IERG 5350 Assignment 3: Value Function Approximation in RL

2020-2021 Term 1, IERG 5350: Reinforcement Learning. Department of Information Engineering, The Chinese University of Hong Kong. Course Instructor: Professor ZHOU Bolei. Assignment author: PENG Zhenghao, SUN Hao, ZHAN Xiaohang.

Student Name	Student ID
Wang Wanli	1155160517

Welecome to the assignment 3 of our RL course.

You need to go through this self-contained notebook, which contains many TODOs in part of the cells and has special <code>[TODO]</code> signs. You need to finish all TODOs. Some of them may be easy such as uncommenting a line, some of them may be difficult such as implementing a function. You can find them by searching the <code>[TODO]</code> symbol. However, we suggest you to go through the notebook step by step, which would give you a better understanding of the content.

You are encouraged to add more code on extra cells at the end of the each section to investigate the problems you think interesting. At the end of the file, we left a place for you to optionally write comments (Yes, please give us rewards so we can keep improving the assignment!).

Please report any code bugs to us via github issue.

We will cover the following knowledege in this assignment:

- 1. The n-step TD control algorithm
- 2. Linear function as value approximator
- 3. Feature construction
- 4. Neural network based function approximation
- 5. The basic usage of Pytorch

In the first section of notebook, we build a basic RL pipeline. In the second section, we implement the linear function as approximator and also introduces feature construction technique. In the third section, we implement a simple neural network simply using Numpy package.

- **Before starting, make sure you have installed the following packages:
 - 1. Python 3
 - 2. Jupyter Notebook
 - 3. Gym
 - 4. gym[atari], install via pip install 'gym[atari]'
 - 5 Numpy
 - 6. Pytorch, please refer to official website https://pytorch.org (https://pytorch.org) for installation guide

Section 1: Basic Reinforcement Learning Pipeline

(5 / 100 points)

In this section, we will prepare several functions for evaulation, training RL algorithms. We will also build an AbstractTrainer class used as a general framework which left blanks for different function approximation methods.

In [73]:

```
import gym
import numpy as np
import torch
from utils import *
import torch
import torch.nn as nn
```

In [74]:

```
# Run this cell without modification
def evaluate(policy, num episodes=1, seed=0, env name='FrozenLake8x8-v0',
             render=False):
    """This function evaluate the given policy and return the mean episode
    :param policy: a function whose input is the observation
    :param num episodes: number of episodes you wish to run
    :param seed: the random seed
    :param env name: the name of the environment
    :param render: a boolean flag indicating whether to render policy
    :return: the averaged episode reward of the given policy.
    env = gym. make (env_name)
    env. seed (seed)
    rewards = []
    if render: num episodes = 1
    for i in range (num episodes):
        obs = env.reset()
        act = policy(obs)
        ep reward = 0
        while True:
            obs, reward, done, info = env. step(act)
            act = policy(obs)
            ep reward += reward
            if render:
                env. render()
                wait (sleep=0.05)
            if done:
                break
        rewards. append (ep reward)
    if render:
        env. close()
    return np. mean (rewards)
```

In [75]:

```
# Run this cell without modification
def run(trainer cls, config=None, reward threshold=None):
    """Run the trainer and report progress, agnostic to the class of trainer
    :param trainer cls: A trainer class
    :param config: A dict
    :param reward threshold: the reward threshold to break the training
    :return: The trained trainer and a dataframe containing learning progress
    assert inspect. isclass (trainer cls)
    if config is None:
        config = {}
    trainer = trainer_cls(config)
    config = trainer.config
    start = now = time.time()
    stats = []
    for i in range(config['max iteration'] + 1):
        stat = trainer.train()
        stats.append(stat or {})
        if i % config['evaluate_interval'] == 0 or \
                i == config["max_iteration"]:
            reward = trainer.evaluate(config.get("evaluate num episodes", 50))
            print("(\{:.1f\}s,+\{:.1f\}s) \setminus tIteration \{\}, current mean episode "
                  "reward is {}. {}".format(
                time. time() - start, time. time() - now, i, reward,
                \{k: round(np.mean(v), 4) for k, v in \}
                 stat.items()} if stat else ""))
            now = time.time()
        if reward threshold is not None and reward > reward threshold:
            print("In {} iteration, current mean episode reward {:.3f} is "
                  "greater than reward threshold \{\}. Congratulation! Now we "
                  "exit the training process.". format(
                i, reward_threshold))
            break
    return trainer, stats
```

In [76]:

```
# Solve TODOs and remove "pass"
default config = dict(
    env name="CartPole-v0",
    max iteration=1000,
    max_episode_length=1000,
    evaluate interval=100,
    gamma=0.99,
    eps=0.3,
    seed=0
)
class AbstractTrainer:
    """This is the abstract class for value-based RL trainer. We will inherent
    the specify algorithm's trainer from this abstract class, so that we can
    reuse the codes.
    def __init__(self, config):
        self.config = merge_config(config, default_config)
        # Create the environment
        self.env_name = self.config['env_name']
        self. env = gym. make (self. env name)
        if self.env_name == "Pong-ram-v0":
            self.env = wrap_deepmind_ram(self.env)
        # Apply the random seed
        self. seed = self. config["seed"]
        np. random. seed (self. seed)
        self. env. seed (self. seed)
        # We set self.obs dim to the number of possible observation
        # if observation space is discrete, otherwise the number
        # of observation's dimensions. The same to self.act dim.
        if isinstance (self. env. observation space, gym. spaces. box. Box):
            assert len(self.env.observation space.shape) == 1
            self.obs dim = self.env.observation space.shape[0]
            self.discrete obs = False
        elif isinstance (self. env. observation space,
                        gym. spaces. discrete. Discrete):
            self.obs dim = self.env.observation space.n
            self.discrete_obs = True
        else:
            raise ValueError("Wrong observation space!")
        if isinstance (self. env. action space, gym. spaces. box. Box):
            assert len(self.env.action space.shape) == 1
            self.act_dim = self.env.action_space.shape[0]
        elif isinstance (self. env. action space, gym. spaces. discrete. Discrete):
            self.act dim = self.env.action space.n
        else:
            raise ValueError("Wrong action space!")
        self.eps = self.config['eps']
        # You need to setup the parameter for your function approximator.
        self.initialize parameters()
```

```
def initialize parameters (self):
   self.parameters = None
   raise NotImplementedError(
        "You need to override the "
        "Trainer._initialize_parameters() function.")
def process_state(self, state):
    """Preprocess the state (observation).
   If the environment provides discrete observation (state), transform
    it to one-hot form. For example, the environment FrozenLake-v0
   provides an integer in [0, ..., 15] denotes the 16 possible states.
   We transform it to one-hot style:
   original state 0 \rightarrow one-hot vector [1, 0, 0, 0, 0, 0, 0, \dots]
   original state 1 \rightarrow one-hot vector [0, 1, 0, 0, 0, 0, 0, \dots]
   original state 15 \rightarrow one-hot vector [0, \ldots, 0, 0, 0, 0, 1]
   If the observation space is continuous, then you should do nothing.
   if not self. discrete obs:
       return state
   else:
        new_state = np. zeros((self. obs_dim,))
        new state[state] = 1
   return new state
def compute values (self, processed state):
    """Approximate the state value of given state.
   This is a private function.
   Note that you should NOT preprocess the state here.
   raise NotImplementedError ("You need to override the "
                              "Trainer.compute values() function.")
def compute_action(self, processed_state, eps=None):
    """Compute the action given the state. Note that the input
   is the processed state."""
   values = self.compute values(processed state)
   assert values. ndim == 1, values. shape
   if eps is None:
        eps = self.eps
   if np. random. uniform() <eps:
        action =self.env.action space.sample()
   else:
        action = np. argmax (values)
   return action
def evaluate(self, num episodes=50, *args, **kwargs):
    """Use the function you write to evaluate current policy.
   Return the mean episode reward of 50 episodes."""
   policy = lambda raw_state: self.compute_action(
        self.process state(raw state), eps=0.0)
   result = evaluate(policy, num_episodes, seed=self.seed,
                      env name=self.env name, *args, **kwargs)
   return result
```

