

On the topological property of dynamic transaction graph

Master Thesis
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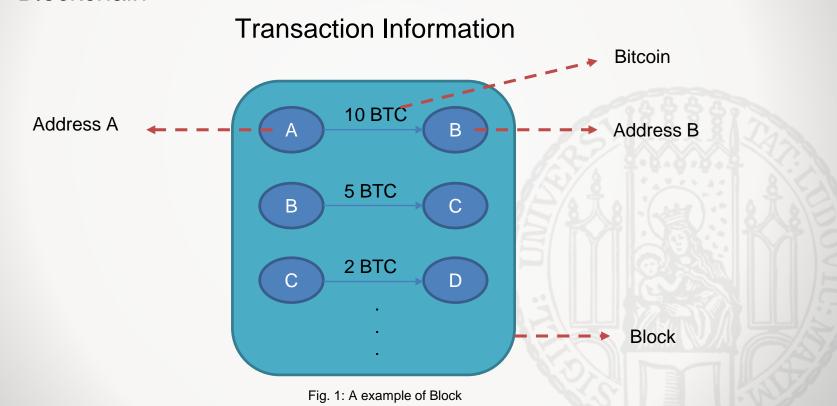
Motivation

- Bitcoin transaction
- Cryptocurrencies
- Dynamic graph
- Money laundering



Background

Blockchain



Background

Blockchain

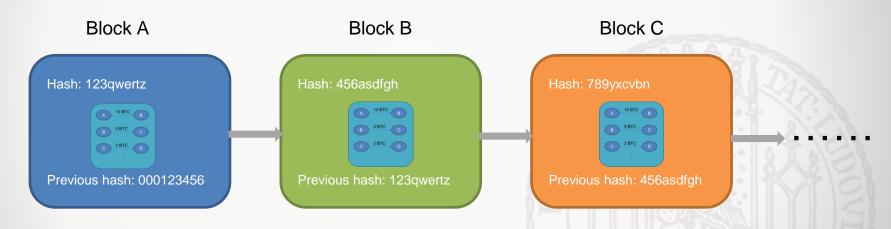


Fig. 2: A example of a BlockChain with three blocks; A,B and C. The system produces a new block every 10 minutes.

A. Learning Graph Representations

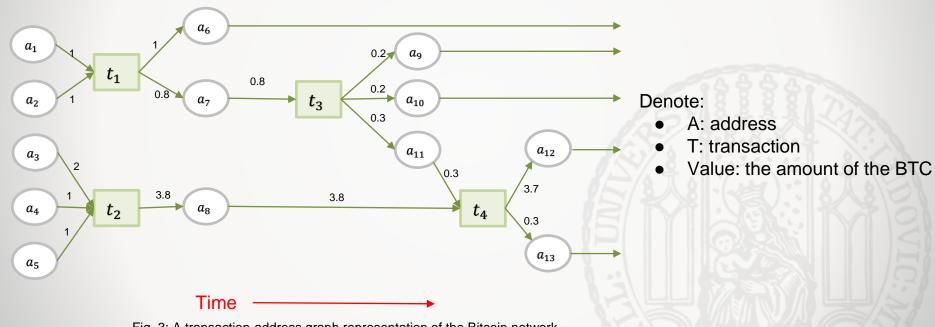


Fig. 3: A transaction-address graph representation of the Bitcoin network.

A. Learning Graph Representations

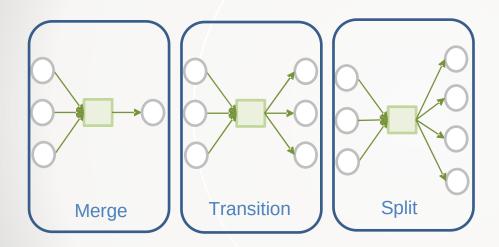


Fig. 4: Merge ($C_{3 \to 1}$), Transition ($C_{3 \to 3}$) and Split ($C_{3 \to 4}$) chainlets for 3 inputs. ¹

Denote:

 $C_{x \rightarrow y}$ refers to x inputs and y outputs:

- Merge: x > y
- Transaction: x = y
- Split: x < y

A. Learning Graph Representations

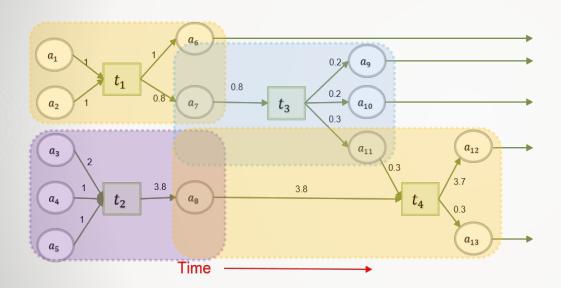


Fig. 5: A transaction-address graph representation of the Bitcoin network. ¹

Occurrence and Amount Matrices:

$$O = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 0.8 \\ 0 & 6.1 & 0 \\ 4 & 0 & 0 \end{bmatrix}$$

Occurence matrices:

- · Row: the amount of the input
- · Column: the amount of the output
- Value: the amount of the transaction with specific input and output

Amount matrics:

- Row: the amount of the input
- · Column: the amount of the output
- Value: the value of the transfered BTC with specific input and output

A. Learning Graph Representations

Graph Filtration (FL):

 Given the amount and occurrence information, a natural combination of them entails filtering the occurrence matrix with user defined thresholds on amounts, or filtering the amount matrix with user defined thresholds on occurrences.

Algorithm 1 FL: Graph Filtration

```
Input: \mathcal{G}: Blockchain graph, time t, \epsilon_{1,\dots,S}: set of S filtration scales.

1: for \epsilon \in \epsilon_{1,\dots,S} do

2: \mathcal{O}^{\epsilon} \leftarrow [] // initialize occurrence matrix

3: for chainlet \mathbb{C}_{i \to j} \in \mathcal{G}_t do

4: for each scale \epsilon \in \epsilon_{1,\dots,S} do
```

5: if
$$\epsilon \leq amout(\mathbb{C}_{i \to j})$$
 then

6:
$$\mathcal{O}_{ij}^{\epsilon} \leftarrow 1 + \mathcal{O}_{ij}^{\epsilon}$$

7: **return** $x_t = [\mathcal{O}^{\epsilon_1 \dots \epsilon_S}]$ // concatenated occ. matrices

A. Learning Graph Representations: Absorbing random walks (ARW)

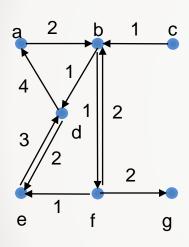


Fig. 6: A visualization	of the transaction	network.
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n	$L_{arw}(n)$
Α	0,09
В	0,06
С	0,24
D	0,08
Е	0,12
F	0,12
G	0,27

Tab. 1: The corresponding 1-ARW-betweenness centrality scores. To calculate the scores, the 1-ARW algorithm (Algorithm 1) is used, which will be explained further on.

$$N = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{2} \\ \frac{1}{7} & \frac{1}{7} & \frac{1}{7} & \frac{1}{7} & \frac{1}{7} & \frac{1}{7} & \frac{1}{7} \end{bmatrix}$$

Matrix 1: The matrix representation of the ARW neighbor-probability distributions.

1

B. Learning Topological Representations

Betti number:

the kth Betti number refers to the number of k-dimensional holes on a topological surface.

Examples:

 b_0 is the number of connected components;

 b_1 is the number of one-dimensional or "circular" holes;

b₂ is the number of two-dimensional "voids" or "cavities".



$$b_0 = 1 \text{ OD loop}$$

$$b_1 = 0 \text{ 1D loop}$$

$$b_2 = 0 \text{ 2D loop}$$



$$b_0 = 1 \text{ OD loop}$$

$$b_1 = 1 \text{ 1D loop}$$

$$b_2 = 1 \text{ 2D loop}$$



$$b_0 = 1 \text{ OD loop}$$

$$b_1 = 2 \text{ 1D loop}$$

$$b_2 = 1 \text{ 2D loop}$$



$$b_0 = 1 \text{ OD loop}$$

$$b_1 = 0 \text{ 1D loop}$$

$$b_2 = 1 \text{ 2D loop}$$

B. Learning Topological Representations

1. Betti Sequences for a Blockchain Network.

- 1) $a' = \log(1 + a/10^8)$, a is an amount of Satoshis.
- 2) a k-th q-quantile, k = 0, 1, ..., q, is the amount Q(k) such that:

$$\sum_{i=1}^{\tau} \mathbb{1}_{y_i < Q(k)} \approx \frac{\tau k}{q} \text{ and } \sum_{i=1}^{\tau} \mathbb{1}_{y_i > Q(k)} \approx \frac{\tau(q-k)}{q}$$

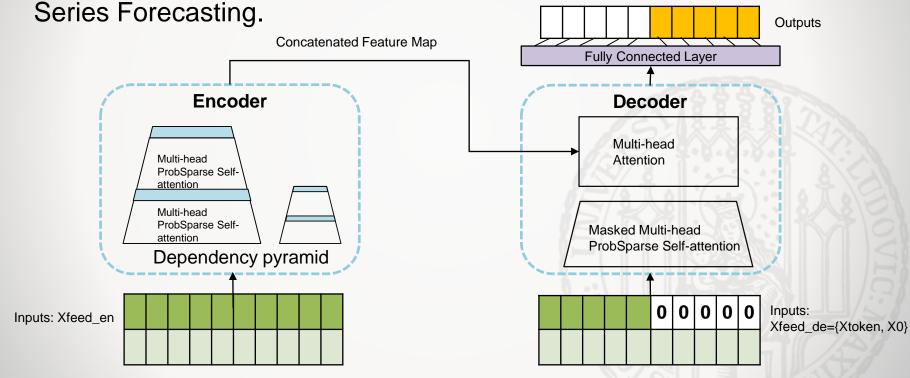
$$d_{ij} = \sqrt{\sum_{k=0}^{q} [Q_i(k) - Q_j(k)]^2}$$

- 3) construct a sequence of scales 1 < 2 < ... < S covering a range of distances during the entire 365- day period, the filtration of VR complexes $VR_1 \subseteq VR_2 \subseteq ... \subseteq VR_S$.
- 4) $x_t = \{\beta_0(\epsilon_1), \ldots, \beta_0(\epsilon_S), ; \beta_1(\epsilon_1), \ldots, \beta_1(\epsilon_S)\}$

2. Betti derivatives.

1)
$$\Delta^{\ell} \beta_{p} (\epsilon_{k}) = \Delta^{\ell-1} \beta_{p} (\epsilon_{k+1}) - \Delta^{\ell-1} \beta_{p} (\epsilon_{k})$$

C. Informer: a beyond efficient Transformer for Long Sequence Time-

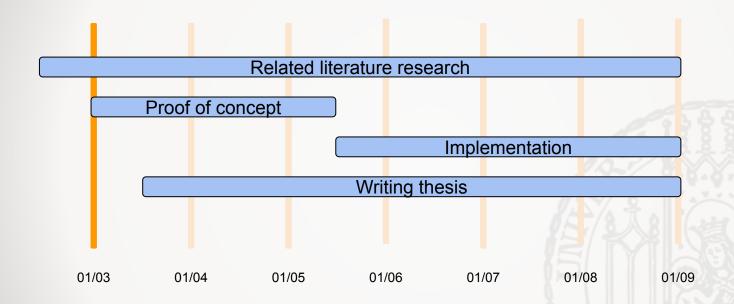


^{1.} Haoyi el.all, Informer: Beyond Efficient Transformer for Long Sequence Time-Series Forecasting. AAAI, 2021.

Research goals

- Research the persistent effectiveness of the topological properties on the Blockchain after the explosion of the cryptocurrency trading.
- Explore the benefit of the Informer on the dynamic transaction graph.
- Examine the dynamic property of topological features on transaction graph.

Time plan



Thank you



Backup



1-ARW-betweeness centrality

Algorithm 1 1-ARW-betweenness centrality algorithm

Require: A graph G = (V, E), and a restart probability α .

Ensure: The 1-ARW-betweenness centrality score for all nodes in V

1:

- for ∀u ∈ V do
- Define C = {u} as the set of central nodes
- Define Q = V\{u\} as the set of query nodes
- Let P_{TT} be the ARW transition matrix of the transient nodes T = V\C under the 1-ARWbetweenness centrality assumptions (Definition 3.7).
- 6: Calculate the fundamental matrix $\mathbf{F} = (I \mathbf{P}_{TT})^{-1}$
- 7: Calculate the expected length of the ARW $\mathbf{L} = \begin{pmatrix} \mathbf{F} \\ \mathbf{0} \end{pmatrix} \mathbf{1}^{|T|}$
- 8: Calculate the 1-ARW-betweenness centrality $L_{arw}(u) = \frac{1}{|V|} \sum_{i=1}^{|V|} \mathbf{L}(i)$

9

10: return The 1-ARW-betweenness centrality score $L_{arw}(u)$, $\forall u \in V$

Feature Engineering

Covariance matrix of features.

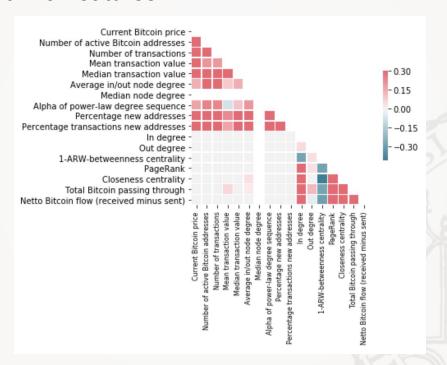
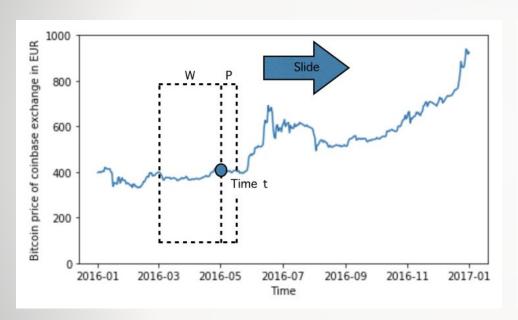


Fig. 6: Covariance matrix of features. 1

Data preparation

A. Window sampling



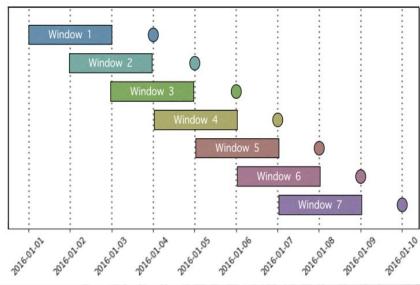


Fig. 11: Visualization of a single window, with w the window size and p the prediction interval. $^{\,1}$

Fig. 12: Example of 7 windows (squares) and their prediction points (circles) ¹

ChainLet

Model	Predictors
Baseline M_0	Price lag 1, Price lag 2, Price lag 3
Model 1	Price lag 1, Price lag 2, Price lag 3,
	# Trans lag 1 , $#$ Trans lag 2, $#$ Trans lag 3
Model 2	Price lag 1, Price lag 2, Price lag 3, Split Pattern lag 1,
	Split Pattern lag 2, Split Pattern lag 3
	Cluster 8 lag 1, Cluster 8 lag 2, Cluster 8 lag 3
Model 3	Price lag 1, Price lag 2, Price lag 3,
	$\mathbb{C}_{1\to7} \operatorname{lag} 1$, $\mathbb{C}_{1\to7} \operatorname{lag} 2$, $\mathbb{C}_{1\to7} \operatorname{lag} 3$
Model 4	Price lag 1, Price lag 2, Price lag 3, $\mathbb{C}_{1\to7}$ lag 1, $\mathbb{C}_{1\to7}$ lag 2,
	$\mathbb{C}_{1\to7}$ lag 2, $\mathbb{C}_{6\to1}$ lag 1, $\mathbb{C}_{6\to1}$ lag 2, $\mathbb{C}_{6\to1}$ lag 3
Model 5	Price lag 1, Price lag 2, Price lag 3,
	$\mathbb{C}_{1\to7} \operatorname{lag} 1$, $\mathbb{C}_{1\to7} \operatorname{lag} 2$, $\mathbb{C}_{1\to7} \operatorname{lag} 2$, $\mathbb{C}_{6\to1} \operatorname{lag} 1$,
	$\mathbb{C}_{6\to 1}$ lag 2, $\mathbb{C}_{6\to 1}$ lag 3, $\mathbb{C}_{3\to 3}$ lag 1, $\mathbb{C}_{3\to 3}$ lag 2, $\mathbb{C}_{3\to 3}$ lag 3

Table 1: Model description for Bitcoin price (response) and varying predictors. ¹



ChainLet

Model	Predictors
Baseline M_0	Price lag 1, Price lag 2, Price lag 3
Model 1	Price lag 1, Price lag 2, Price lag 3,
	# Trans lag 1 , $#$ Trans lag 2, $#$ Trans lag 3
Model 2	Price lag 1, Price lag 2, Price lag 3, Split Pattern lag 1,
	Split Pattern lag 2, Split Pattern lag 3
	Cluster 8 lag 1, Cluster 8 lag 2, Cluster 8 lag 3
Model 3	Price lag 1, Price lag 2, Price lag 3,
	$\mathbb{C}_{1\to7} \text{ lag } 1, \mathbb{C}_{1\to7} \text{ lag } 2, \mathbb{C}_{1\to7} \text{ lag } 3$
Model 4	Price lag 1, Price lag 2, Price lag 3, $\mathbb{C}_{1\to7}$ lag 1, $\mathbb{C}_{1\to7}$ lag 2,
	$\mathbb{C}_{1\to7} \operatorname{lag} 2$, $\mathbb{C}_{6\to1} \operatorname{lag} 1$, $\mathbb{C}_{6\to1} \operatorname{lag} 2$, $\mathbb{C}_{6\to1} \operatorname{lag} 3$
Model 5	Price lag 1, Price lag 2, Price lag 3,
	$\mathbb{C}_{1\to7} \operatorname{lag} 1$, $\mathbb{C}_{1\to7} \operatorname{lag} 2$, $\mathbb{C}_{1\to7} \operatorname{lag} 2$, $\mathbb{C}_{6\to1} \operatorname{lag} 1$,
	$\mathbb{C}_{6\to 1}$ lag 2, $\mathbb{C}_{6\to 1}$ lag 3, $\mathbb{C}_{3\to 3}$ lag 1, $\mathbb{C}_{3\to 3}$ lag 2, $\mathbb{C}_{3\to 3}$ lag 3

Model 1 Model 2 Model 3 Model 4 Model 5 Model 5 Model 5 Model 5 Model 5 Model 6 Model 7 Model 7 Model 7 Model 8 Model 8 Model 9 Model

Table. 2: Model description for Bitcoin price (response) and varying predictors. $^{\rm 1}$

Fig. 7: % Change (decrease) in RMSE compared to the baseline model. $^{\mathrm{1}}$

ChainLet

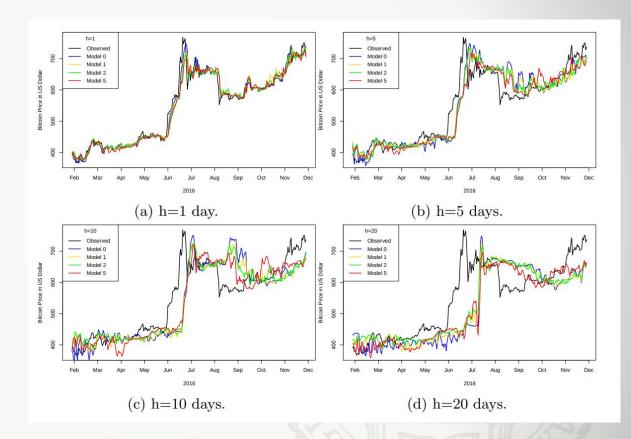


Fig. 8: Price prediction for 2016 with 1, 5, 10 for 20 day horizons. 1

ChainNet

Result

1. RMSE

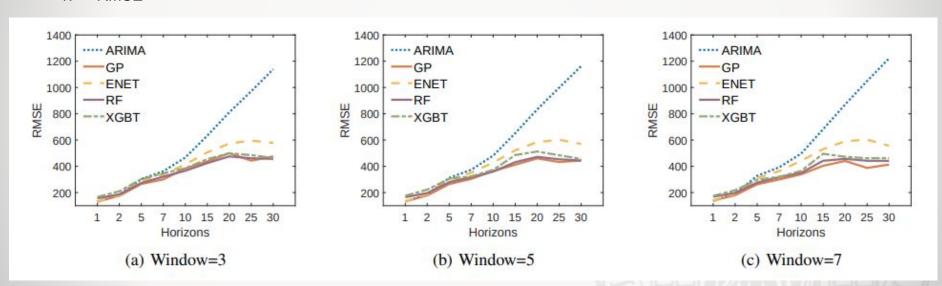


Fig. 9: RMSE of sliding window based predictions of 2017 Bitcoin prices in different window and horizon values.

ChainNet

Conclusion

1. RMSE

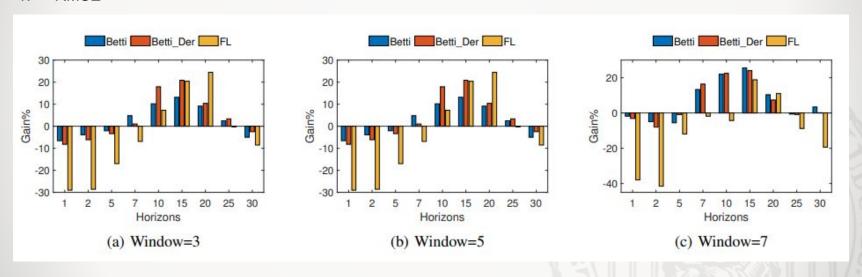


Fig. 10: Random Forest Performance. 1

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Englischer Titel der Arbeit: Topological features on the dynamic transaction graph