# Dynamics

Bill Wang*

Last updated August 20, 2021

Forces are an important part of physics. Here we will discuss some common forces and associated scenarios regarding forces.

## Contents

## Vocabulary



## §1 Introduction

While we use ***kinematics*** to describe the motion of any object mathematically, we use ***dynamics*** to describe *what* causes motion (or more precisely, *what causes motion to change?*)

> *Remark 1.* Note that forces and dynamics are different, but are fundamentally the same. Dynamics is, at its core, the study of forces.

## §2 Laws of Motion

### §2.1 First Law

**Direct Translation of First Law** Every body persists in its state of being at rest or moving uniformly straight forward, except insofar as it is compelled to change its state by force impressed.

This is confusing. Let's simplify it:

**Theorem 2 (First Law of Motion)**
An object in motion stays in motion and an object at rest stays at rest if no forces are acting on it.

- "Moving uniformly" means *uniform motion* with constant velocity
  - An object "at rest" is also in uniform motion with $v = 0$
- As long as an object moves in uniform motion, it must be that $F_{\text{net}} = 0$

**Example 3**
Common examples:

- A hockey puck sliding on very smooth ice has gravity and normal force, but the net force is zero
- A car traveling on a highway at 100 km/h has many forces acting on it, but the net force is zero

### §2.2 Second Law

**Direct Translation of Second Law** The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.

The *alteration of motion* means ***acceleration***. Again, in easier terms:

**Theorem 4 (Second Law of Motion)**
Force is equal to mass times acceleration.

In other words,

$$F_{\text{net}} = \Sigma F = ma.$$

## §3 Third Law

**Direct Translation of Third Law** To every action there is always opposed an equal reaction: the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.

**Theorem 5 (Third Law of Motion)**
For every action force on an object $B$ due to another object $A$, there is a reaction force which is equal in magnitude but opposite in direction, on object $A$, due to object $B$:

$$F_{\text{AB}} = -F_{\text{BA}}.$$

These direct translations are for *clarity*. Often, we find the "basic ideas" to be ambiguous – we need the direct source to make sure we understand the law. But for now, stick with the theorems.

- "Contrary parts" means the action and reaction forces act on different objects
- Third law is the natural consequence of the first and second law. Action/reaction forces are *internal* forces.

## §4 Forces

**Force**
A ***force*** is the interaction between the objects.

- When there is interaction, then forces are created
- A "push" or a "pull"

### §4.1 Categories

There are two broad categories of forces:

- ***Contact forces*** act between two objects that are in contact with one another
- ***Non-contact forces*** act between two objects without them touching each other. They are also called "action-at-a-distance" force

### §4.2 Center of Mass

If the net force on an object is zero ($\Sigma F = 0$) then the object is in a ***state of equilibrium***.

- ***Dynamic equilibrium***: the object is moving relative to us
- ***Static equilibrium***: the object is not moving relative to us

## §5 Common Forces

Common everyday forces that we encounter in include:

- ***Weight*** (gravitational force) $w$ (or $F_G$)
- ***Normal force*** $N$
- ***Friction*** (static $f_s$ and kinetic $f_k$)
- ***Tension*** $T$
- ***Applied force*** $F_a$
- ***Spring force*** $F_e$
- ***Drag*** $D$ (fluid resistance, then again in fluid mechanics)
- ***Buoyant force*** $B$ (discussed in fluid mechanics)
- ***Electrostatic force*** $F_E$ (discussed in E & M exam)
- ***Magnetic force*** $F_M$ (discussed E & M exam)

We won't cover all of these forces in this unit. But as you'll soon realize, **forces are everywhere**. If you don't understand forces, the rest of physics will make no sense.

## §5.1 Gravity

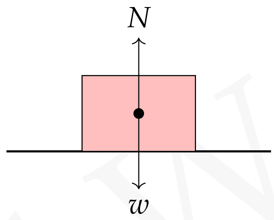
**Gravity**
*Gravity* is the force of attraction between all objects with mass.

