The numbers 0.1 and 0.9 are not crucial. Similar fits, essentially identical to visual inspection and with only small changes in parameter values, are obtained by using, for instance,  $(0.1O + A)^4$  or  $(0.05O + 0.9A)^4$  as the open probability (computations not shown). We note that the receptor must be able to pass  $\text{Ca}^{2+}$  current when all four subunits are in the O state, as it is known to allow  $\text{Ca}^{2+}$  current even in the absence of cytosolic  $\text{Ca}^{2+}$  (7).

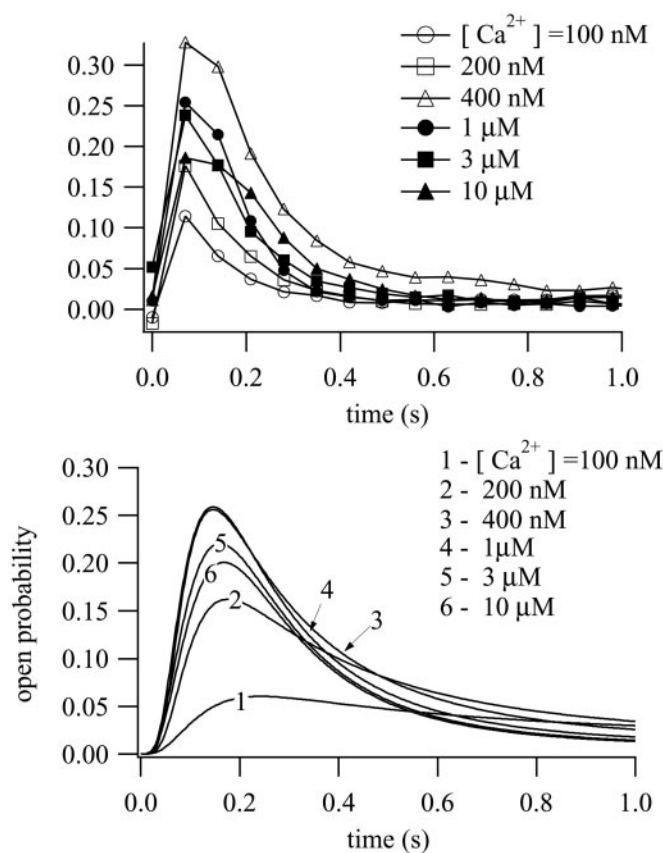
The model closely follows the ideas of Taylor (7, 19, 23). Once  $\text{IP}_3$  is bound, there is a competition between the intrinsic inactivation to state S and the  $\text{Ca}^{2+}$ -induced activation to state A. It is broadly consistent also with the scheme proposed by Hajnóczky and Thomas (24), in which the binding of  $\text{IP}_3$  is modulated by  $\text{Ca}^{2+}$  binding, and binding of activating and inactivating  $\text{Ca}^{2+}$  happens sequentially. One difference between the model of ref. 24 and that presented here is that we assume that  $\text{Ca}^{2+}$  can bind directly to the inactivating site without first having to bind to the activating site. Another difference is that in ref. 24, the positive feedback action of  $\text{Ca}^{2+}$  on the receptor is mediated by the modulation of  $\text{IP}_3$  binding alone, with no contribution from an activating binding site. Although our model includes the modulation of  $\text{IP}_3$  binding by  $\text{Ca}^{2+}$ , the excitable nature of  $\text{Ca}^{2+}$  release results from the activation by  $\text{Ca}^{2+}$  (i.e., the O to A transition).

One major difference between our model and that of Taylor (19) is that we assume that an inactivating  $\text{Ca}^{2+}$  can bind even when  $\text{IP}_3$  is already bound; i.e., the binding of  $\text{IP}_3$  offers no protection to the inactivating site, as proposed in ref. 19. If, following Taylor, we assume the reverse, that  $\text{Ca}^{2+}$  cannot inactivate an IPR that already has  $\text{IP}_3$  bound [i.e., we set  $\phi_5(c) \equiv 0$ ], then we are unable to obtain acceptable fits to the dynamic data (computations not shown). Of course, we hasten to emphasize that this by no means rules out Taylor's hypothesis. We discuss this difference in more detail later.