In [77]:

```
# Run this cell without modification
class TestTrainer(AbstractTrainer):
    """This class is used for testing. We don't really train anything."""
    def compute values (self, state):
        return np. random. random_sample(size=self.act_dim)
    def initialize parameters(self):
        self.parameters = np.random.random_sample(size=(self.obs_dim, self.act_dim))
t = TestTrainer(dict(env name="CartPole-v0"))
obs = t.env.observation space.sample()
processed = t.process_state(obs)
assert processed. shape == (4, )
assert np. all (processed == obs)
# Test compute action
values = t. compute values (processed)
correct_act = np. argmax(values)
assert t.compute action(processed, eps=0) == correct act
print("Average episode reward for a random policy in 500 episodes in CartPole-v0: ",
      t. evaluate (num_episodes=500))
```

Average episode reward for a random policy in 500 episodes in CartPole-v0: 22.068

Section 2: Linear function approximation

In this section, we implement a simple linear function whose input is the state (or the processed state) and output is the state-action values.

First, we implement a LinearTrainer class which implements (1). Linear function approximation and (2). n-step semi-gradient method to update the linear function.

Then we further implement a LinearTrainerWithFeatureConstruction class which processs the input state and provide polynomial features which increase the utility of linear function approximation.

We refer the Chapter 9.4 (linear method), 9.5 (feature construction), and 10.2 (n-step semi-gradient method) of the RL textbook to you.

In this section, we leverage the n-step semi-gradient. What is the "correct value" of an action and state in one-step case? We consider it is $r_t + \gamma Q(s_{t+1}, a_{t+1})$ and thus lead to the TD error $TD = r_t + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)$. In n-step case, the target value of Q is:

$$Q(s_t,a_t) = \sum_{i=t}^{t+n-1} \gamma^{i-t} r_i + \gamma^n Q(s_{t+n},a_{t+n})$$

We follow the pipeline depicted in Chapter 10.2 (page 247) of the textbook to implement this logic. Note that notation of the time step of reward is different in this assignment and in the textbook. In textbook, the reward R_{t+1} is the reward when apply action a_t to the environment at state s_t . In the equation above the r_t has exactly the same meaning. In the code below, we store the states, actions and rewards to a list during training. You need to make sure the indices of these list, like the tau in actions[tau] has the correct meaning.

After computing the target Q value, we need to derive the gradient to update the parameters. Consider a loss function, the Mean Square Error between the target Q value and the output Q value:

$$ext{loss} = rac{1}{2} [\sum_{i=t}^{t+n-1} \gamma^{i-t} r_i + \gamma^n Q(s_{t+n}, a_{t+n}) - Q(s_t, a_t)]^2$$

Compute the gradient of Loss with respect to the Q function:

$$rac{d ext{loss}}{d Q} = -(\sum_{i=t}^{t+n-1} \gamma^{i-t} r_i + \gamma^n Q(s_{t+n}, a_{t+n}) - Q(s_t, a_t))$$

According to the chain rule, the gradient of the loss w.r.t. the parameter (W) is:

$$rac{d ext{loss}}{dW} = -(\sum_{i=t}^{t+n-1} \gamma^{i-t} r_i + \gamma^n Q(s_{t+n}, a_{t+n}) - Q(s_t, a_t)) rac{dQ}{dW}$$

To minimize the loss, we only need to descent the gradient:

$$W = W - lr rac{d \mathrm{loss}}{dW}$$

wherein lr is the learning rate. Therefore, in conclusion the update rule of parameters is:

$$W = W + lr(\sum_{i=t}^{t+n-1} \gamma^{i-t} r_i + \gamma^n Q(s_{t+n}, a_{t+n}) - Q(s_t, a_t)) rac{dQ}{dW}$$

In the following codes, we denote $G=\sum_{i=t}^{t+n-1}\gamma^{i-t}r_i+\gamma^nQ(s_{t+n},a_{t+n})$ and will compute dQ/dW according to the form of the approximator.

Section 2.1: Basics

(30 / 100 points)

We want to approximate the state-action values. That is, the expected return when applying action a_t in state s_t . Linear methods approximate state-action value function by the inner product between a parameter matatrix W and the input state vector s:

$$v(s, W) = W^T s$$

Note that $W \in \mathbb{R}^{(O,A)}$ and $s \in \mathbb{R}^{(O,1)}$, wherein O is the observation (state) dimensions, namely the self. obs_dim in trainer and A is the action dimension, namely the self. act_dim in trainer. Each action corresponding to one state-action values Q(s,a).

Note that you should finish this section purely by Numpy without calling any other packages.

