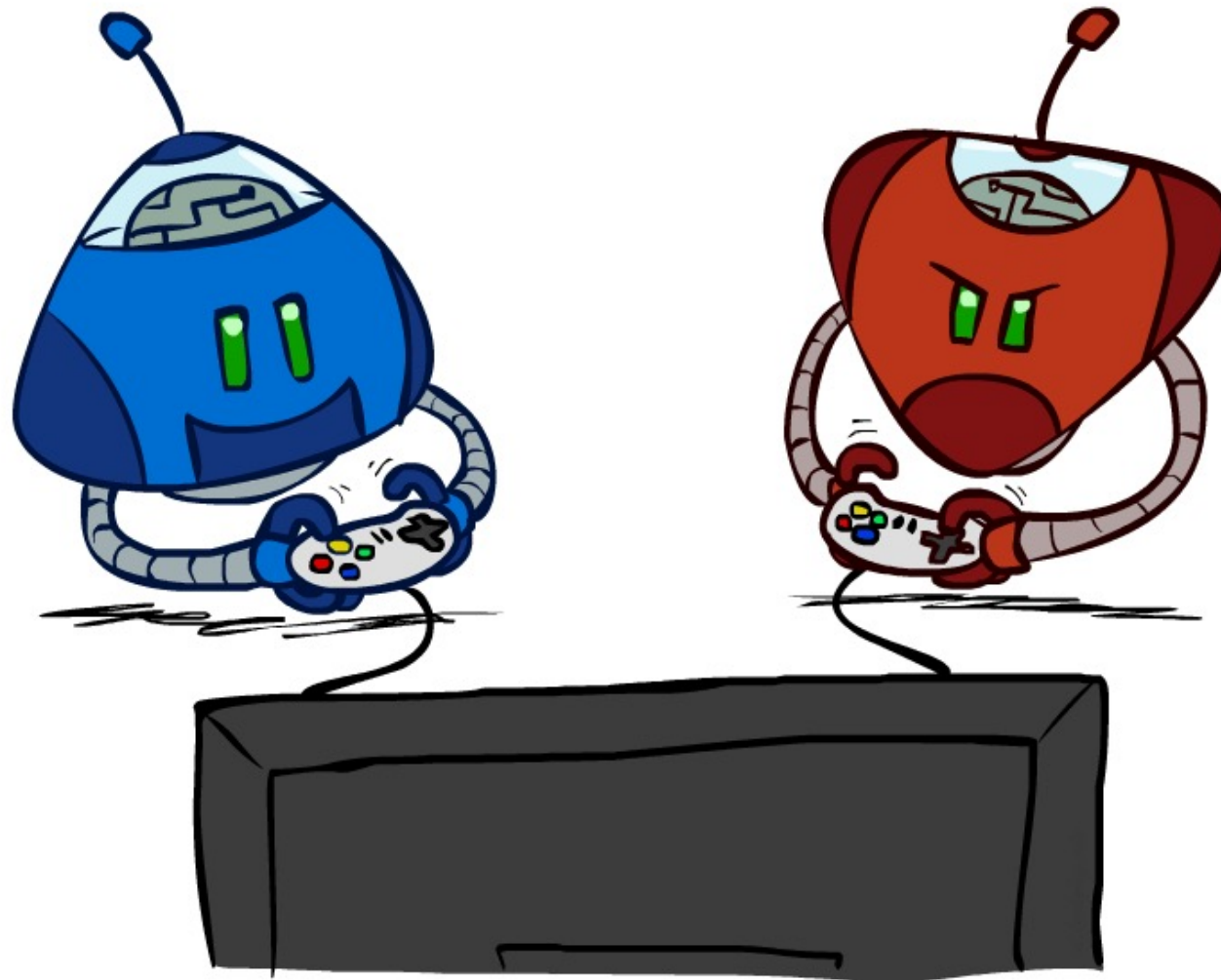


Ve492: Introduction to Artificial Intelligence

Search in Games; Adversarial Search



Paul Weng

UM-SJTU Joint Institute

Slides adapted from <http://ai.berkeley.edu>, AIMA, UM, CMU

Announcements

❖ Homework 1: Search

- ❖ Part I AND Part II due Sun. May 23 at 11:59pm (Extension).
- ❖ Part I through Online Judge.
- ❖ Part II through Canvas -- submit pdf

❖ Project 1: Search

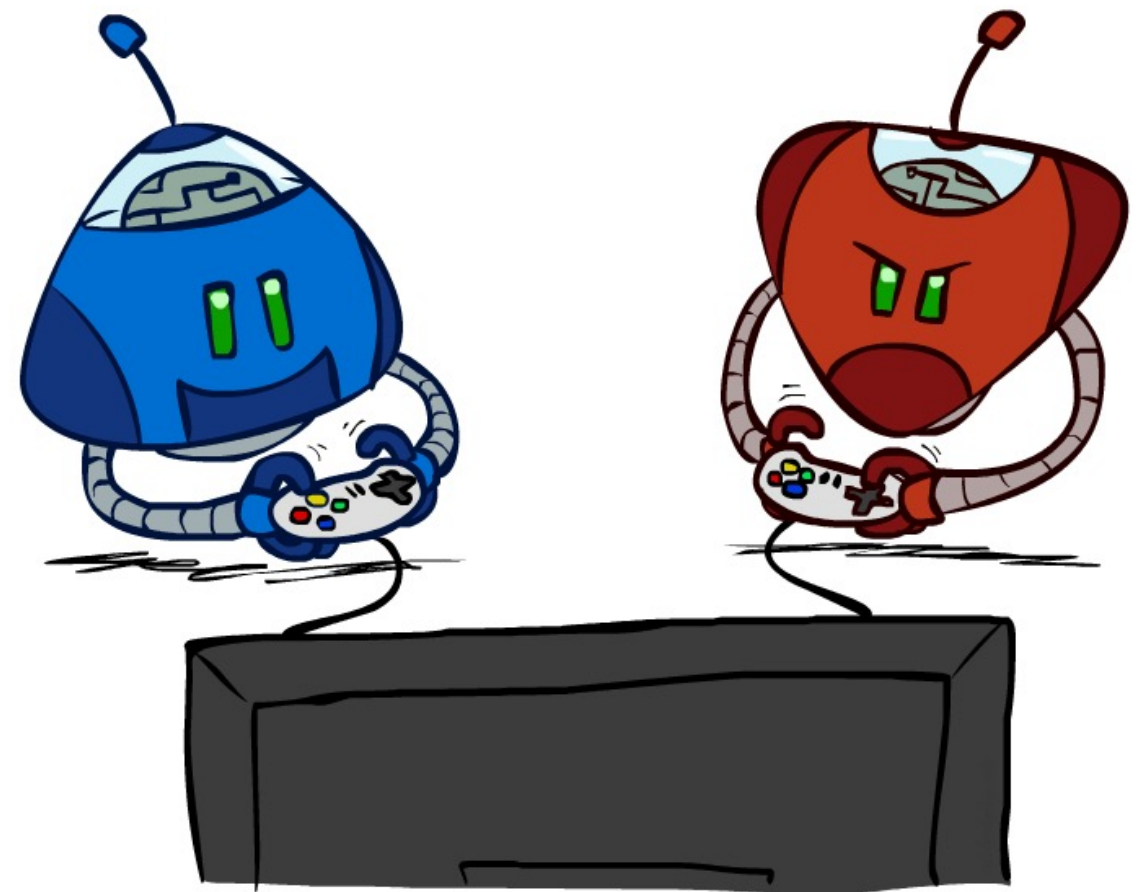
- ❖ Released today. Due Mon May 31 at 11:59pm
- ❖ Start early and ask questions. It's longer than most!

❖ OH

- ❖ Mon 9am-11am, JI 406
- ❖ Yunpeng Jiang: Thu 1pm-3pm, JI 326B
- ❖ Zhengjie JI: Wed 8pm-10pm, JI 326B

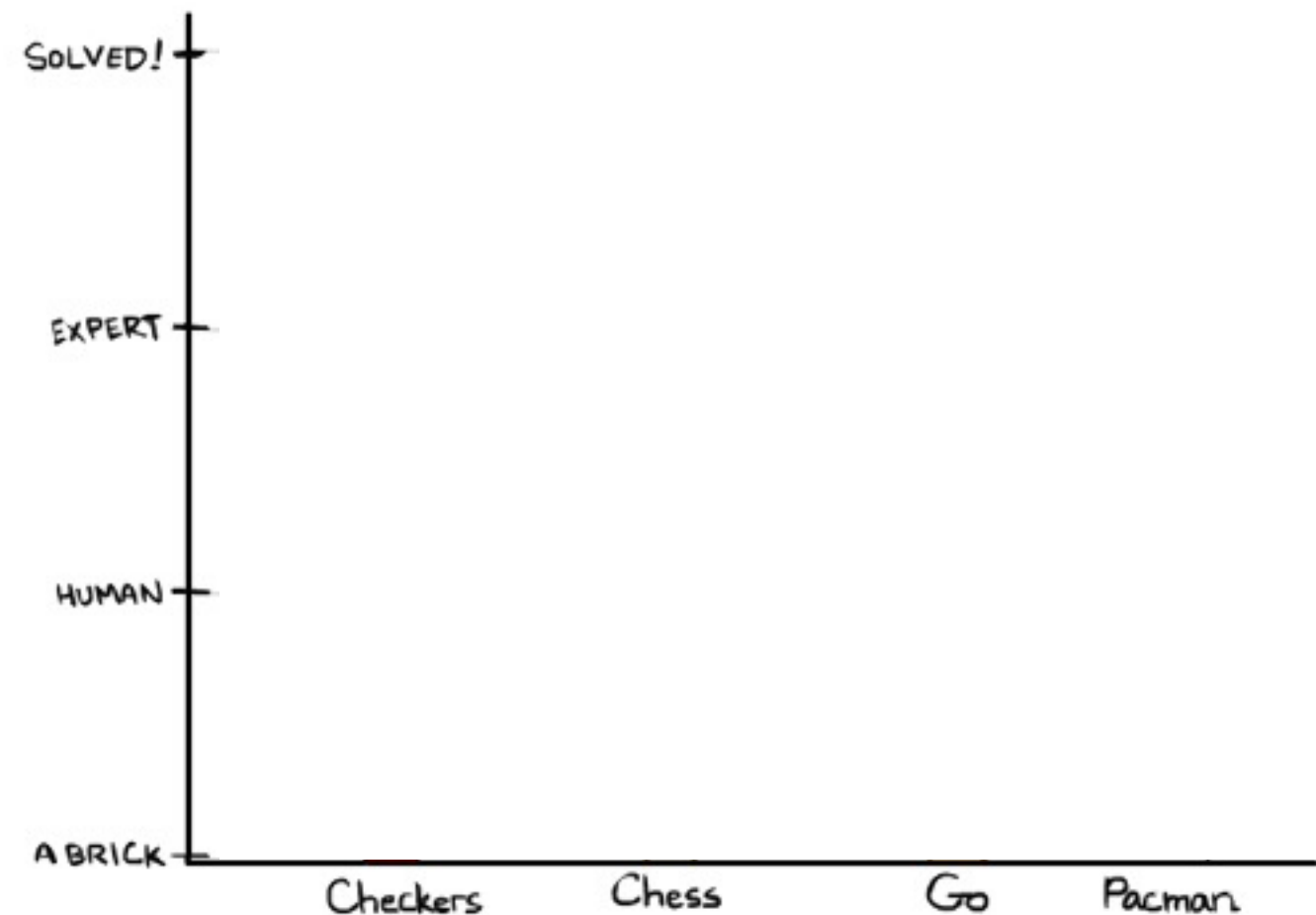
Outline

- ❖ Introduction
- ❖ Game formalization
- ❖ Games with an adversary

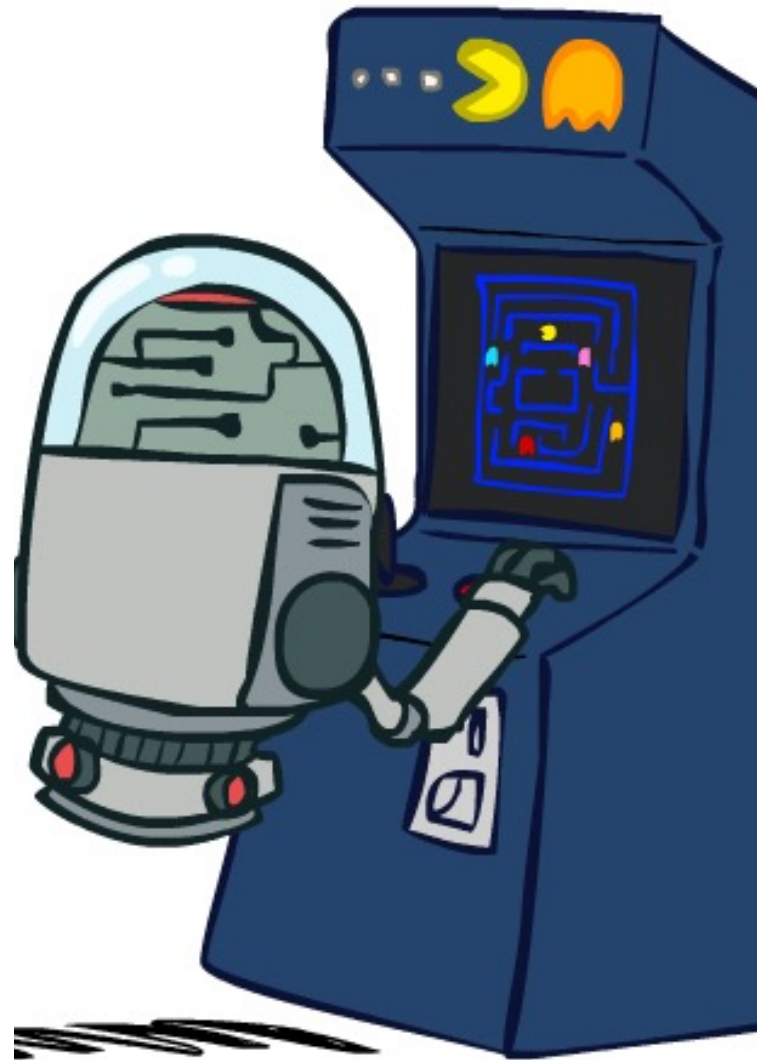


Game Playing State-of-the-Art

- ❖ Checkers: 1950: First computer player. 1994: First computer champion: Chinook ended 40-year-reign of human champion Marion Tinsley using complete 8-piece endgame. 2007: Checkers solved!
- ❖ Chess: 1997: Deep Blue defeats human champion Gary Kasparov in a six-game match. Deep Blue examined 200M positions per second, used very sophisticated evaluation and undisclosed methods for extending some lines of search up to 40 ply. Current programs are even better, if less historic.
- ❖ Go: 2016: Human champions are now losing against machines. In go, $b > 300$! Classic programs use pattern knowledge bases, but recent advances use Monte Carlo (randomized) expansion methods and learned evaluation function.
- ❖ Pacman

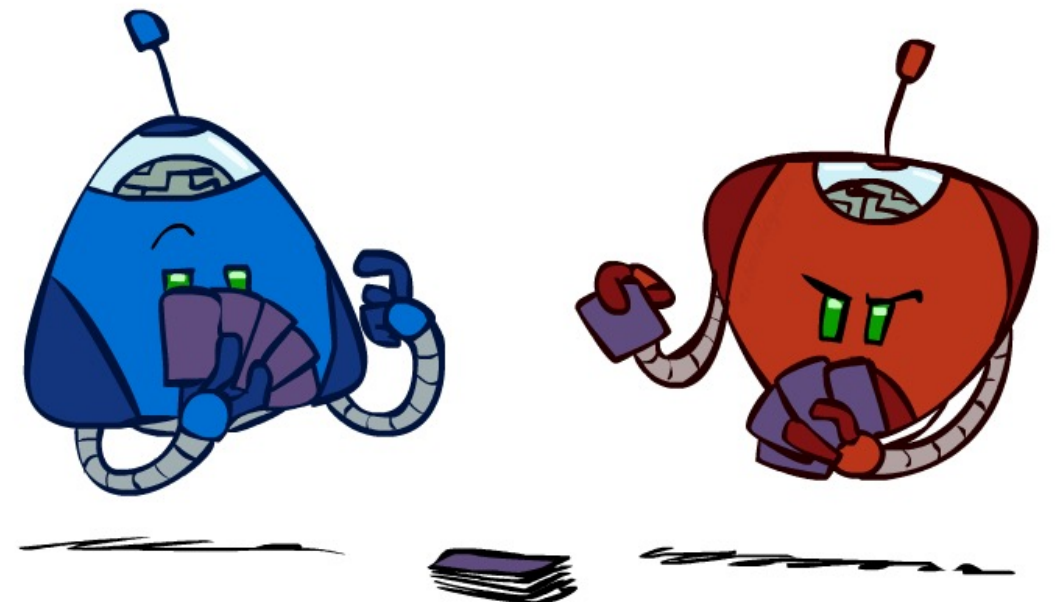
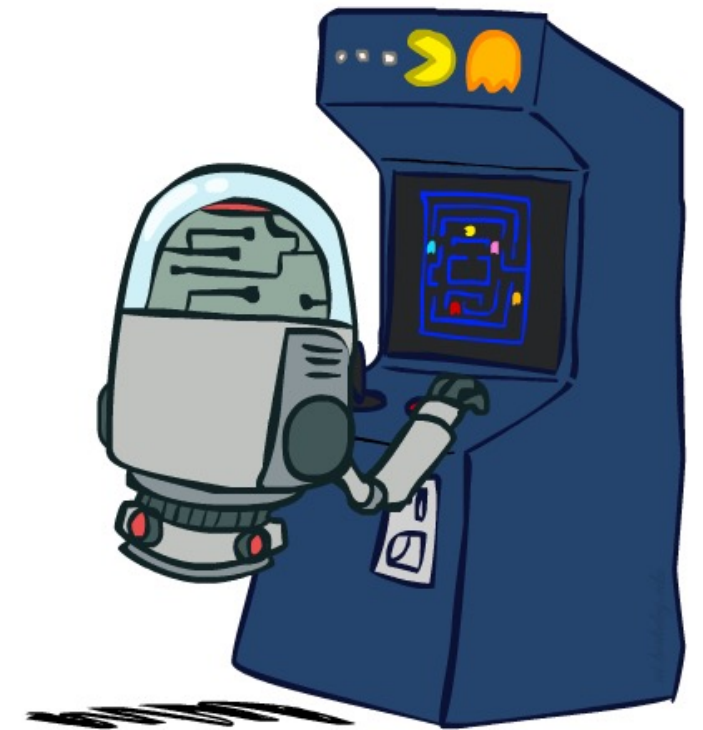


Game Formalization

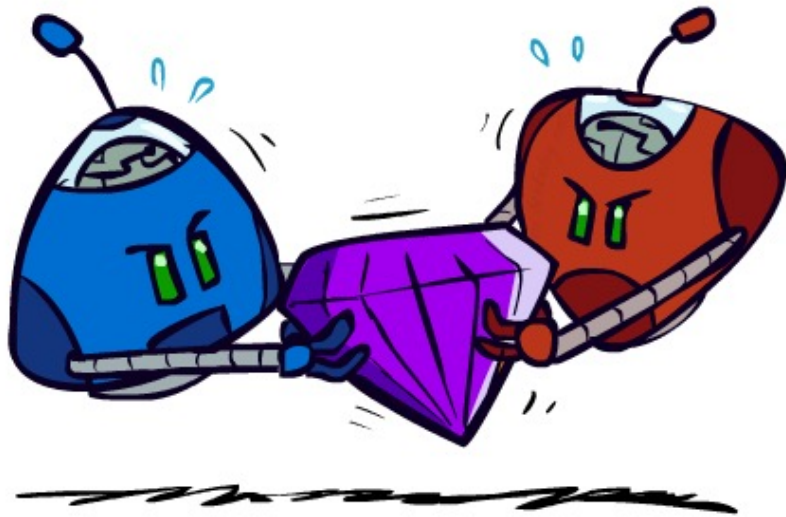


Games

- ❖ Many variety of games
- ❖ Properties:
 - ❖ Deterministic or stochastic?
 - ❖ Perfect information (fully observable)?
 - ❖ One, two, or more players?
 - ❖ Turn-taking or simultaneous?
 - ❖ Zero sum?
- ❖ Goal: which strategy to play?



