第三章算法分析

1. 编写程序实现归并排序算法 MergeSortL 和快速排序算法 QuickSort;

主要代码如下，完整代码见压缩包内 python 文件。

MergeSortL:

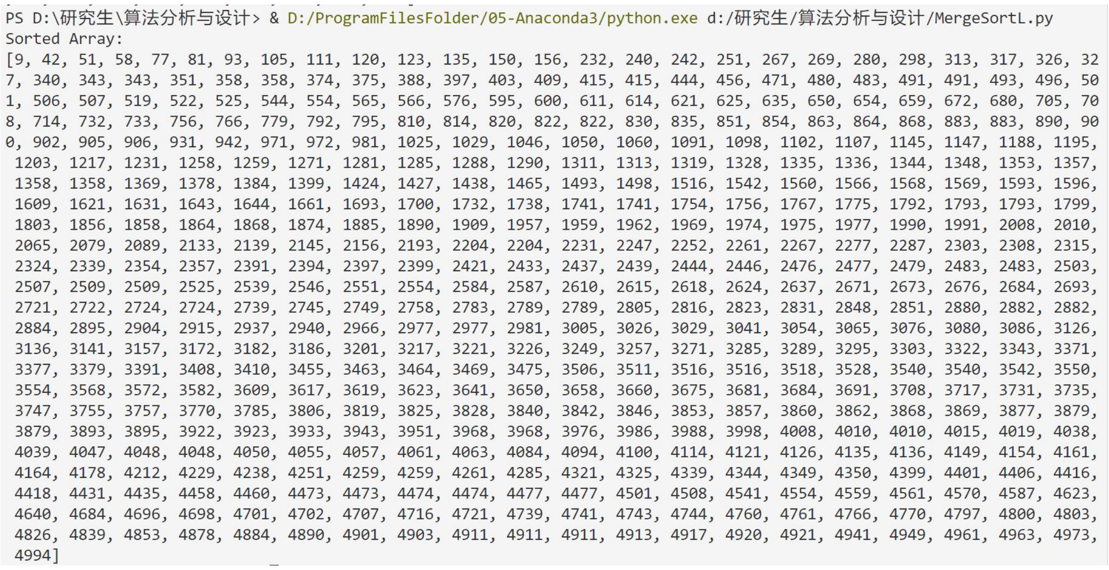
```
class ListNode: # 存储 index
    def __init__(self, val=0, next=None):
        self.val = val
        self.next = next

def insert_to_list(list_head, array, index):
    current_node = ListNode(val = index)
    # print("=========================" )
    # print(index)
    # print(array[index])
    # print_list(list_head)
    if (list_head == None):
        return current_node
    current_point = list_head
    last_node = None

    while(current_point != None):
        compared_index = current_point.val
        # print(array[compared_index])
        if (array[compared_index] > array[index]):
            if (last_node == None):
                current_node.next = list_head
                return current_node
            last_node.next = current_node
            current_node.next = current_point
            return list_head
        else:
            last_node = current_point
            current_point = current_point.next

    last_node.next = current_node
    return list_head

def insertion_sort(array, start, end):
    list_head = None
    for i in range(start, end + 1):
        list_head = insert_to_list(list_head, array, i)
        # print_list(list_head)
    return list_head
```

```
def merge_l(node_a, node_b, array):

    new_head = None
    current_node = None
    i = node_a
    j = node_b
    while i and j:
        i_num = array[i.val]
        j_num = array[j.val]

        if i_num <= j_num:
            # tmp_i_next = i.next
            if (new_head == None):
                new_head = i
            else :
                current_node.next = i
            current_node = i
            i = i.next
        else:
            if (new_head == None):
                new_head = j
            else :
                current_node.next = j
            current_node = j
            j = j.next
    if i:
        current_node.next = i
    if j:
        current_node.next = j

    return new_head

def merge_sort_l(array, start, end):
    if end - start + 1 < 16:
        return insertion_sort(array, start, end)
    else:
        mid = (start + end) >> 1
        node_a = merge_sort_l(array, start, mid)
        node_b = merge_sort_l(array, mid + 1, end)
        return merge_l(node_a, node_b, array)
```

QuickSort:

```
def quicksort(array, start, end):
    if start < end:
        pi = partition(array, start, end)
        quicksort(array, start, pi - 1)
        quicksort(array, pi + 1, end)

def partition(array, low, high):
    pivot = array[high]
    i = low - 1
    for j in range(low, high):
        if array[j] < pivot:
            i += 1
            array[i], array[j] = array[j], array[i]
    array[i + 1], array[high] = array[high], array[i + 1]
    return i + 1
```

