Deep Learning

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Summer 2021

Neural Network Winter and Revival

- While Machine Learning was flourishing, there was a Neural Network winter (late 1990's until late 2000's)
- Around 2010 there was a revival which made neural networks again extremely popular; it was restarted by Geoffrey Hinton, Yann LeCun, and Yoshua Bengio
- Yann LeCun (New York University and Facebook Artificial Intelligence Research) and Yoshua Bengio (Université de Montréal) are two world-class researchers who never stopped working with neural networks. Geoffrey Hinton (co-inventor of the MLP, Boltzmann machines and others) (Google and University of Toronto) says it all got re-started with the 2006 paper "A fast learning algorithm for deep belief nets" by Hinton, Osindero, and Teh
- In Europe: Juergen Schmidhuber at IDSIA
- Deep networks achieved best results on many tasks/datasets

Schmidhuber: “Deep Learning Conspiracy”

Juergen Schmidhuber

Kai Yu

Yu Kai, head of Baidu's Institute of Deep Learning (IDL), demonstrates the smart bike project, DuBike, at the company's headquarters in Beijing. Photo: Simon Song

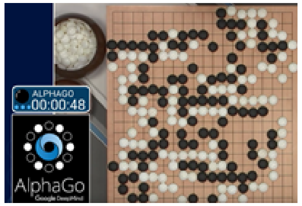
What Belongs to Deep Learning

1. In general: Multi Layer Perceptrons (Neural Networks) with many large hidden layers
2. Any Recurrent Neural Network (RNN) network, in particular LSTMs and GRUs (separate slides); Transformers
3. Convolutional Neural Networks (CNNs)
4. Deep Reinforcement Learning (separate slides)
5. Representation Learning, including representations for words, entities, predicates, samples
6. Deep Generative Models: VAEs, GANs (separate slides)

Artificial Intelligence

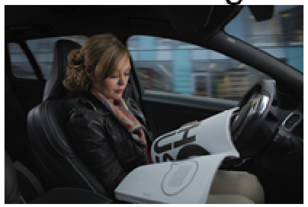
- *Creating machines that perform functions that require intelligence when performed by people (Kurzweil, 1990)*

Games



Q&A

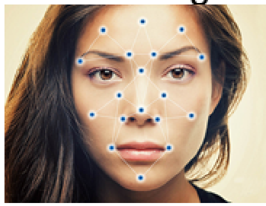
Auton. Driving



Drones, Robots

Translation

Face Recognition



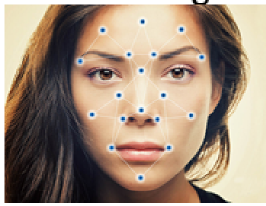
Speech Recognition

?

Deep Learning

- Deep Learning is the reason for the emerging huge interest in AI
  - Convolutional DL
  - Recurrent DL
  - Reinforcement DL
  - Generative Adversarial Networks (GANs)

Face Recognition

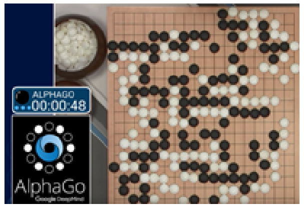


Translation

Speech Recognition

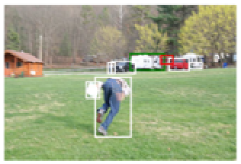
CycleGan

Games



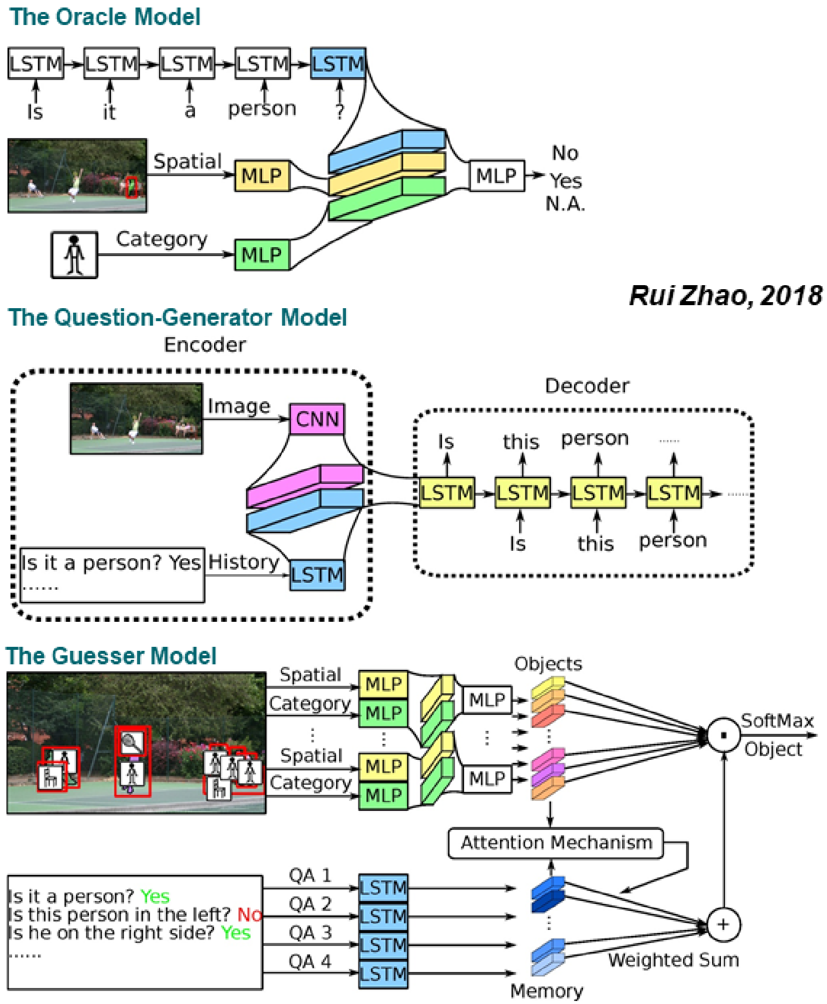
Student Magic: Visual Q&A

"I spy with my little eye ..."