- Near the surface of Earth, use $g = 9.81\,\mathrm{m/s^2}$ (or $g = 10\,\mathrm{m/s^2}$ for the AP exam)
- You may be asked to find the value of $g$ on some "unknown planet".
- $w$ always points *down* (the direction of $w$ is how down is defined)

## §5.2 Normal Force

In the following figure,

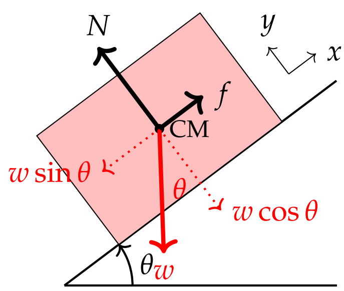
$$w = mg = -N.$$




- A force a surface exerts on another object that it is in contact with
- Always *perpendicular* to the contact surface
- **Special case**: When an object is on a horizontal surface with no additional applied force, the magnitude of the normal force is equal to the magnitude of the weight of the object, i.e. $N = w$

## §5.3 Normal Force on a Slope

For this case, we label the $x$-axis to be along the slope, and $y$-axis to be perpendicular to the slope.




- If on a slope: $N = w\cos\theta$
  - $N$ decreases as ramp angle $\theta$ increases
- $w$ has a component along the ramp $w\sin\theta$ that wants to slide the block down.
- Friction force $f$ opposes the motion
  - Be careful: if the block is moving *up* the ramp with an applied force, then $f$ will point *down* the ramp

## §5.4 Friction

**Friction**

***Friction*** is a force that opposes the sliding of two surface against one another.

- Always act in a direction that opposes motion or attempted motion
- Depends on:
  - The force the two surfaces are pressed against each other
  - The smoothness of the surfaces, which itself depends on
  - The material(s) the surfaces are made of
  - The use of lubricants

**Static Friction**

***Static friction*** between the two surfaces is when there is no relative motion between them

- Increases with increasing applied force
- Maximum when the object is just about to move

**Theorem 10 (Static Friction)**

The static friction is

$$f_s \leq \mu_s N,$$

where $\mu_s$ is the coefficient of static friction and $N$ is the normal force.

| Quantity | Symbol | SI Unit |
|---|---|---|
| Magnitude of maximum static friction | $f_s$ | N |
| Static friction coefficient | $\mu_s$ | no units |
| Magnitude of normal force | $N$ | N |

**Kinetic Friction**

***Kinetic friction*** between two surfaces is when they are moving relative to each other.

$f_k$ is constant along the path of movement as long as $N$ stays constant.

**Theorem 12 (Kinetic Friction)**

The kinetic friction is

$$f_k = \mu_k N,$$

where $\mu_k$ is the coefficient of kinetic friction and $N$ is the normal force.

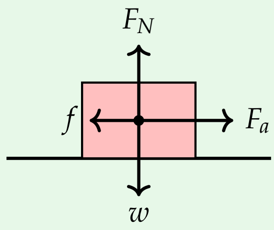
| Quantity | Symbol | SI Unit |
|---|---|---|
| Magnitude of kinetic friction | $f_k$ | N |
| Kinetic friction coefficient | $\mu_k$ | no units |
| Magnitude of normal force | $N$ | N |

## §5.5 Static and Kinetic Coefficients of Friction

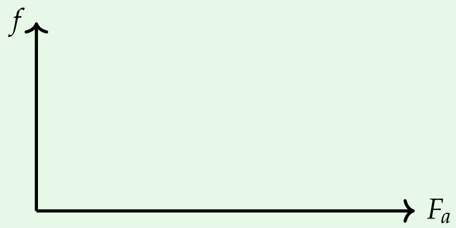
Kinetic friction coefficient is always lower than the static coefficient, otherwise nothing will ever move:

$$\mu_k \leq \mu_s.$$

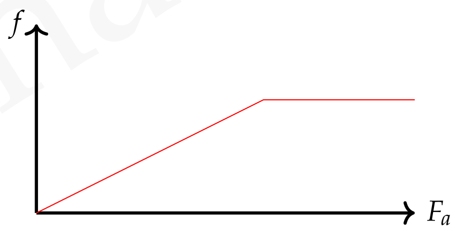
**Example 13**
Consider a simple case of a box being pulled along a level floor.




The free-body diagram is simple. How do the magnitudes of the applied force $F_a$ and friction $f$ compare?




