Kinematics

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The starting point of physics is usually kinematics. Similar to the distance-rate-time questions most are used to, here we will discuss important definitions and formulas that will be used throughout physics.

Contents

Vocabulary

§1 Introduction

Kinematics
Kinematics is the study of an object's motion in terms of its displacement, velocity, and acceleration.

It is likely you have heard of kinematics before. For example, $d = rt$ (rate problems) is a part of kinematics at a constant velocity. However, in physics we deal with acceleration as well.

§1.1 Intuition

In kinematics, a lot of things will seem obvious, like if you are going faster on the same path, your time will decrease. **Trust your intuition**. In physics, some things are counterintuitive, both kinematics is (usually) not one of those things.

§2 1D Kinematics

§2.1 Position, Distance, & Displacement

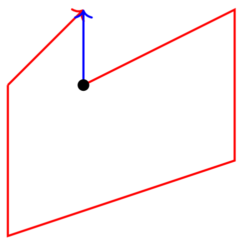
To avoid any misconceptions, I will define three important terms.

Position
Position is an object's relation to a coordinate axis system.

Distance
Distance is a scalar that represents the total amount travelled by an object.

Displacement
Displacement is an object's change in position.

In the following figure, the red arrow represents *distance* and blue arrow represents *displacement*.



Even though both started at the black dot and ended in the same position, *distance* is the total amount, and displacement is just the difference between the final position and the initial position. Distance is a ***scalar*** whereas displacement is a ***vector***. Since displacement means change in position, it is usually denoted Δs[1] or Δd, where s, d is the ***spacial location*** of the object. Horizontal displacement is written as Δx and vertical displacement is written as Δy. The magnitude of the displacement vector is the *net distance* travelled. The SI unit for displacement is the ***meter***.

Theorem 5 (Displacement Components)
If an object is displaced Δx horizontally and Δy vertically, then it is displaced a total of

$$\Delta d = \sqrt{(\Delta x)^2 + (\Delta y)^2}.$$

Example 6
Travelling along a single axis, a car starts 10 m from the origin. The car then moves 8 m directly away from the origin and then turns around and moves 12 m back toward the origin. Determine the final position of the car, the distance the car travelled, and the displacement of the car.

Solution. The car goes from the 10 m mark to the 18 m mark then to the 6 m mark, so the final position is $\boxed{6\,\text{m}}$. The car travels $8\,\text{m} + 12\,\text{m} = \boxed{20\,\text{m}}$, so that is the distance. The car ends up at 6 m when it starts from 10 m, so its displacement is $6\,\text{m} - 10\,\text{m} = \boxed{-4\,\text{m}}$. □

Example 7
An infant crawls 5 m east, then 3 m north, then 1 m east. Find the magnitude of the infant's displacement.

Solution. The infant crawls $5\,\text{m} + 1\,\text{m} = 6\,\text{m}$ east and 3 m north, and using the formula we get

$$\Delta d = \sqrt{6^2 + 3^2} \approx \boxed{6.7\,\text{m}}.$$

□

§2.2 Speed and Velocity

[1] p is not used because it is reserved for ***momentum***.

Average Speed

Average speed[a] is equal to the total distance divided by the total time.

[a] You may have noticed in the formulas that average speed is written as $\overline{\text{speed}}$. This is another notation, which I used to save space, but on the AP exam, average speed is usually written out. In fact, $\overline{\textbf{quantity}}$ just means average quantity.

A car's speedometer doesn't care about direction. Therefore, *speed* doesn't care about direction either – it is a *scalar*.

Average Velocity

Average velocity is equal to the change in displacement divided by the change in time.

This is sometimes written as

$$\overline{v} = \frac{\Delta s}{\Delta t}.$$

Average velocity is a *vector*. Furthermore, **velocity is speed with a direction**. However, the magnitude of average velocity is **not** average speed. This is because the magnitude of displacement is not distance.

§2.3 Zero Acceleration Examples

Example 10

If the infant in this example completes his journey in 20 seconds, find the magnitude of his average velocity.

Solution. Since the displacement is 6.7 m, the magnitude of his average velocity is

$$\overline{v} = \frac{\Delta s}{\Delta t} = \frac{6.7\,\text{m}}{20\,\text{s}} = \boxed{0.34\,\text{m/s}}.$$

$\square$

Example 11

A runner completes a race in 1 minute and 18 seconds around a circular track of length 500 m. Find her average speed and the magnitude of her average velocity.