In [78]:

```
# Solve the TODOs and remove `pass`
# Build the algorithm-specify config.
linear approximator config = merge config(dict(
    parameter std=0.01,
    learning_rate=0.01,
   n=3.
), default config)
class LinearTrainer (AbstractTrainer):
    def __init__(self, config):
        config = merge_config(config, linear_approximator_config)
        # Initialize the abstract class.
        super(). init (config)
        self. max episode length = self.config["max episode length"]
        self.learning_rate = self.config["learning_rate"]
        self.gamma = self.config["gamma"]
        self.n = self.config["n"]
    def initialize parameters (self):
        std = self.config["parameter_std"]
        self. parameters = np. random. randn(self. obs dim, self. act dim)*std
        print("Initialize parameters with shape: {}.".format(self.parameters.shape))
    def compute values (self, processed state):
        ret=processed state.dot(self.parameters)
        assert processed_state.ndim == 1, processed_state.shape
        return ret
    def train(self):
        s = self.env.reset()
        processed s = self.process state(s)
        processed states = [processed s]
        rewards = [0.0]
        actionss =self.compute_action(processed_s)
        actions = [actionss]
        T = float("inf")
        for t in range (self. max episode length):
            if t < T:
                next_state, reward, done, _ = self.env.step(actionss)
                processed s = self.process state(next state)
                processed states. append (processed s)
                rewards. append (reward)
                if done:
                    T=t+1
                else:
                    next act = self.compute action(processed s)
                    actionss=next act
                    actions.append(next act)
            tau = t - self.n + 1
            if tau >= 0:
                gradient = self.compute gradient(
                    processed states, actions, rewards, tau, T
```

```
self.apply_gradient(gradient)
        if tau = T - 1:
            break
def compute_gradient(self, processed_states, actions, rewards, tau, T):
    """Compute the gradient"""
   n = self.n
   if tau>=0:
        target=0
        for i in range (tau+1, min(T, tau+n)+1):
            target+=np. power(self. gamma, i-tau-1)*rewards[i]
   if tau + n < T:
        q values next = self.compute values(processed states[tau + n])
        target += q_values_next[actions[tau + n]]
   EstimatedQ=self.compute_values(processed_states[tau])
   errors=target-EstimatedQ[actions[tau]]
   loss grad = np. zeros((self. act dim, 1))
   loss_grad[actions[tau]]=1
   value_grad=np. expand_dims (processed_states[tau], axis=1)
   assert loss grad. shape == (self. act dim, 1)
   assert value_grad.shape == (self.obs_dim, 1)
   gradient =errors*value_grad*loss_grad. T
   return gradient
def apply_gradient(self, gradient):
   assert gradient. shape == self. parameters. shape, (
        gradient. shape, self. parameters. shape)
   self.parameters+= self.learning_rate * gradient
```

In [79]:

```
# Run this cell without modification
# Build the test trainer.
test trainer = LinearTrainer(dict(parameter std=0.0))
# Test self.parameters.
assert test trainer.parameters.std() == 0.0, \
    "Parameters should subjects to a normal distribution with standard" \
    "deviation config['parameter_std'], but you have {}." \setminus
    "". format(test_trainer.parameters.std())
assert test trainer.parameters.mean() == 0, \
    "Parameters should subjects to a normal distribution with mean 0. " \
    "But you have {}.". format(test_trainer.parameters.mean())
# Test compute_values
fake state = test trainer.env.observation space.sample()
processed_state = test_trainer.process_state(fake_state)
assert processed state. shape == (test trainer.obs dim, ), processed state. shape
values = test_trainer.compute_values(fake_state)
assert values.shape == (test_trainer.act_dim, ), values.shape
# Test compute_gradient
tmp gradient = test trainer.compute gradient(
    [processed_state]*10, [test_trainer.env.action_space.sample()]*10, [0.0]*10, 2, 5)
assert tmp gradient. shape == test trainer. parameters. shape
test trainer. train()
print("Now your codes should be bug-free.")
```

Initialize parameters with shape: (4, 2). Now your codes should be bug-free.

In [80]:

```
# Run this cell without modification
linear_trainer, _ = run(LinearTrainer, dict(
    max_iteration=10000,
    evaluate_interval=1000,
    parameter_std=0.01,
    learning_rate=0.01,
    n=3,
    env_name="CartPole-v0"
))
# It's OK to see bad performance
```

```
Initialize parameters with shape: (4, 2).
(0.0s, +0.0s)
                Iteration 0, current mean episode reward is 9.18.
(0.9s, +0.9s)
                Iteration 1000, current mean episode reward is 9.68.
(1.8s, +0.9s)
                Iteration 2000, current mean episode reward is 9.72.
(2.6s, +0.8s)
                Iteration 3000, current mean episode reward is 9.78.
(3.6s, +0.9s)
                Iteration 4000, current mean episode reward is 9.84.
(4.5s, +0.9s)
                Iteration 5000, current mean episode reward is 9.84.
(5.3s, +0.9s)
                Iteration 6000, current mean episode reward is 9.84.
(6.2s, +0.9s)
                Iteration 7000, current mean episode reward is 9.82.
(7.1s, +0.9s)
                Iteration 8000, current mean episode reward is 9.82.
(8.0s, +0.9s)
                Iteration 9000, current mean episode reward is 9.84.
(9.0s, +0.9s)
                Iteration 10000, current mean episode reward is 9.84.
```

In [81]:

Average episode reward for your linear agent in CartPole-v0: 10.0

You will notice that the linear trainer only has 8 trainable parameters and its performance is quiet bad. In the following section, we will expand the size of parameters and introduce more features as the input to the system so that the system can learn more complex value function.

