ASE 2020-21 Advanced Notes

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Contents

§ 1 Problems

Here is a short list of problems from the previous lessons taught, to make up for the fact we didn't get to go over all of them. If you would like a hint, please email me at dylanyu@gmail.com.

Problem 1. Let r_1 and r_2 be the roots of the quadratic $x^2 - 5x + 6$. Let the parabola $x^2 - 5x + 6$ go through the x-axis at $(p_1, 0)$ and $(p_2, 0)$. What is $r_1 + r_2 - p_1 - p_2$?

Problem 2. Find the center and radius of the circle with equation

$$x^2 + y^2 = 6x + 8y.$$

At what points does it intersect the x-axis and y-axis?

Problem 3 ($*$). A **tangent line** of a circle is a line that intersects a circle at exactly one point. What is the equation of a tangent line that is tangent to a circle with equation $x^2 + y^2 = 6x + 8y$ at $(0, 0)$? This requires you know that a tangent line is perpendicular to the line that goes through the tangent point and the center of the circle.

Problem 4. Find the equation of the line in which all the points on the line are **equidistant** (meaning same distance) from the points $(1, 2)$ and $(3, 4)$.

*The ASE playlist can be found here.

Problem 5 (∗). Let the vertices of a rectangle be at the points $(-1,-1),(-1,1),(3,1),(3,-1)$. Find the coordinates of the intersection of the diagonals.

Problem 6. Find $\frac{x}{y}$ if $2^{2^x}=2^{2^y}$.

Problem 7. If $3^{9x}=\frac{9^{24}}{81^14}$.

Problem 8. Find all x that satisfy $4^x-5\cdot 2^x+4=0$.

Problem 9 (MathLeague Sprint 11323/27). How many ordered pairs of real numbers (a,b) satisfy the equations $|ab|=\sqrt{3}$ and $\frac{a}{b}=\frac{b}{a}$?

Problem 10. A bacteria colony doubles its population every 10 seconds. After x seconds, the population increases from 1 bacterium to 2^{20} bacteria. What is x?

Problem 11 (MathLeague Target 11322/2). Find the units digit of 147^{148}.

Problem 12 (∗). Find the last two digits of 6^{2020}.

Problem 13 (∗). Compute

$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots+\frac{1}{\sqrt{99}+\sqrt{100}}.$$