Solution. The length of the rack is 500 m, and it takes her 78 s, so

$$\overline{v} = \frac{\Delta s}{\Delta t} = \frac{500\,\text{m}}{78\,\text{s}} = \boxed{6.4\,\text{m/s}}.$$

She is displaced a distance of 0, so the average velocity has magnitude $\boxed{0\,\text{m/s}}$. $\square$

Example 12

Is it possible to move with constant speed but not constant velocity? Is it possible to move with constant velocity but not constant speed?

Solution. If you turn but maintain the same speed, then you are changing direction, implying your velocity is not constant, but your speed is. However, if velocity is constant, then speed and direction are also constant, implying there is constant speed. So the answer to the first question is $\boxed{\text{yes}}$ and the second is $\boxed{\text{no}}$. $\square$

§2.4 Acceleration

Average Acceleration
Average acceleration is the change in velocity divided by the change in time.

In other words,

$$\text{average acceleration} = \frac{\text{change in velocity}}{\text{change in time}}.$$

The units of acceleration are $\mathrm{m/s^2}$. An increase in speed is a positive acceleration, whereas a decrease in speed is a negative acceleration (***deceleration***). ***Instantaneous acceleration*** is the speed at a specific time. The formula for it requires calculus, however.

Example 14
A car is travelling in a straight line along a highway at a constant speed of 80 miles per hour for 10 seconds. Find its acceleration.

Solution. Constant speed implies the acceleration is $\boxed{0}$. □

§2.4.1 Uniform Accelerated Motion

Now let's get into **real** kinematics. The simplest motion in AP physics is constant velocity, and after that is constant acceleration. If the acceleration is constant, then average acceleration equals instantaneous acceleration, i.e. $\bar{a} = a$. Furthermore, let's only consider 1D motion. Thus, there are only two directions – positive and negative. This means we can jut use $+$ and $-$ to express direction (later on, in 2D motion, we will need north, south, west, east as we are used to).

§2.4.2 Fundamental Quantities and Notation

Three of the fundamental quantities are the ones we just discussed – displacement (Δx), velocity (v), and acceleration (a). However, we also have an initial velocity v_0 – the one we start with. We will let the starting time be 0, so $\Delta t = t - 0 = t$. This matches with the AP equation sheet. Thus, we have **five** fundamental quantities: $\Delta x, v_0, v, a$, and t.

§2.4.3 The Big Five

These five quantities are related by the *Big Five* equations:

Theorem 15 (Big Five)
Using our definitions of $\Delta x, v_0, v, a$, and t above:

Big Five #	Equation	Missing Variable
1	$\Delta = \bar{v}t$	a
2	$\boxed{v = v_0 + at}$	Δx
3	$\boxed{\Delta x = v_0 t + \frac{1}{2}at^2}$	v
4	$\Delta x = vt - \frac{1}{2}at^2$	v_0
5	$\boxed{v^2 = v_0^2 + 2a\Delta x}$	t

Big Five #2, 3, and 5 are listed in the AP equation sheet.

§2.4.4 Free Fall

Free Fall
An object is in ***free fall*** if the only acceleration is due to gravity.

The gravitational acceleration of Earth is a constant of $9.81\,\text{m/s}^2$, but it is recommended in AP Physics exams that you use $10\,\text{m/s}^2$. Note that the gravitational acceleration vector, denoted g, points *downwards*.

§2.5 1D Kinematics Examples

Example 17
On a planet that has no atmosphere, a rocket 14.2m tall is resting on its launch pad. Free fall acceleration on the planet is $4.45\frac{m}{s^2}$. A ball is dropped from the top of the rocket with zero initial velocity.

- What is the speed of the ball right as it is dropped?
- What is the speed of the ball right before it hits the ground?
- How long does it take for the ball to reach the ground?

Solution. This is problem with ***constant acceleration*** present, so we can only use the boxed equations

- The speed of the ball right as it is dropped is zero. This is because before the ball is dropped, the ball is motionless (i.e. zero velocity), then right afterwards, the ball starts *accelerating*, but since time is zero, its velocity is still zero.
- We can use the equation $v_f^2 = v_0^2 + 2ad$ for this question. Let's define down as *upwards* in this case. That would mean the acceleration would be a positive value, since the ball is accelerating down. $v_f = \sqrt{v_0^2 + 2ad} = \sqrt{2 \cdot 4.45 \cdot 14.2} = 11.242\frac{m}{s}$
- To find the time it takes for the ball to hit the ground we can use the equation $d = v_0 t + \frac{1}{2}at^2$. $a = -4.45\frac{m}{s^2}$ in this case, and we are given $d = 14.2$m. Furthermore, the velocity right as the ball is dropped is zero, so we know $v_0 = 0\frac{m}{s}$. Plugging in our values, we find that $14.2 = \frac{1}{2}(4.45)t^2$, and $t = 2.526$s.