Zero-Sum Games



Zero-Sum Games

- ❖ Agents have opposite utilities
- ❖ Pure competition:
 - ❖ One **maximizes**
 - ❖ The other **minimizes**



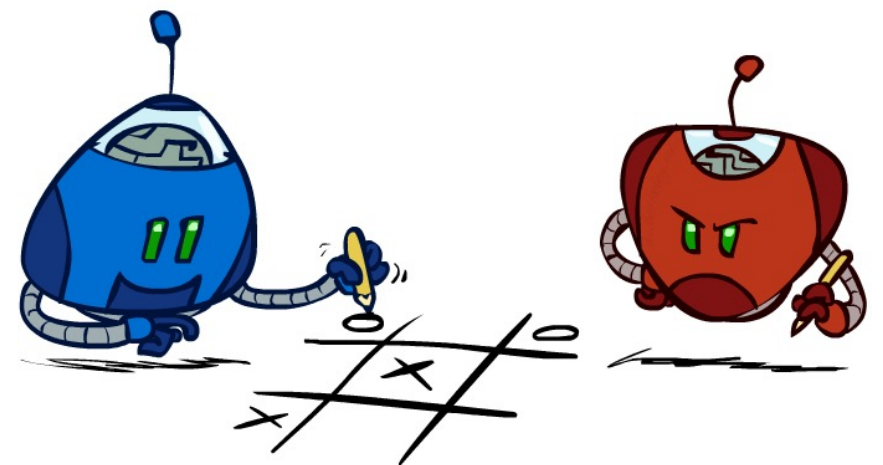
General Games

- ❖ Agents have **independent** utilities
- ❖ Cooperation, indifference, competition, shifting alliances, and more are all possible

Classic Games

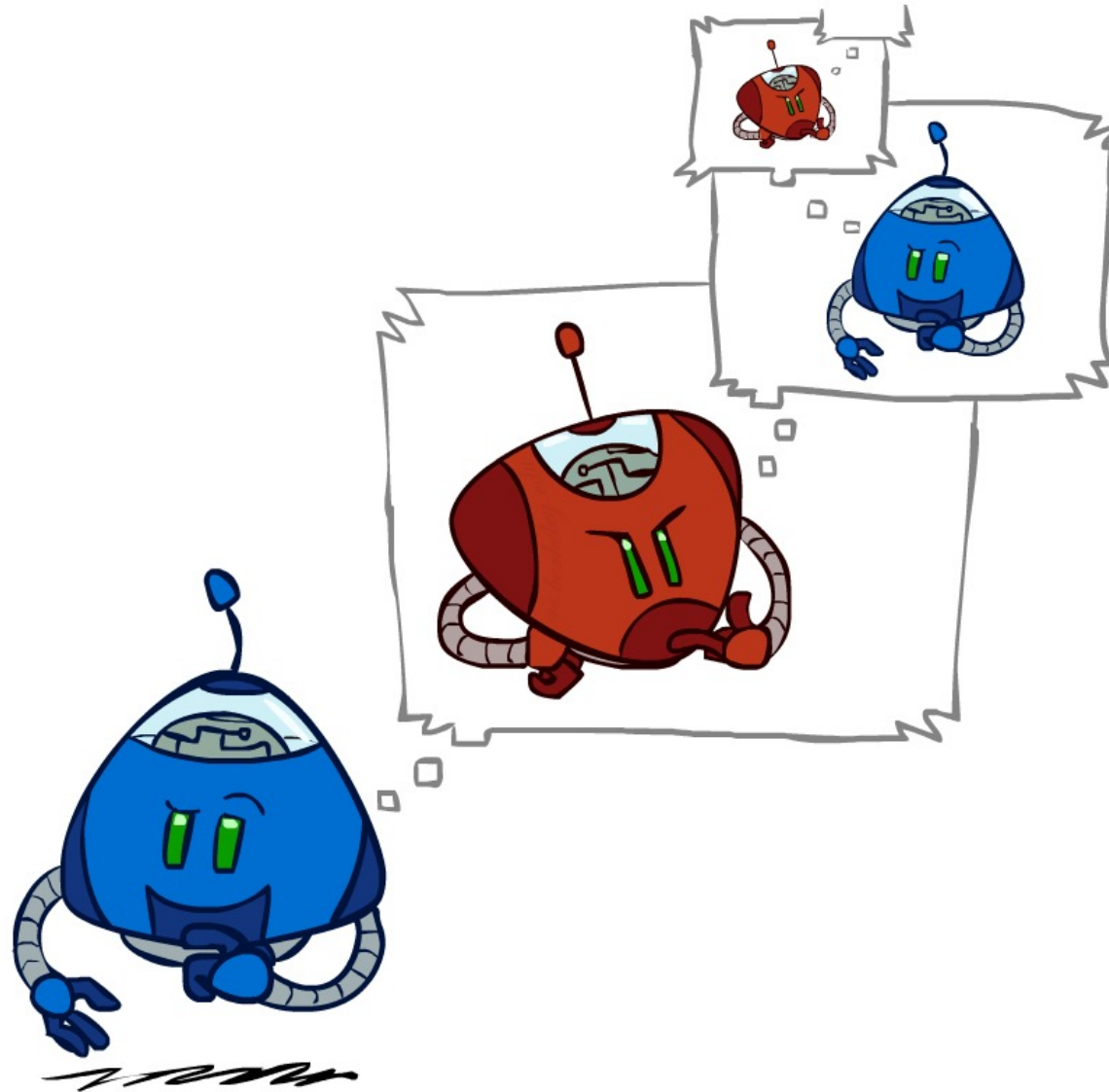
- ❖ Classic games are deterministic, observable, two-player, turn-taking, zero-sum
- ❖ Game formulation:
 - ❖ **Initial state and States:** $s_0 \in S$
 - ❖ **Players:** $Player(s) \in \{1, \dots, N\}$ indicates whose move it is
 - ❖ **Actions:** $a \in Action(s)$ for player on move
 - ❖ **Transition model:** $Succ(s, a) \in S$
 - ❖ **Terminal test:** $Terminal-Test(s)$ boolean
 - ❖ **Terminal values:** $Utility(s, p)$ for player p

Or just $Utility(s)$ for player making the decision at root
 - ❖ **Solution for a player:** strategy or policy defines how to choose an action in a state.



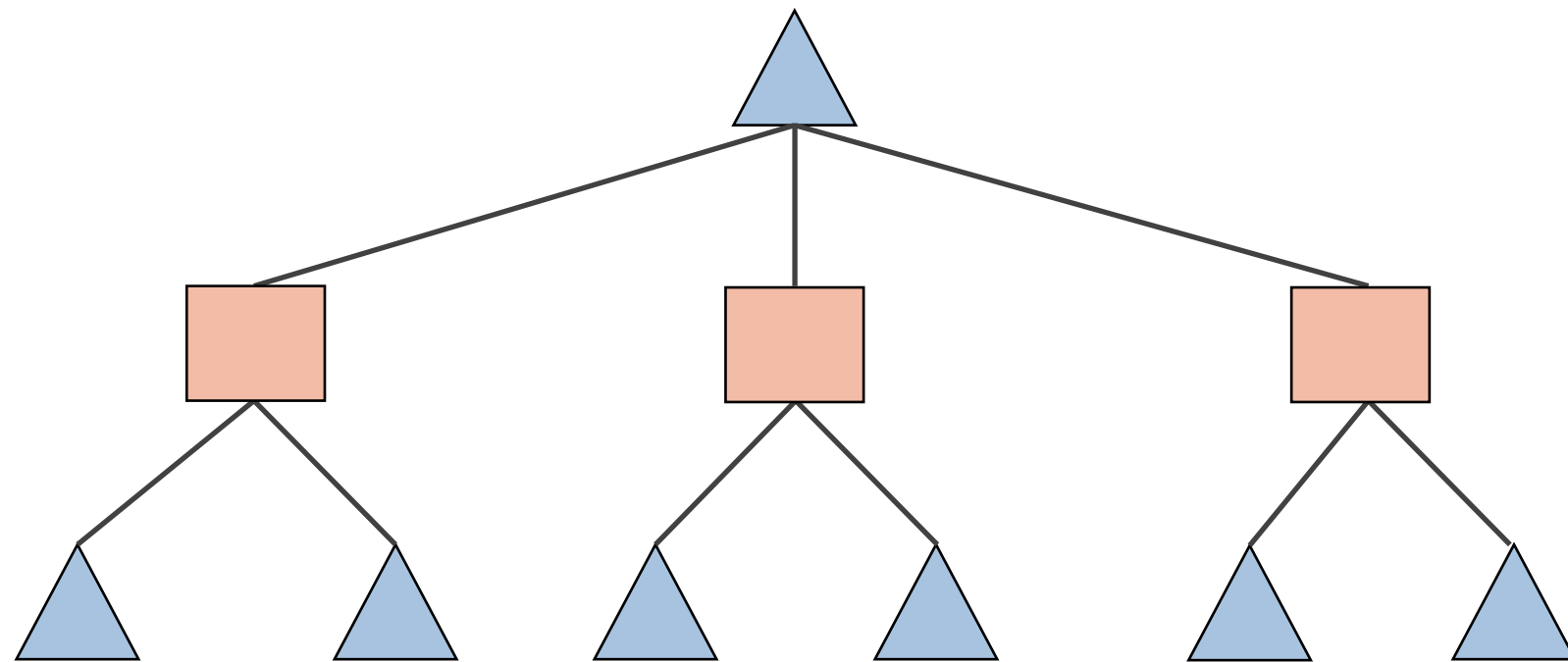
Games with an Adversary

Adversarial Search



Game Trees

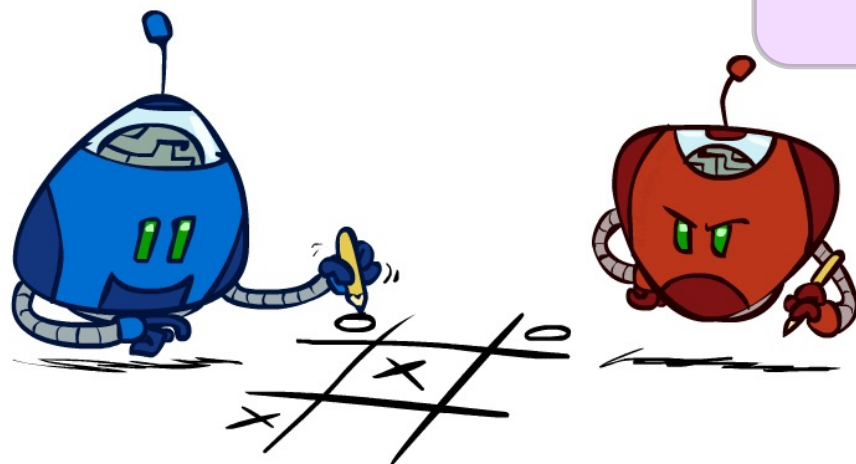
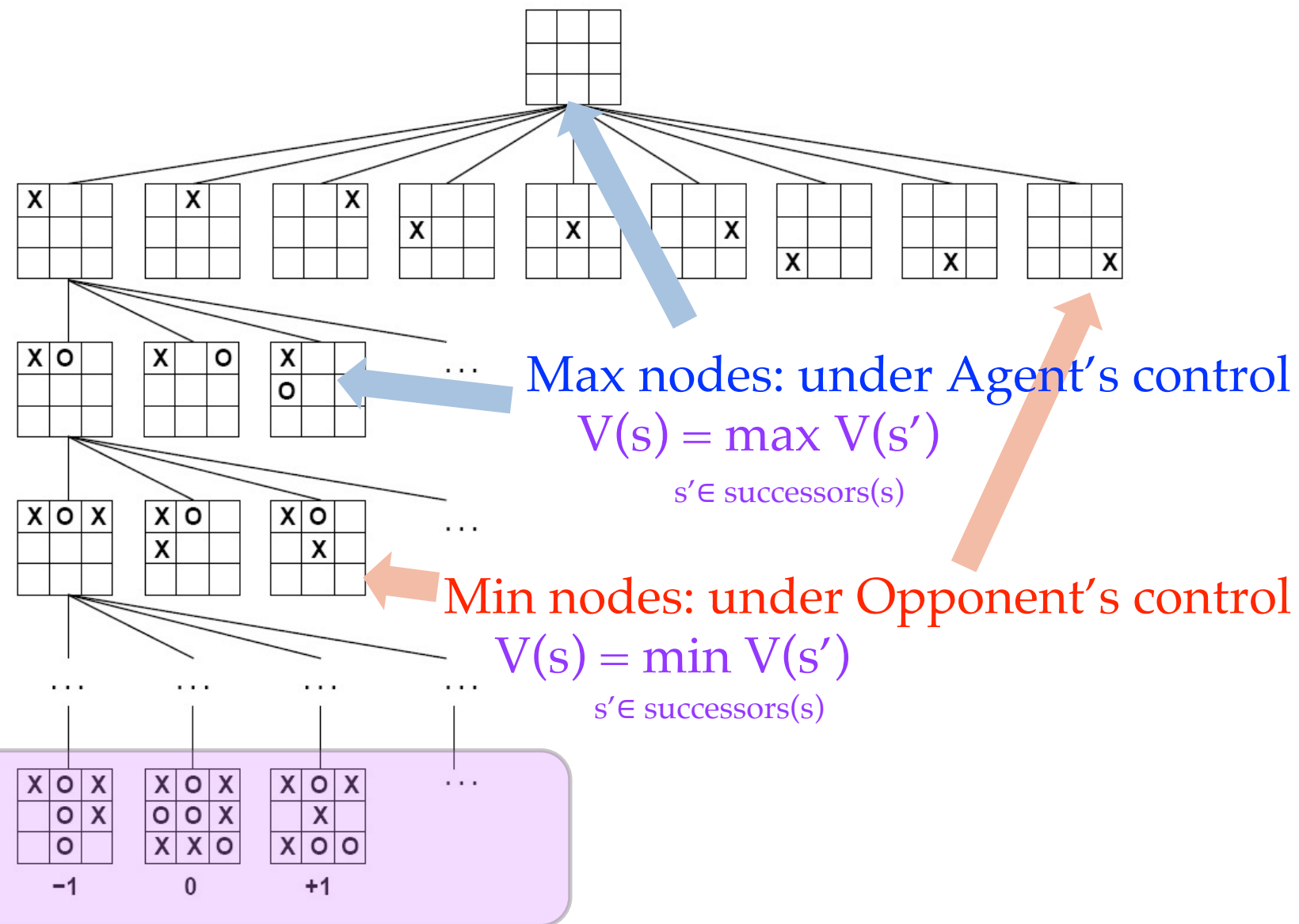
- ❖ Search one agent's actions from a state, search the competitor's actions from those resulting states , etc...



Game Tree of Tic-Tac-Toe

This is a zero-sum game,
the best action for X is
the worst action for O
and vice versa
How do we define
best and worst?

Instead of taking the
max utility at every
level, alternate max
and min



Example: Small Pacman

Max nodes: under Agent's control

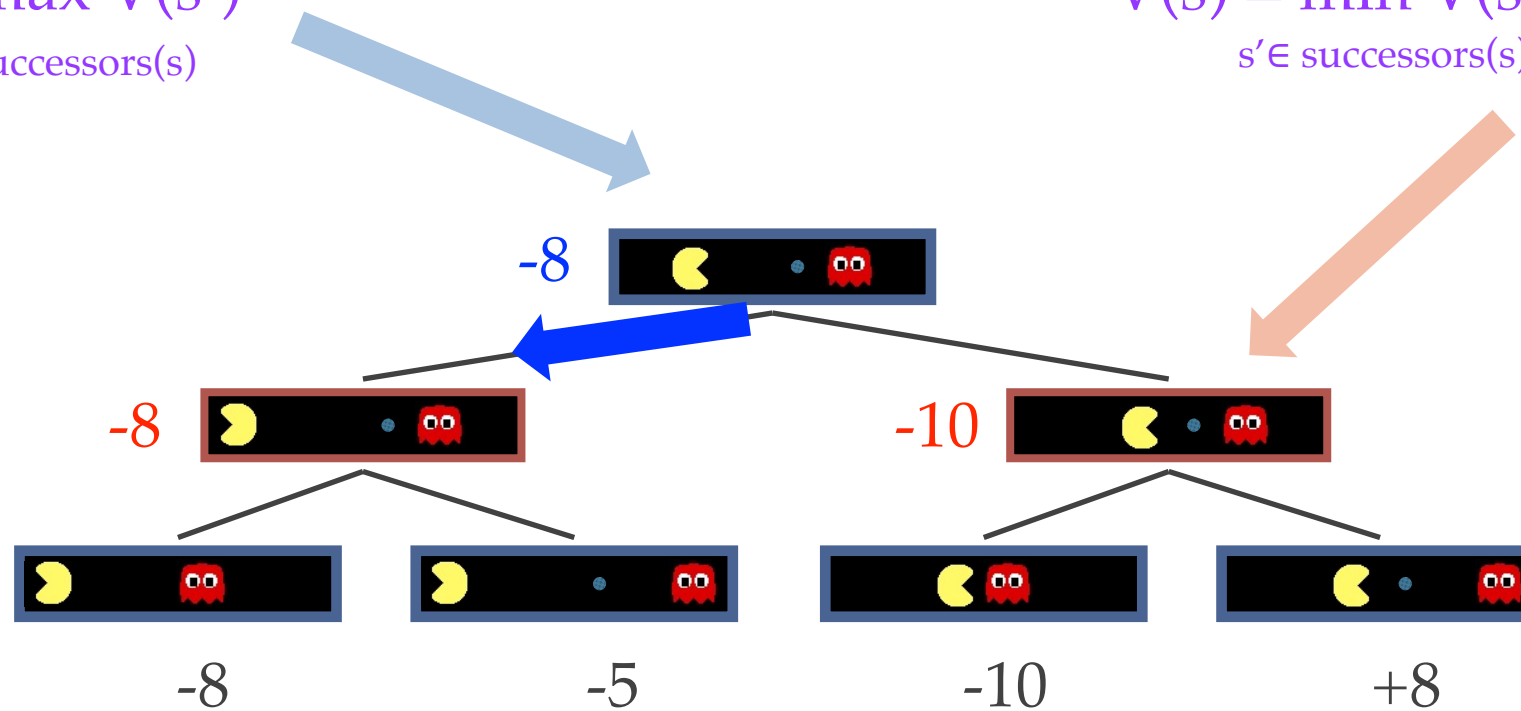
$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