*Solution.* At first, the box is static because the force used is not enough. Thus, we have $f = F_a$ at first – they match each other. Then, when the $F_a$ crosses the threshold, $f$ can no longer increase (beyond $\mu_k N$). Thus, the graph is:




□

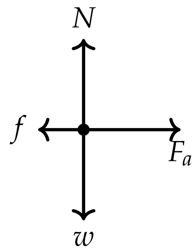
## §5.6 Tension in a Cable

**Tension**
*Tension* is the force exerted on and by a cable, rope, or string.

- You can't push on a rope
- Assume the cable/rope/string to be mass less
- Force can change direction when used with pulleys

# §6 Free Body Diagrams

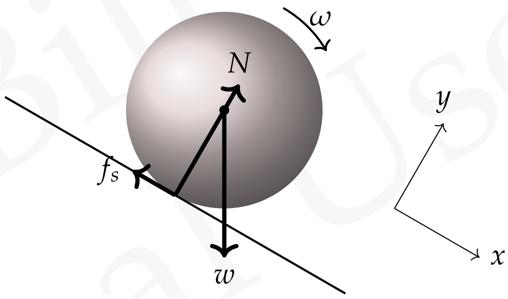
In basic physics, for ***rectilinear motion*** (no *rotational motion*), FBDs are usually drawn by assuming that all forces acting at the CM, represented by the "big dot". For example:




We can still use this method for rectilinear motion. However, this is not entirely correct. We notice that:

- Gravitational force $w$ acts at the CM, but
- Normal force $N$, friction $f$ and applied force $F_a$ act at the point of contact

Instead, forces should be drawn at the point where they are applied. For example, a sphere rolling down a ramp should have weight $w$, normal force $N$ and static friction $f_s$ acting on it:




Once the FBD is drawn, decide on the axes to help you solve the motion. One of the axes should line up with the direction of motion. This guarantees that the *other* axis will not have any net force.

# §7 Connected Bodies

- The objects are connected by a cable or a solid linkage with negligible mass
- All objects (usually) have the same acceleration
- Require multiple free-body diagrams

## §7.1 Solving Connected Bodies Problems

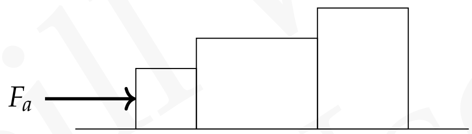
To solve a connected-bodies problem, you can follow these procedures:

1. Draw a FBD on each of the objects
2. Sum all the forces on all the objects along the direction of motion
   - Direction of motion is usually very obvious
   - All internal forces should cancel and do not figure into the acceleration of the system
3. Compute the acceleration of the entire system using Newton's second law
   - Remember that (usually) every object has the same acceleration!
4. Go back to the FBD of each of the objects and compute the unknown forces (usually tension)

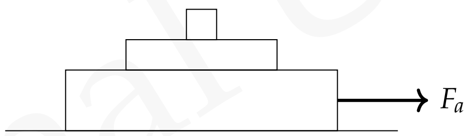
## §7.2 Problem Types

Some problem types of connected bodies problem may be like this:

- Multiple objects pressed against one another:




- Multiple objects stacked on top of one another:



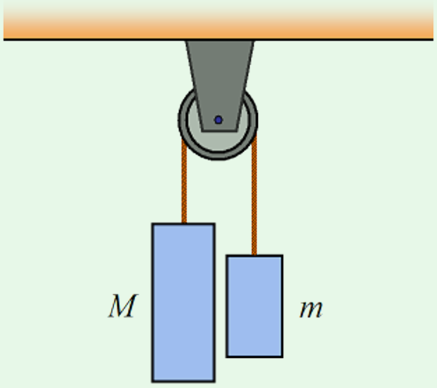

# §8 Pulleys

**Atwood Machine**

An **Atwood machine** is made of two objects connected by a rope that runs over a pulley. The pulley allows the direction of force and direction of motion to change between two objects.

**Example 16**

The object below has a mass of $M$ and the object on the right has a mass of $m$.




(a) What is the acceleration of the masses?

(b) What is the tension in the rope?