□

Example 18
A stone is thrown vertically upwards with a speed of $20.0\,\text{m/s}$.

- How fast is the stone moving after 2.5 seconds?
- What is the acceleration of the stone at its apex?
- What is the velocity of the stone at its apex?
- What is the maximum height the stone reaches?

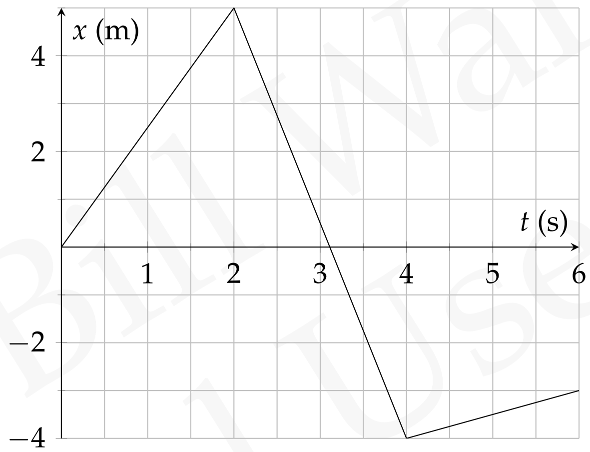
Solution. This is again, a constant acceleration problem. Since the location of the experiment is not specified, we assume the experiment takes place on Earth, where the acceleration of gravity in free fall is $-9.98\frac{m}{s^2}$.

- We use the equation $v_f = v_0 + at$ to do this equation. $v_f = 20 - 9.8(2.5) = -4.5\frac{m}{s}$ (Note: The negative sign indicates the stone is moving downwards).
- The acceleration while the stone is in free fall is always $-9.8\frac{m}{s^2}$.
- The velocity of the stone while it's at its apex is $0\frac{m}{s}$ because the stone is switching directions.
- We use the equation $v_f^2 = v_0^2 + 2ad$ to solve this. The velocity when the stone is at its apex is $0\frac{m}{s}$, $0 = 20^2 + 2(-9.8)d$, so $d = 20.408$m.

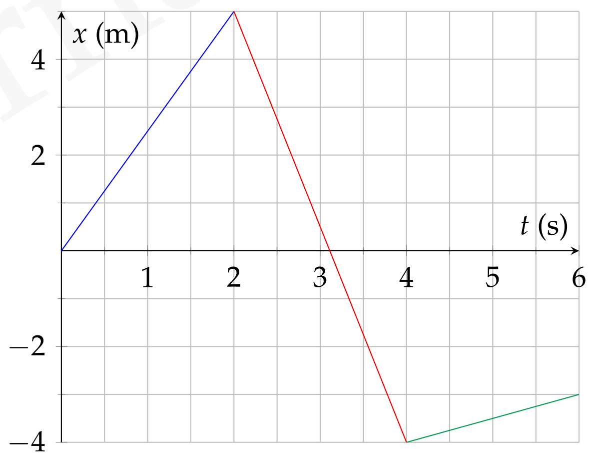
□

§2.6 Kinematics with Graphs

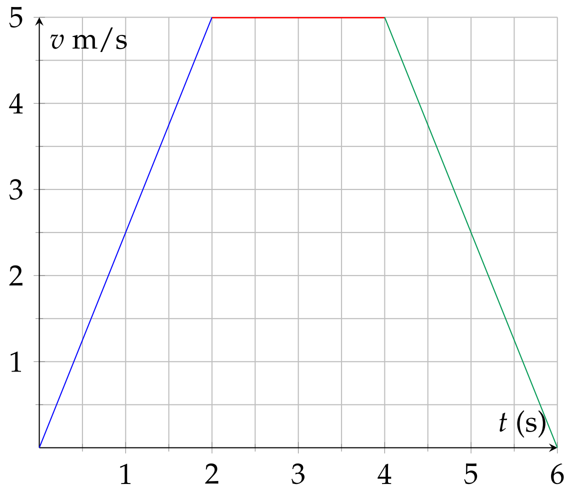
Let's try to handle some kinematic equations graphically. The two most popular graphs in kinematics are distance-time graphs and velocity-time graphs. The following is a position-time graph:



Let's investigate the three parts individually:



The blue part shows an increasing slope, which we can easily calculate. The red part shows a decreasing slope, and the green part still shows an increasing slope, but not as quickly as the blue part. This slope is actually the **velocity**. Now let's try a velocity-time graph:



Here, the slope of the graph is the **acceleration**, and the area under the curves is **displacement**. Positive acceleration occurs in the blue part, no acceleration (i.e. constant velocity) occurs in the red part, and negative acceleration occurs in the green part. As time passes, you will become more familiar with graphs.