$s' \in \text{successors}(s)$

Min nodes: under Opponent's control

$$V(s) = \min_{s' \in \text{successors}(s)} V(s')$$

$s' \in \text{successors}(s)$



Terminal States:

$$V(s) = \text{known}$$

Adversarial Search with Minimax

- ❖ Deterministic, zero-sum games:
 - ❖ Tic-tac-toe, chess, checkers
 - ❖ One player maximizes result
 - ❖ The other minimizes result
- ❖ Minimax search:
 - ❖ State-space search tree
 - ❖ Players alternate turns
 - ❖ Computes each node's **minimax value**: best achievable utility against rational (optimal) adversary

Minimax Implementation

```
function minimax-decision(s) returns action
  return an action a in Actions(s) with the highest
    min-value(Succ(s,a))
```

$\text{Succ}(s,a) \rightarrow s'$

```
function max-value(s) returns value
  if Terminal-Test(s) then return Utility(s)
  initialize v =  $-\infty$ 
  for each a in Actions(s):
    v = max(v, min-value(Succ(s,a)))
  return v
```

```
function min-value(s) returns value
  if Terminal-Test(s) then return Utility(s)
  initialize v =  $+\infty$ 
  for each a in Actions(s):
    v = min(v, max-value(Succ(s,a)))
  return v
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Max nodes: under Agent's control

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

$s' \in \text{successors}(s)$

Min nodes: under Opponent's control

$$V(s) = \min_{s' \in \text{successors}(s)} V(s')$$

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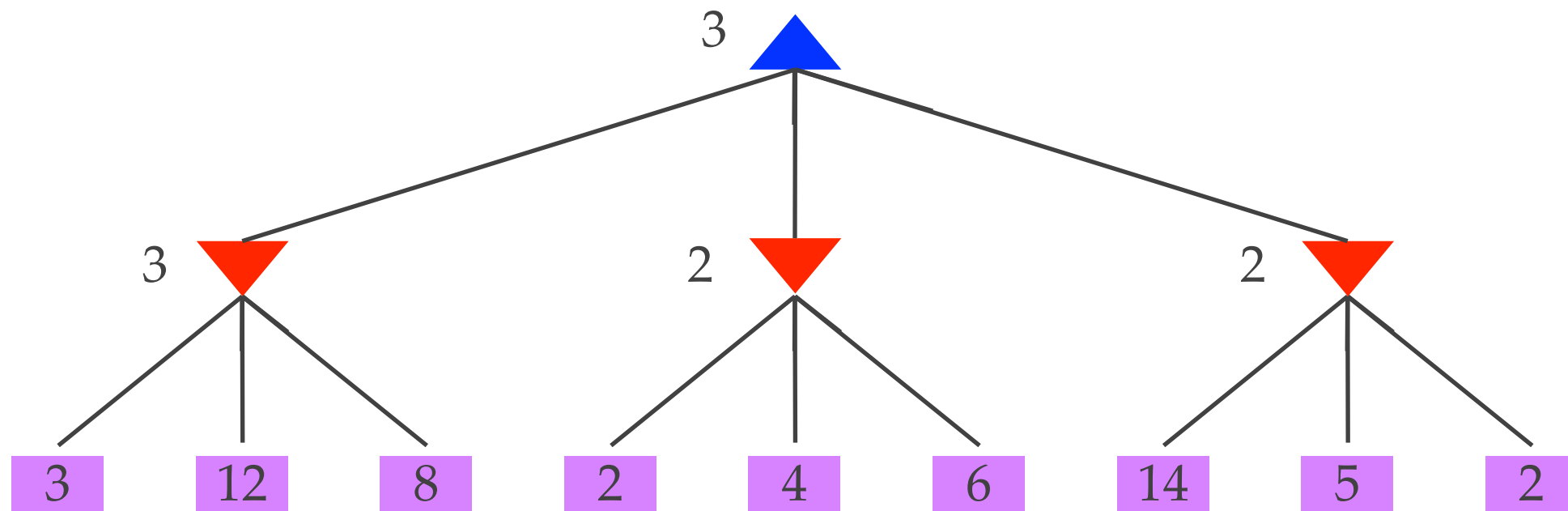
Alternative Implementation

```
function minimax-decision(s) returns an action  
  return an action a in Actions(s) with the highest  
    value(Succ(s,a))
```