Section 2.2: Linear Model with Feature Construction

(15 / 100 points)

In [82]:

```
linear fc config = merge config(dict(
    polynomial_order=1,
), linear_approximator_config)
def polynomial_feature(sequence, order=1):
    import itertools
    sample_size =len(sequence)
    orderinfor = [orderse for orderse in range(order + 1)]
    orderinfor = orderinfor * sample size
    weightfactor = list(set(list(itertools.permutations(orderinfor, sample size))))
    weightfactor = sorted(weightfactor, key=lambda x: sum(x))
    output = sorted([np.prod(np.power(sequence, weightfactor[knums])) for knums in range(len(we
ightfactor))])
    """Construct the order-n polynomial-basis feature of the state.
   Refer to Chapter 9.5.1 of the textbook. We expect to get a
    vector of length (n+1) k as the output.
    Example:
    When the state is [2, 3, 4], the first order polynomial feature
    of the state is [
        1,
        2,
        3,
        4,
        2 * 3 = 6,
        2 * 4 = 8,
        3 * 4 = 12,
        2 * 3 * 4 = 24
    It's OK for function polynomial() to return values in different order.
    # [TODO] finish this function.
   return output
assert sorted(polynomial_feature([2, 3, 4])) == [1, 2, 3, 4, 6, 8, 12, 24]
assert len(polynomial_feature([2, 3, 4], 2)) == 27
assert len(polynomial feature([2, 3, 4], 3)) == 64
class LinearTrainerWithFeatureConstruction(LinearTrainer):
    """In this class, we will expand the dimension of the state.
   This procedure is done at self. process state function.
    The modification of self.obs dim and the shape of parameters
    is also needed.
    def __init__(self, config):
        config = merge_config(config, linear_fc_config)
        # Initialize the abstract class.
        super(). init (config)
        self.polynomial_order = self.config["polynomial_order"]
        # Expand the size of observation
        self.obs dim = (self.polynomial order + 1) ** self.obs dim
```

```
# Since we change self.obs_dim, reset the parameters.
self.initialize_parameters()

def process_state(self, state):
    """Please finish the polynomial function."""
    processed = polynomial_feature(state, self.polynomial_order)
    processed = np. asarray(processed)
    assert len(processed) == self.obs_dim, processed.shape
    return processed
```

In [83]:

```
# Run this cell without modification
linear_fc_trainer, _ = run(LinearTrainerWithFeatureConstruction, dict(
    max_iteration=10000,
    evaluate_interval=1000,
    parameter_std=0.01,
    learning_rate=0.001,
    polynomial_order=1,
    n=3,
    env_name="CartPole-v0"
), reward_threshold=195.0)
assert linear_fc_trainer.evaluate() > 20.0, "The best episode reward happening " \
    "during training should be greater than the random baseline, that is greather than 20+."

# This cell should be finished within 10 minitines.
```

```
Initialize parameters with shape: (4, 2).
Initialize parameters with shape: (16, 2).
(0.3s, +0.3s) Iteration 0, current mean episode reward is 9.48.
(6.3s, +6.0s)
                Iteration 1000, current mean episode reward is 9.94.
(12.5s, +6.2s)
                Iteration 2000, current mean episode reward is 10.86.
(19.3s, +6.8s)
                Iteration 3000, current mean episode reward is 15.18.
(27.2s, +7.9s)
                Iteration 4000, current mean episode reward is 13.7.
               Iteration 5000, current mean episode reward is 15.46.
(37.1s, +10.0s)
(49. 1s, +12. 0s)
                Iteration 6000, current mean episode reward is 17.64.
                Iteration 7000, current mean episode reward is 23.4.
(61. 3s, +12. 2s)
(73. 0s, +11. 6s)
                Iteration 8000, current mean episode reward is 23.7.
(91.4s, +18.4s)
                Iteration 9000, current mean episode reward is 53.62.
(107.5s, +16.1s) Iteration 10000, current mean episode reward is 27.12.
```

In [84]:

Average episode reward for your linear agent with feature construction in CartPol e-v0: 27.0

Section 3: Multi-layer Perceptron as the approximiator

In this section, you are required to implement a single agent MLP using purely Numpy package. The differences between MLP and linear function are (1). MLP has a hidden layer which increase its representation capacity (2). MLP can leverage activation function after the output of each layer which introduce not linearity.

Consider a MLP with one hidden layer containing 100 neurons and activation function f(). We call the layer that accepts the state as input and output the activation **hidden layer**, and the layer that accepts the activation as input and produces the values **output layer**. The activation of the hidden layer is:

$$a(s_t) = f(W_h^T s_t)$$

obvious the activation is a 100-length vector. The output values is:

$$Q(s_t) = f(W_o^T a(s_t))$$

wherein W_h, W_o are the parameters of hidden layer and output layer, respectively. In this section we do not add activation function and hence f(x) = x.

Moreover, we also introduce the gradient clipping mechanism. In on-policy learning, the norm of gradient is prone to vary drastically, since the output of Q function is unbounded and it can be as large as possible, which leads to exploding gradient issue. Gradient clipping is used to bound the norm of gradient while keeps the direction of gradient vector unchanged. Concretely, the formulation of gradient clipping is:

$$g_{clipped} = g_{original} rac{c}{\max(c, ext{norm}(g))}$$

wherein c is a hyperparameter which is $config["clip_norm"]$ in our implementation. Gradient clipping bounds the gradient norm to c if the norm of original gradient is greater than c. You need to implement this mechanism in function $apply_gradient$ in the following cell.