§3 2D Kinematics

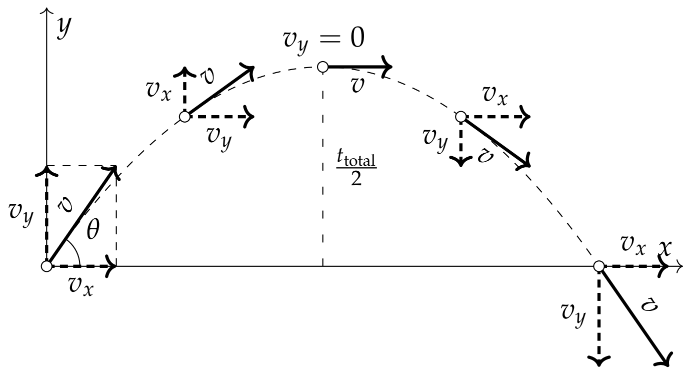
2D Kinematics is very similar 1D Kinematics, with the exception that we are solving things in the x and y direction now. However, one thing to keep in mind is that horizontal and vertical motion is *always* calculated separately.

§3.1 Vertical Motion

- An object is in ***free fall*** when gravity is the only force acting on it.
  - Objects that are falling under the influence of gravity are in free fall.
  - Objects that are *rising* can be in free fall if the only force on them is gravity.
- The acceleration of objects in free-fall is g.
  - On Earth $g_{\text{Earth}} = 9.81\ \text{m/s}^2$
  - Other planets, moons, asteroids, comets, etc. have their own gravity. Don't use g_{Earth} for them.
- If an object lands at the same height it was launched from, the rising time is equal to the falling time.

§3.2 Parabolic Motion

We don't always throw a ball straight up – sometimes we through it horizontally as well. The following is a diagram of what ***parabolic motion*** looks like:



The ***launch angle*** is the angle θ the object in motion is launched from with respect to the x-axis. Let's look at the Big Five for parabolic motion:

Theorem 19 (Big Five of Parabolic Motion)
Let's split the parabolic motion into horizontal and vertical motion. Then we have:

Horizontal Motion	Vertical Motion
$\Delta x = v_{0x}t$	$\Delta y = v_{0y}t + \frac{1}{2}(-g)t^2$
$v_x = v_{0x}$	$v_y = v_{0y} + (-g)t$
$a_x = 0$	$a_y = -g$
	$v_y^2 = v_{0y}^2 + 2(-g)\Delta y$

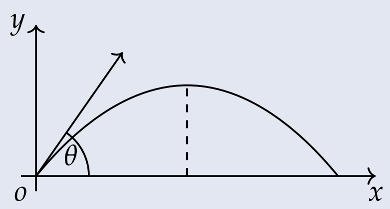
Furthermore:

Theorem 20 (Pythagorean Theorem for Velocity)
Let the horizontal component of velocity be v_x and let the vertical component by v_y. Then the velocity at that point is

$$v = \sqrt{v_x^2 + v_y^2}.$$

This applies to **all** vectors, but it is especially useful for velocity here.

Theorem 21 (Highest Point of Parabolic Motion)
If an object in parabolic motion starts and ends at the same position, and it takes t seconds to reach that same position, then it is at its highest point at $\frac{t}{2}$ seconds and the vertical velocity at that point is 0.



§3.3 2D Kinematics Examples

Example 22
A football is kicked from a ground-level tee at an angle of 37° with an initial velocity of 20.0 m/s. Calculate the following:

- The maximum height the ball reaches
- The total time the ball is in the air
- How far away from the kicker the ball hits the ground

Solution. It is important to remember to calculate actions occurring in the x direction separately from actions in the y direction. Height and time, for example, is solely a y-direction factor. Total distance traveled, however, depends on both x and y.

- The velocity in the vertical direction is $20 \cdot \sin(37^\circ) = 12.036$ m/s. Now, we can use the equation $v_f^2 = v_0^2 + 2ad$. We know $v_f = 0$ because the y-velocity of the ball when it reaches its maximum height is zero. The v_0 is what we calculated, 12.036 m/s, and $a = -9.8\frac{m}{s^2}$ because we are on Earth. $0 = 12.036^2 - 2 \cdot 9.8 \cdot d$, $d = 7.391$m.
- We can use the equation $v_f = v_0 + at$ to calculate the total time it takes to reach the apex, then multiply by 2 to find the total time in air. Again, $v_f = 0$ at the apex, $v_0 = 12.036$, and $a = -9.8\frac{m}{s^2}$, so $t = \frac{12.036}{9.8} = 1.228$ s, and the total time in air is $1.228 \cdot 2 = 2.456$ s.
- The total distance traveled is the total time in air multiplied by the horizontal velocity. The horizontal velocity can be calculated as $20 \cdot \cos(37^\circ)$, and the total time in air is 2.456 s, so the total distance traveled is $(2.456)(20\cos(37^\circ)) = 39.234$m.