## Results and Discussion

The parameter values were determined by fitting the model to time courses of  $\text{Ca}^{2+}$  release from type-2 IPR at a fixed  $[\text{Ca}^{2+}]$  and  $[\text{IP}_3]$  (18) and to type-2 IPR steady-state data at a single  $[\text{IP}_3]$  (25). For each time course (indexed by  $j$ ), the net  $\text{Ca}^{2+}$  efflux ( $e_j$ ) is measured at a sequence of time points,  $t_i$ . Dufour *et al.* (18) did not measure the receptor open probability directly but measured instead net  $\text{Ca}^{2+}$  release. This release will be proportional to the instantaneous open probability but with an unknown constant of proportionality. We made the simplest assumption, that this constant of proportionality is equal to one. To determine the objective function for each set of parameter values, we solved the differential equations (by using a forward Euler method with a time step of 0.001 s) to determine the model solution  $m_j(t_i)$  at each of these time points and for each time course. The objective function corresponding to the time courses is then  $\sum_{i,j} (e_j(t_i) - m_j(t_i))^2$ .

In addition to the sum of squares shown above, the objective function included a sum of squares of differences between the model steady state and the data of (25) from type-2 IPR. The steady-state data of ref. 25 have open probabilities that reach as high as 0.5, a figure inconsistent with our dynamic data, because in the dynamic responses the open probabilities reached after the initial peak are approximately a factor of 10 smaller than the peak values. This difference is not possible if the steady-state open probability reaches as high as 0.5. However, there is extreme variation in the published open



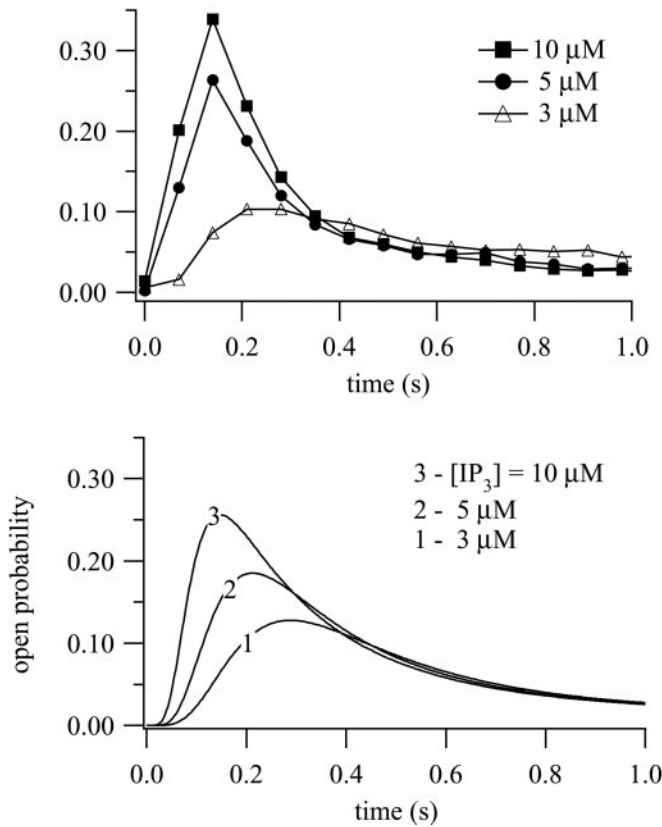
**Fig. 4.** (Upper) Data obtained from superfusion of  $^{45}\text{Ca}^{2+}$ -loaded hepatic microsomes. At  $t = 0.10 \mu\text{M}$ ,  $\text{IP}_3$  and the indicated  $[\text{Ca}^{2+}]$  were added to the stimulus buffer and the  $\text{Ca}^{2+}$  flux measured as a function of time. Full details are given in ref. 18. (Lower)  $\text{Ca}^{2+}$  dose responses calculated from the model. Initial conditions were  $R = 1$ ; all other receptor states were zero. At  $t = 0$ ,  $10 \mu\text{M}$  of  $\text{IP}_3$  was added, with the indicated  $\text{Ca}^{2+}$  concentrations. Best-fit parameter values are:  $k_1 = 0.64 \text{ s}^{-1} \cdot \mu\text{M}^{-1}$ ,  $k_{-1} = 0.04 \text{ s}^{-1}$ ,  $k_2 = 37.4 \text{ s}^{-1} \cdot \mu\text{M}^{-1}$ ,  $k_{-2} = 1.4 \text{ s}^{-1}$ ,  $k_3 = 0.11 \text{ s}^{-1} \cdot \mu\text{M}^{-1}$ ,  $k_{-3} = 29.8 \text{ s}^{-1}$ ,  $k_4 = 4 \text{ s}^{-1} \cdot \mu\text{M}^{-1}$ ,  $k_{-4} = 0.54 \text{ s}^{-1}$ ,  $L_1 = 0.12 \mu\text{M}$ ,  $L_3 = 0.025 \mu\text{M}$ ,  $L_5 = 54.7 \mu\text{M}$ ,  $l_2 = 1.7 \text{ s}^{-1}$ ,  $l_4 = 1.7 \text{ s}^{-1} \cdot \mu\text{M}^{-1}$ ,  $l_6 = 4707 \text{ s}^{-1}$ ,  $l_{-2} = 0.8 \text{ s}^{-1}$ ,  $l_{-4} = 2.5 \mu\text{M}^{-1} \cdot \text{s}^{-1}$ , and  $l_{-6} = 11.4 \text{ s}^{-1}$ .