In [85]:

```
# Solve the TODOs and remove `pass`
# Build the algorithm-specify config.
mlp trainer config = merge config(dict(
    parameter std=0.01,
    learning rate=0.01,
   hidden dim=100,
    n=3,
    clip_norm=1.0,
    clip gradient=True
), default config)
class MLPTrainer(LinearTrainer):
    def __init__(self, config):
        config = merge_config(config, mlp trainer config)
        self.hidden dim = config["hidden dim"]
        super(). init (config)
    def initialize parameters (self):
        # [TODO] Initialize self.hidden_parameters and self.output_parameters,
        # which are two dimensional matrices, and subject to normal
        # distributions with scale config["parameter std"]
        std = self.config["parameter_std"]
        self.hidden gradient = np.random.randn(self.obs dim, self.hidden dim)*std
        self.output_gradient = np.random.randn(self.hidden_dim, self.act_dim)*std
    def compute values (self, processed state):
        """[TODO] Compute the value for each potential action. Note that you
        should NOT preprocess the state here."""
        assert processed_state.ndim == 1, processed_state.shape
        activation = self.compute_activation(processed_state)
        values=activation.dot(self.output gradient)
        return values
    def compute activation (self, processed state):
        activation=processed state.dot(self.hidden gradient)
        return activation
    def compute gradient(self, processed states, actions, rewards, tau, T):
        n = self.n
        if tau>=0:
            target=0
            for i in range (tau+1, min(T, tau+n)+1):
                target+=np. power (self. gamma, i-tau-1)*rewards[i]
        if tau + n < T:
            q values next = self.compute values(processed states[tau + n])
            target += q values next[actions[tau + n]]
        EstimatedQ=self.compute values(processed states[tau])
        errors=target-EstimatedQ[actions[tau]]
        cur state = processed states[tau]
```

```
cur_statehidden=np. expand_dims (cur_state, axis=1)
        cur_stateoutput=np. expand_dims(cur_state, axis=0)
        loss grad = np. zeros((self. act dim, 1))
        loss grad[actions[tau]]=1
        hidden gradient =errors*cur statehidden*(np. dot(self.output gradient, loss grad)). T
        output_gradient =errors*np.dot((np.dot(cur_stateoutput, self.hidden_gradient)).T,loss_gr
ad. T)
        assert np. all(np. isfinite(output gradient)), \
            "Invalid value occurs in output_gradient! {}".format(
                output gradient)
        assert np. all(np. isfinite(hidden_gradient)), \
            "Invalid value occurs in hidden_gradient! {}".format(
                hidden_gradient)
        return [hidden gradient, output gradient]
    def apply_gradient(self, gradients):
        """Apply the gradientss to the two layers' parameters."""
        assert len(gradients) == 2
        hidden_gradient, output_gradient = gradients
        assert output gradient. shape == (self. hidden dim, self. act dim)
        assert hidden_gradient.shape == (self.obs_dim, self.hidden_dim)
        # [TODO] Implement the clip gradient mechansim
        # Hint: when the old gradient has norm less that clip_norm,
        # then nothing happens. Otherwise shrink the gradient to
          make its norm equal to clip_norm.
        normsoutput_gradient=np. linalg. norm(output_gradient, ord=2)
        normshidden_gradient=np.linalg.norm(hidden_gradient, ord=2)
        if self.config["clip_gradient"]:
            clip norm = self.config["clip norm"]
            if normsoutput_gradient>clip_norm:
                output_gradient=output_gradient*clip_norm/max(clip_norm, normsoutput_gradient)
            if normshidden_gradient>clip_norm:
                hidden_gradient=hidden_gradient*clip_norm/max(clip_norm, normshidden_gradient)
        minusshidden, minussoutput=0.00000, 0.00000
        self.hidden gradient+= self.learning rate * (hidden gradient+minusshidden)
        self.output gradient+= self.learning rate * (output gradient+minussoutput)
```

```
In [86]:
# Run this cell without modification
print ("Now let's see what happen if clip gradient is not enable!")
try:
    failed mlp trainer, = run(MLPTrainer, dict(
        max iteration=3000,
        evaluate interval=100,
        parameter_std=0.01,
        learning rate=0.001,
        hidden dim=100,
        clip gradient=False, # <<< Gradient clipping is OFF!
        env_name="CartPole-v0"
    ), reward threshold=195.0)
    print ("We expect to see bad performance (<195)."
          "The performance without gradient clipping: {}."
          "". format(failed_mlp_trainer.evaluate()))
except AssertionError as e:
    print(traceback.format exc())
    print ("Infinity happen during training. It's OK since the gradient is not bounded.")
finally:
    print("Try next cell to see the impact of gradient clipping.")
Now let's see what happen if clip gradient is not enable!
(0.1s, +0.1s)
                Iteration 0, current mean episode reward is 34.48.
(2.4s, +2.3s)
                Iteration 100, current mean episode reward is 92.5.
(5.2s, +2.8s)
                Iteration 200, current mean episode reward is 86.76.
(7.2s, +2.0s)
                Iteration 300, current mean episode reward is 77.26.
```

```
(9.5s, +2.3s)
                Iteration 400, current mean episode reward is 72.92.
(12.8s, +3.3s)
                Iteration 500, current mean episode reward is 69.36.
(14.7s, +1.8s)
                Iteration 600, current mean episode reward is 66.78.
(16.4s, +1.7s)
                Iteration 700, current mean episode reward is 61.02.
(18.1s, +1.8s)
                Iteration 800, current mean episode reward is 59.52.
                Iteration 900, current mean episode reward is 55.76.
(19.7s, +1.6s)
(21.4s, +1.6s)
                Iteration 1000, current mean episode reward is 52.74.
(22.8s, +1.4s)
                Iteration 1100, current mean episode reward is 52.5.
                Iteration 1200, current mean episode reward is 52.32.
(24.3s, +1.5s)
(25.7s, +1.4s)
                Iteration 1300, current mean episode reward is 51.92.
(27.1s, +1.4s)
                Iteration 1400, current mean episode reward is 46.44.
(28.3s, +1.2s)
                Iteration 1500, current mean episode reward is 45.28.
(29.7s, +1.4s)
                Iteration 1600, current mean episode reward is 38.92.
(31.0s, +1.2s)
                Iteration 1700, current mean episode reward is 40.02.
(32.2s, +1.2s)
                Iteration 1800, current mean episode reward is 33.62.
(33.3s, +1.1s)
                Iteration 1900, current mean episode reward is 33.58.
(34.2s, +0.9s)
                Iteration 2000, current mean episode reward is 33.18.
(35.2s, +1.0s)
                Iteration 2100, current mean episode reward is 33.28.
(36.1s, +0.9s)
                Iteration 2200, current mean episode reward is 33.28.
(37.2s, +1.0s)
                Iteration 2300, current mean episode reward is 33.28.
(38.1s, +0.9s)
                Iteration 2400, current mean episode reward is 33.28.
(39.0s, +0.9s)
                Iteration 2500, current mean episode reward is 32.66.
(40.0s, +1.0s)
                Iteration 2600, current mean episode reward is 33.58.
(41.1s, +1.2s)
                Iteration 2700, current mean episode reward is 33.32.
(42.0s, +0.9s)
                Iteration 2800, current mean episode reward is 33.58.
(43.0s, +0.9s)
                Iteration 2900, current mean episode reward is 32.92.
(44. 1s, +1. 2s)
                Iteration 3000, current mean episode reward is 33.54.
We expect to see bad performance (<195). The performance without gradient clippin
g: 33.54.
Try next cell to see the impact of gradient clipping.
```