□

Example 23
A soccer player kicks a rock horizontally off a 40.0 m high cliff into a pool of water. If the player hears the sound of the splash 3.00 s later, what was the initial speed given to the rock? Assume the speed of sound in air to be 343 m/s.

Solution. We can find the time it takes for the rock to land on the pool with $d = v_0 t + \frac{1}{2}at^2$, and since the rock was kicked horizontally off the cliff, the initial vertical velocity is zero, so we have $40 = \frac{1}{2}(9.8)t^2$. Solving the equation gives us $t = 2.857$s.

Sound travels at the shortest distance, so since we heard the sound of the splash 3 seconds later, we know the rock traveled a total distance of $(3 - 2.857) \cdot 343 = 49$m away from the soccer player (Note: This distance is the *direct* distance. It is NOT the vertical or horizontal distance; it's the Pythagorean sum of those two). The cliff was 40m high, so the horizontal distance that the rock traveled was $\sqrt{49^2 - 40^2} = 28.302$m.

Thus, we know the horizontal distance is 28.302, and we know the rock reaches that distance in $t = 2.857$ because that is the time it takes for the rock to fall, so the horizontal speed (and also initial speed since the rock was kicked horizontally off the cliff) will be $v_h = \frac{28.302}{2.857} = 9.906\frac{m}{s}$ □

Example 24
A projectile is launched with an initial speed of 60.0 m/s at an angle of 30.00° above the horizontal. The projectile lands on a hillside 4.00s later. Neglect air friction.

- What is the projectile's velocity at the highest point of its trajectory?

- What is the displacement of the rocket?

Solution. We again need to remember to work the x and y components separately.

- At the highest point of its trajectory, the projectile does not have any y velocity – all the projectile's velocity is in the x-direction. The velocity in the x direction can be calculated as $60\cos(30°) = 51.962\frac{m}{s}$
- To calculate displacement, we first need to calculate the displacement in the x and y directions, then use Pythagorean theorem to find the net displacement. We are told the projectile lands after 4 seconds, and we know the velocity in the x-direction is 51.962, so the distance in the x-direction is $4 \cdot 51.962 = 207.846$m. We can calculate the distance in the y-direction by using the equation $d = v_0 t + \frac{1}{2}gt^2$. The initial velocity, v_0, is $v_0 = 60 \cdot \sin(30°) = 30\frac{m}{s}$. The time in air is 4s, and the acceleration of gravity is $-9.8\frac{m}{s^2}$, so the distance traveled in the y-direction is $d = 30 \cdot 4 + \frac{1}{2}(-9.8)(4)^2 = 41.6$m. This means the net displacement is $\sqrt{41.6^2 + 207.846^2} = 211.968$m.

□

§3.4 Frames of Reference

- Relative motion problems can be solved by changing your frame of reference:
  1. Instead of seeing the problem from a 3rd person point of view, put yourself in the situation.
     - Velocities that are directed in opposite directions in the 3rd person point of view will add.
     - Velocities that are in the same direction in the 3rd person point of view will subtract.
  2. Calculate the time in the 1st person point of view.
  3. Use the time to calculate distances in the 3rd person point of view.
- Relative motion problems can be solved by graphing.
- Relative motion problems can be solved by solving a system of equations.

§4 Parting Notes

§4.1 General Ideas

- Motion in one dimension does not effect the motion in the other dimension.
  - Thus, kinematic equations can only be applied to one dimension at a time.
  - The only variable that can be applied to each dimension is time.
- Each dimension (X, Y) has its own set of kinematic variables. Use a subscript to distinguish them, ie: v_{fx} and v_{fy}.
- Distances, velocities, and accelerations can be **combined as vectors** by using the Pythagorean Theorem.
- Distances, velocities, and accelerations can be **decomposed** (broken down) into component vectors by using trigonometric equations.

§4.2 Solving 2D Problems

1. Draw a diagram.
2. Define a positive X direction and a positive Y direction. Label those directions clearly with arrows: $\longrightarrow X$ and $\uparrow Y$
   - X and Y directions must be 90° to each other.
   - X and Y don't have to be right and up. Instead, choose directions that correspond with the motion described in the problem.
3. Indicate in words what portion of motion your are considering, (like "motion from launch to the peak of the flight.")
4. Fill out a chart, including signs and units, of the five kinematics variables for each direction. Remember time is the only variable that can be used in any dimension.