probabilities of IPR, with peak values ranging from around 0.01 or less (27) to 0.5 or greater (25, 26, 28). The reasons for these differences are unclear. Thus, we chose to retain the qualitative nature of the type-2 IPR data of ref. 25 but to scale the data by a factor of 1/30 to force consistency with our dynamic data. We estimated the data of ref. 25 by eye from figure 7B (the curve labeled R-T2) of that paper.

A minimum of the objective function was then found by Powell's method (29). Typical parameters determined by the fit are shown in the caption to Fig. 4. The objective function was not sensitive to the precise parameter values. We also ensured that  $K_1 = L_1 L_2$ ,  $K_2 = L_3 L_4$ , and  $K_4 = L_5 L_6$ , so that the reaction scheme satisfies detailed balance.

Experimental and model  $\text{Ca}^{2+}$  dose responses are shown in Fig. 4. At  $t = 0$ , various concentrations of  $\text{Ca}^{2+}$  were added, all at  $[\text{IP}_3] = 10 \mu\text{M}$ . These time courses are faster than those in ref. 23 but agree well with those of refs. 16 and 19; the experiments of ref. 23 were in permeabilized cells, whereas those of refs. 16 and 18 were in microsome preparations, which may explain these differences. Another possible explanation of these differences is the different loading state of the stores. Our time courses agree well also with those of ref. 17; Parker *et al.* used *in vivo* pulses of  $\text{IP}_3$  release in *Xenopus* oocytes to





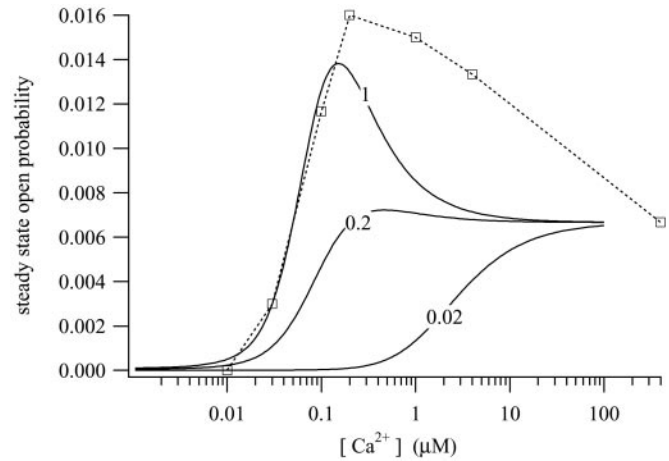
**Fig. 5.** (Upper) Data obtained from superfusion of <sup>45</sup>Ca<sup>2+</sup>-loaded hepatic microsomes. At  $t = 0.0$ ,  $Ca^{2+}$  and the indicated  $[IP_3]$  were added to the stimulus buffer and the  $Ca^{2+}$  flux measured as a function of time. Full details are given in ref. 18. (Lower) Model  $IP_3$  dose responses. Initial conditions were  $R = 1$ ; all other receptor states were zero (i.e., low  $Ca^{2+}$ , low  $IP_3$ ). At  $t = 0.04$ ,  $\mu M$   $Ca^{2+}$  was added, together with the indicated  $[IP_3]$ .