In [87]:

```
# Run this cell without modification

print("Now let's see what happen if clip gradient is not enable!")
mlp_trainer, _ = run(MLPTrainer, dict(
    max_iteration=3000,
    evaluate_interval=100,
    parameter_std=0.01,
    learning_rate=0.001,
    hidden_dim=100,
    clip_gradient=True, # <<< Gradient clipping is ON!
    env_name="CartPole-v0"
), reward_threshold=195.0)

assert mlp_trainer.evaluate() > 195.0, "Check your codes. " \
    "Your agent should achieve {} reward in 200 iterations." \
    "But it achieve {} reward in evaluation."
# In our implementation, the task is solved in 200 iterations.
```

```
Now let's see what happen if clip gradient is not enable!
(0.1s, +0.1s)
                Iteration 0, current mean episode reward is 34.6.
(2.1s, +2.0s)
                Iteration 100, current mean episode reward is 93.24.
(4.6s, +2.5s)
                Iteration 200, current mean episode reward is 92.18.
(7.2s, +2.6s)
                Iteration 300, current mean episode reward is 96.08.
(9.6s, +2.3s)
                Iteration 400, current mean episode reward is 101.72.
(11.9s, +2.3s)
                Iteration 500, current mean episode reward is 104.36.
(14.3s, +2.5s)
                Iteration 600, current mean episode reward is 108.14.
(16.9s, +2.6s)
                Iteration 700, current mean episode reward is 109.38.
(19.6s, +2.8s)
                Iteration 800, current mean episode reward is 122.66.
(22.9s, +3.3s)
                Iteration 900, current mean episode reward is 144.06.
(26.5s, +3.6s)
                Iteration 1000, current mean episode reward is 185.58.
                Iteration 1100, current mean episode reward is 191.54.
(31.0s, +4.4s)
(35.6s, +4.6s)
                Iteration 1200, current mean episode reward is 197.4.
In 1200 iteration, current mean episode reward 197.400 is greater than reward thre
shold 195.0. Congratulation! Now we exit the training process.
```

In [88]:

Average episode reward for your MLP agent with gradient clipping in CartPole-v0: 200.0

Interesting right? The gradient clipping technique makes the training converge much faster!

Section 4: Implement Deep Q Learning in Pytorch

(50 / 100 points)

In this section, you will get familiar with the basic logic of pytorch, which lay the ground for further learning. We will implement a MLP similar to the one in Section 3 using Pytorch, a powerful Deep Learning framework. Before start, you need to make sure using <code>pip install torch</code> to install it.

If you are not familiar with Pytorch, we suggest you to go through pytorch official quickstart tutorials:

- 1. guickstart (https://pytorch.org/tutorials/beginner/deep_learning_60min_blitz.html)
- 2. tutorial on RL (https://pytorch.org/tutorials/intermediate/reinforcement_q_learning.html)

Different from the algorithm in Section 3, we will implement Deep Q Network (DQN) in this section. The main differences are concluded as following:

DQN requires an experience replay memory to store the transitions. A replay memory is implemented in the following ExperienceReplayMemory class. It can contain a certain amount of transitions: $(s_t, a_t, x_t, s_{t+1}, done_t)$. When the memory is full, the earliest transition is discarded to store the latest one.

The introduction of replay memory increase the sample efficiency (since each transition might be used multiple times) when solving complex task, though you may find it learn slowly in this assignment since the CartPole-v0 is a relatively easy environment.

DQN is an off-policy algorithm and has difference when computing **TD** error, compared to Sarsa. In Sarsa, the TD error is computed as:

$$(r_t + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t))$$

wherein the next action a_{t+1} is the one the policy selects. However, in traditional Q learning, it assume the next action is the one that maximizes the action values and use this assumption to compute the TD:

$$(r_t + \gamma \max_{a_{t+1}} Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t))$$

DQN has make delayed update target network, which is another difference even compared to the traditional Q learning. DQN maintains another neural network called target network that has identical structure of the Q network. After a certain amount of steps has been taken, the target network copies the parameters of the Q network to itself. Normally, the update of target network is much less frequent than the update of the Q network. The Q network is updated in each step.

The reason to leverage the target network is to stabilize the estimation of TD error. In DQN, the TD error is evaluated as:

$$(r_t + \gamma \max_{a_{t+1}} Q^{target}(s_{t+1}, a_{t+1}) - Q(s_t, a_t))$$

The Q values of next state is estimated by the target network, not the Q network that is updating. This mechanism can reduce the variance of gradient because the estimation of Q values of next states is not influenced by the update of the Q network.

In the engineering aspect, the differences between DQNTrainer and the previous MLPTrainer are:

- 1. DQN uses pytorch model to serve as the approximator. So we need to rewrite the initialize_parameter function to build the pytorch model. Also the train function is changed since the gradient optimization is conducted by pytorch, therefore we need to write the pytorch pipeline in train.
- 2. DQN has replay memory. So we need to initialize it, feed data into it and take the transitions out.

- 3. Thank to the replay memory and pytorch, DQN can be updated in a batch. So you need to carefully compute the Q target via matrix computation.
- 4. We use Adam optimizer to conduct the gradient optimization. You need to get familiar with how to compute the loss and conduct backward propagation

In [89]:

```
# Solve the TODOs and remove `pass`

from collections import deque
import random

import torch.nn as nn

class ExperienceReplayMemory:
    def __init__(self, capacity):
        self.memory = deque(maxlen=capacity)
    def push(self, transition):
        self.memory.append(transition)
    def sample(self, batch_size):
        return random.sample(self.memory, batch_size)
    def __len__(self):
        return len(self.memory)
```

In [90]:

```
# Solve the TODOs and remove `pass`
class PytorchModel(nn. Module):
    def __init__(self, input_shape, num_actions):
        super(PytorchModel, self).__init__()
        self. obs_dim, self. hidden_dim=input_shape[0], 100
        self. action_value= nn. Sequential(nn. Linear(self. obs_dim, self. hidden_dim), nn. ReLU(True)
), nn. Linear(self. hidden_dim, num_actions))
            nn. init. normal_(self. action_value[0]. weight, std=0.1) # 使用标准差是 0.1 的正态分布初始
化
            nn. init. normal_(self. action_value[2]. weight, std=0.1) # 使用标准差是 0.1 的正态分布初始
化
            def forward(self, obs):
            return self. action_value(obs)

# Test
assert isinstance(PytorchModel((3,), 7).action_value, nn. Module)
```