$d_x =$	$d_y=$
$v_{ix} =$	$v_{iy} =$
$v_{fx} =$	$v_{fy} =$
$a_x =$	$a_y =$
$t =$	

5. Pick an dimension (X, Y) that will allow you to use a kinematic equation that has only **ONE** unknown variable.
   - Remember: **Do not mix X and Y variables in the same equation!**
6. Manipulate the equation to isolate the unknown variable (if needed).
7. Plug in the numbers.
8. Write your answer with units. Add it to the table above.
9. Continue to solve for variables until you have determined the variable you want.

§4.3 Horizontally Launched Projectiles

- A projectile launched horizontally has an initial vertical velocity of zero: $v_{iy} = 0$ m/s.
- Since gravity is the only force acting on the object while it is in free-fall, $a_y = 9.8$ m/s^2 downward, and $a_x = 0$ m/s^2.
- Be sure to double-check your positive and negative signs to make sure they correspond with the diagram you drew.

§4.4 Vertically Launched Projectiles

- If the vertical velocity is $v_{iy} = v$, then the highest a ball can reach from its original position (if it is thrown vertically) is $\frac{v_i^2}{2g}$.
- The time to reach the highest position is $\frac{v}{g}$ and the time to return back to the original position is $\frac{2v}{g}$.

- The height (in **feet**) above the original position after t seconds is given by $-16t^2 + 176t$. Furthermore, if you start from a height of h then the height above the original position is $-16t^2 + 176t + h$.

§4.5 Projectiles At an Angle

- Use trigonometry to determine the initial X and Y velocities.
- Since gravity is the only force acting on the object while it is in free-fall, $a_y = 9.81$ m/s^2 downward, and $a_x = 0$ m/s^2.
- Be sure to double-check your positive and negative signs to make sure they correspond with the diagram you drew.
- Determine the time to the top of the path first. At the top of a projectile's path, v_y = 0 m/s.
- Rarely, you will need to solve the equation $d = v_i t + \frac{1}{2}at^2$ for t when neither v_i nor a are zero.
  - This can be done by using the quadratic equation.
  - Alternately, the problem can be broken into a rising component and a falling component and the times can be added.

§5 Problems

§5.1 1D Kinematics

Problem 1. You are being chased by velociraptors. If you know that it takes a velociraptor 4 seconds to run 100 meters, what is the minimum speed you need to drive (in meters per second) in order to not be eaten?

Problem 2. George is riding his bicycle to the east at 10 m/s. There is a car, 100 meters away, coming toward him at 30 m/s. How long does George have to react in order to avoid a collision?

Problem 3. Ernest, a police officer, is in pursuit of a car that has been stolen by the notorious criminal Bert. Bert is 5 km ahead of the police car. Bert drives 30 m/s, and the Ernest is catching up by driving 45 m/s.

Problem 4. Bob is flying 100 m/s in a plane directly above a road. He sees a car driving in the same direction, 550 meters in front of him. It takes the plane 7.25 seconds to pass the car. How fast is the car driving?

Problem 5. A car starts from a stop at a traffic light that has just turned green. The car accelerates at a rate of 3 m/s^2. How fast is the car traveling after 5 seconds?

Problem 6. You are riding your bike at a speed of 4 m/s when you see a large dog behind you. In an effort to outrun the dog, you accelerate to 20 m/s over the course of the next 8 seconds. What was your rate of acceleration?

Problem 7. Lauren is walking at 2 m/s when she is startled by Benny walking behind her. Over the next two seconds, she starts to run at 6 m/s. What was her acceleration?

Problem 8. You are in a plane that is taking off. It starts from a stop and accelerates at a rate of 5 m/s^2 until it leaves the ground 675 meters away.

(a) What is the velocity that the plane is traveling with when it leaves the ground?

(b) How long does it take the plane to travel the distance of the runway?

Problem 9. How far does a baseball travel if it is airborne for 10 seconds, traveling at a speed of 70 m/s (horizontally)?

Problem 10. Kirk and McCoy step off the edge of a 50-m high cliff on an alien planet. If gravity causes them to accelerate at 2.3 m/s^2,

(a) What is their speed when they hit the water below?

(b) How long are they falling toward the water?

Problem 11. You step off the 5m high-dive into a pool.

(a) How long does it take you to hit the water?

(b) What is the speed that you hit the water at?

Problem 12. In a track-and-field, an athlete runs exactly once around an oval track, a total distance of 500 m. Find the runner's displacement for the race.

Problem 13. You ride your bicycle in a straight line for a distance of 73 meters in 12.5 second. What is your average speed?

Problem 14. A monkey is on the ground. He jumps at a velocity of 10 m/s directly up.

(a) What is the time that it takes the monkey to reach the top his jump?

(b) How high does he jump?

