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Principles of Machine Learning

The Three Perspectives

Principles of Machine Learning

Springer

Part II Frameworks

6 Symbolic Framework

Contents

About the Symbolic Framework

- Symbols refer to all advanced symbols invented and understood by humans, including signs, numbers, characters, and words.
- Based on symbols, human has also created many systems composed of symbols, logical relationships between symbols, and their processing methods, referred to as symbolic systems.
- Symbolic systems are the advanced form of representation, thinking, communication, and dissemination.
- Various natural languages have their symbolic systems, and many disciplines, such as mathematics, physics, chemistry, logic, and computer science, have also established different symbolic systems.
- The symbolic framework is another theoretical framework of machine learning based on symbolic systems.

Symbolism

- Symbolism is also known as logicism, the school of psychology, or computerism.
- Symbolism is based on the principles of physical symbology and the principle of limited rationality.
- It views human thinking as a combination of symbols, uses symbols and their relationships to represent information and knowledge, and completes human cognition and learning through the storage, extraction, reasoning, and transformation of symbols.
- Symbolists invent specific algorithms to process these symbols, solve problems, and derive new knowledge.
- “Logic Theorist”, world’s first symbolic AI program, was presented in 1956.

Symbolic Learning Theory

What is Symbolic Learning Theory

"A theory that attempts to explain how imagery works in performance enhancement. It suggests that imagery develops and enhances a coding system that creates a mental blueprint of what has to be done to complete an action."

https://dictionary.apa.org/symbolic-learning-theory

Physical Symbol System

What is Physical Symbol System

"A physical symbol system consists of a set of entities, called symbols, which are physical patterns that can occur as components of another type of entity called an expression (or symbol structure). Thus, a symbol structure is composed of several instances (or tokens) of symbols related in some physical way (such as one token being next to another). ... ... A physical symbol system is a machine that produces through time an evolving collection of symbol structures. Such a system exists in a world of objects wider than just these symbolic expressions themselves."

https://dl.acm.org/doi/pdf/10.1145/360018.360022

GOFAI

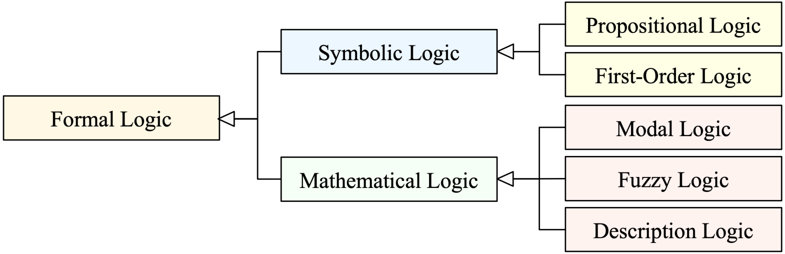
- GOFAI (good old-fashioned artificial intelligence) was appeared in 1985. Where “good old-fashioned” means “even though it’s old-fashioned, it’s still good.”
- The period around 1985 was a time of rapid development of computer processing power, which provided good computational power for statistical learning and multi-layer neural networks.
- It was also from this period that symbolic AI, which do not require high computational power, gradually declined.
- One more reason is that symbolic AI can only handle problems that can be represented as physical symbols, while vision, hearing, and speech are difficult to represent with symbols.

Types of Formal Logic

Formal logic abstracts logic into a symbolic language, giving logic certain mathematical properties and allowing mathematical operations.

Formal logic can be divided into symbolic logic and mathematical logic.

- Symbolic logic includes propositional logic and first-order logic.
- Mathematical logic includes modal logic, fuzzy logic, and description logic.



Symbolic Logic

Symbolic logic, also known as classical logic.

Be a development of formal logic employing a special symbols.

Be capable of manipulation in accordance with precise rules.

Consisting of *propositional logic* and *first-order logic*.

Main symbols in logic.

Category	Symbol	Brief Explanation
Connectives	$\neg$	Logical negation
	$\wedge$	Logical conjunction
	$\vee$	Logical disjunction
	$\Rightarrow$	Implication
	$\Leftrightarrow$	Material equivalence
	$\models$	Entails
	$\not\models$	Not valid
Quantifiers	$\forall$	Universal quantifier
	$\exists$	Existential quantifier
Equality	$=$	Equal to

Propositional Logic

Propositional logic is a formal system for dealing with statements that have a value of true or false, and for constructing rules for proving theorems.

$$
\begin{array}{rcl}
\mathit{Sentence} & \rightarrow & \mathit{AtomicSentence} \mid \mathit{ComplexSentence} \\
\mathit{AtomicSentence} & \rightarrow & \mathit{True} \mid \mathit{False} \mid P \mid Q \mid R \mid \ldots \\
\mathit{ComplexSentence} & \rightarrow & < \mathit{Sentence} > \mid [\ \mathit{Sentence}\ ] \\
 & \mid & \neg\ \mathit{Sentence} \\
 & \mid & \mathit{Sentence} \land \mathit{Sentence} \\
 & \mid & \mathit{Sentence} \lor \mathit{Sentence} \\
 & \mid & \mathit{Sentence} \Rightarrow \mathit{Sentence} \\
 & \mid & \mathit{Sentence} \Leftrightarrow \mathit{Sentence} \\
\mathit{OPERATOR\ PRECEDENCE} & : & \neg, \land, \lor, \Rightarrow, \Leftrightarrow
\end{array}
$$

First-Order Logic

$$
\begin{aligned}
\mathit{Sentence} \;&\rightarrow\; \mathit{AtomicSentence} \mid \mathit{ComplexSentence} \\
\mathit{AtomicSentence} \;&\rightarrow\; \mathit{Predicate} \mid \mathit{Predicate}(\mathit{Term}, \ldots) \mid \mathit{Term} = \mathit{Term} \\
\mathit{ComplexSentence} \;&\rightarrow\; < \mathit{Sentence} > \mid [\ \mathit{Sentence}\ ] \mid \neg\ \mathit{Sentence} \\
&\mid\; \mathit{Sentence} \land \mathit{Sentence} \mid \mathit{Sentence} \lor \mathit{Sentence} \\
&\mid\; \mathit{Sentence} \Rightarrow \mathit{Sentence} \mid \mathit{Sentence} \Leftrightarrow \mathit{Sentence} \\
&\mid\; \mathit{Quantifier}\ \mathit{Variable}, \ldots\ \mathit{Sentence} \\
\mathit{Term} \;&\rightarrow\; \mathit{Function}(\mathit{Term}, \ldots) \mid \mathit{Constant} \mid \mathit{Variable} \\
\mathit{Quantifier} \;&\rightarrow\; \forall \mid \exists \\
\mathit{Constant} \;&\rightarrow\; A \mid X_1 \mid \mathit{John} \mid \ldots \\
\mathit{Variable} \;&\rightarrow\; a \mid x \mid s \mid \ldots \\
\mathit{Predicate} \;&\rightarrow\; \mathit{True} \mid \mathit{False} \mid \mathit{After} \mid \mathit{Loves} \mid \mathit{Raining} \mid \ldots \\
\mathit{Function} \;&\rightarrow\; \mathit{Mother} \mid \mathit{LeftLeg} \mid \ldots \\
\mathit{OPERATOR\ PRECEDENCE} \;&:\; \neg, =, \land, \lor, \Rightarrow, \Leftrightarrow
\end{aligned}
$$

First-Order Logic

First-order logic is a formal system built on top of propositional logic.

Can use quantified variables of non-logical objects and statements containing variables.

$$Mary \in FemalePersons$$
$$John \in MalePersons$$
$$SisterOf(Mary, John)$$
$$FemalePersons \subset Persons$$
$$MalePersons \subset Persons$$
$$\forall x, x \in Persons \Rightarrow [\forall y, HasMother(x, y) \Rightarrow y \in FemalePersons]$$

Examples of first-order predicate logic expressions.

Propositional Logic vs. First-Order Logic

Comparison of propositional logic and first-order logic.

Formal Language	Logic Scope	Logical Value
Propositional Logic	Facts	True/False/Unknown
First-Order Logic	Facts, objects, relations	True/False/Unknown

- Propositional logic is also known as zeroth-order logic, a term that facilitates comparison with first-order logic.
- Propositional logic uses connectives only.
- First-order logic uses not only connectives, but also quantifiers, equality, and predicates.
- First-order logic is also known as predicate logic, or first-order predicate calculus.

Modal Logic

Modal Logic can be formalized as a 3-tuple $\mathcal{M} = \langle W, R, v \rangle$, where: the W is a set of possible worlds; R is a binary relation called accessibility on W; and v is a valuation function which assigns a truth value to each pair of an atomic formula and a world.

$$\textit{if } v(w, P) \textit{ then } w \vDash P$$

$$w \vDash \neg P \textit{ if and only if } w \nvDash P$$

$$w \vDash (P \wedge Q) \textit{ if and only if } w \vDash P \textit{ and } w \vDash Q$$

$$w \vDash \Box P \textit{ if and only if for every element } u \textit{ of } W, \textit{ if } wRu \textit{ then } u \vDash P$$

$$u \vDash \Diamond P \textit{ if and only if for some element } u \textit{ of } W, \textit{ it holds that } wRu \textit{ and } u \vDash P$$

$$\vDash P \textit{ if and only if } w* \vDash P$$

Some researchers have combined modal logic with neural networks and proposed connectionist modal logic.

Fuzzy Logic

Fuzzy logic is a form of many-valued logic that is formed by assigning truth values to propositions.

The standard set of truth values is [0, 1], where: 0 represents “totally false”, 1 represents “totally true”, and any real value between 0 and 1 represents “partially true”.

Fuzzy logic are mathematical methods for representing fuzzy and imprecise information, with the ability to recognize, characterize, operate, interpret, and utilize data and information that are fuzzy and lack certainty.

Fuzzy logic and fuzzy sets have been widely used in the field of machine learning, known as fuzzy machine learning, such as fuzzy clustering, fuzzy neural networks, and fuzzy deep neural networks.

Description Logic

Description logic is a set of formalized knowledge representation languages.

And, it is a decidable fragment of first-order logic, and a subset of first-order logic.

Description Logic	First-Order Logic
Individual	Constant
Concept	Unary predicate
Role	Binary predicate

Symbol	Description	Example
$\top$	Universal concept	$\top$
$\bot$	Empty concept	$\bot$
$\sqcap$	Concept conjunction	$C \sqcap D$
$\sqcup$	Concept disjunction	$C \sqcup D$
$\neg$	Concept Negation	$\neg C$
$\forall$	Universal Quantifier	$\forall R, C$
$\exists$	Existential Quantifier	$\exists R, C$
$\sqsubseteq$	Concept Inclusion	$C \sqsubseteq D$
$\equiv$	Concept Equivalence	$C \equiv D$
$\doteq$	Concept Definition	$C \doteq D$
$:$	Concept Assertion	$a : C$
$:$	Role Assertion	$(a, b) : R$

Description Logic

$$
\begin{aligned}
Coure &\sqsubseteq \forall enrolls.Student \sqcap \geq 5\ entrolls \sqcap \leq 50\ enrolls \sqcap \\
&\quad \forall taughtby.(Professor \sqcup GradStudent) \sqcap 1\ taughtby \\
AdvCourse &\sqsubseteq Course \sqcap \forall enrolls.(GradStudent \sqcup \neg UndStudent) \sqcap \leq 20\ enrolls \\
BasCourse &\sqsubseteq Course \sqcap \forall taughtby.Professor \\
GradStudent &\sqsubseteq Student \sqcap \forall degree.String \sqcap \geq 1\ dgree \\
UndStudent &\sqsubseteq Student
\end{aligned}
$$

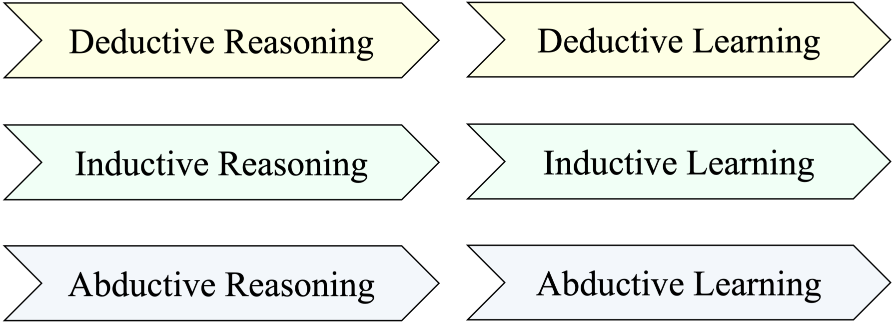
Description logic is used in AI to describe and interpret concepts known as terminological knowledge in the application domain. It provides a logical formalization method for ontology and the semantic web.

Some researchers are also engaged in research combining description logic with neural symbols.

Logical Learning Theory

- Logical learning theory was formally proposed by Joseph Rychlak in 1981.
- The core concept is logical form and its argumentation. Logical form refers to manifestation of logic, and argumentation is carried out by logical reasoning.
- The main features is completeness, consistency, decidability, and expressiveness.
- Human thinking can be divided into concrete and abstract thinking.
- Logical learning is most suitable for abstract thinking, but machine learning belong to concrete thinking.
- The theory mainly studies psychology from a teleological perspective rather than machine learning.
- But the view that the cognitive process is a logical process rather than a mechanical one is worth pondering and exploring for researchers engaged in AI, especially ML.

Logical Learning Modes



Deductive Learning

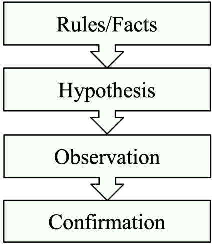
Deductive learning originates from deductive reasoning.

Deductive reasoning is to draw individual conclusion from known general premise.

Deductive learning is to propose hypothesis from facts/rules, and confirm it by observation.

Three types of deductive reasoning.

	Modus ponens	Modus tollens	Syllogism
Major premise	$P \to Q$	$P \to Q$	$P \to Q$
Minor premise	P	$\neg Q$	$Q \to R$
Conclusion	Q	$\neg P$	$P \to R$



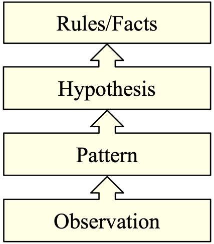
Example: All humans are mortal; Socrates is human; therefore Socrates will die.

Inductive Learning

Inductive learning originates from inductive reasoning.

Inductive reasoning is from individual instance to derive general conclusion.

Inductive learning is from observation, to form hypothesis and use it to learn general facts/rules.



Two major types of inductive reasoning.

	Premise	Conclusion
Generalization	Proportion Q of the sample has attribute a.	So, the proportion Q of the population has attribute a.
Prediction	Proportion Q of observed members of group G have had attribute a.	So, there is a probability corresponding to Q that other members of group G will have attribute a when next observed.

Example: All swans we have seen are white, therefore all swans are white.

Abductive Learning

Abductive learning originates from abductive reasoning.

Abductive reasoning starts from observed facts and seeks their most likely premises.

Abductive learning is to start from observed facts, seek optimal hypotheses, and generate new knowledge.

Abductive reasoning.

Abduction	Reasoning Rule
Fact	Q
Hypothesis	$P \to Q$
Premise	P

- The surprising fact, C, is observed.
- But if A were true, C would be a matter of course.
- Hence, there is reason to suspect that A is true.

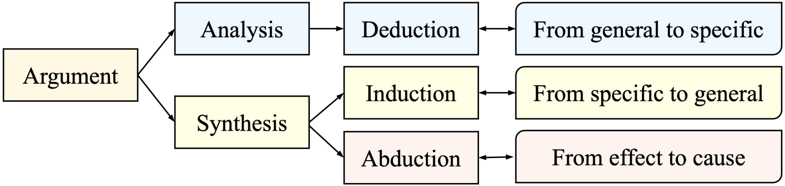
Example: The theory of biological evolution, proposed by Darwin.

Deductive vs. Inductive vs. Abductive

An intrinsic connection between argumentation methods and reasoning patterns.

Argumentation is a series of arguments, presented as argument outlines.

Argumentation is used to provide reasons for them to accept its conclusion.



The intrinsic connection between argumentation and reasoning modes.

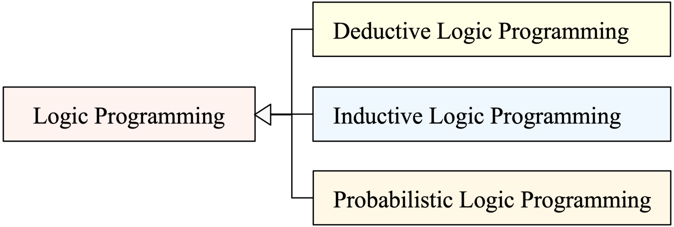
Deductive vs. Inductive vs. Abductive

Deduction	Rule	All beans in this bag are white	Specific conclusion (always true)
	Case	These beans are from this bag	
	Result	These beans are white	
Induction	Case	These beans are from this bag	General conclusion (maybe true)
	Result	These beans are white	
	Rule	All beans in this bag are white	
Abduction	Rule	All beans in this bag are white	Best explanation (maybe true)
	Result	These beans are white	
	Case	These beans are from this bag	

Examples and characteristics of the three major reasoning modes.

Logic Programming Methods

Logic programming (LP) is a programming paradigm based on formal logic.



LP $=$ Logic $+$ Computational Control.

Deductive Logic Programming

The representative of deductive logic programming (DLP) is Prolog language.

Prolog stands for "programming in logic", originates from first-order logic.

The program in Prolog is a set of facts and rules, and a rule is represented as:

$$H :- B_1, \ldots, B_n. \quad \text{It means: } H \Leftarrow B_1 \wedge \cdots \wedge B_n.$$

```
likes(bill, car).
animal(X) :- cat(X).
bird(X)   :- animal(X), has(X, feather).
```

Its reasoning adopts forward chaining and backward chaining, initiated by "query".

DLP = Logic + Computational Control + Facts and Rules.

Inductive Logic Programming

Inductive logic programming (ILP) is a supervised learning method that uses logic programming to represent background knowledge, samples, and hypotheses.

ILP can include additional information in the learning problem.

Proposition	Expression
Necessity	$B \not\models E^+$
Sufficiency	$B \wedge h \models E^+$
Weak consistency	$B \wedge h \not\models$ false
Strong consistency	$B \wedge h \wedge E^- \not\models$ false

Let B be background knowledge and serve as a logical inference, E^+ and E^- be positive and negative examples. A correct hypothesis h is a logical proposition in the table.

$$\text{ILP} = \text{Logic} + \text{Computational Control} + \text{Statistics.}$$

Probabilistic Logic Programming

Probabilistic logic learning (PLL) is to combine probability, logic, and learning.
Probabilistic logic programming (PLP) is to combine programming with probability.

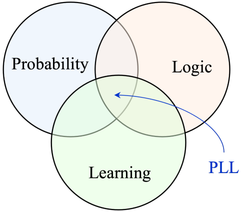
$$\text{PLP} = \text{Logic} + \text{Computational Control} + \text{Probability}$$

ProbLog (Probabilistic Prolog) is formed by extending probability on Prolog, the facts:

$$F = \{p_1 :: f_1, p_2 :: f_2, \ldots, p_n :: f_n\}.$$

For a fact set $F' \subseteq F$, its probability distribution is:

$$P_F(F') = \prod_{f_i \in F'} p_i \prod_{f_i \in F \setminus F'} (1 - p_i).$$



About the Rule Learning

Rule learning:

also known as rule-based learning, or rule-based machine learning.

It includes association rule learning, decision trees, and random forests constructed from multiple decision trees.

Association Rule Learning:

also known as association analysis, used to discover strong rules between variables in large transaction datasets.

New rules will be generated as the analysis increases.

Its goal is to simulate the feature extraction of human brain, and improve the ability to abstract its associations from new unclassified data.

Association Rule Learning

Definition: (Association Rule)

Association rule (AR) can be defined as a 3-tuple, AR $= \langle I, T, R \rangle$. Where: $I = \{i_1, i_2, \ldots, i_m\}$ is an itemset, $\forall i_i \in \{1, 0\}$; $T = \{t_1, t_2, \ldots, t_n\}$ is a transaction dataset, $\forall t_i \subseteq I$; $R : X \Rightarrow Y$ is a rule, $X, Y \subseteq I$, and $X \cap Y = \emptyset$.

A supermarket's transaction dataset T contains 6 items and 4 transactions:

Trans	butter	bread	egg	milk	beer	diaper
1	0	1	1	1	0	0
2	1	0	1	1	0	0
3	1	1	0	1	0	0
4	0	0	0	0	1	1

An instance rule derived from the T is:

$\{$*butter, egg, bread*$\} \Rightarrow \{$*milk*$\}$.

And, a rule of "Beer and Diapers" story:

$\{$*diaper*$\} \Rightarrow \{$*beer*$\}$.

Association Rule Discovery

Association rule discovery is to select useful rules from all possible association rules.

Support: includes the support of antecedent and the support of association rule.

$$\begin{aligned} \text{supp}(X) &= \frac{|t \in T; X \subseteq t|}{|T|} = P(X). \\ \text{supp}(X \Rightarrow Y) &= \text{supp}(X \cup Y) = P(X \cap Y). \end{aligned}$$

Confidence: only measures for association rules.

$$\text{conf}(X \Rightarrow Y) = \frac{\text{supp}(X \Rightarrow Y)}{\text{supp}(X)} = \frac{\text{supp}(X \cup Y)}{\text{supp}(X)} = \frac{P(X \cap Y)}{P(X)} = P(Y|X).$$

Association Rule Synthesis

There are many methods for association rule synthesis.

The task of rule synthesis is usually decomposed into the following two subtasks:

- Frequent itemset synthesis, that is, discover frequent itemset from many transactions in the transaction dataset.
- Strong rules synthesis, extract strong rules based on the discovered frequent itemset.

The computational cost required to synthesize frequent itemset is much greater than the computational overhead required to synthesize strong rules.

Apriori algorithm is a famous method that greatly reduces the amount of computation:

https://en.wikipedia.org/wiki/Apriori_algorithm

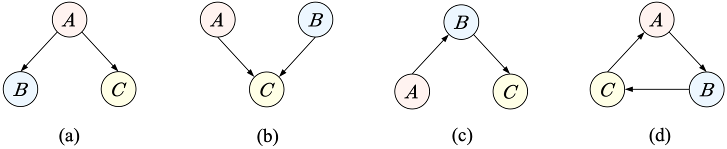
About the Causal Learning

There are several terms related to causal learning: causality, cause and effect, and causal relation.

> "Causality refers to causal relations, i.e. the relations between causes and effects."

We will discuss on causal relation, causal inference, and causal learning that are closely related to machine learning.

Causal Relations



(a) Common-cause: cause A leads to effects B and C respectively, i.e., one cause leads to several effects.

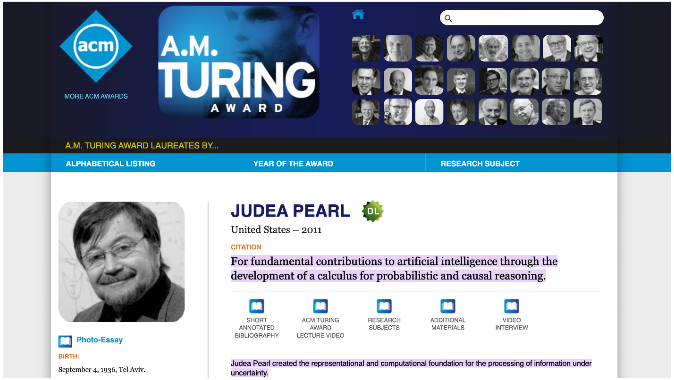
(b) Common-effect: causes A and B both lead to the same effect C, i.e., several causes lead to one effect.

(c) Causal chains: cause A leads to effect B, and B in turn causes effect C, i.e., one cause leads to one effect, and that effect is the cause of another effect.

(d) Causal homeostasis: A, B, and C are interrelated, i.e., several causal relations form a cycle, constituting a stable state.

Causal Inference

Judea Pearl is a pioneer and advocate of causal inference.



Judea Pearl: Turing Award for Probabilistic and Causal Reasoning in 2011.

Causal Inference

Level		Activity	Typical Questions	Examples
1	Association $P(y\|x)$	Seeing	• What is? • How would seeing X change my belief in Y?	• What does a symptom tell me about a disease? • What does a survey tell us about the election results?
2	Intervention $P(y\|do(x),z)$	Doing, Intervening	• What if? • What if I do X?	• What if I take aspirin, will my headache be cured? • What if we ban cigarettes?
3	Counterfactuals $P(y_x\|x',y')$	Imagining, Retrospection	• Why? • Was it X that caused Y? • What if I had acted differently?	• Was it the aspirin that stopped my headache? • Would Kennedy be alive had Oswald not shot him? • What if I had not been smoking the past two years?

Three-level hierarchy of causal inference.
Questions at level i $(i = 1, 2, 3)$ can be answered only if level j $(j > i)$ is available.

Levels of Causal Models

Model	Predict in i.i.d. setting	Predict under distr. shift/intervention	Answer counter-factual questions	Obtain physical insight	Learn from data
Mechanical/physical	yes	yes	yes	yes	?
Structural causal	yes	yes	yes	?	?
Causal graphical	yes	yes	no	?	?
Statistical	yes	no	no	no	yes

- mechanistic/physical model: characterized by differential equations, provides a comprehensive description of the system and can complete the first four tasks.
- structural causal model: abstracted from physical realism, composed of a set of causal variables and a set of causal conditions, supporting the first three tasks.
- causal graphical model: is a graphical model that uses nodes and edges in the graph to represent causal relations, supporting the first two tasks.
- statistical model: besides being able to predict in an i.i.d. environment and learn from data, does not support the other three tasks.

About the Neuro-Symbolic Learning

In recent years, hybrid machine learning has attracted the attention of some researchers and become a new research approach.

The neuro-symbolic learning combines neural networks and symbolic methods.

With the rapid development of AI and ML, some very complex tasks, such as Chess, Go, Texas Hold'em, medical diagnosis, and protein structure prediction, artificial intelligence has surpassed human intelligence.

However, there are some simple tasks that even toddlers, chicks, or cubs can complete, but AI is hard to handle.

Moravec's Paradox

> "It is comparatively easy to make computers exhibit adult level performance on intelligence tests or playing checkers, and difficult or impossible to give them the skills of a one year old when it comes to perception and mobility."

Moravec thought, the perception and action skills are all biological instincts, which have evolved in the process of natural selection. In this process, natural selection tends to evolve and optimize these skills. The older the skill, the longer the evolution time. The evolution time of human abstract thinking is relatively short.

He concluded from this that the difficulty of reverse engineering human skills is roughly proportional to the time of evolution of the skill.

The View on Moravec's Paradox

Human advanced cognitive activities are complex processes, which are often formalized and symbolized for ease of communication and dissemination. So, they are easy for computers to represent and reason.

Chess, Go, and poker are created by humans, and same do AI. So, AI can defeat human intelligence in those games.

However, the perception and action skills rely on intuition and instinct. Intuition is a direct feeling without conscious reasoning; while instinct is the inherent inclination towards a particular complex behaviour.

The feature of intuition and instinct is difficult to formalize and symbolize, so it is not surprising that the effective methods using AI to process intuition and instinct has not yet been found.

Thinking Fast and Slow: System 1 and System 2

System 1 (Fast Thinking)	System 2 (Slow Thinking)
Relying on intuition, memory, and habits, and making judgments or reactions quickly.	Requiring focused attention, deep rational and logical thinking, and making decisions after careful consideration.
Be often unconscious, fast but not pursuing accuracy, but sometimes it is influenced by the rule of thumb that seeing is believing, and the illusions it produces often lead to wrong choices.	Be conscious, relying on long-term learned knowledge and accumulated experience, analyzing and solving problems through attention and logical thinking, and making complex decisions. It is not prone to errors, but it will be influenced by the intuitive judgment of system 1.

Neuro-Symbolic Approaches

The two theoretical frameworks of machine learning:

- The connectionist framework represented by neural networks.
- The symbolic framework with logical reasoning capabilities.

Strength and Weakness of the two frameworks:

- Deep neural networks are very effective for certain unstructured data such as vision, speech, and natural language, but their ability to handle abstract problems such as logic and reasoning is very limited.
- Symbolic learning is powerful in logical reasoning, but it is clumsy in handling large amounts of unstructured data.

Neural-symbolic learning methods are the product of combining the strengths of these two theoretical frameworks.

Thank You