measure the rate of  $Ca^{2+}$  release as a function of time, although not at constant  $[Ca^{2+}]$ . Experimental and model  $IP_3$  dose responses are shown in Fig. 5. At  $t = 0$ ,  $IP_3$  was added at various concentrations, all at  $0.4 \mu M$   $Ca^{2+}$ . As  $[IP_3]$  increases, so does the response maximum. The model time courses show a clear latent period of around 20 ms before the open probability begins to rise, even at the highest  $[IP_3]$ , similar to that seen experimentally (17, 30, 31).

The steady-state open probability of the model for various  $[IP_3]$  is shown in Fig. 6. At lower  $[IP_3]$ , the steady-state curve is not bell-shaped, and at all  $[IP_3]$ , the open probability remains nonzero as  $[Ca^{2+}]$  gets large.

Preincubation by  $Ca^{2+}$  or  $IP_3$  attenuates the responses. After 0.3 s of exposure to  $0.4 \mu M$   $Ca^{2+}$ , in the absence of  $IP_3$ , the response maximum is greatly decreased (computations not shown), in agreement with ref. 16. Similarly, after 0.3 s of exposure to  $1 \mu M$   $IP_3$ , in the absence of  $Ca^{2+}$ , the response maximum is again greatly decreased (computations not shown). Although the time scales are different, this result is similar to the results of Marchant and Taylor (23).

In Fig. 7, we plot the various  $\phi$  functions (note the different scale for  $\phi_4$ ). By comparing  $\phi_2$ ,  $\phi_1$ , and  $\phi_5$ , we see that the binding of  $IP_3$  proceeds faster than the inactivation by  $Ca^{2+}$ . Activation by  $Ca^{2+}$  is much faster than either, with  $\phi_4$  being about an order of magnitude larger than all the other rate functions for all  $c > 0.1$ . This fast activation rate was a persistent feature of the fits and is consistent with previous modeling work (8, 9, 11, 14). Thus our model supports the



**Fig. 6.** Steady-state open probability of the IPR, measured at  $[IP_3] = 0.02$ ,  $0.2$ , and  $1 \mu M$ . The symbols are the experimental data of ref. 25, from type-2 IPR, measured at  $[IP_3] = 1 \mu M$ . The data were estimated by eye from figure 7B of ref. 25.