(c) What is the total time that the monkey was in the air?

Problem 15. A car is traveling along a straight highway at a speed of 20 m/s. The driver steps on the gas pedal and 3 seconds later, the car's speed is 32 m/s. Find its average acceleration.

Problem 16. Spotting a police car ahead, the driver of the car in the previous example slows from 32 m/s to 20 m/s in 2 seconds. Find the car's average acceleration.

Problem 17. An object with an initial velocity of 4 m/s moves along a straight axis under constant acceleration. Three seconds later, its velocity is 14 m/s. How far did it travel during this time?

Problem 18. A car that's initially traveling at 10 m/s accelerates uniformly for 4 seconds at a rate of 2 m/s^2, in a straight line. How far does the car travel during this time?

Problem 19. A rock is dropped off a cliff that's 80 m high. If it strikes the ground with an impact velocity of 40 m/s, what acceleration did it experience during its descent?

Problem 20. A rock is dropped from an 80-meter cliff. How long does it take to reach the ground?

Problem 21. A baseball is thrown straight upward with an initial speed of 20 m/s. How high will it go?

Problem 22. If an object is thrown straight upward with an initial speed of 8 m/s and takes 3 seconds to strike the ground, from what height was the object thrown?

§5.2 2D Kinematics

Problem 23. An object is thrown horizontally off a cliff with an initial speed of 10 m/s. How far will it drop in 4 seconds assuming it does not hit the ground first?

Problem 24. From a height of 100 m, a ball is thrown horizontally with an initial speed of 15 m/s. How far does it travel horizontally in the first 2 seconds?

Problem 25. A projectile is traveling in a parabolic path for a total of 6 seconds. How does its horizontal velocity 1 s after launch compare to its horizontal velocity 4 s after launch?

Problem 26. An object is projected upward with a 30° launch angle and an initial speed of 40 m/s. How long will it take for the object to reach the top of its trajectory? How high is this?

Problem 27. An object is projected upward with a 30° launch angle and an initial speed of 60 m/s. For how many seconds will it be in the air? How far will it travel horizontally before returning to its original height?

Problem 28. You throw a rock at 18 m/s directly toward the horizon while standing at the edge of a 25 meter tall cliff.

(a) How long does it take the rock to hit the ground?

(b) How far does the rock land from the bottom of the cliff?

(c) What is the speed that the rock hits the ground at?

(d) What is the angle of impact of the rock?

Problem 29. A pitcher throws a curve-ball at 20 m/s toward home plate, perfectly horizontal. The ball leaves his hand 1.5 meters above the ground.

(a) How far does the ball go?

(b) With what velocity (magnitude and direction) does the ball hit the ground?

Problem 30. Frodo and Sam are 87 meters from a group of Orcs, but are cursed with shorter legs. They are running away from the Orcs at 3 m/s, but the Orcs are overtaking them by running at 7 m/s.

(a) How long do Frodo and Sam have to talk about the shire before they are caught again by the Orcs?

(b) How far do the Orcs have to run to catch Frodo and Sam?

Problem 31. A cannon is placed on a wall, 25 meters above the surrounding area. The cannon shoots cannonballs at a speed of 350 m/s, and is aimed at the horizon.

(a) What is the distance that the cannonball travels?

(b) What is the speed that the cannonball hits the ground with?

(c) What is the angle of impact for the cannonball?

Problem 32. You are following the road to town in order to meet with the ruler who has a strange affection for the color green. However, you think that you might never make it to your destination, because it is a hefty 12,000 meters away. For the first 3,000 meters you travel at an average rate of 1 meter per second. You then meet a brainless companion who likes to dance and sing, which only slows you down to a speed of 0.5 meters per second for the next 3,000 meters. Then you meet a heartless man as well as a fearful king, which causes your speed to drop to 0.25 m/s for the remaining portion of the trip.

(a) How long does the first part of the trip take?

(b) How long does it take to sing and dance your way through the second 3000 meters?

(c) How long does the final part of the trip take?

(d) What is the total time that the trip to see the ruler take?

Problem 33. A cannon is placed on level ground. It is aimed 25 degrees above horizontal. The cannonball leaves the cannon with an initial speed of 300 m/s.

(a) What is the horizontal component of the initial velocity (v_{ix})?

(b) What is the vertical component of the initial velocity (v_{iy})?

(c) What is the time it takes for the cannonball to reach its maximum height?

(d) What is the maximum height of the cannonball?

(e) What is the total time of flight for the cannonball?

(f) How far from the cannon does the cannonball land?

Problem 34. Kay is attempting to kick a football through the field-goal posts. She kicks the ball at 18 m/s at a 45° angle to the ground. She is 20 meters from the goal-post.