```
function value(s) returns a value  
  if Terminal-Test(s) then return Utility(s)  
  if Player(s) = MAX then return  $\max_{a \in \text{Action}(s)}$  value(Succ(s,a))  
  if Player(s) = MIN then return  $\min_{a \in \text{Action}(s)}$  value(Succ(s,a))
```

Minimax Example

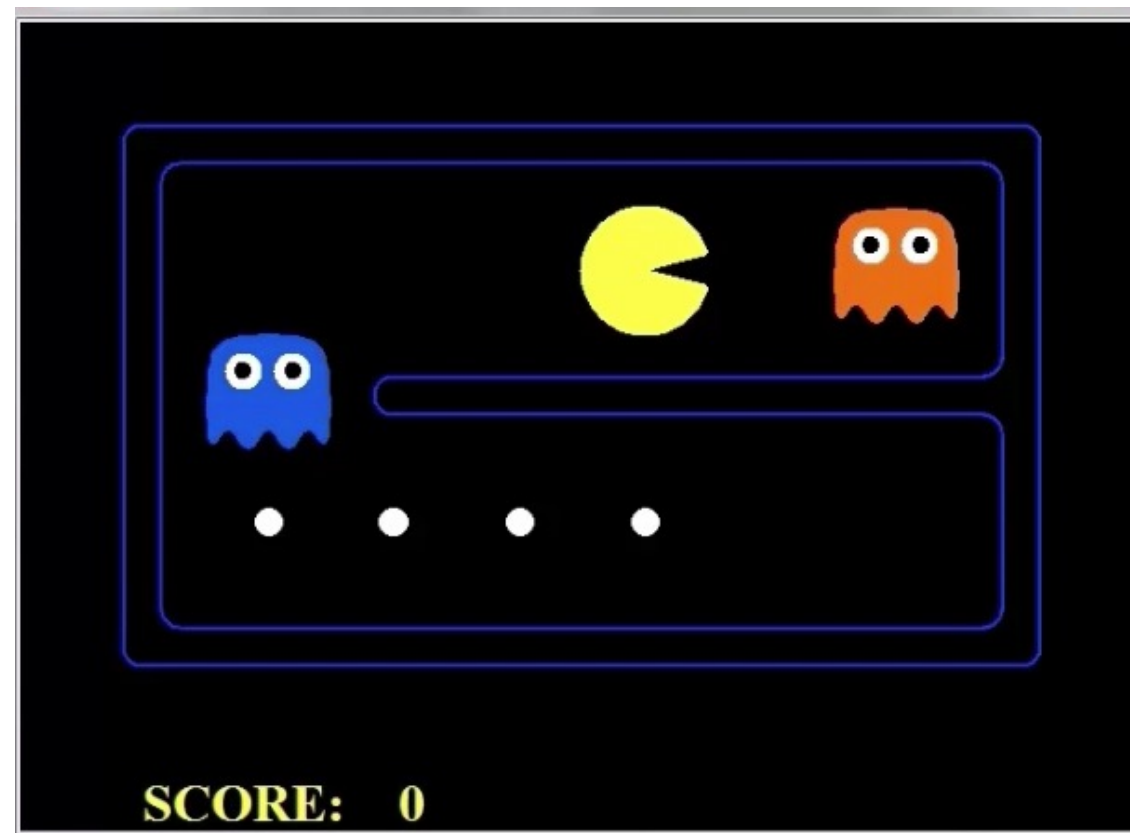


- ❖ What kind of search is Minimax Search?

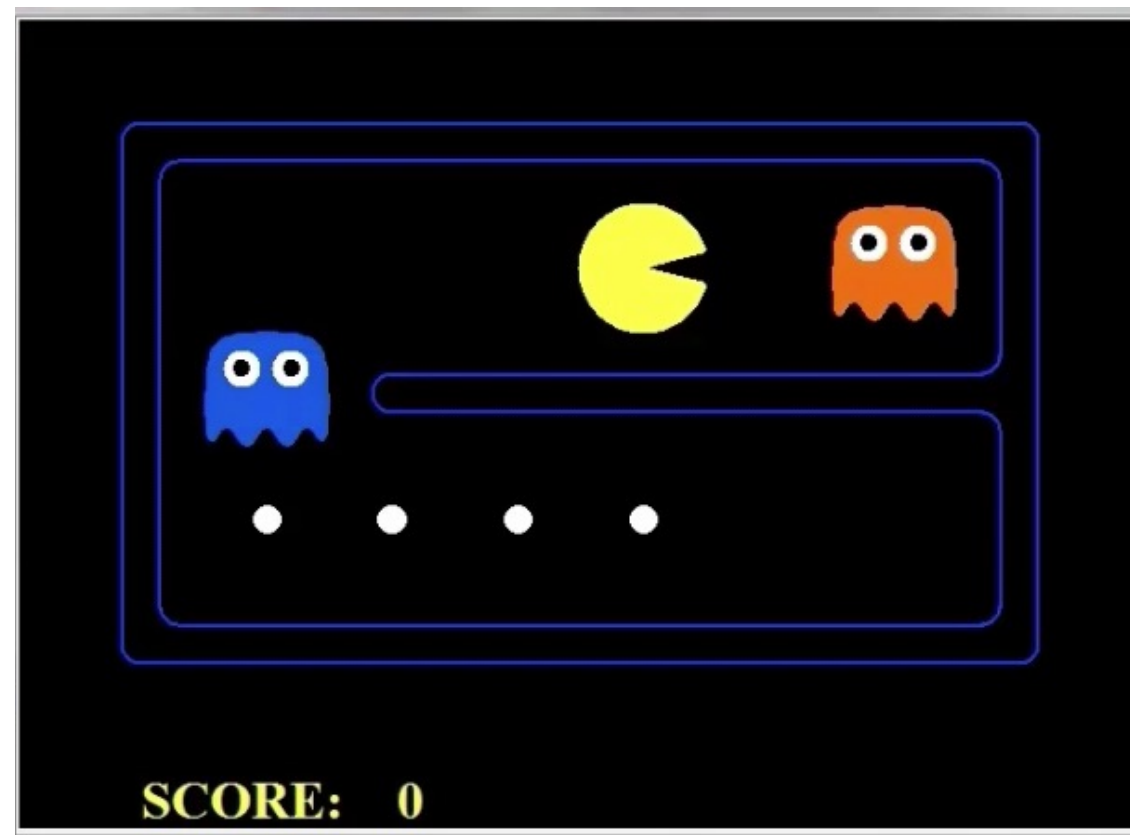
Minimax Properties

- ❖ Multi-agent decision-making requires a model of other agents
- ❖ Here, adversary assumed to be rational
- ❖ Minimax is optimal against perfect opponent
- ❖ Is Minimax still optimal otherwise?

Example: Adversarial Opponents

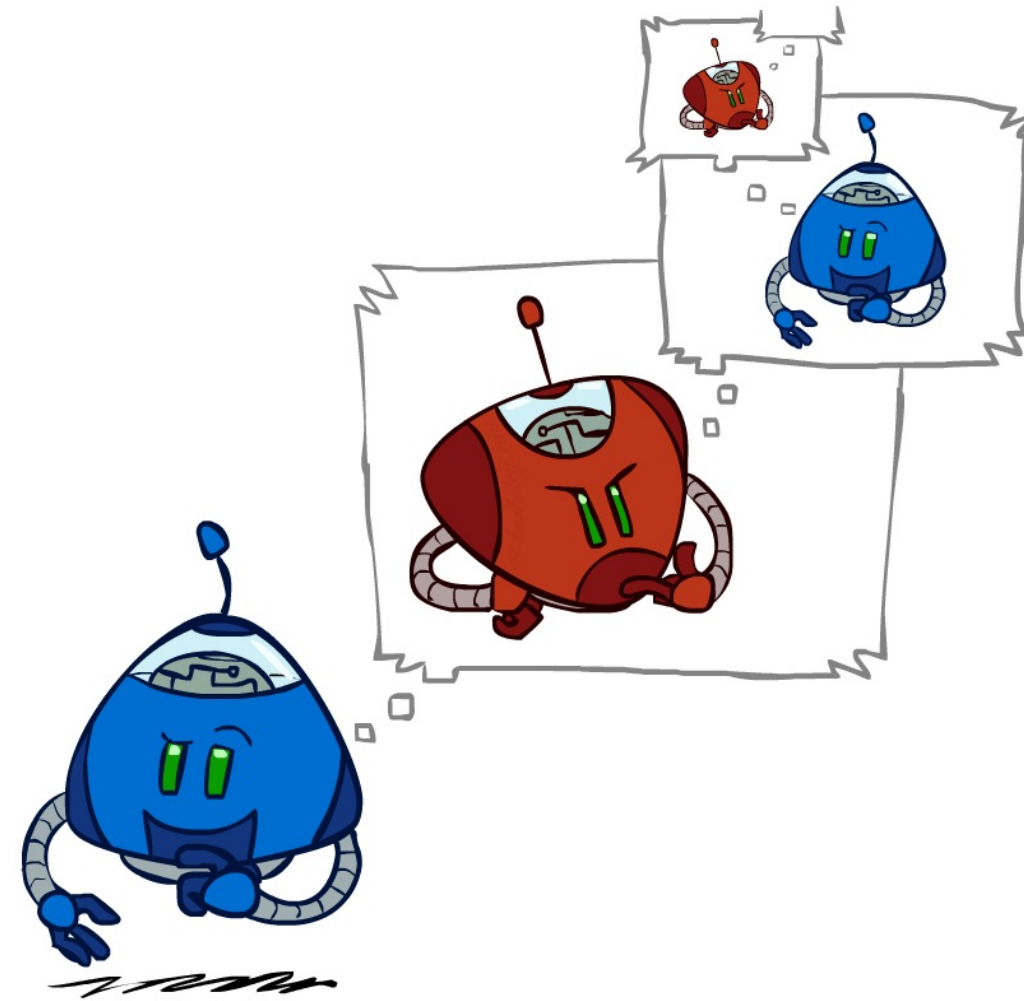


Example: Random Opponents



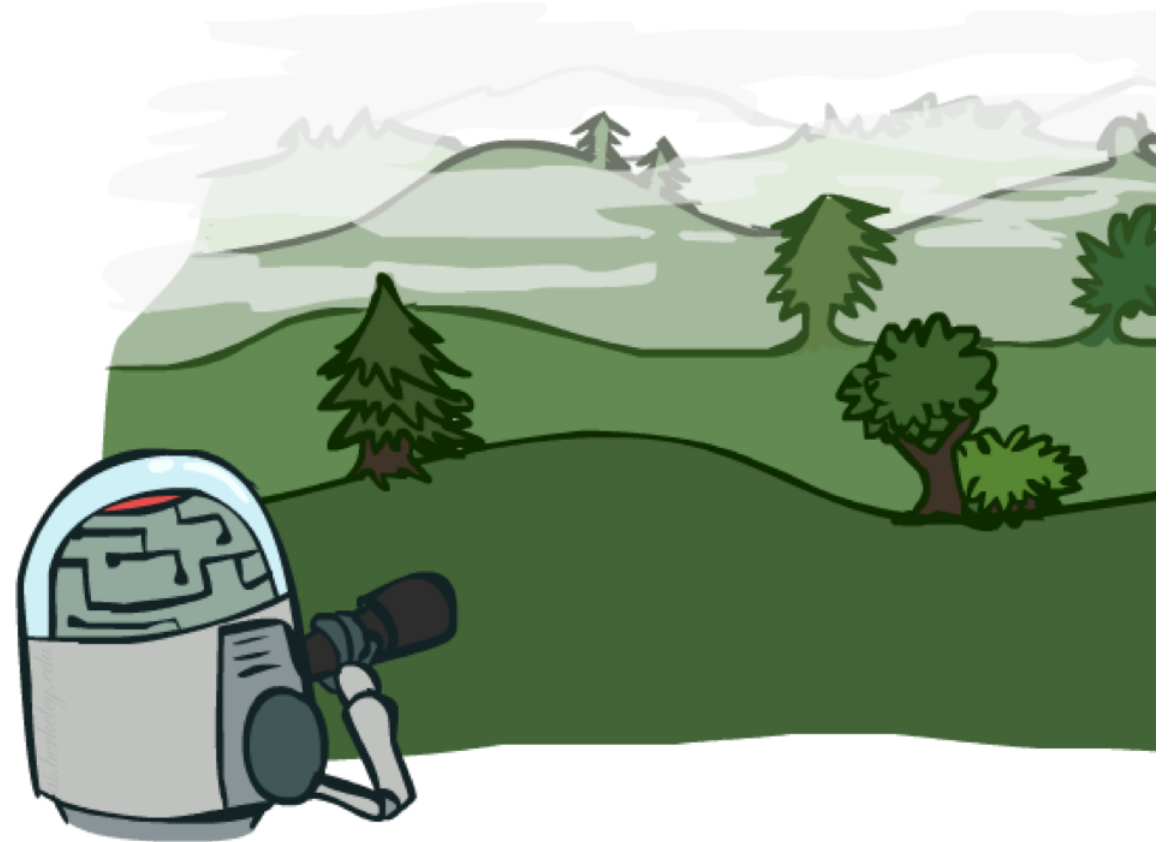
Minimax Efficiency

- ❖ How efficient is minimax?
 - ❖ Just like (exhaustive) DFS
 - ❖ Time: $O(b^m)$
 - ❖ Space: $O(bm)$
- ❖ E.g., for chess, $b \approx 35$, $m \approx 100$
 - ❖ Exact solution is completely infeasible
 - ❖ Humans can't do this either, so how do we play chess?



Resource Limits

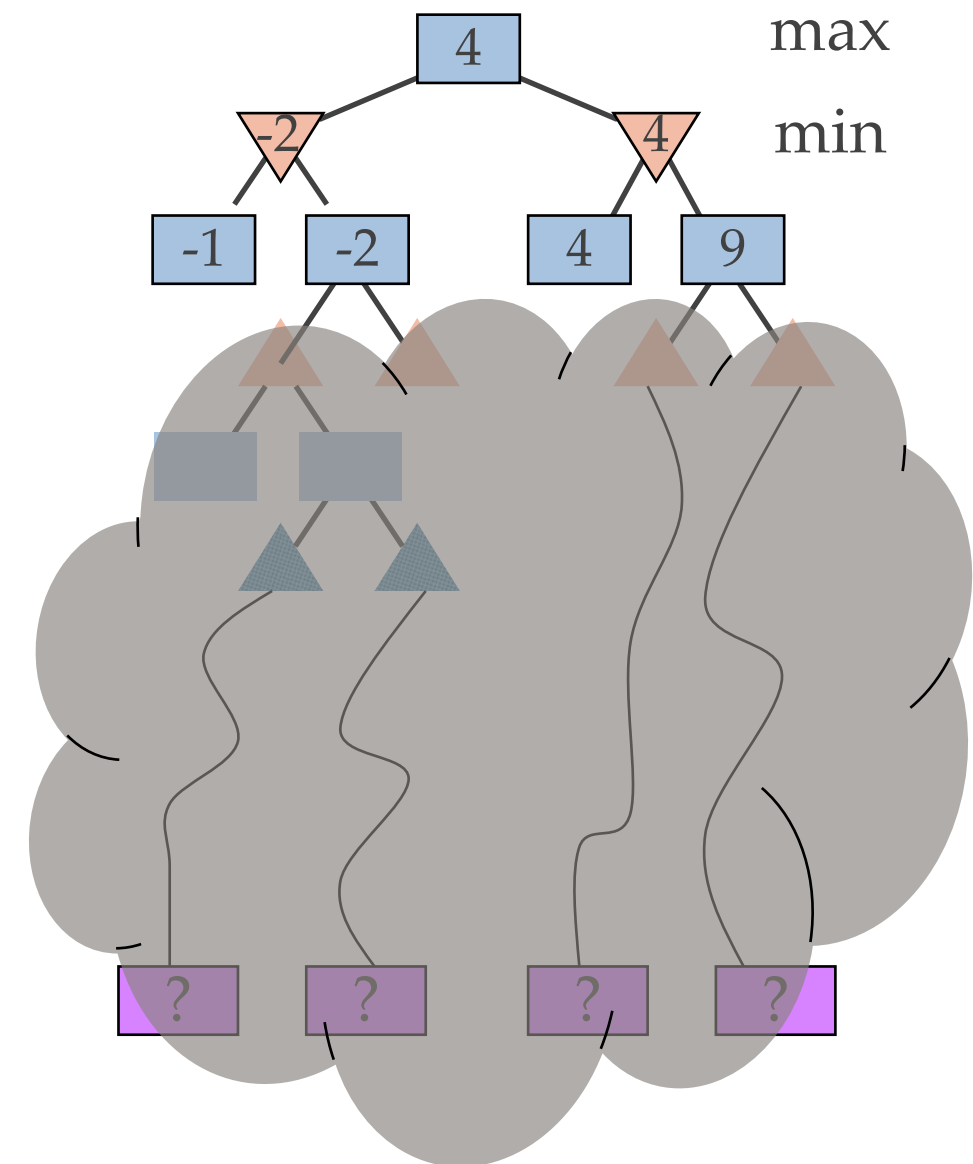
Problem: In realistic games, we cannot search to leaves!



Solution 1: Bounded lookahead

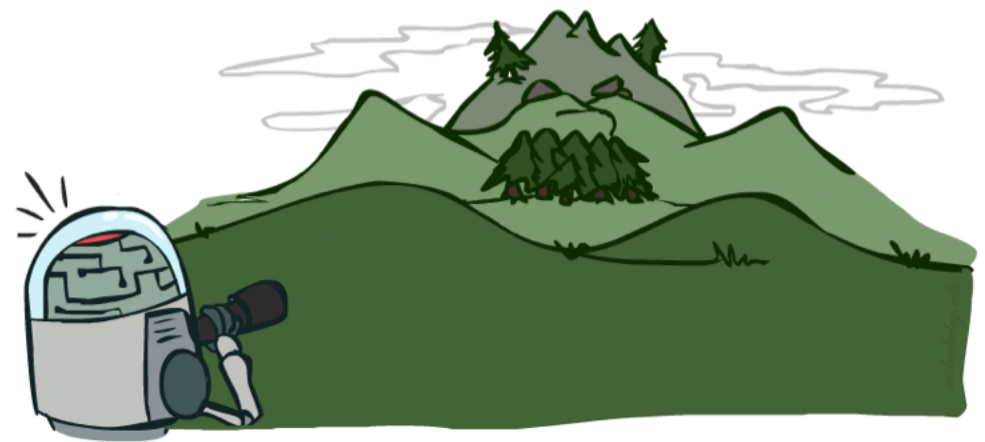
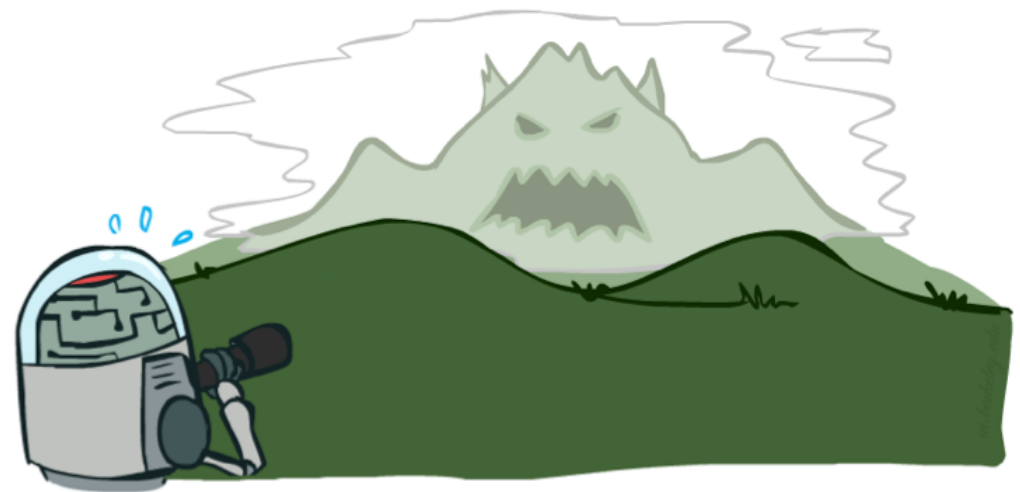
❖ Idea:

- ❖ Search only to a preset **depth limit** or **horizon**
- ❖ Use an **evaluation function** for non-terminal positions
- ❖ Guarantee of optimal play is gone
- ❖ More plies make a BIG difference
- ❖ Use iterative deepening for an anytime algorithm