In [91]:

```
# Solve the TODOs and remove `pass`
pytorch config = merge config(dict(
    memory size=50000,
    learn start=5000.
   batch size=32,
    target update freq=500, # in steps
    learn_freq=1, # in steps
), mlp trainer config)
def to_tensor(x):
    """A helper function to transform a numpy array to a Pytorch Tensor"""
    if isinstance(x, np.ndarray):
        x = torch. from numpy(x). type(torch. float32)
    assert isinstance(x, torch. Tensor)
    if x. \dim() == 3 \text{ or } x. \dim() == 1:
        x = x. unsqueeze (0)
    assert x.dim() == 2 or x.dim() == 4, x.shape
    return x
class DQNTrainer(MLPTrainer):
    def init (self, config):
        config = merge_config(config, pytorch_config)
        self.learning_rate = config["learning_rate"]
        super().__init__(config)
        self.criterion = nn.MSELoss()
        self.memory = ExperienceReplayMemory(config["memory size"])
        self.learn_start = config["learn_start"]
        self.batch_size = config["batch_size"]
        self.target_update_freq = config["target_update_freq"]
        self.clip norm = config["clip norm"]
        self.step since update = 0
        self. total step = 0
         self.initialize parameters()
    def initialize parameters (self):
        input shape = self.env.observation space.shape
        n actions = self.env.action space.n
        self.network, self.target network = PytorchModel(input shape, n actions), PytorchModel(i
nput shape, n actions)
        self. network. eval()
        self. network. share memory()
        self. target network. load state dict(self. network. state dict())
        self. target network. eval()
        self. optimizer = torch. optim. Adam(self. network. parameters(), lr=self. learning rate)
        self. loss = nn. MSELoss()
    def compute values (self, processed state):
        q eval = self.network(processed state)
        values=q eval.cpu().data.numpy()
        return values
    def train(self):
        s = self.env.reset()
```

```
processed s = self.process state(s)
       act = self.compute action(processed s)
       stat = {"loss": []}
       for t in range (self. max episode length):
           next_state, reward, done, _ = self.env.step(act)
           next processed s = self.process state(next state)
           x, x_dot, theta, theta_dot =next_processed_s
           r1 = (self. env. x threshold - abs(x)) / self. env. x threshold - 0.8
           r2 = (self.env.theta_threshold_radians - abs(theta)) / self.env.theta_threshold_radi
ans -0.5
           reward = r1 + r2
           self.memory.push((processed_s, act, reward, next_processed_s, done))
           ##############继续选择状态并且产生动作
           processed_s = next_processed_s
           act = self.compute action(next processed s)
           self.step_since_update += 1
           self. total step += 1
           if done:
               break
           if t % self.config["learn_freq"] != 0:
               continue
           if len(self.memory) < self.learn_start:</pre>
               continue
           elif len(self.memory) == self.learn start:
               print("Current memory contains {} transitions, "
                     "start learning!".format(self.learn_start))
           batch = self.memory.sample(self.batch size)
           state batch = to tensor(np.stack([transition[0] for transition in batch]))
           action batch = to tensor(np.stack([transition[1] for transition in batch])).view(-1
, 1). long()
           reward_batch = to_tensor(np.stack([transition[2] for transition in batch]))
           next_state_batch = torch.stack([transition[3] for transition in batch])
           done_batch = to_tensor(np. stack([transition[4] for transition in batch]))
           # 計算現有 eval net 和 target net 得出 Q value 的落差
           Q t =self.network(state batch).gather(1, action batch)
           Q t=torch. squeeze(Q t)
           reward batch=reward batch.view(self.batch size,)
           Q_t_plus_one =self.target_network(next_state_batch).detach().max(1)[0].view(self.bat
ch size,)
           Q_target = reward_batch +self.gamma* Q_t_plus_one
           assert isinstance(Q_t_plus_one, torch. Tensor)
           assert Q t plus one. dim() == 1
           assert Q target. shape == (self. batch size,)
           assert Q t. shape == Q target. shape
           # Update the network
           self. optimizer. zero grad()
           loss =self.criterion(Q_t, Q_target)
           loss value = loss.item()
           stat['loss']. append(loss_value)
           loss. backward()
```

In [92]:

```
# Run this cell without modification
# Build the test trainer.
test_trainer = DQNTrainer({})
# Test compute values
fake state = test trainer.env.observation space.sample()
processed_state = test_trainer.process_state(fake_state)
assert processed_state.shape == (test_trainer.obs_dim, ), processed_state.shape
values = test_trainer.compute_values(processed_state)
assert values. shape == (test trainer.act dim, ), values. shape
test trainer. train()
print("Now your codes should be bug-free.")
_ = run(DQNTrainer, dict(
    max iteration=20,
    evaluate interval=10,
    learn start=100,
    env name="CartPole-v0",
))
print("Test passed!")
```

```
Now your codes should be bug-free.

(0.1s, +0.1s) Iteration 0, current mean episode reward is 9.46. {'loss': nan, 'e pisode_len': 9.0}

Current memory contains 100 transitions, start learning!

(0.4s, +0.2s) Iteration 10, current mean episode reward is 9.48. {'loss': 0.028 8, 'episode_len': 27.0}

(0.9s, +0.6s) Iteration 20, current mean episode reward is 22.02. {'loss': 0.011 9, 'episode_len': 10.0}

Test passed!
```

In [93]:

```
# Run this cell without modification
pytorch_trainer, pytorch_stat = run(DQNTrainer, dict(
    max iteration=2000,
    evaluate interval=10,
    learning_rate=0.01,
    clip_norm=10.0,
    memory_size=50000,
    learn_start=1000,
    eps=0.1,
    target_update_freq=1000,
    batch size=32,
    env_name="CartPole-v0",
), reward_threshold=195.0)
reward = pytorch trainer.evaluate()
assert reward > 195.0, "Check your codes." \
    "Your agent should achieve {} reward in 1000 iterations." \
    "But it achieve {} reward in evaluation.".format(195.0, reward)
# Should solve the task in 10 minutes
```