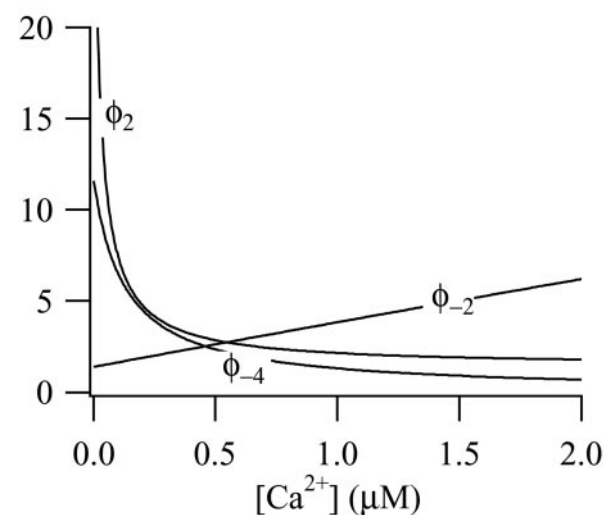
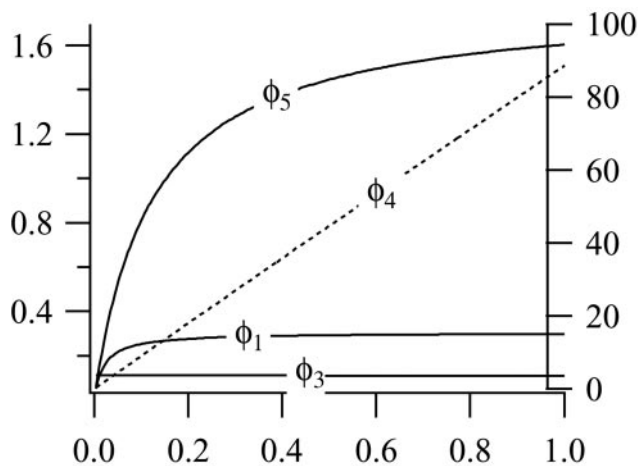
theory that the dynamic responses are mediated by the fast activation and slower inactivation of the IPR by  $Ca^{2+}$ .

Our model predicts that  $Ca^{2+}$ -independent inactivation of the IPR by  $IP_3$  has a time constant of around 10 s ( $\phi_3 \approx 0.1 s^{-1}$ ), which is slower than the figure of 250 ms of ref. 23 but faster than the figure of 30 s of ref. 32. Our rate of  $Ca^{2+}$ -dependent IPR inactivation ( $\phi_5 \approx 1.7$  at  $[Ca^{2+}] = 10 \mu M$ , giving a time constant of 588 ms) is slower than the figure of 50 ms of ref. 19 but is in excellent agreement with the figure of 580 ms (at  $10 \mu M$   $Ca^{2+}$ ) of ref. 16. Our speed of the reversal of  $Ca^{2+}$ -induced IPR inactivation (around 800 ms) is slower than the figure of 400 ms of ref. 19. The rate of  $IP_3$  dissociation from the IPR ( $2 \leq \phi_{-2} \leq 7$ , approximately, giving a time constant of 140–500 ms) agrees well with the figure of 450 ms of ref. 33, but the rate of  $IP_3$  binding ( $k_2 = 37.4 \mu M \cdot s^{-1}$ ) is slower than the figure of  $120 \mu M \cdot s^{-1}$  of ref. 33. Because of the different model structures, it is more difficult to compare our results to those of ref. 34; however, Moraru *et al.* get a  $Ca^{2+}$  binding on-rate of  $10 \mu M \cdot s^{-1}$ , compared to our value of  $k_4 = 4 \mu M \cdot s^{-1}$ , and an  $IP_3$  binding on-rate of  $250 \mu M \cdot s^{-1}$ , compared to our value of  $k_2 = 37.4 \mu M \cdot s^{-1}$ . It thus appears that some of the rate constants predicted by our model lie within the experimentally observed ranges, whereas for others the deviations are greater. Differences in experimental method may account for some of these deviations, although it is highly likely that variability in the fitting procedure also plays an important role. The physiological effects of differences in the parameter values are even less clear. Not until such dynamic models are constructed for all the different receptor subtypes will we be able to study the functional importance of any differences in rate constants between receptor subtypes.