- ❖ Example:
 - ❖ Suppose we have 100 seconds, can explore 10K nodes / sec
 - ❖ So can check 1M nodes per move
 - ❖ For chess, $b \approx 35$ so reaches about depth 4 – not so good



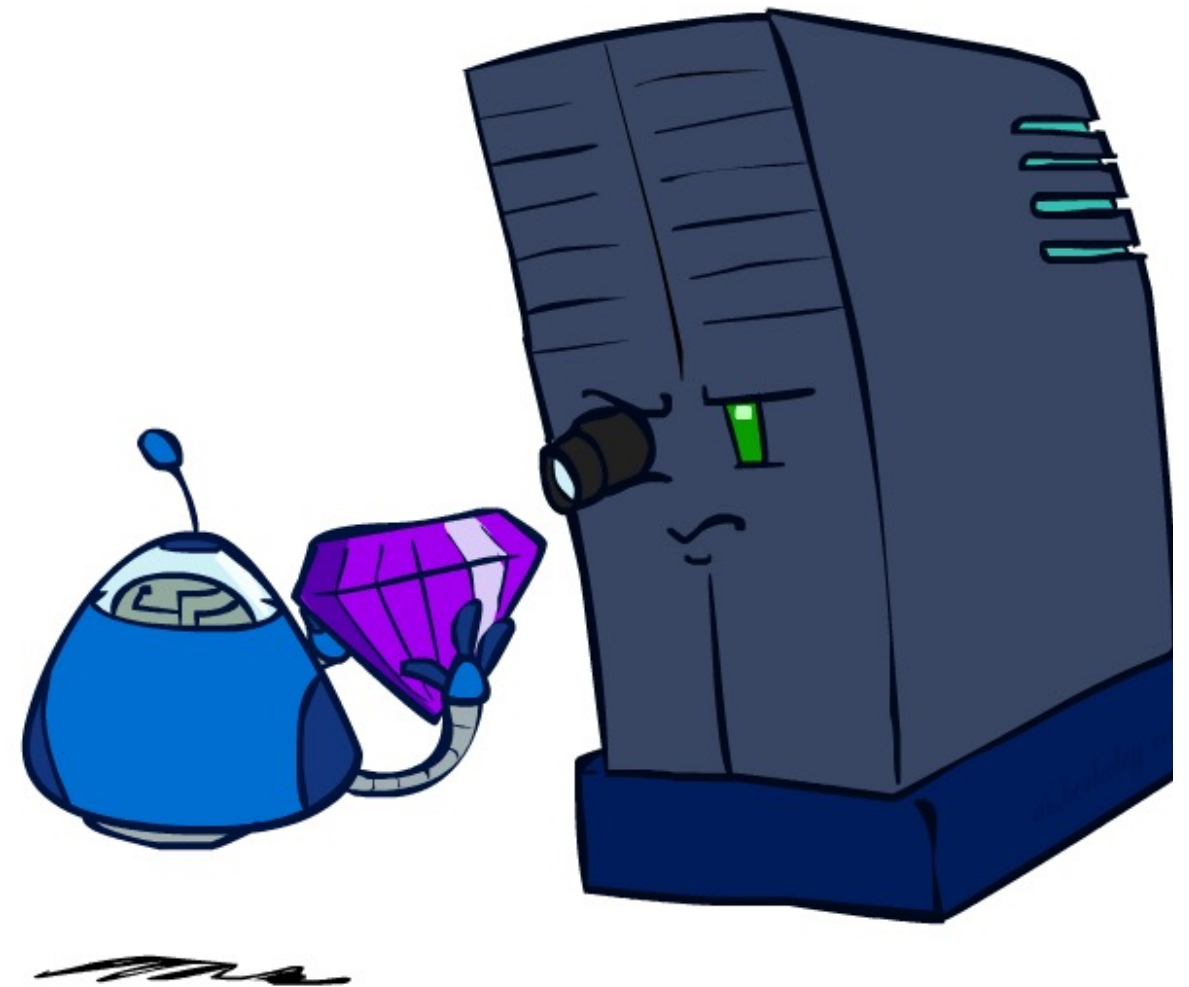
Depth Matters

- ❖ Evaluation functions are always imperfect
- ❖ Deeper search => better play (usually)
- ❖ Deeper search gives same quality of play with a less accurate evaluation function
- ❖ An important example of the tradeoff between complexity of features and complexity of computation



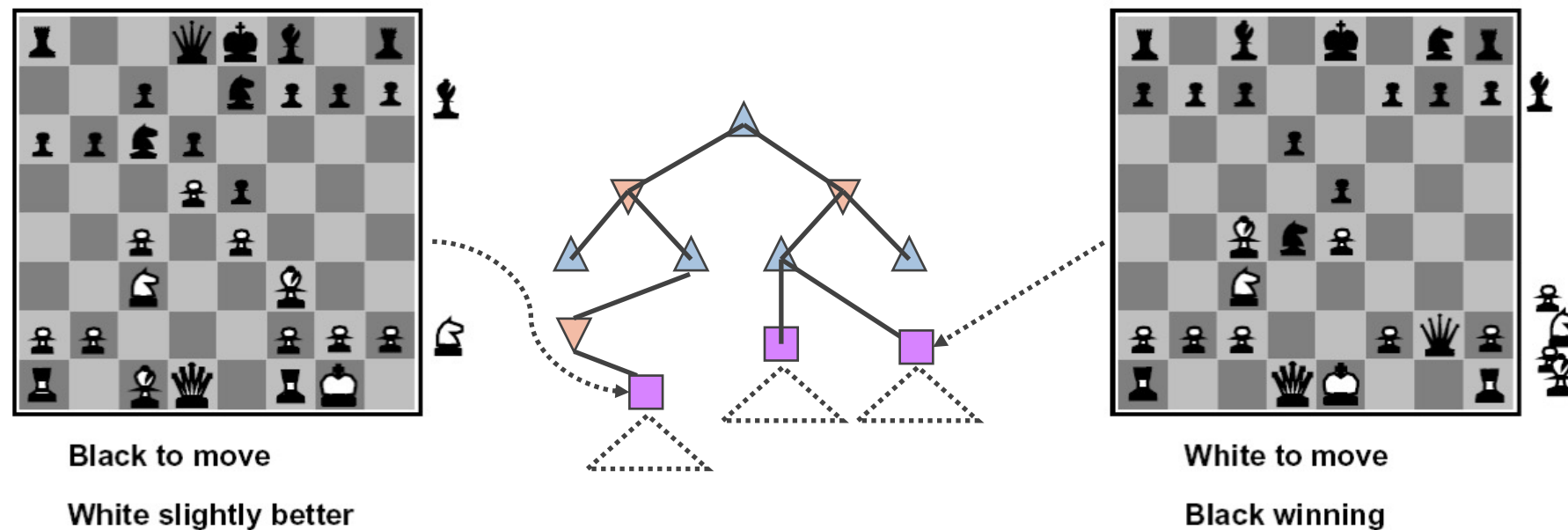
Adversarial Search

Evaluation Functions



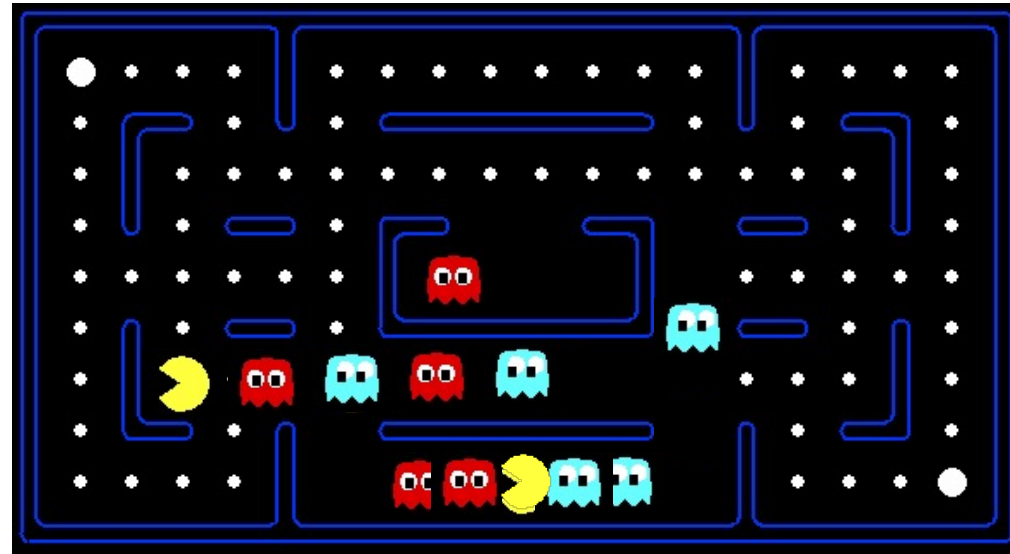
Evaluation Functions

- ❖ Evaluation functions score non-terminals in depth-limited search

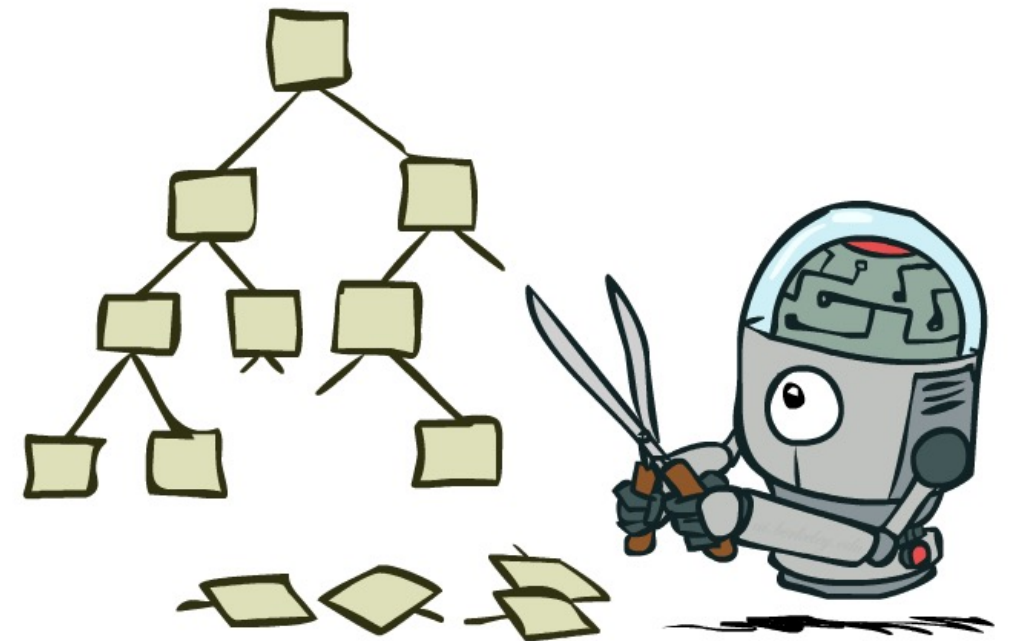


- ❖ Ideal function: returns the actual minimax value of the position
- ❖ In practice: typically weighted linear sum of features:
 - ❖ $EVAL(s) = w_1 f_1(s) + w_2 f_2(s) + \dots + w_n f_n(s)$
 - ❖ E.g., $w_1 = 9$, $f_1(s) = (\text{num white queens} - \text{num black queens})$, etc.
- ❖ Terminate search only in **quiescent** positions, i.e., no major changes expected in feature values

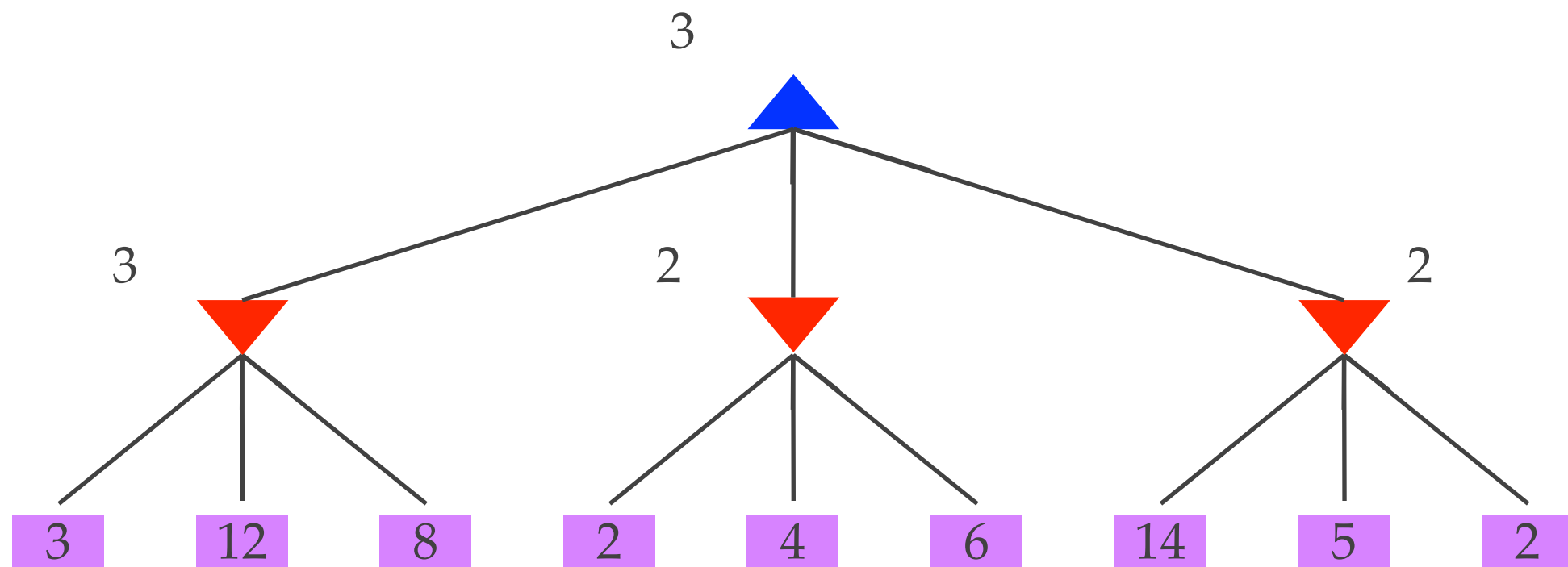
Evaluation for Pacman



Solution 2: Game Tree Pruning

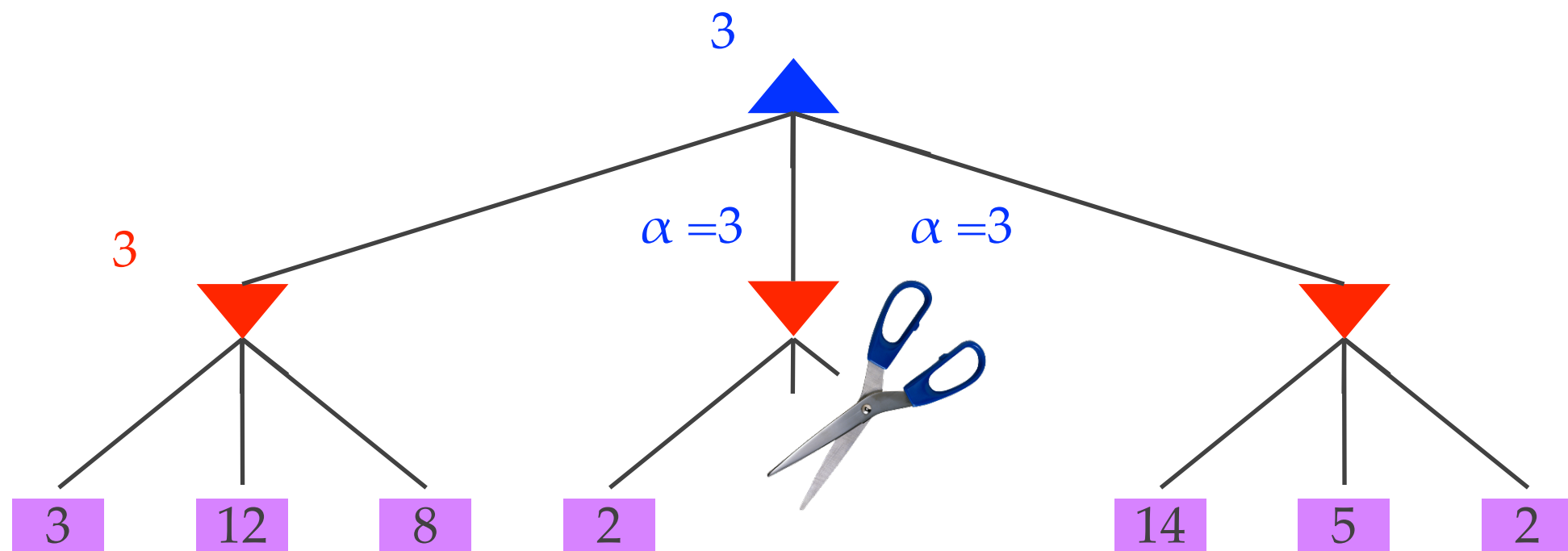


Intuition: prune the branches that can't be chosen



Alpha-Beta Pruning Example

α = best option so far from any
MAX node on this path

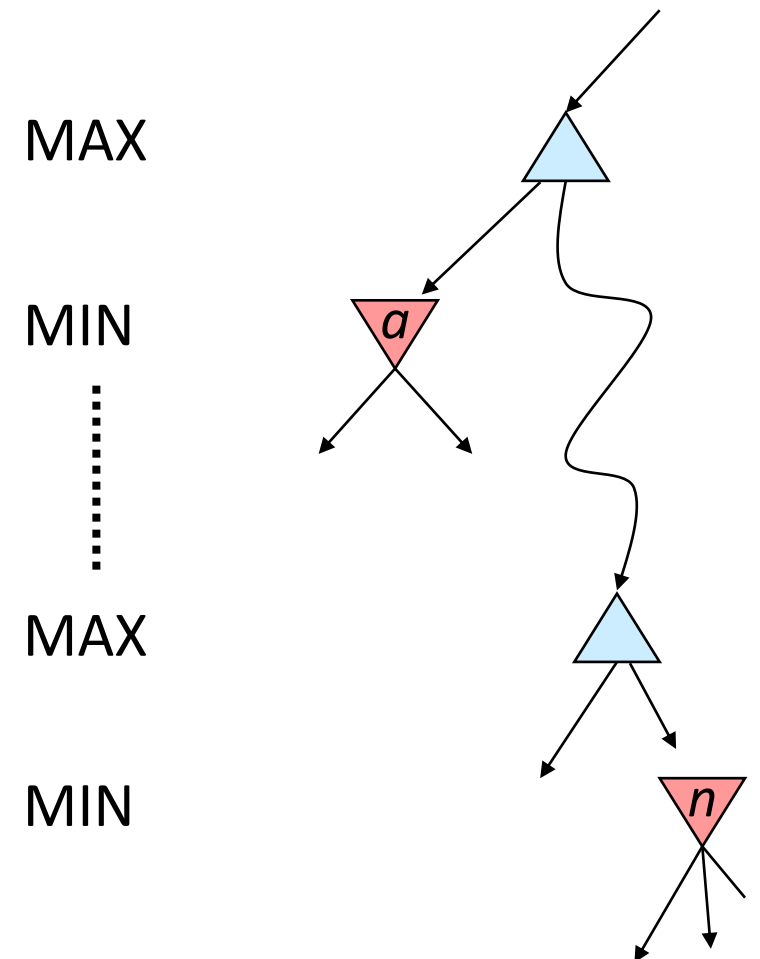


We can prune when: min node
won't be higher than 2, while