MergeSortL 产生 500 个随机数（取值范围 1-5000）的排序结果：



```
PS D:\研究生\算法分析与设计> & D:/ProgramFilesFolder/05-Anaconda3/python.exe d:/研究生/算法分析与设计/MergeSortL.py
Sorted Array:
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 4994]
```
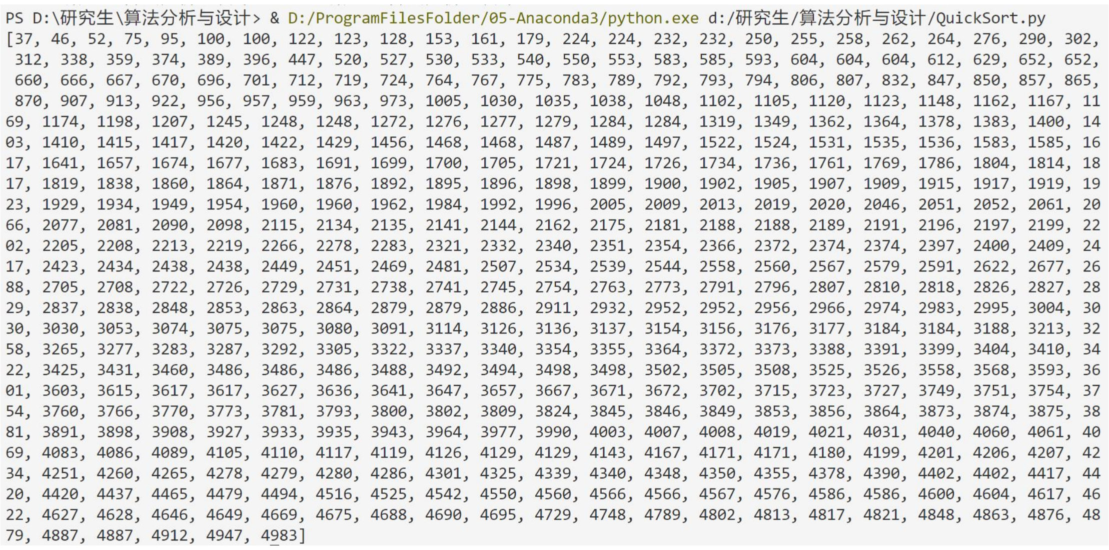

QuickSort 产生 500 个随机数（取值范围 1-5000）的排序结果：



```
PS D:\研究生\算法分析与设计> & D:/ProgramFilesFolder/05-Anaconda3/python.exe d:/研究生/算法分析与设计/QuickSort.py
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79, 4887, 4887, 4912, 4947, 4983]
```

2. 用长分别为 10000、30000、50000、80000、100000、200000 的 6 个数组(可用机器随机产生)的排列来统计这两种算法的时间复杂性；

执行时间的单位为：秒
随机数范围为 1-300000

	10000	30000	50000	80000	100000	200000
MergeSortL	0.01369	0.05854	0.10203	0.19005	0.26170	0.72831
QuickSort	0.01141	0.03777	0.06931	0.10264	0.15116	0.32312

可以发现 QuickSort 确实比 MergeSortL 快，那当然也会比没优化的 MergeSort 快
而且随着数据量的增大，这种差异越发明显

3．讨论归并排序算法 MergeSort 的空间复杂性。

归并排序由分解与合并两部分组成，如果用 S(n)表示排序 n 个数所用的总空间。
由分治：
分解的时候，MergeSort(low, mid) 和 MergeSort(mid+1, high)分别用 S(n/2)
归并的时候，Merge(low, mid, high) 空间复杂度为 O(n)，如果使用辅助数组，则约为 2n
则有：

$$S(n)=\max\{S(n / 2),O(n)\}$$

即：

$$S(n)<=S(n / 2)+O(n)$$

递归推导得：

$$S(n)<=S(n/2) + O(n)<=S(1)+O(n/2\text{^}k)+...+O(n/2)+O(n)$$

由推导进一步得：

$$S(n)<=S(1)+O(2n)=O(n)$$

又有存储数组长度为 n，则有 S(n)>=O(n)
综上得：

$$S(n)=O(n)$$

4．说明算法 PartSelect 的平均时间复杂性为$O(n)$。

提示：假定数组中的元素各不相同，且第一次划分时划分元素v是第i小元素的概率为$1/n$。因为 Partition 中的 case 语句所要求的时间都是$O(n)$，所以，存在常数c，使得算法 PartSelect 的平均时间复杂度$C_A^k(n)$可以表示为

$$C_A^k(n)\le cn+\frac{1}{n}\left(\sum_{1\le i<k}C_A^{k-i}(n-i)+\sum_{k<i\le n}C_A^k(i-1)\right)$$

令$R(n)=\max\limits_k(C_A^k(n))$，取$c\ge R(1)$，试证明$R(n)\le 4cn$。

证明如下：（下页）

令 $R(n) = \max(C_A^k(n))$. 设 $k=k_n$ 时取之. 则 $R(n)$ 有.

$$R(n) \leq cn + \frac{1}{n}\left(\sum_{1\leq i<k} C_A^{k-1}(n-i) + \sum_{k<i\leq n} C_A^{k}(i-1)\right)$$

取 $c \geq R(1)$ 当 $n=1$ 时. 取 $c\geq$年. 则 $R(1) \leq 4c$.

$n=2$ 时. $R(2) \leq 2c + \frac{1}{2}R(1) \leq 2c + 2c = 4c$ 成立.

假设结论对 $n=k-1$ 成立 则对 $n=k$ 有

$$R(k) \leq ck + \frac{1}{k}R(k-1) + \cdots + R(k-k_n+1) + R(k_n) + \cdots + R(k-1))$$

$$\leq ck + \frac{4c}{k}(k-1) + \cdots + (k-k_n+1) + k_n \cdots (k-1)$$

$$\leq ck + \frac{4c}{k}\left(\frac{(k_n-1)(2k-k_n)}{2} + \frac{(k-k_n)(k_n+k-1)}{2}\right)$$

$$\leq ck - \frac{4c}{k}\left(-\left(\frac{k+1}{2}\right)^2 - \frac{k^2-3k}{2}\right)$$

$$= ck + c\left(\frac{3k^2-4k+1}{k}\right)$$

$$\leq ck + c(3k-3)$$

$$\leq 4ck$$

归纳可得. $R(n) \leq 4cn$ 成立.

故 时间复杂度为 $O(n)$