(a) What are the initial vertical and horizontal velocities of the football?

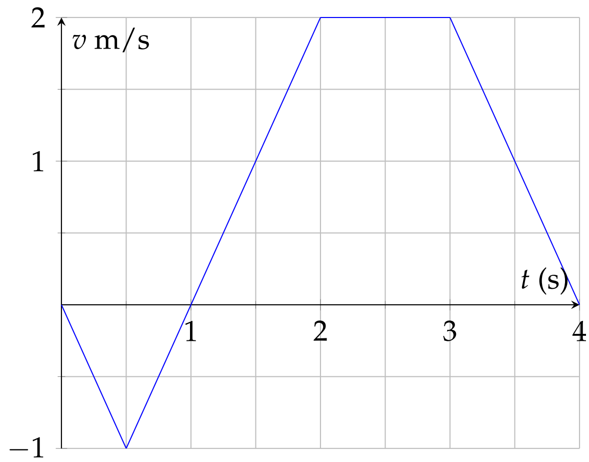
(b) How long does it take the football to travel the distance to the goal post? (Hint: does this depend on the vertical direction or the horizontal direction?)

(c) What is the height of the football when it passes the goal-post?

(d) Assuming the football is kicked straight, does she score 3 points for her team?

Problem 35. You're standing in a freight train, and have no way to see out. If you have to lean to stay on your feet, what, if anything, does that tell you about the train's velocity? Explain.

Problem 36. Consider the following figure:

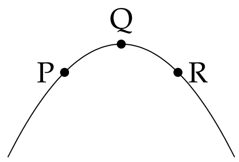


1. The graph above shows the velocity versus time for an object moving in a straight line. At what time after $t = 0$ does the object again pass through its initial position?

Problem 37. A body moving in the positive x direction passes the origin at time $t = 0$. Between $t = 0$ and $t = 1$ second, the body has a constant speed of 24 meters per second. At $t = 1$ second, the body is given a constant acceleration of 6 meters per second squared in the negative x direction. What is the position x of the body at $t = 11$ seconds?

Problem 38. A diver initially moving horizontally with speed v dives off the edge of a vertical cliff and lands in the water a distance d from the base of the cliff. How far from the base of the cliff would the diver have landed if the diver initially had been moving horizontally with speed $2v$?

Problem 39. Consider a figure:



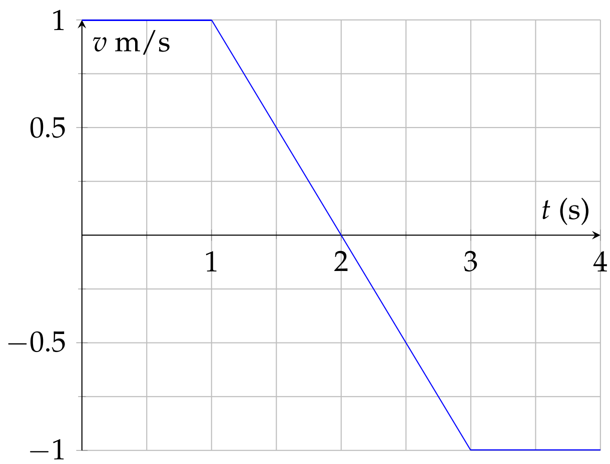
A ball is thrown and follows the parabolic path shown above. Air friction is negligible. Point Q is the highest point on the path. Points P and R are the same height above the ground.

1. How do the speeds of the ball at the three points compare?
2. What direction is the acceleration of the ball at point P?

Problem 40. The velocity of a projectile at launch has a horizontal component v_h and a vertical component v_v. Air resistance is negligible. When the projectile is at the highest point of its trajectory, what is the vertical and horizontal components of its velocity and the vertical component of its acceleration?

Problem 41. A target T lies flat on the ground 3 m from the side of a building that is 10 m tall, as shown above. A student rolls a ball off the horizontal roof of the building in the direction of the target. Air resistance is negligible. What is the horizontal speed with which the ball must leave the roof if it is to strike the target?

Problem 42. Consider a figure:



The graph above shows velocity v versus time t for an object in linear motion. What is a possible graph of position x versus time t for this object?

Problem 43. An object is thrown with velocity v from the edge of a cliff above level ground. Neglect air resistance. In order for the object to travel a maximum horizontal distance from the cliff before hitting the ground, the throw should be at what angle?

Problem 44. A whiffle ball is tossed straight up, reaches a highest point, and falls back down. Air resistance is not negligible. Which of the following statements are true?

I. The ball's speed is zero at the highest point.

II. The ball's acceleration is zero at the highest point.

III. The ball takes a longer time to travel up to the highest point than to fall back down.