parent max has seen
something larger in another
branch

The order of generation matters: more pruning
is possible if good moves come first

Alpha-Beta Pruning

- ❖ General configuration (MIN version)
 - ❖ We're computing the MIN-VALUE at some node n
 - ❖ We're looping over n 's children
 - ❖ n 's estimate of the childrens' min is dropping
 - ❖ Who cares about n 's value? MAX
 - ❖ Let α be the best value that MAX can get at any choice point along the current path from the root
 - ❖ If n becomes worse than α , MAX will avoid it, so we can stop considering n 's other children (it's already bad enough that it won't be played)
- ❖ MAX version is symmetric



Alpha-Beta Implementation

α : MAX's best option on path to root
 β : MIN's best option on path to root

def max-value(state, α , β):

 initialize $v = -\infty$

 for each successor of state:

$v = \max(v, \text{min-value}(\text{successor}, \alpha, \beta))$

 if $v \geq \beta$ return v

$\alpha = \max(\alpha, v)$

 return v

def min-value(state, α , β):

 initialize $v = +\infty$

 for each successor of state:

$v = \min(v, \text{max-value}(\text{successor}, \alpha, \beta))$

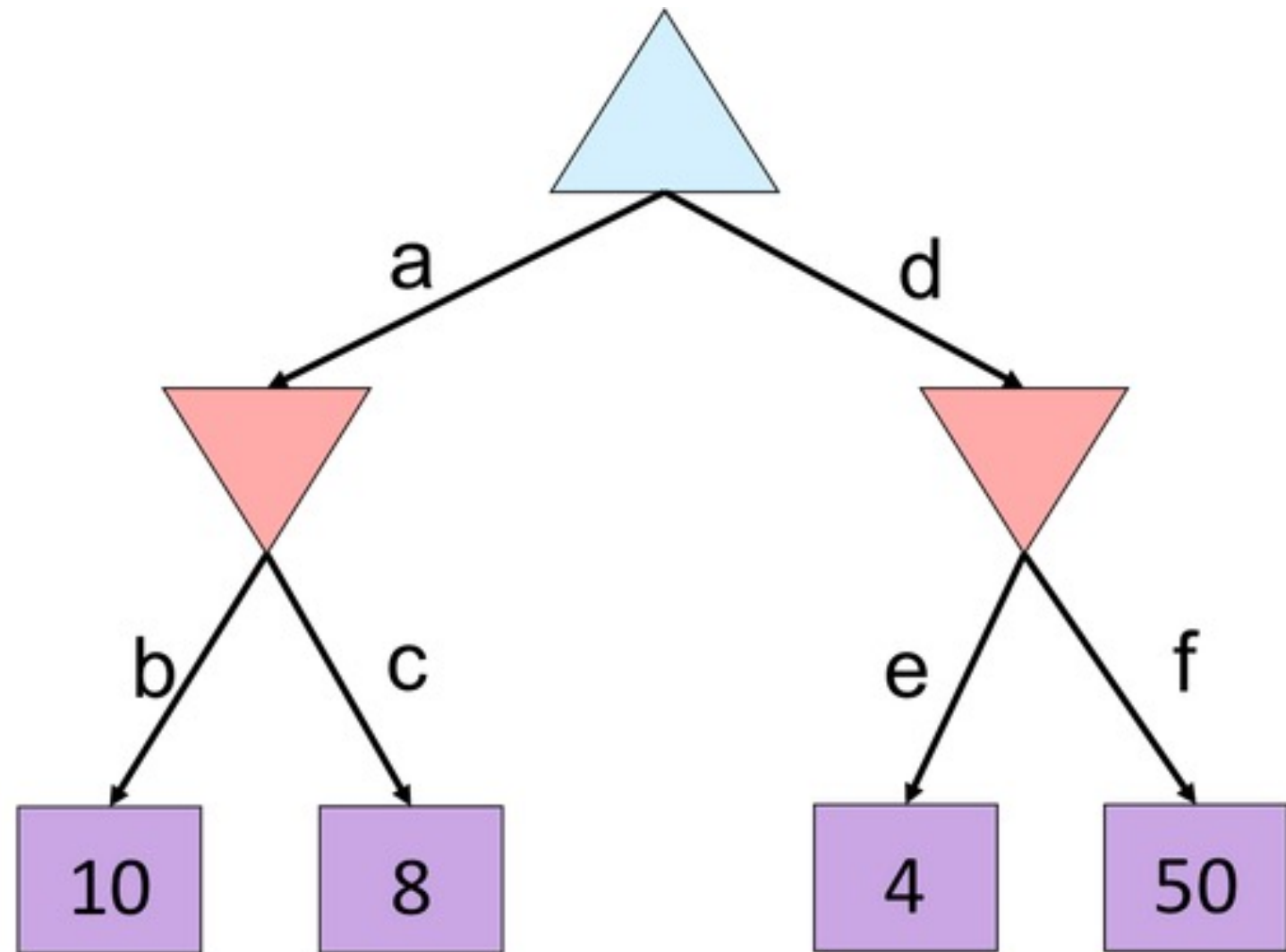
 if $v \leq \alpha$ return v

$\beta = \min(\beta, v)$

 return v

Minimax Example

- ❖ What is the value of the blue triangle?

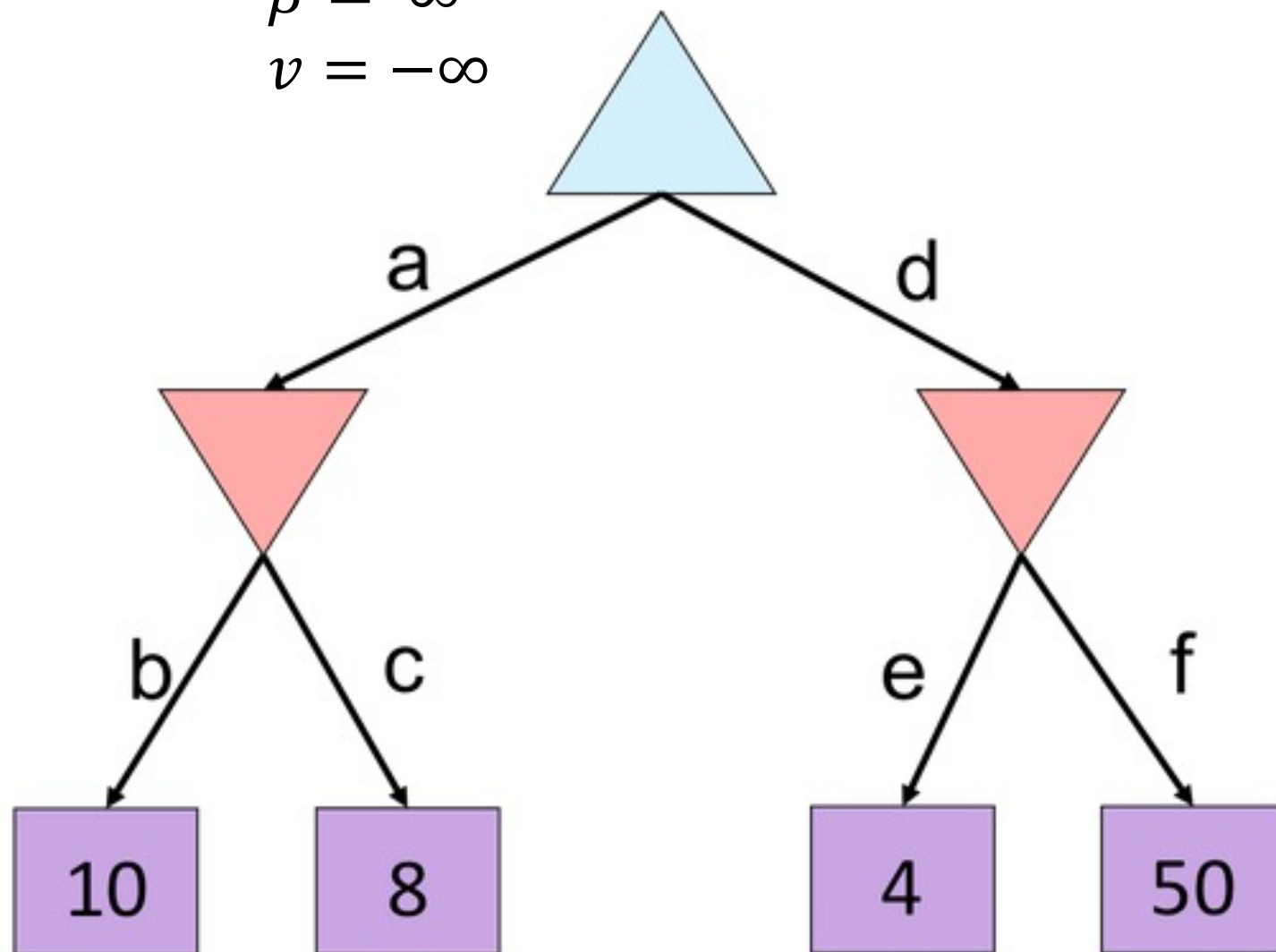


Alpha-Beta Small Example

$$\alpha = -\infty$$

$$\beta = \infty$$

$$v = -\infty$$



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def max-value(state,  $\alpha$ ,  $\beta$ ):
