

Machine Learning
Summer 2021
Exercise Sheet 5

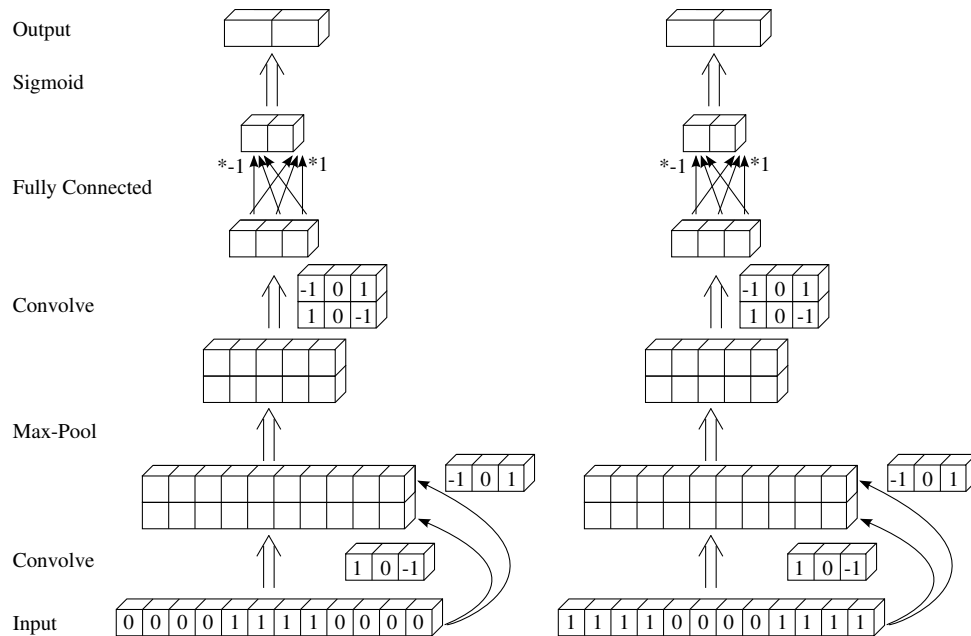
Exercise 5-1 Convolutional Neural Networks

In this exercise we address a convolutional neural network (CNN) with one-dimensional input. While two-dimensional CNNs can be used for example for grayscale images, one-dimensional CNNs could be used for time-series such as temperature or humidity readings. Concepts for the 1D-case are equivalent to 2D networks. We interpret data in our network as three-dimensional arrays where a row denotes a feature map, a column denotes a single dimension of the observation, and the depth of the array represents different observations. As we will only work with a single input vector, the depth will always be one.

Let the following CNN be given:

- Input I : Matrix of size $1 \times 12 \times 1$. We therefore have an input with twelve dimensions consisting of a single feature map.
- First convolutional layer with filters $F_0^1 = (-1, 0, 1)$ and $F_1^1 = (1, 0, -1)$ that generates two output feature maps from a single input feature map. Use *valid* mode for convolutions.
- Max-pooling layer with stride 2 and filter size 2. Note that max-pooling pools each feature map separately.
- Convolutional layer with convolutional kernel $F_0^2 = ((-1, 0, 1), (1, 0, -1))$ of size $2 \times 3 \times 1$.
- Fully connected layer that maps all inputs to two outputs. The first output is calculated as the negative sum of all its inputs, and the second layer is calculated as the positive sum of all its inputs.
- Sigmoidal activation function

Calculate the response of the CNN for the two inputs $(0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0)$ and $(1, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1)$.



Exercise 5-2 Basis Functions of Neural Networks

Given a test vector \mathbf{x}_i , the output of a neural network is defined as

$$f(\mathbf{x}_i) = \sum_{h=0}^{M_\phi-1} w_h \phi_h(\mathbf{x}_i, \mathbf{v}_h).$$

The weights of the neurons can be learned by employing the back-propagation rule with sample-based gradient descent. In the lecture neural networks with sigmoid neurons have been introduced, but it is possible to employ different basis functions:

- Which properties do these basis functions have to fulfill?
- Can a linear combination $\phi(\mathbf{x}_i, \mathbf{v}_h) = z_h = \sum_{j=0}^M v_{h,j} x_{i,j}$ be suitable for this?
- Is the number of parameters for $\phi(\mathbf{x}_i, \mathbf{v}_h)$ limited? Could several different basis functions be used for the same neural network?

Exercise 5-3 PyTorch - Convolutional Neural Network

On the course website you will find an ipython notebook leading you through the implementation of a Convolutional Neural Network in PyTorch for classifying handwritten digits from the MNIST dataset. Fill in the missing parts and compare the results to the ones from the Feedforward Neural Network we used last week. What do you observe?