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Machine Learning

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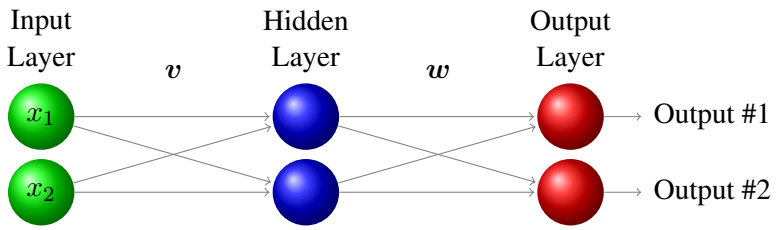
Exercise Sheet 4

Exercise 4-1 Backpropagation

In order to classify a two-dimensional input $\mathbf{x} = (x_1, x_2)^T$ into two classes, we will use a 2-layer feed forward network, i.e, one hidden layer and one output layer with two units on each layer. As activation function, we use the sigmoid function in both layers, $\sigma(arg) = (1 + e^{-arg})^{-1}$. We will use the sum-of-squares error function, i.e.

$$\mathcal{L} = \frac{1}{2} \sum_{k=1}^{N} (\hat{y}_k - y_k)^2$$



The architecture is shown without biases. In the following use also biases for the input ($x_0 = 1$), resp., for the hidden layer:

(a) Derive the update rule for the weights $\boldsymbol{w}$ connecting the neurons of the hidden layer with the neurons of the output layer.

(b) Derive the update rule for the weights $\boldsymbol{v}$ connecting the neurons of the input layer with the neurons of the hidden layer.

Exercise 4-2 Convolutions

Given the following 5x5 input image with one channel:

5	5	2	5	5
5	5	2	5	5
7	7	5	7	7
5	5	2	5	5
5	5	2	5	5

Let's assume we have the following 3x3 filters:

1	0	-1
2	0	-2
1	0	-1

1	2	1
0	0	0
-1	-2	-1

0	1	0
1	-4	1
0	1	0

After the filter size has been chosen (here: 3×3), we also have to define a *stride* and a *padding*. The former one controls the amount of shifting the filter at a time, i.e., how the filter convolves around the input volume. The latter one controls the spatial dimension of the output, e.g., a zero padding of size k indicates that the input volume is padded with k zeros around the border of the image such that the input volume can be preserved. The most prominent options for padding are called 'same' and 'valid': 'same' results in a padding of the input such that the output has the same volume as the original input, whereas the option 'valid' means that there is no padding at all.

(a) Apply the given filters (by cross-corellation) to the above dataset. The formula is given by:

$$S_{i,j} = (K \star X)_{i,j} = \sum_{m=0}^{2} \sum_{n=0}^{2} K_{m,n} X_{i+m,j+n}.$$

where S is the output, K denotes the kernel with dimensions $m \times n$ and X indicates the input of dimensions $i \times j$.

For this exercise use 'valid' padding and a stride of one.

(b) Look at the structure of the filters. What do they do?

Exercise 4-3 PyTorch - Convolutional Neural Network

On the course website you will find an ipython notebook leading you through the implementaion of a Convolutional Neural Network in PyTorch for classifying handwritten digits from the MNIST dataset. Fill in the missing parts and compare the results to the ones from the Feedforward Neural Network we used last week. What do you observe?