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        if  $v \geq \beta$  return  $v$ 
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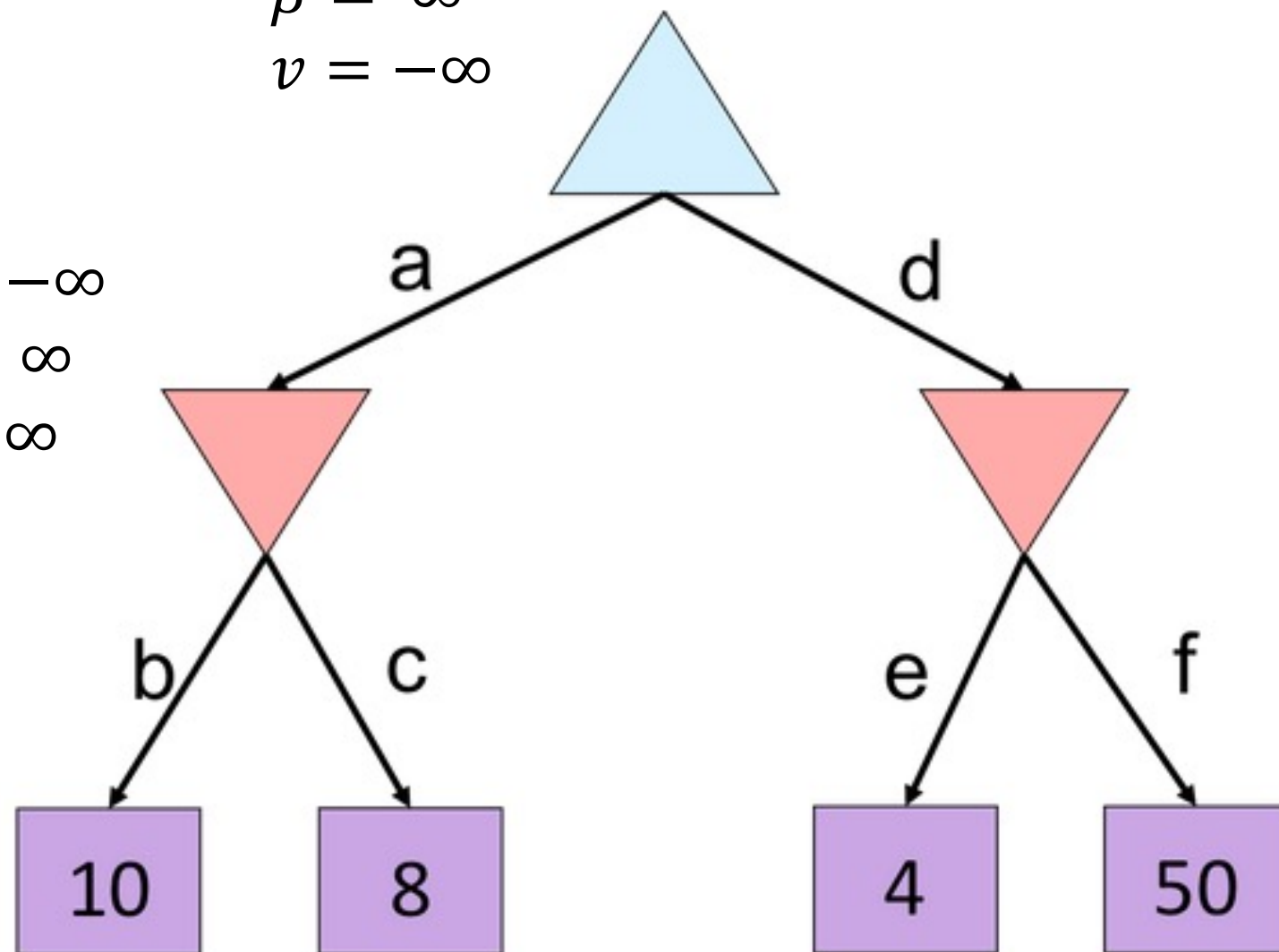
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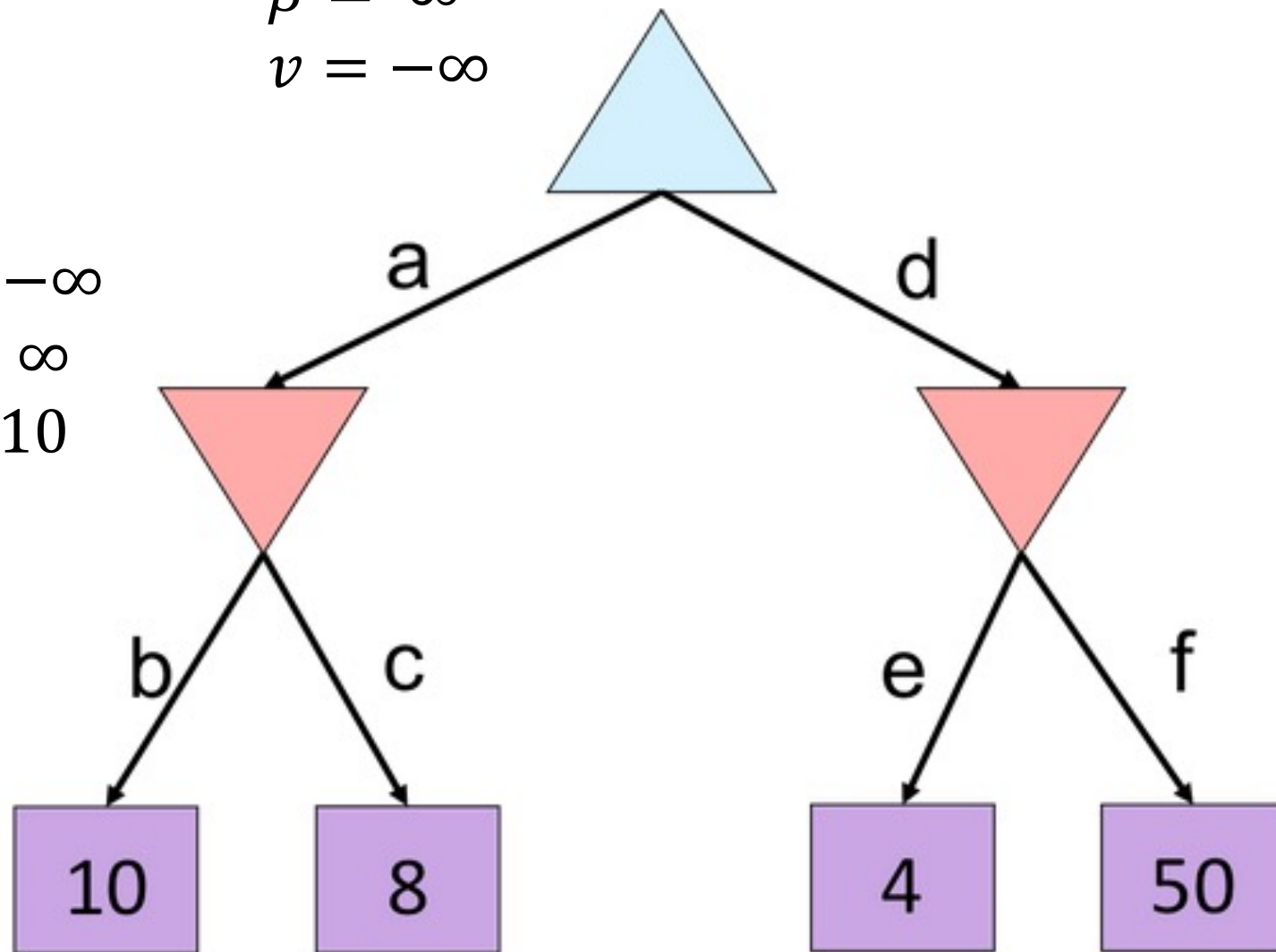
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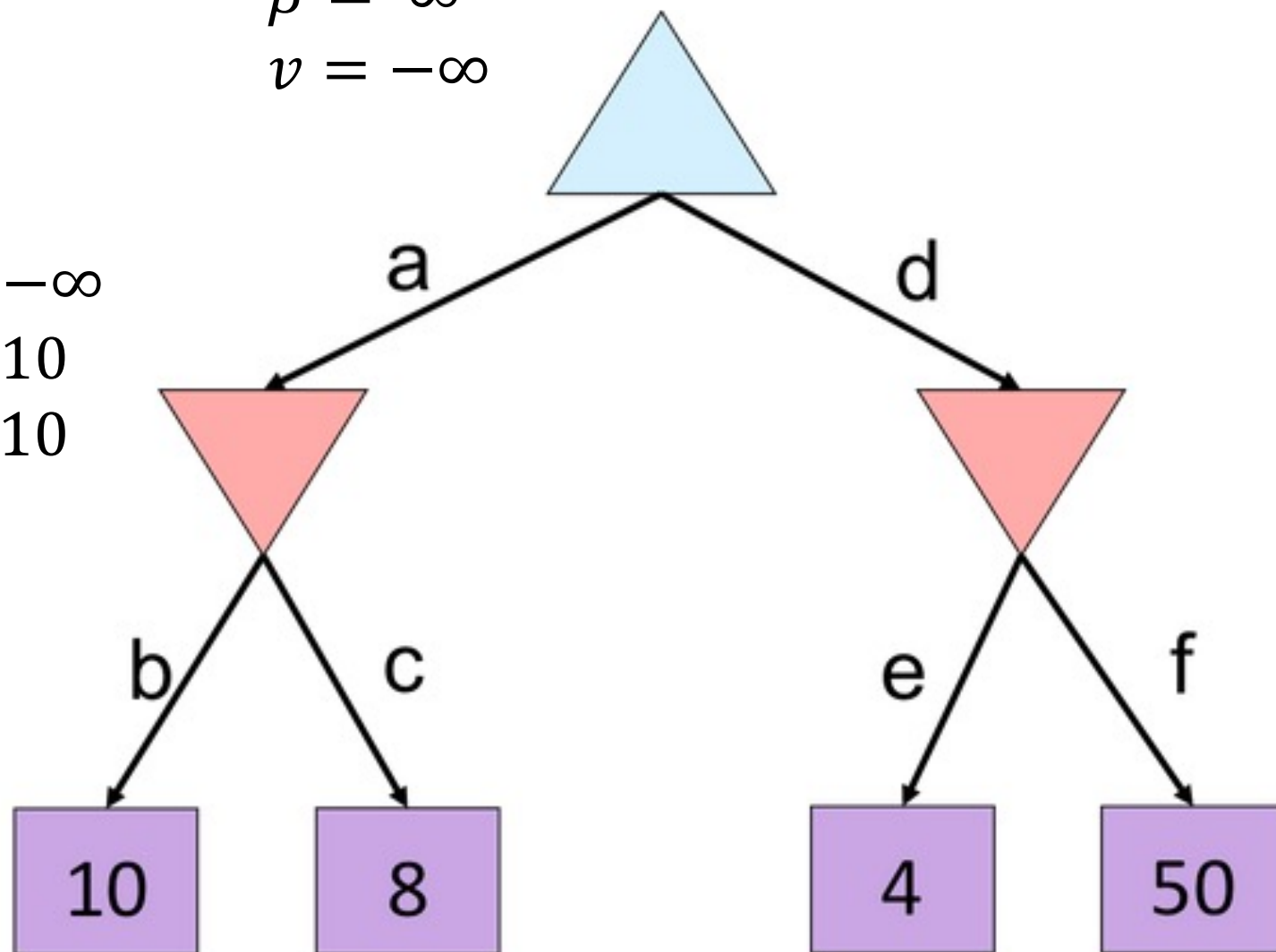
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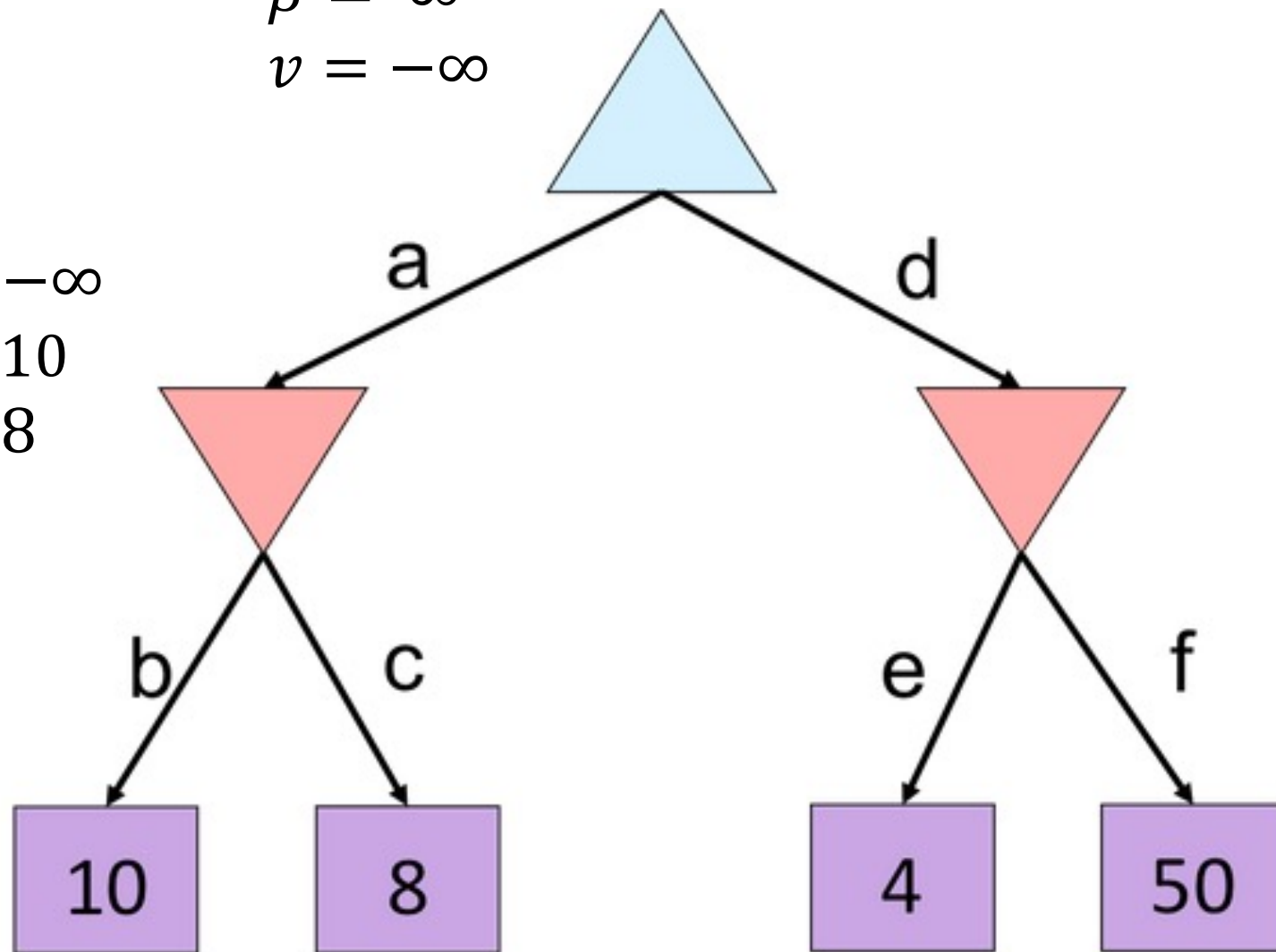
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$$\beta = 10$$

$$v = 8$$



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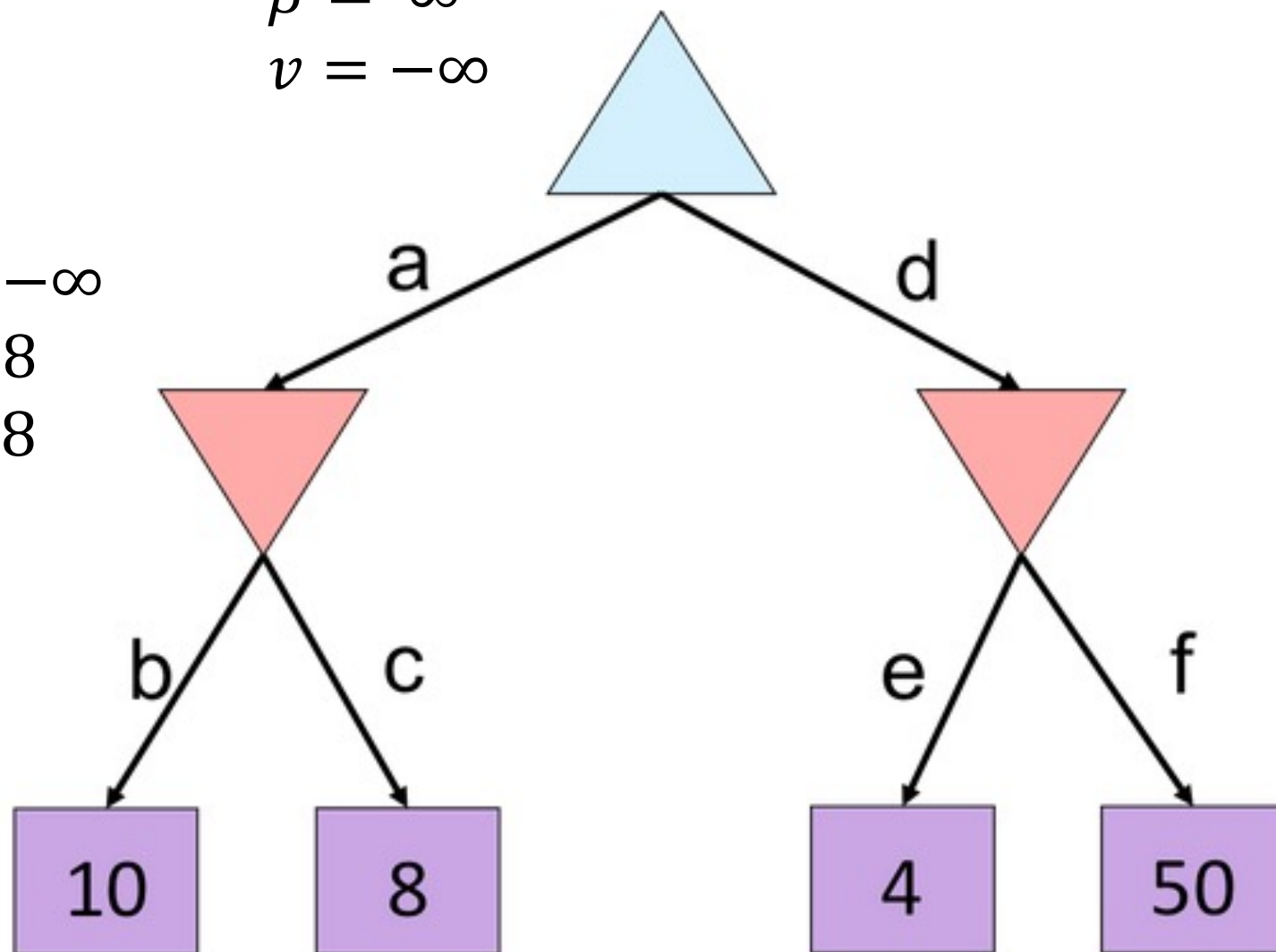
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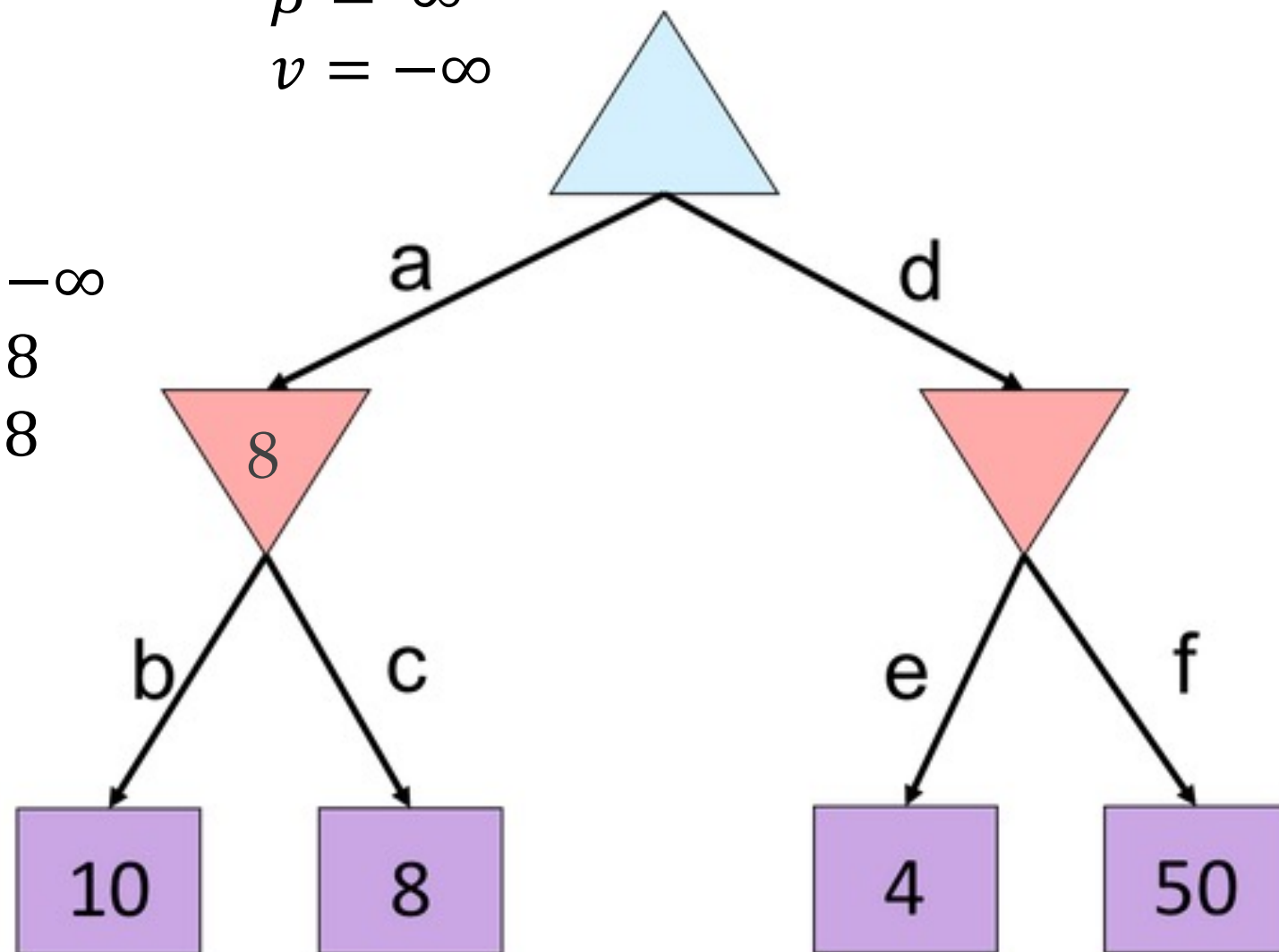
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```
    initialize  $v = +\infty$ 
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         $v = \min(v, \text{value}(\text{successor}, \alpha, \beta))$ 
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        if  $v \leq \alpha$  return  $v$ 
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         $\beta = \min(\beta, v)$ 
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Alpha-Beta Small Example

$$\alpha = -\infty$$

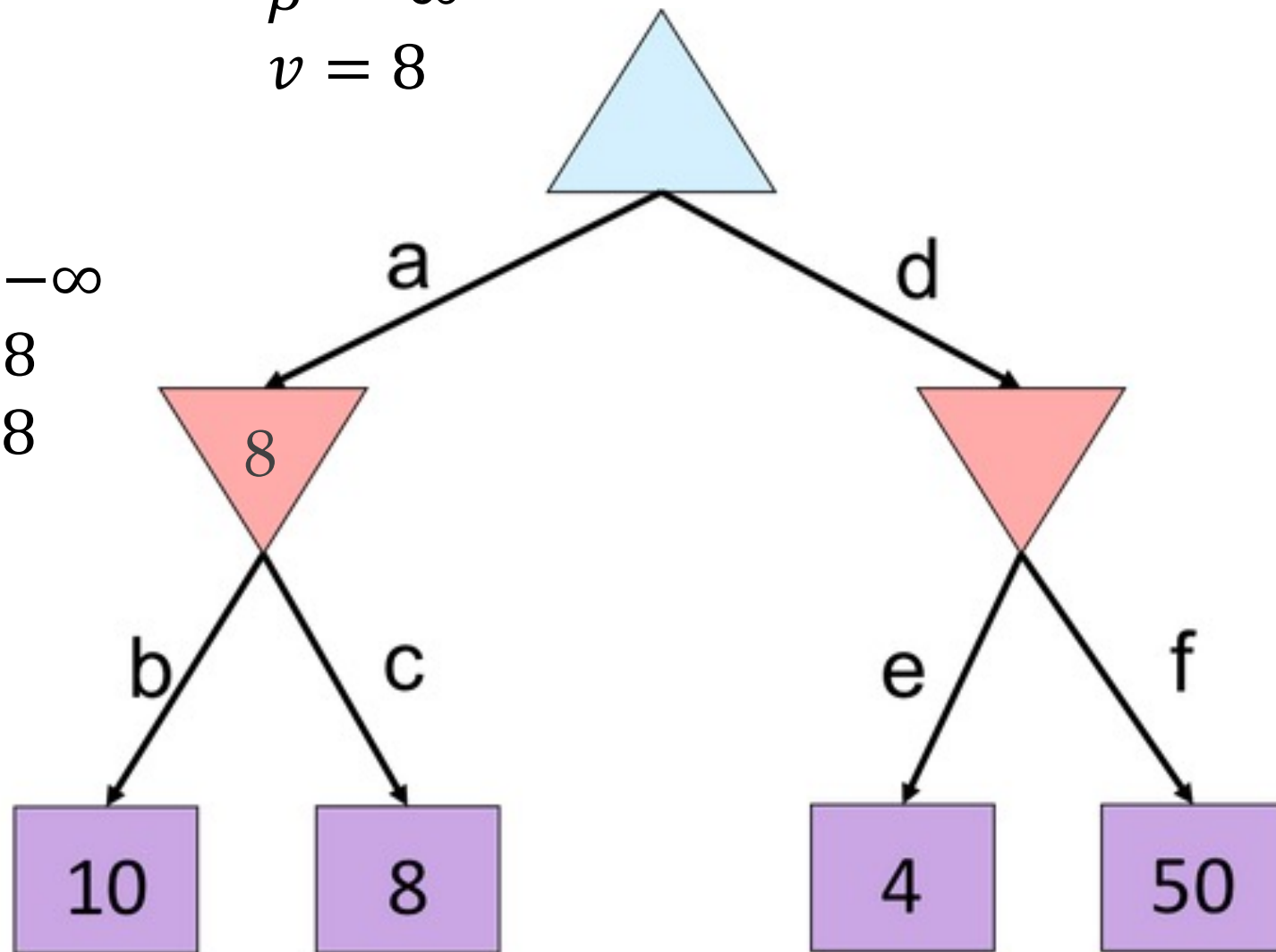
$$\beta = \infty$$

$$v = 8$$

$$\alpha = -\infty$$

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def max-value(state,  $\alpha$ ,  $\beta$ ):
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        if  $v \geq \beta$  return  $v$ 
