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28.05.2021

Machine Learning

Summer 2021

Exercise Sheet 7

Exercise 7-1 Curse of Dimensionality vs. Kernel Trick

a) Explain the term *curse of dimensionality*.
When does it occur, how can it be avoided?

b) Explain the term *Kernel Trick*.
How can it be used, what is its connection to the *curse of dimensionality*?

Exercise 7-2 Kernel - Feature Mapping

In this exercise we want to compute the explicit representation of some kernels.

(a) The homogeneous quadratic kernel $K(x, y) = \langle x, y \rangle^2$ defined on the 2-dimension real vector space.

(b) The gaussian radial basis function kernel $K(x, y) = \exp\left(-\gamma \|x - y\|^2\right)$ for $x, y \in \mathbb{R}$ and $\gamma > 0$.
Hint: Use the power series expansion of the exponential function: $\exp(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!}$.

Exercise 7-3 Kernel Combinations

In order to use a custom kernel $k(\mathbf{x}, \mathbf{y})$ for $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, it must be shown that it is indeed a valid kernel. We can do that by expressing the explicit mapping of the implicit basis transformations: $k(\mathbf{x}, \mathbf{y}) = \phi(\mathbf{x})^T \phi(\mathbf{y})$. Another popular method of showing the validity of a kernel is representing a kernel, $k(\mathbf{x}, \mathbf{y}) = k_1(\mathbf{x}, \mathbf{y}) \circ k_2(\mathbf{x}, \mathbf{y})$, as a combination of valid kernels combined through valid basis operations.

Show that for a valid kernel $k_l(\mathbf{x}, \mathbf{y})$, where $l \in \mathbb{N}_+$, the following combinations are valid:

[label=)]**Scaling**: For $a > 0$: $k(\mathbf{x}, \mathbf{y}) := a \cdot k_1(\mathbf{x}, \mathbf{y})$ **Sum**: $k(\mathbf{x}, \mathbf{y}) := k_1(\mathbf{x}, \mathbf{y}) + k_2(\mathbf{x}, \mathbf{y})$ **Linear combination**: For $w \in \mathbb{R}_+^d$: $k(\mathbf{x}, \mathbf{y}) := \sum_{l=1}^{d} w_l \cdot k_l(\mathbf{x}, \mathbf{y})$ **Product**: $k(\mathbf{x}, \mathbf{y}) := k_1(\mathbf{x}, \mathbf{y}) \cdot k_2(\mathbf{x}, \mathbf{y})$ **Power**: For a $p \in \mathbb{N}_+ : k(\mathbf{x}, \mathbf{y}) := (k_1(\mathbf{x}, \mathbf{y}))^p$