*Solution.* The tension in the rope is $T$, so the left mass has an acceleration of $a = \frac{Mg-T}{M}$. The right mass has an acceleration of $a = \frac{T-mg}{m}$. Since these two accelerations have to be equal,

$$\frac{Mg - T}{M} = \frac{T - mg}{m},$$

$$T = \boxed{\frac{2Mmg}{M+m}}.$$

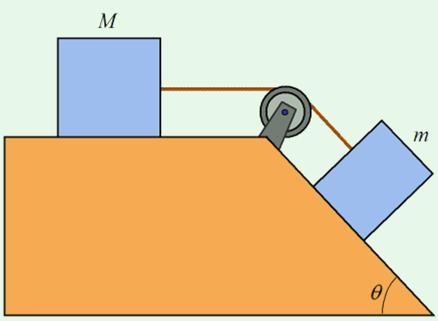
Plugging $T$ back in, we get

$$a = \boxed{\frac{(M-m)g}{M+m}}.$$

□

**Example 17**

Two blocks of mass $m$ and $M$ are connected via pulley with a configuration as shown. The coefficient of static friction is $\mu_s$, between blocks and surface. What is the maximum mass $m$ so that no sliding occurs?



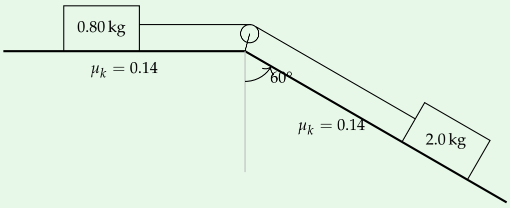

*Answer.* $m = \frac{Mg\mu_s}{g\sin\theta - g\cos\theta\mu_s}$. □

*Remark 18.* When there are multiple pulleys involved, we have to remember that tension force is distributed evenly along the cable.

More typically, an Atwood machine problem is one where two objects are sliding on a surface. These surfaces may have (or may not) have friction.

**Example 19**

In this example, two blocks are connected by a mass-less string over a friction-less pulley as shown in the diagram.




(a) Determine the acceleration of the blocks.

(b) Calculate the tension in the string.

(c) If the string broke, for what minimum value of the coefficient of static friction would the 2.0 kg block not begin to slide?

*Answer.*

(a) 2.259 $m/s^2$

(b) 2.905 N

(c) 0.577

□

**Example 20**

A helicopter holding a 70-kilogram package suspended from a rope 5.0 meters long accelerates upward at a rate of $5.2\frac{m}{s^2}$. Neglect air resistance on the package.

(a) Determine the tension in the rope.

(b) When the upward velocity of the helicopter is 30 meters per second, the rope is cut and the helicopter continues to accelerate upward at $5.2\frac{m}{s^2}$. Determine the distance between the helicopter and the package 2.0 seconds after the rope is cut.

*Answer.*

(a) 1050 N

(b) 35 m

□

**Example 21**

An empty sled of mass 25kg slides down a muddy hill with a constant speed of 2.4 m/s. The slope of the hill is inclined at an angle of 15° with the horizontal.

(a) Calculate the time it takes the sled to go 21 m down the slope.

(b) Calculate the frictional force on the sled as it slides down the slope.

(c) Calculate the coefficient of friction between the sled and the muddy surface of the slope.

*Answer.*

(a) 8.75s

(b) 63.4 N

(c) 0.27

□

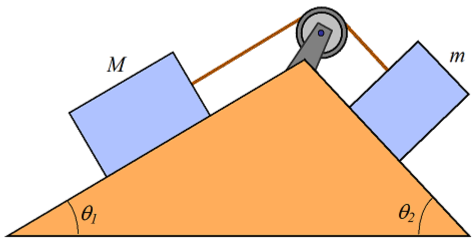
## §9 Problems

**Problem 1.** You are holding an 85 kg trunk at the top of a ramp that slopes from a moving van to the ground, making an angle of 35° with the ground. You lose your grip and the trunk begins to slide.

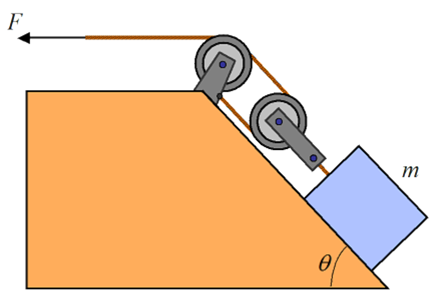
- If the coefficient of friction between the trunk and the ramp is 0.42, what is the acceleration of the trunk?
- If the trunk slides 1.3 m before reaching the bottom of the ramp, for what time interval did it slide?

**Problem 2.** A 55 kg person is standing on a scale in an elevator. If the scale is calibrated in *newtons*, what is the reading on the scale when the elevator is not moving? If the elevator begins to accelerate upward at $0.75\,\mathrm{m/s^2}$, what will be the reading on the scale?