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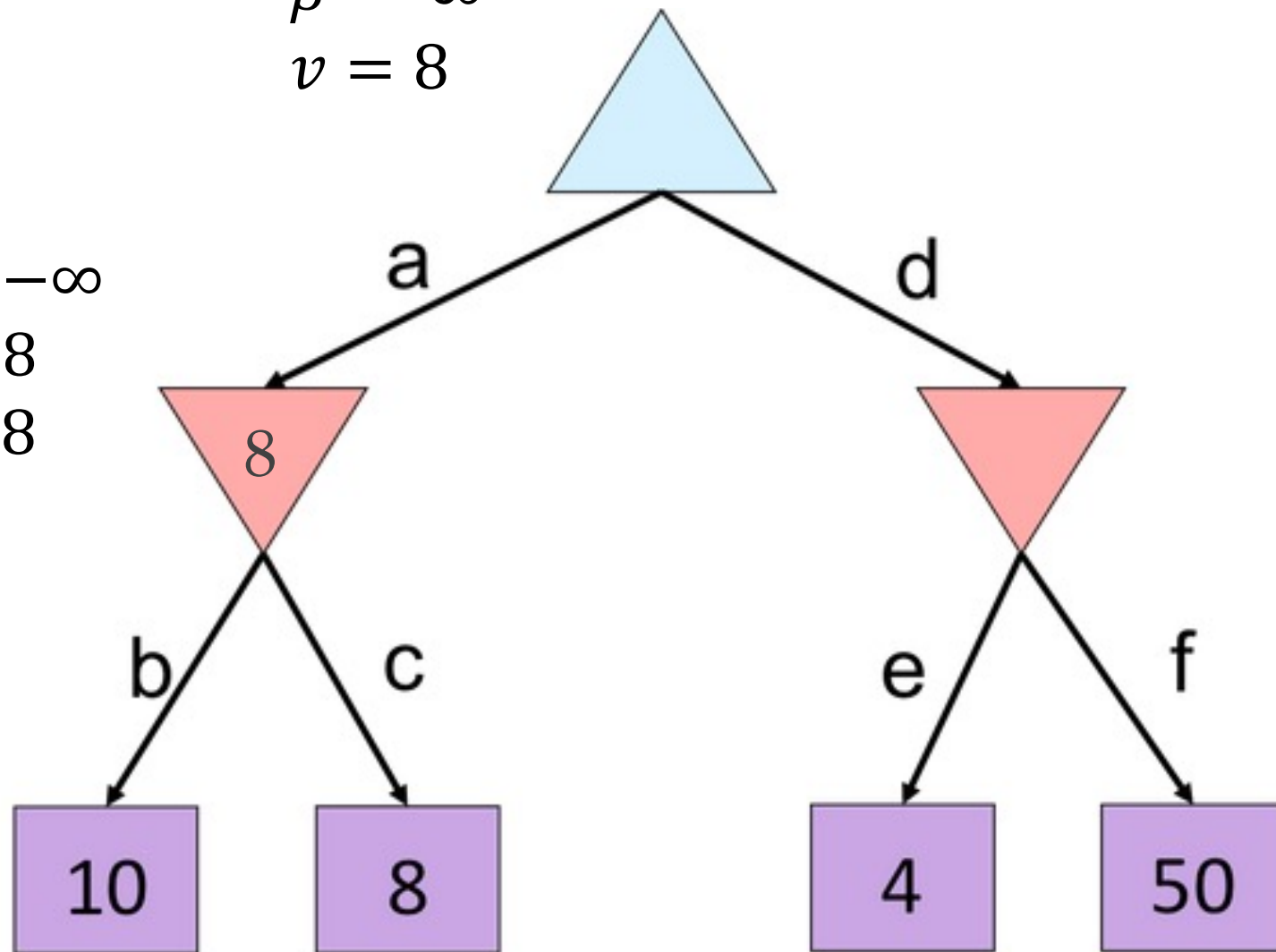
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Alpha-Beta Small Example

$$\begin{aligned}\alpha &= 8 \\ \beta &= \infty \\ v &= 8\end{aligned}$$

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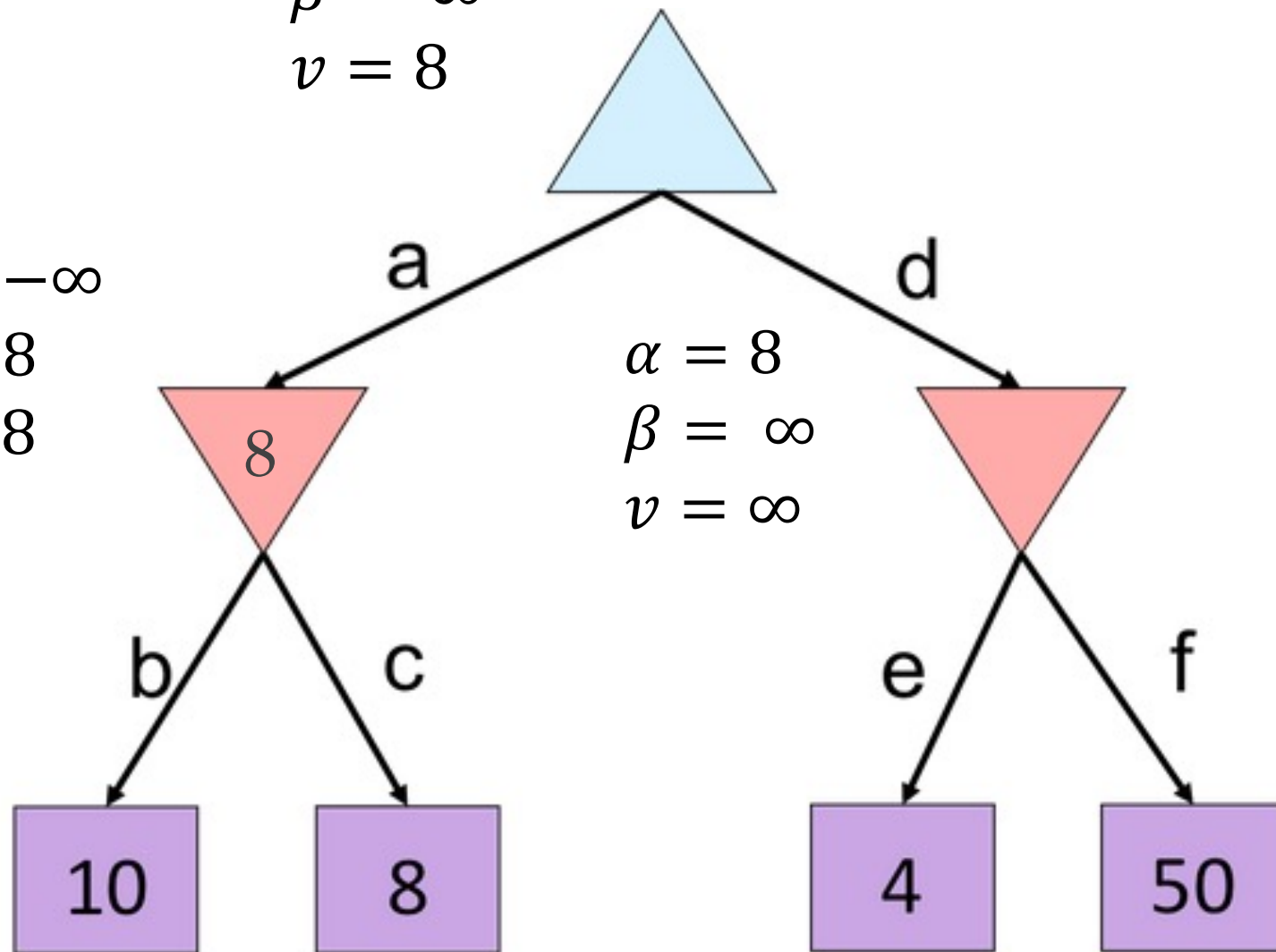
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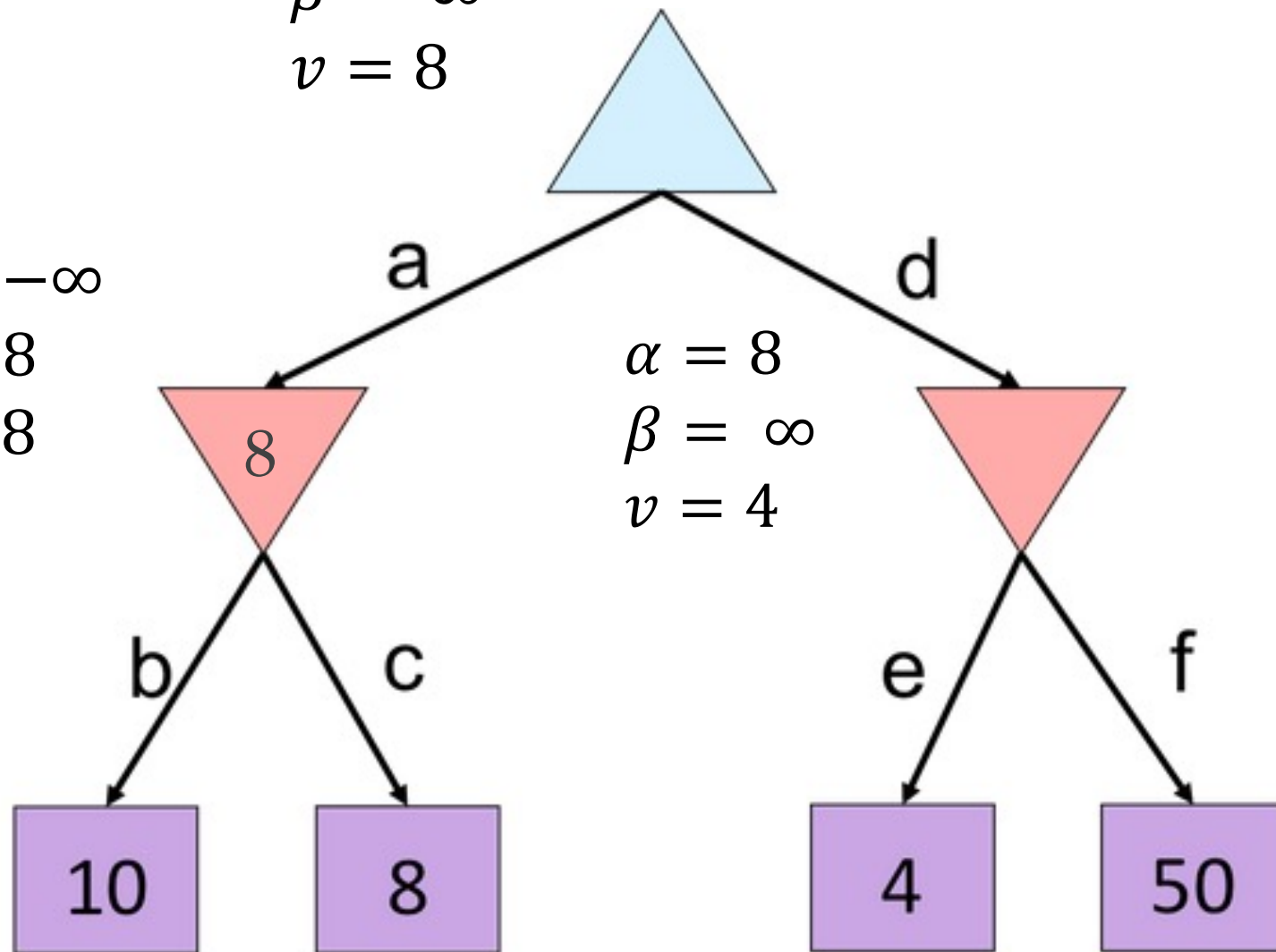
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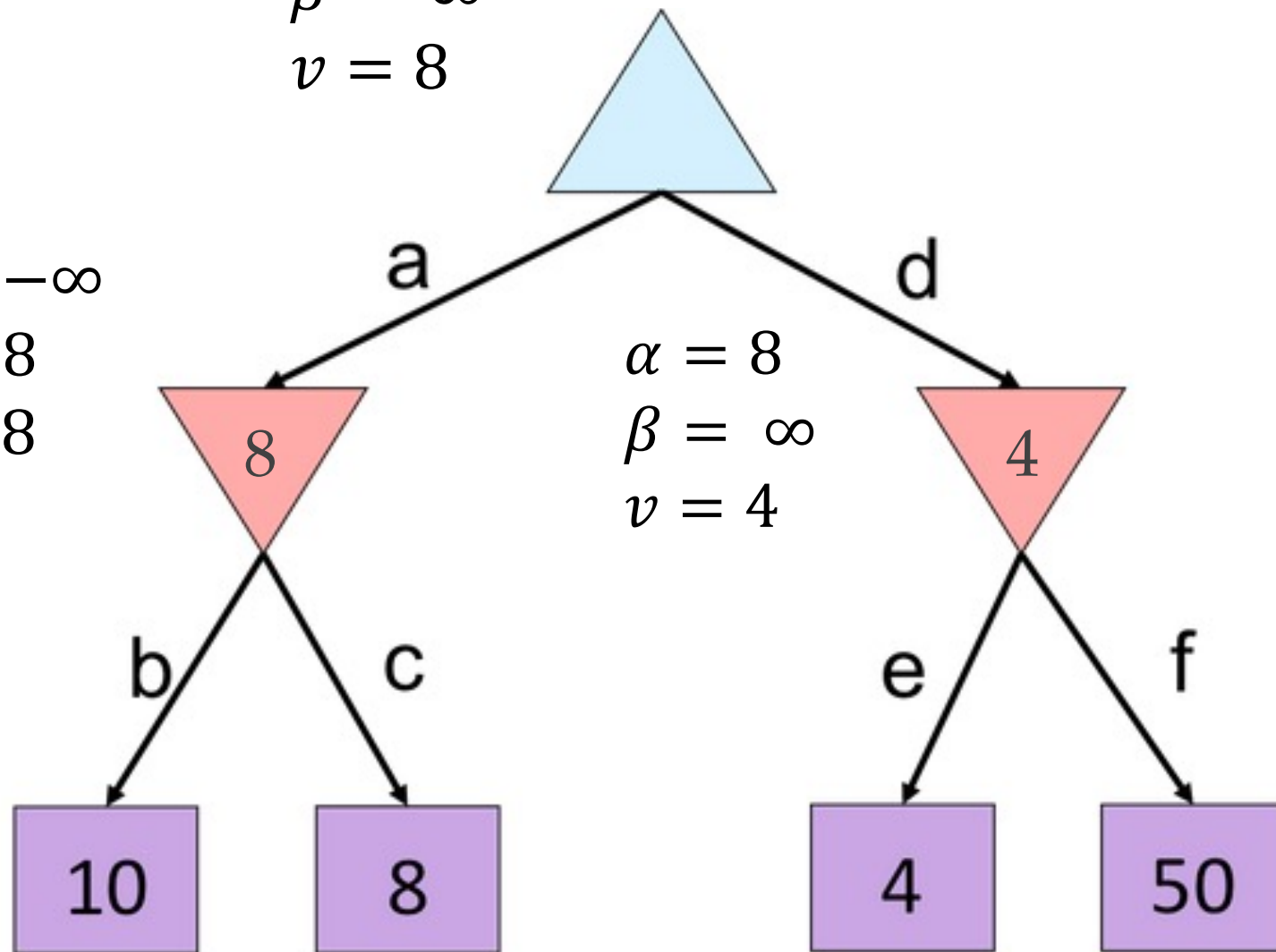
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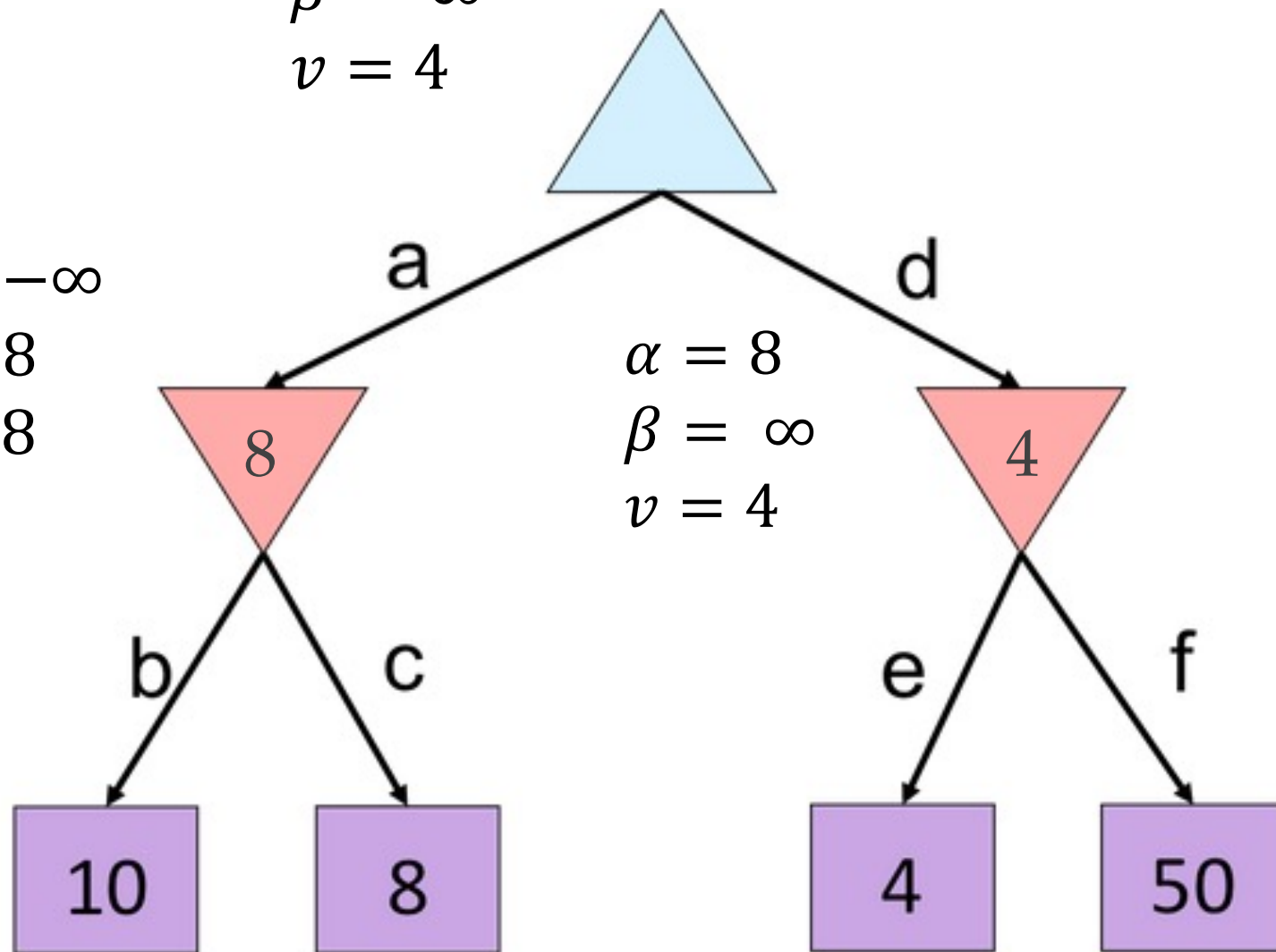
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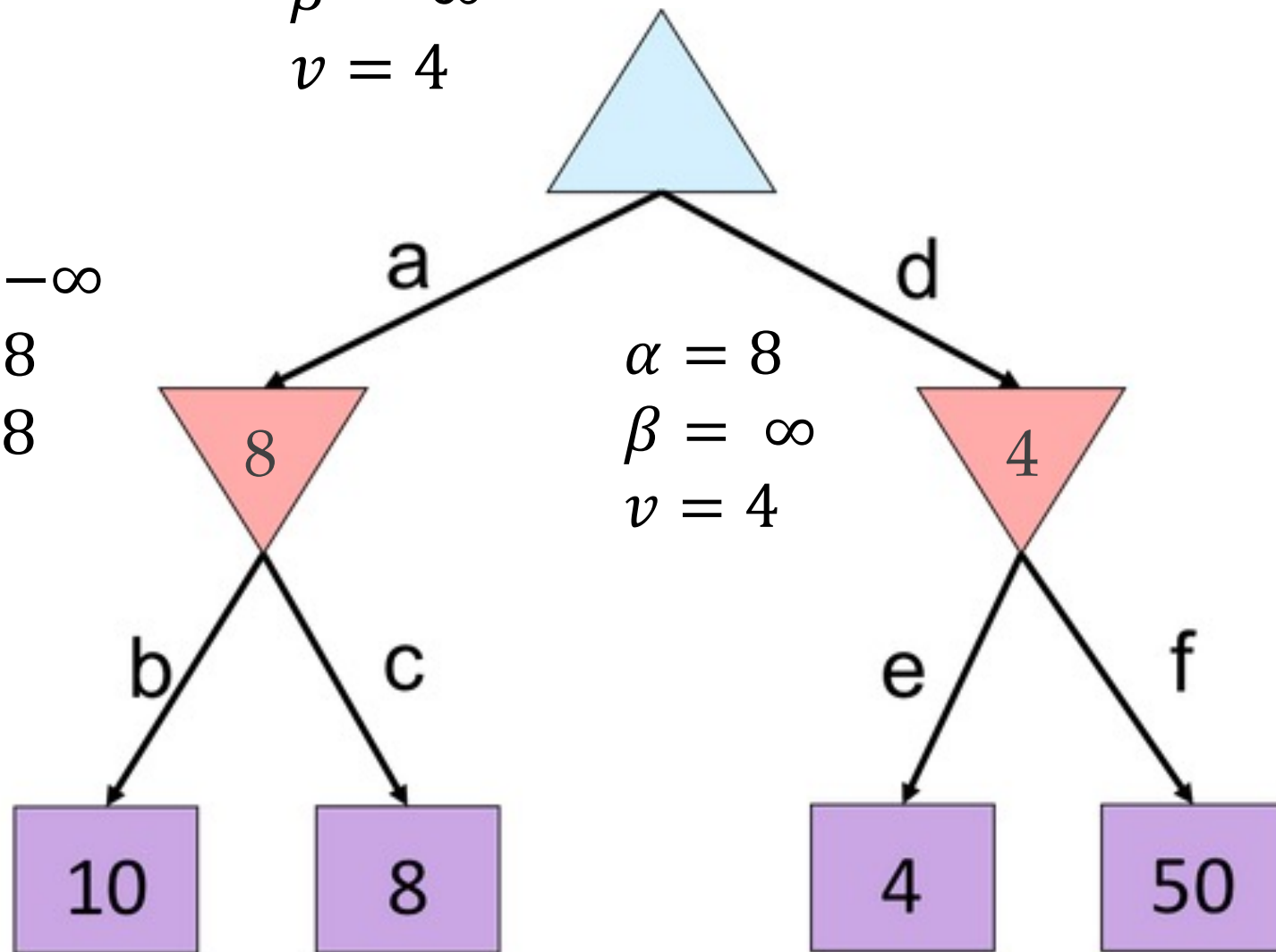
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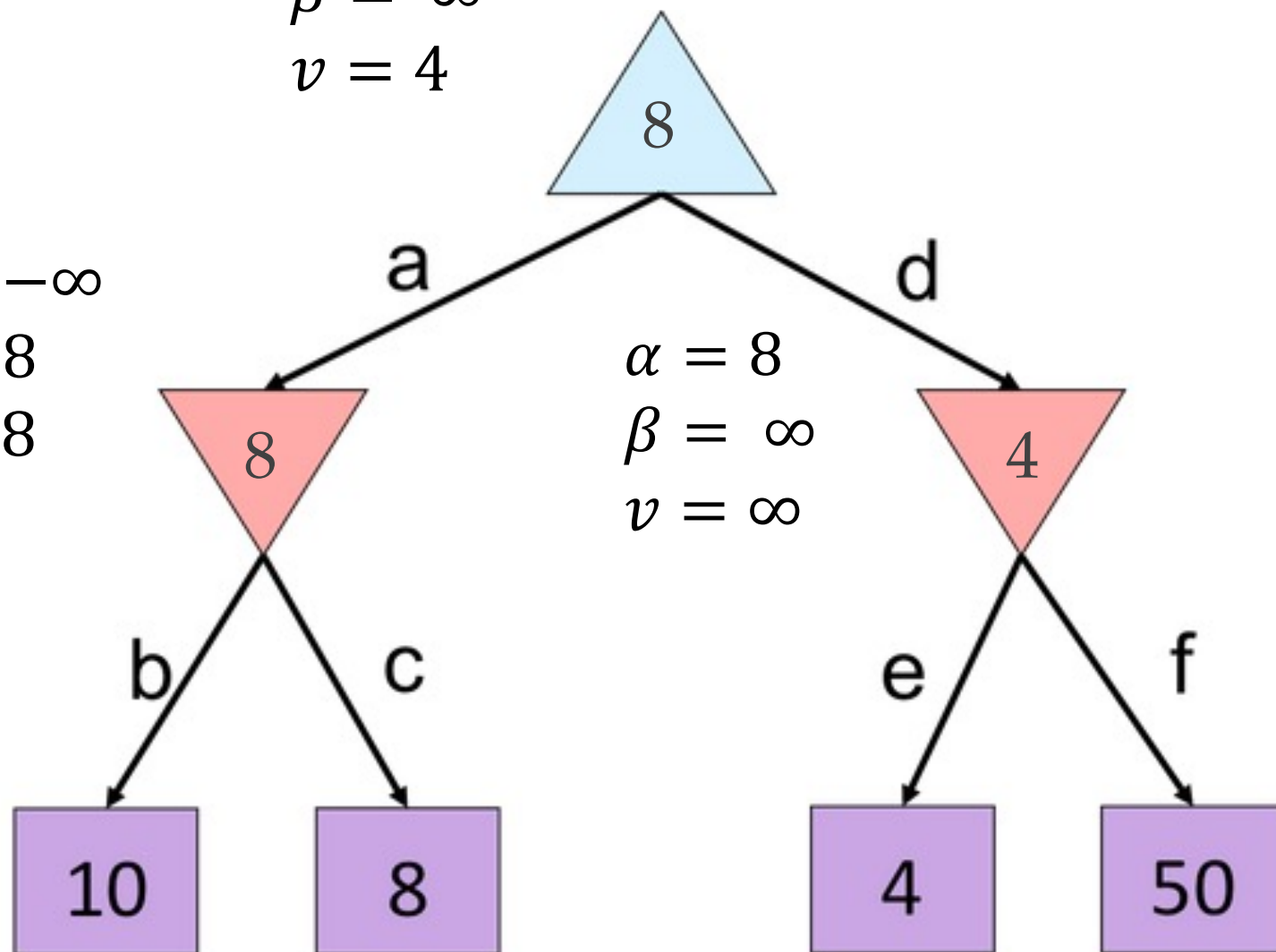
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Quiz: Minimax

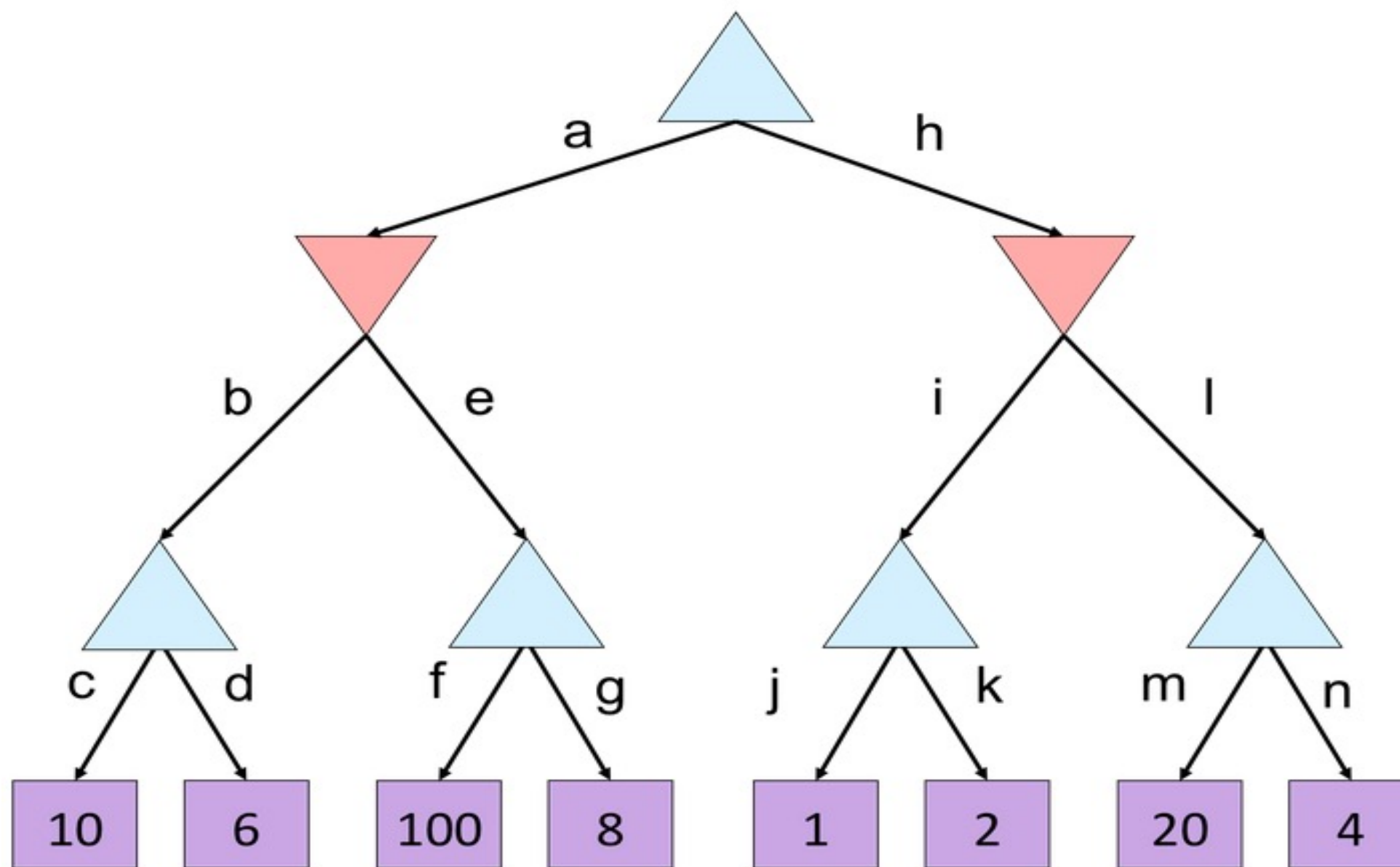
❖ What is the value of the root node?

A. 10

B. 100

C. 2

D. 4



Quiz: Alpha Beta

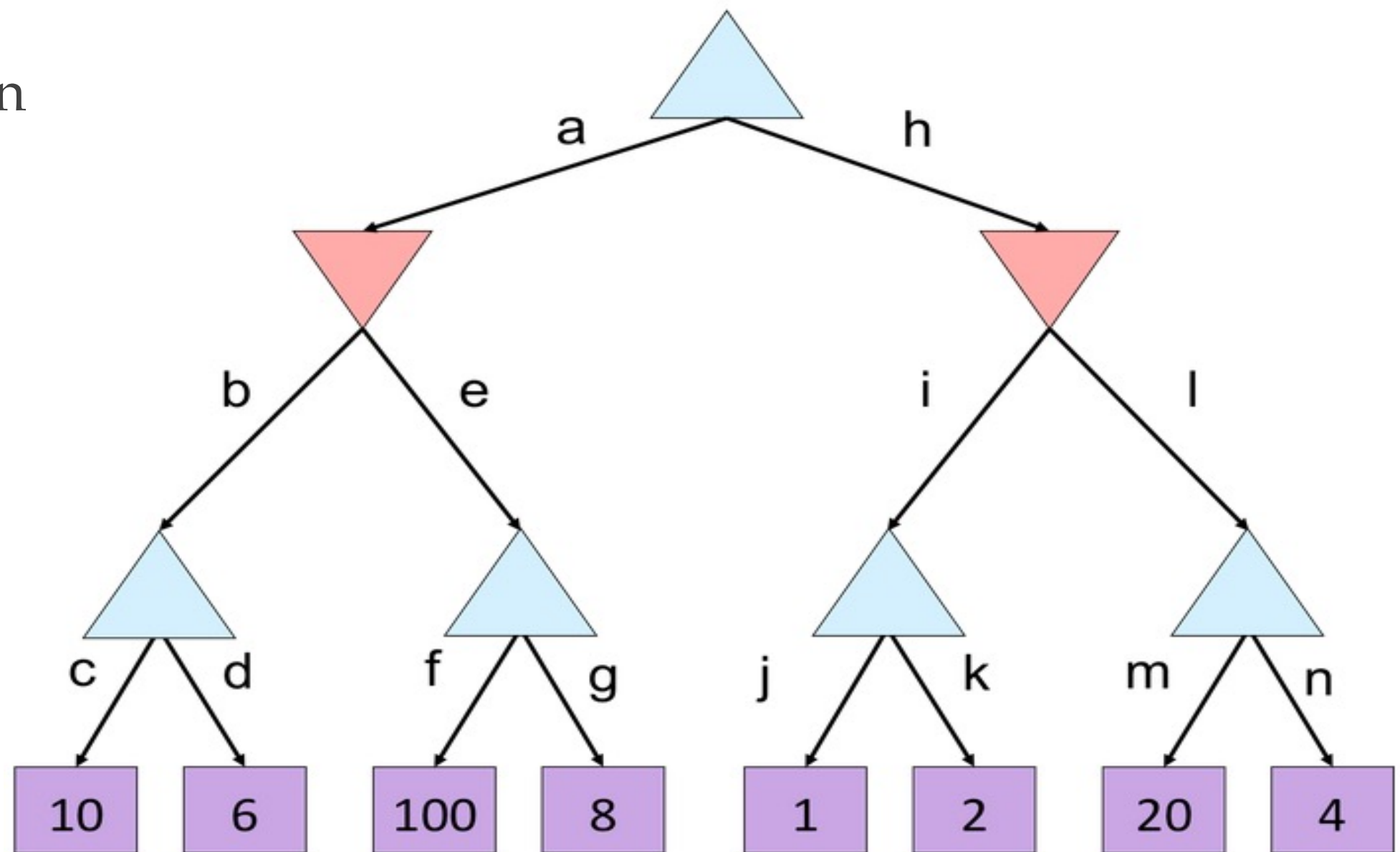
❖ Which branches are pruned?

A. e, l

B. g, l, m, n

C. g, k, n

D. g, n



Alpha-Beta Pruning Properties

- ❖ **Theorem:** This pruning has **no effect** on minimax value computed for the root!
- ❖ Good child ordering improves effectiveness of pruning
 - ❖ Iterative deepening helps with this
- ❖ With “perfect ordering”:
 - ❖ Time complexity drops to $O(b^{m/2})$
 - ❖ Doubles solvable depth!
 - ❖ 1M nodes / move \Rightarrow depth=8, respectable
- ❖ This is a simple example of **metareasoning** (computing about what to compute)