Because  $\phi_2$  is a decreasing function of  $c$ , the rate of  $IP_3$  binding is decreased by higher  $[Ca^{2+}]$ . However, the steady-state  $[IP_3]$  for half-maximal binding can be calculated to be

$$\frac{F_2(k_{-1} + l_{-2} + \phi_1)}{(k_{-1} + l_{-2}(1 + F_4^{-1} + \phi_3/k_{-3}) + \phi_5 F_4^{-1}},$$

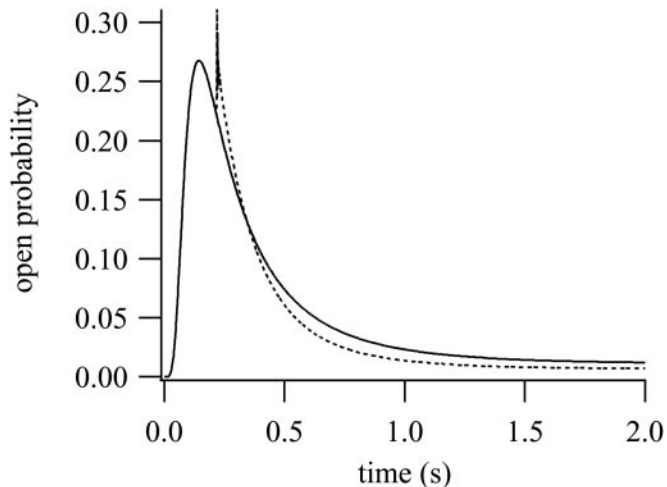
where  $F_i = \phi_{-i}/\phi_i$ . For the best-fit parameters, this equation is a decreasing function of  $c$  when  $c > 0.1 \mu M$ . Thus, when  $c > 0.1 \mu M$ , the steady-state sensitivity of the IPR to  $IP_3$  increases as  $c$  increases, even though the rate of  $IP_3$  binding decreases. Hence, steady-state measurements of the  $Ca^{2+}$  dependence of  $IP_3$  bind-



**Fig. 7.** The rate functions. The dotted curve,  $\phi_4$ , is relative to the right-hand axis, whereas all the other curves are relative to the left-hand axis. All functions, except  $\phi_2$ , are in units of  $s^{-1}$ .  $\phi_2$  has units of  $\mu M^{-1}s^{-1}$ .

ing give no information on the *dynamic* effects of  $Ca^{2+}$  on  $IP_3$  binding.

It has been proposed that the binding of  $IP_3$  renders the IPR unable to bind  $Ca^{2+}$  to the inactivating site (19), the evidence for this assertion being principally that the addition of a large concentration of  $Ca^{2+}$  does not inactivate the receptor once



**Fig. 8.** The effect of the addition of high  $[Ca^{2+}]$  after addition of  $IP_3$ . Solid curve:  $10 \mu M$   $IP_3$  and  $500 nM$   $Ca^{2+}$  were added at  $t = 0$ . Dotted curve: same as the solid curve, except that at  $t = 0.22$ ,  $30 \mu M$  of  $Ca^{2+}$  was added.

$IP_3$  is already bound. We simulate this experiment in Fig. 8. The solid line is the control response to the addition of  $10 \mu M$   $IP_3$  and  $500 nM$   $Ca^{2+}$ . The dotted line is the result of adding (in addition to the control stimulation)  $30 \mu M$   $Ca^{2+}$  at  $t = 0.22$ . After the second addition of  $Ca^{2+}$ , the open probability initially rises as the receptor activates but then relaxes along much the same time course as the control response. The initial spike on the second addition of  $Ca^{2+}$  is very fast and unlikely to be easily detectable in superfusion studies. The results shown in Fig. 8 are qualitatively similar to those of ref. 19, even though our model assumes that  $Ca^{2+}$  can inactivate an IPR that has already bound  $IP_3$ . In addition, we investigated a model in which  $\phi_5$  was set to zero (i.e., modeling the assumption that the receptor cannot be inactivated by  $Ca^{2+}$  once it has bound  $IP_3$ ). We were unable to obtain good fits of this model to the time-course data (computations not shown). Although this result does not imply that such a model cannot be made to fit the time-course data, in combination with the simulation of Fig. 8, it suggests that inactivation by  $Ca^{2+}$ , even when  $IP_3$  is bound, is an important part of the response.

We conclude that, because of the saturating kinetics of  $Ca^{2+}$  activation and inactivation, the response to the addition of large amounts of  $Ca^{2+}$  may not be sufficient to determine whether the binding of  $IP_3$  shields the  $Ca^{2+}$ -inactivating site.

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