**Problem 3.** Two blocks of mass $m$ and $M$ are connected via pulley with a configuration as shown. The coefficient of static friction between the left block and the surface is $\mu_{s,1}$, and the coefficient of static friction between the right block and the surface is $\mu_{s,2}$. Formulate a mathematical inequality for the condition that no sliding occurs. There may be more than one inequality.




**Problem 4.** A block of mass $m$ is pulled, via two pulleys as shown, at constant velocity along a surface inclined at angle $\theta$. The coefficient of kinetic friction is $\mu_k$, between block and surface. Determine the pulling force $F$. Ignore the mass of the pulleys.




**Problem 5.** A small moving block collides with a large block at rest. Which of the following is true of the forces the blocks apply to each other?

(a) The small block exerts twice the force on the large block compared to the force the large block exerts on the small block.

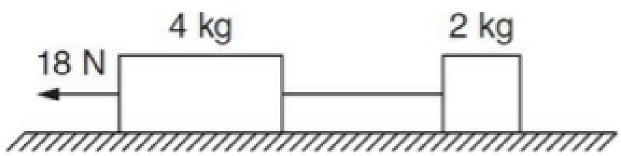
(b) The small block exerts half the force on the large block compared to the force the large block exerts on the small block.

(c) The small block exerts exactly the same amount of force on the large block that the large block exerts on the small block.

(d) The large block exerts a force on the small block, but the small block does not exert a force on the large block.

(e) The small block exerts a force on the large block, but the large block does not exert a force on the small block.

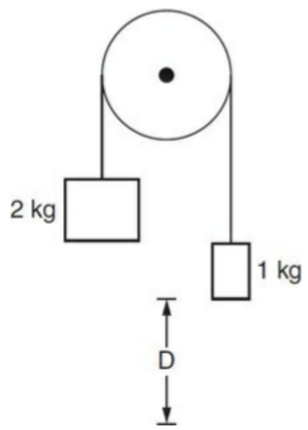
**Problem 6.** Two blocks, 4.0 kg and 2.0 kg, are connected by a string. An applied force $F$ of magnitude 18 N pulls the blocks to the left.




(a) What is the acceleration of the 4.0 kg block?

(b) What is the tension in the string between the blocks?

**Problem 7.** A system consists of two blocks having masses of 2 kg and 1 kg. The blocks are connected by a string of negligible mass and hung over a light pulley, and then released from rest.




(a) What is the acceleration of the 2 kg block?

(b) What is the speed of the 2 kg block after it has descended a distance $D$?

**Problem 8.** A weight of magnitude $W$ is suspended in equilibrium by two cords, one horizontal and one slanted at an angle of 60° from the horizontal, as shown.

(a) Which of the following statements is true?

(i) The tension in the horizontal cord must be greater than the tension in the slanted cord.

(ii) The tension in the slanted cord must be greater than the tension in the horizontal cord.

(iii) The tension is the same in both cords.

(iv) The tension in the horizontal cord equals the weight $W$.

(v) The tension in the slanted cord equals the weight $W$.

(b) The tension in the horizontal cord is

(i) equal to the tension in the slanted cord

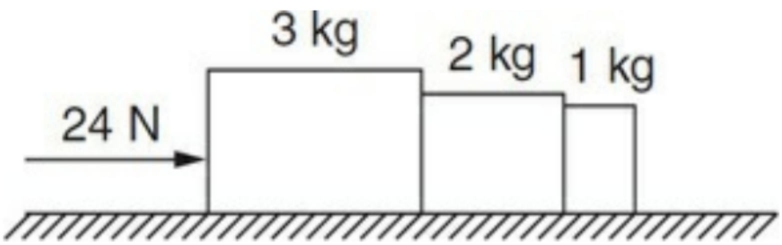
(ii) one-third as much as the tension in the slanted cord

(iii) one-half as much as the tension in the slanted cord

(iv) twice as much as the tension in the slanted cord

(v) three times as much as the tension in the slanted cord

**Problem 9.** Three blocks of mass 3 kg, 2 kg, and 1 kg are pushed along a horizontal frictionless plane by a force of 24 N to the right, as shown.




(a) What is he acceleration of the 2 kg block?

(b) What is the force that the 2 kg block exerts on the 1 kg block?

**Problem 10.** A ball is thrown straight up into the air, encountering air resistance as it rises. What forces, if any, act on the ball as it rises?

(a) A decreasing gravitational force and an increasing force of air resistance

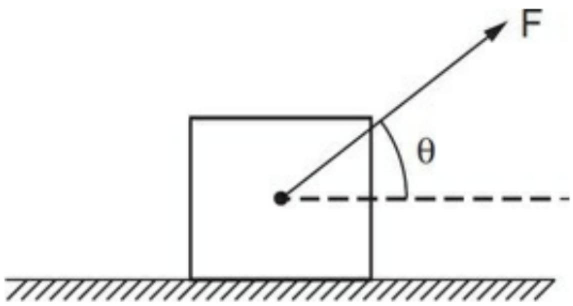
(b) An increasing gravitational force and an increasing force of air resistance

(c) A decreasing gravitational force and a decreasing force of air resistance

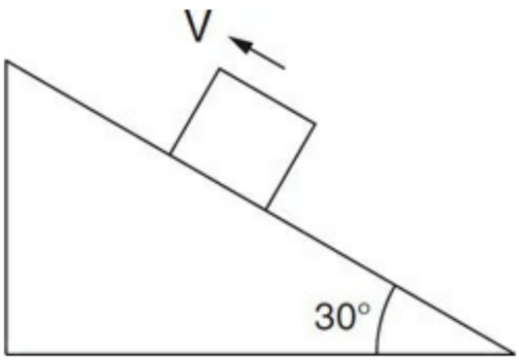
(d) A constant gravitational force and an increasing force of air resistance

(e) A constant gravitational force and a decreasing force of air resistance

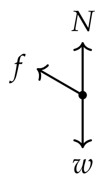
**Problem 11.** A force of magnitude $F$ pulls up at an angle $\theta$ to the horizontal on a block of mass $m$. The mass remains in contact with the level floor and the coefficient of friction between the block and the floor is $\mu$. What is the frictional force between the floor and the block?



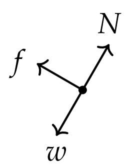

**Problem 12.** A 1 kg block is sliding up a rough 30° incline and is slowing down with an acceleration of $-6\,\mathrm{m/s^2}$. The mass has a weight $w$, and encounters a frictional force $f$ and a normal force $N$. The direction up the ramp is positive.



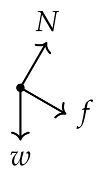

(a) Which of the following free body diagrams best represents the forces acting on the block as it slides up the plane?



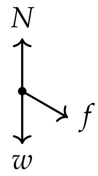

(i)

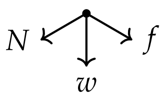



(ii)




(iii)

(iv) 


(v) 


(b) What is the magnitude of the frictional force $f$ between the block and the plane?

# §A Other Forces

Here are some forces that you don't need to know at this moment, but are good to keep in mind.

## §A.1 Drag

**Drag**

***Drag*** (or ***fluid resistance***) is the force opposing the motion of an object moving in a fluid, with a magnitude of:

$$D = \frac{1}{2}\rho V_\infty^2 A_{\text{ref}} C_d.$$

| Quantity | Symbol | SI Unit |
|---|---|---|
| Magnitude of drag force | $D$ | N |
| Density of the fluid | $\rho$ | kg/m$^3$ |
| Free-stream fluid velocity | $V_\infty$ | m/s |
| Reference area | $A_{\text{ref}}$ | m$^2$ |
| Drag coefficient | $C_d$ | (no unit) |

Drag coefficient depends on the shape and surface smoothness of the object. For blunt objects $A_{\text{ref}}$ is the frontal area; for streamlined objects $A_{\text{ref}}$ is the planform (top-view) area. In AP Physics you are not explicitly asked to know the drag equation. However, you should know that drag (air resistance) depends on the motion of the object and is not a constant.

## §A.2 Terminal Velocity

When we take drag force into account, we understand that the drag force increases as an object speeds up, and therefore a free-falling object does *not* accelerate infinitely. Instead it reaches a ***terminal velocity***.

**Simulating Terminal Velocity** There is no air resistance just as the object *begins* to fall. Acceleration is due to gravity alone.


Drag increases as $v$ increases. Magnitude of acceleration decreases, but the object continues to gather speed.


Terminal velocity is reached when the drag force equals the object's weight. Not net force; no acceleration.