Iteration 0, current mean episode reward is 9.48. {'loss': nan, 'e (0.2s, +0.2s)pisode len': 8.0} (0.4s, +0.2s)Iteration 10, current mean episode reward is 9.48. {'loss': nan, 'episode len': 10.0} (0.5s, +0.1s)Iteration 20, current mean episode reward is 9.48. {'loss': nan, 'episode len': 12.0} (0.6s, +0.1s)Iteration 30, current mean episode reward is 9.48. ('loss': nan, 'episode len': 8.0} (0.8s, +0.2s)Iteration 40, current mean episode reward is 9.48. ('loss': nan, 'episode_len': 8.0} (1.1s, +0.2s)Iteration 50, current mean episode reward is 9.48. {'loss': nan, 'episode len': 8.0} (1.3s, +0.2s)Iteration 60, current mean episode reward is 9.48. {'loss': nan, 'episode len': 13.0} (1.5s, +0.2s)Iteration 70, current mean episode reward is 9.48. {'loss': nan, 'episode_len': 12.0} (1.7s, +0.2s)Iteration 80, current mean episode reward is 9.48. {'loss': nan, 'episode_len': 9.0} (1.9s, +0.2s)Iteration 90, current mean episode reward is 9.48. {'loss': nan, 'episode len': 11.0} Current memory contains 1000 transitions, start learning! 1007 steps has passed since last update. Now update the parameter of the behavior policy. Current step: 1007 (2.4s, +0.5s)Iteration 100, current mean episode reward is 27.38. {'loss': 0.09 95, 'episode len': 8.0} (3.1s, +0.7s)Iteration 110, current mean episode reward is 9.24. {'loss': 0.01 5, 'episode len': 11.0} (3.4s, +0.3s)Iteration 120, current mean episode reward is 9.26. {'loss': 0.006 7, 'episode_len': 7.0} (3.7s, +0.3s)Iteration 130, current mean episode reward is 9.24. {'loss': 0.003 4, 'episode len': 10.0} (4.0s, +0.3s)Iteration 140, current mean episode reward is 9.24. {'loss': 0.000 9, 'episode len': 9.0} (4.3s, +0.3s)Iteration 150, current mean episode reward is 9.24. {'loss': 0.001 4, 'episode len': 12.0} (4.7s, +0.3s)Iteration 160, current mean episode reward is 9.24. {'loss': 0.000 5, 'episode len': 10.0} (4.9s, +0.3s)Iteration 170, current mean episode reward is 9.24. {'loss': 0.000 6, 'episode len': 8.0} (5.3s, +0.3s)Iteration 180, current mean episode reward is 9.24. {'loss': 0.000 3, 'episode_len': 9.0} (5.6s, +0.3s)Iteration 190, current mean episode reward is 9.24. {'loss': 0.000 3, 'episode_len': 9.0} 1007 steps has passed since last update. Now update the parameter of the behavior policy. Current step: 2014 (5.9s, +0.3s)Iteration 200, current mean episode reward is 9.24. {'loss': 0.029 9, 'episode len': 20.0} (6.2s, +0.3s)Iteration 210, current mean episode reward is 9.24. {'loss': 0.000 9, 'episode len': 12.0} (6.5s, +0.3s)Iteration 220, current mean episode reward is 9.24. {'loss': 0.000 4, 'episode len': 8.0} (6.8s, +0.3s)Iteration 230, current mean episode reward is 9.24. {'loss': 0.000 3, 'episode len': 10.0} (7.1s, +0.3s)Iteration 240, current mean episode reward is 9.24. {'loss': 0.000 2, 'episode len': 9.0} (7.4s, +0.3s)Iteration 250, current mean episode reward is 9.24. {'loss': 0.000 2, 'episode len': 10.0} (7.7s, +0.3s)Iteration 260, current mean episode reward is 9.24. {'loss': 0.000 2, 'episode len': 9.0} Iteration 270, current mean episode reward is 9.24. {'loss': 0.000 (8.0s, +0.3s)2, 'episode len': 7.0}

```
(8.3s, +0.3s)
                Iteration 280, current mean episode reward is 9.24. {'loss': 0.000
2, 'episode len': 10.0}
(8.6s, +0.3s)
                Iteration 290, current mean episode reward is 9.24. {'loss': 0.000
1, 'episode len': 7.0}
1004 steps has passed since last update. Now update the parameter of the behavior
policy. Current step: 3018
(8.9s, +0.3s)
                Iteration 300, current mean episode reward is 9.24. ('loss': 0.015
2, 'episode_len': 11.0}
(10.1s, +1.1s)
                Iteration 310, current mean episode reward is 35.74. {'loss': 0.00
08, 'episode len': 40.0}
(11.3s, +1.3s)
               Iteration 320, current mean episode reward is 90.4. {'loss': 0.000
6, 'episode len': 10.0}
              Iteration 330, current mean episode reward is 26.94. ('loss': 0.00
(11.9s, +0.6s)
09, 'episode_len': 8.0}
1004 steps has passed since last update. Now update the parameter of the behavior
policy. Current step: 4022
(15.8s, +3.8s)
              Iteration 340, current mean episode reward is 200.0. {'loss': 0.00
26, 'episode len': 199.0}
In 340 iteration, current mean episode reward 200.000 is greater than reward thres
hold 195.0. Congratulation! Now we exit the training process.
```

In [94]:

Average episode reward for your Pytorch agent in CartPole-v0: 200.0

In [95]:

```
# [optional] BONUS!!! Train DQN in "Pong-ram-v0" environment
# Tune the hyperparameter and take some time to train agent
# You need to install gym[atari] first via `pip install gym[atari]`
#
#pytorch_trainer2, _ = run(DQNTrainer, dict(max_episode_length=10000, max_iteration=500, evaluate_interval=10, evaluate_num_episodes=10, learning_rate=0.0001, clip_norm=10.0,
#memory_size=1000000, learn_start=10000, eps=0.02, target_update_freq=10000, learn_freq=4, batch_siz e=32, env_name="Pong-ram-v0"), reward_threshold=-20.0)
#
# This environment is hard to train.
```

In [96]:

```
# [optional] If you have train the agent in Pont-ram-v0, please save the weights so that
# we can restore it. Please include the pong-agent.pkl into the zip.

# import pickle
# with open("pong-agent.pkl", "wb") as f:
# pickle.dump(pytorch_trainer2.network.state_dict(), f)
```

In [97]:

```
#
#print("Average episode reward for your Pytorch agent in Pong-ram-v0: ",
# pytorch_trainer2.evaluate(1, render=True))
#
```

Conclusion and Discussion

In this assignment, we learn how to build several function approximation algorithm, how to implement basic gradient descent methods and how to use pytorch.

It's OK to leave the following cells empty. In the next markdown cell, you can write whatever you like. Like the suggestion on the course, the confusing problems in the assignments, and so on.

If you want to do more investigation, feel free to open new cells via Esc + B after the next cells and write codes in it, so that you can reuse some result in this notebook. Remember to write sufficient comments and documents to let others know what you are doing.

Following the submission instruction in the ass	signment to submit your	assignment to our staff.	Thank you!

In []: