Ludwig-Maximilians-Universitaet Muenchen
Institute for Informatics
Prof. Dr. Volker Tresp
Rajat Koner
Andreas Lohrer
Sebastian Schmoll

Machine Learning

Summer 2021

Exercise Sheet 10

Exercise 10-1 Bayesian Network

Consider the following network, in which a mouse agent is reasoning about the behavior of a cat. The mouse really wants to know whether the cat will attack (A), which depends on whether the cat is hungry (H) and whether the cat is sleepy (S). The mouse can observe two things, whether the cat is sleepy (S) and whether the cat has a collar (C). The cat is more often sleepy (S) when it's either full (f) or starved (v) than when it is peckish (p) and the collar (C) tends to indicate that the cat is not starved. Note that entries are omitted, such as $P(C = \neg c)$, when their complements are given.

$P(C)$

C	P(C)
c	0.4

$P(H \mid C)$

H	C	P
f	c	0.7
v	c	0.1
p	c	0.2
f	¬c	0.2
v	¬c	0.5
p	¬c	0.3

$P(S \mid H)$

S	H	P
s	f	0.9
s	v	0.6
s	p	0.3

$P(A \mid H, S)$

A	H	S	P
a	f	s	0.01
a	f	¬s	0.1
a	v	s	0.4
a	v	¬s	0.9
a	p	s	0.2
a	p	¬s	0.7

(a) Draw the Bayesian network corresponding to the above joint probability distribution on C, H, S, A.

(b) Compute the following probabilities:

- P(A=a, C=c, S=s, H=f)
- P(A=a, C=c, S=s)
- P(C=c, S=s)
- P(A=a | C=c, S=s)

(c) The mouse is trying to figure out whether it should run out its hole and eat the cheese (E) or do nothing (N). If the mouse hides, nothing happens but it stays hungry. If the mouse runs out to eat the cheese and the cat attacks, the mouse dies (which has a low utility). Otherwise, if the mouse tries to eat the cheese and the cat does not attack, it gets to eat tasty cheese (high utility).

Utilities

Cat ready to attack (A)	Mouse's action	Utility
a	E	x
¬ a	E	5
Any	N	-2

- Suppose in the above table x is -10. The mouse sees that the cat has a collar on and is sleepy. What is the utility of trying to eat the cheese? What about doing nothing? Which option should the mouse choose?
- What should the utility of dying (x in the above table) be in order for the mouse to be ambivalent between running for the cheese and doing nothing? Again, the cat is wearing a collar and is sleepy.

(d) You may have noticed that one of the variables in the network is "collar", which according to the CPTs (conditional probability table) causes hunger. However, the real relationship is of correlation, not causation. Introduce a new node O for "owner" and draw a network which better models the true relationship between the variables. C and H should be independent contitioned on O.

Exercise 10-2 Maximum A Posteriori Rule

Consider the lifetime of 2 different types of devices, which we will refer to as type A and type B. The lifetime of a type-A device is exponentially distributed with parameter λ. The lifetime of type-B is exponentially distributed with parameter μ, with $\mu > \lambda > 0$. Assuming you have a box full of devices of the same type, and you would like to know whether they are of type A or B. Assume an *a priori* probability of $1/3$ that the box contains devices of type B.

(a) You observe the value t_1 of the lifetime, T_1, of a device. A MAP decision rule implies that the device is of type A if and only if $t_1 > \alpha$. Assuming that $\mu \geq 2\lambda$, find α. Express your answer in terms of μ and λ.
(*Hint: Exponential distribution is given by* $f(x;\lambda) = \lambda e^{-\lambda x}$ *iff* $x \geq 0$)

(b) Assuming that $\mu \geq 2\lambda$, what is the probability of error of the MAP estimator?

(c) Assume that $\lambda = 2$ and $\mu = 3$. Find the LMS estimate of T_2, the lifetime of another device from the same box, based on observing $T_1 = 2$. Assume that conditioned on the device type, the lifetimes of devices are independent.

Exercise 10-3 Markov Property

For each of the following definitions of the state X_k at time k (for $k = 1, 2, \ldots$), determine whether the Markov property is statisfied by the sequence $X_1, X_2, \ldots$.

A fair six-sided die (with sides labeled 1,2,3,4,5,6) is rolled repeatedly and independently.

(a) Let X_k denote the largest number obtained in the first k rolls. Does the sequence $X_1, X_2 \ldots$ satisfy the Markov property?

(b) Let X_k denote the number of 6's obtained in the first k rolls, up to a maximum of ten. (That is, if ten or more 6's are obtained in the first k rolls, then $X_k = 10$.) Does the sequence $X_1, X_2, \ldots$ satisfy the Markov property?

(c) Let Y_k denote the result of the k^{th} roll. Let $X_1 = Y_1$ and for $k \geq 2$, let $X_k = Y_k + Y_{k-1}$. Does the sequence $X_1, X_2, \ldots$ satisfy the Markov property?

(d) Let $Y_k = 1$ if the k^{th} roll results is an odd number; and $Y_k = 0$ otherwise. Let $X_1 = Y_1$ and for $k \geq 2$ let $X_k = Y_k \cdot X_{k-1}$. Does the sequence $X_1, X_2, \ldots$ satisfy the Markov property?