1 Problems

Problem 1 (MATHCOUNTS). What is the digit in the units place of $(3^3)^5$?

Problem 2. Find the units digit of the following:

$$(972 - 269)(973 - 267)(974 - 214)(999 - 222)^3(42 - 43).$$

Problem 3. What are the only digits a perfect square can end with?

Problem 4. What are the only digits a perfect cube can end with?

Problem 5. If m is a whole number, what are the possible units digits of $2 \cdot 3^m$? What about $6 \cdot 3^m$?

Problem 6. How many positive divisors of 6^{2006} have a units digit of 6?

Problem 7. How many of the following have a units digit of 6?

$$2^1, 2^2, 2^3, \ldots, 2^{99}, 2^{100}$$

Problem 8. Find the units digit of n given that $mn = 21^6$ and m has a units digit of 7.

Problem 9. Alice and her younger brother Bob are both between 10 and 20 years old. The sum of their ages has a units digit of 6 and the difference between their ages is 2. If Bob's age is an even number, how old is Alice?

Problem 10 (MATHCOUNTS). The cube of the three-digit natural number $A7B$ is 10853133. What is $A + B$?

Problem 11. Find the units digit of $1! + 2! + 3! + \ldots + 1000!$.

Problem 12. Find the units digit of 3^{2006}.

Problem 13 (MATHCOUNTS). What is the units digit of $(133^{13})^3$?

Problem 14. Bob is reading a book and notices that the product of the numbers of the two pages his book is open to has a units digit of 6. What is the units digit of the sum of the two page numbers?

Problem 15 (AIME I 2010). Find the remainder when $9 \times 99 \times 999 \times \cdots \times \underbrace{99\cdots9}_{\text{999 9's}}$ is divided by 1000.

Problem 16 (AMC 10B 2010). Positive integers a, b, and c are randomly and independently selected with replacement from the set $\{1, 2, 3, \dots, 2010\}$. What is the probability that $abc + ab + a$ is divisible by 3?