Image	Policy Gradient		Tempered Policy Gradient	
	Is it in left?	No	Is it a person?	No
	Is it in front?	No	Is it a **vehicle**?	Yes
	Is it in right?	Yes	Is it a **truck**?	Yes
	Is it in middle?	Yes	Is it in front of photo?	No
	Is it person?	No	In the left half?	No
	Is it ball?	No	In the middle of photo?	Yes
	Is it bat?	No	Is it to the right photo?	Yes
	Is it **car**?	Yes	Is it in the middle of photo?	Yes
	Status:	Failure	Status:	Success
	Is it in **left**?	No	Is it a giraffe?	Yes
	Is it in front?	Yes	In front of photo?	Yes
	Is it in right?	No	In the **left half**?	Yes
	Is it in middle?	Yes	Is it in the middle of photo?	Yes
	Is it person?	No	Is it **to the left** of photo?	Yes
	Is it giraffe?	Yes	Is it to the right photo?	No
	Is in middle?	Yes	**In the left** in photo?	No
	Is in middle?	Yes	In the middle of photo?	Yes
	Status:	Failure	Status:	Success

Convolutional DL
+ Recurrent DL
+ Reinforcement DL

Talents, Talents Talents!

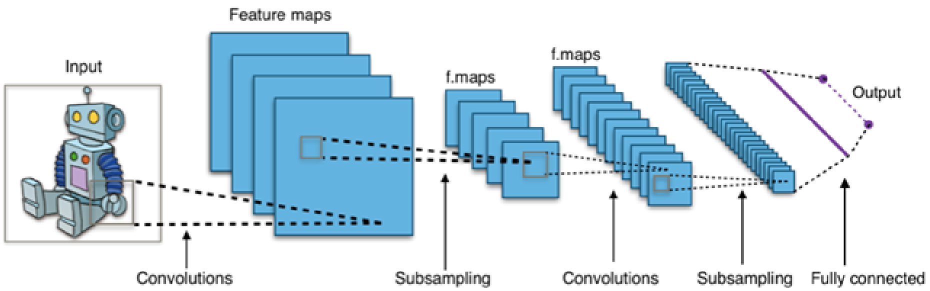


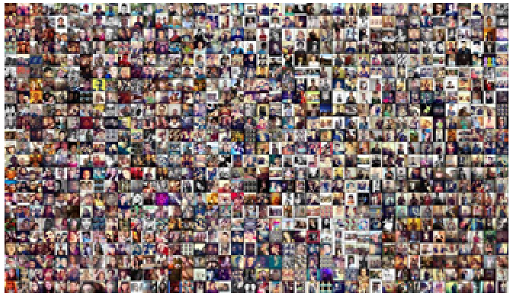
Rui Zhao, 2018

Deep X Technologies behind Artificial Intelligence

- **Deep Learning**; Machine Learning; Data Mining; Statistics
  - More (Labeled) Data
  - Deeper Models
  - New Algorithms
  - End-to-End Training; Differentiable-Computing (no Feature Engineering)
  - Computational Power
  - Community

- **Deep Knowledge: Facts and Models**
  - Huge Document Repositories with Rapid IE / QA (IBM Watson)
  - Maps with GPS for Autonomous Driving
  - Ubiquitous IoT and Big Data in Industry
  - Detailed (Patient) Profiles
  - Web Content, Wikipedia for Humans
  - **Knowledge Graphs for Machines**





Deep Learning Recipe: What Are the Reasons? (Hinton 2013)

1. Take a large data set
2. Take a Neuronal Network with many (e.g., 7) large (z.B. 1000 nodes/layer) layers
3. Optional: Use GPUs
4. Train with Stochastic Gradient Decent (SGD)
5. Except for the output layer use *rectified linear units*: $\max(0, h)$
6. Regularize with *drop-out*
7. Optional: Initialize weights with unsupervised learning
8. If the input is spatial (e.g., a picture), use convolutional networks (*weight sharing*) with *max-pooling*

Important Benefits

- A deep network learns complex application-specific features trained on **many data points (large N)**
- Data are given in some feature space (can be raw pixel images); **no additional feature engineering or basis function design is necessary**. Work with the data as they are, also with **large** M
- A deep architecture can achieve an efficient representation with fewer resources in a hierarchical layered structure
- Composition: In a classifier, an image is analysed by composing low level representations formed in the first processing layers, to generate high level features in the higher layers; a deep generative models composes an image from hierarchical representations

1: Large Data Set

- When decision boundaries are complex, a large data set describes the details of those boundaries
- Details can be captured with a complex (multi-layer) neural network
- "As of 2016, a rough rule of thumb is that a supervised deep learning algorithm will generally achieve acceptable performance with around 5,000 labeled examples per category, and will match or exceed human performance when trained with a dataset containing at least 10 million labeled examples." (Deep Leaning, Goodfellow et al.)

2: Large Networks

- It has been possible to train small to medium size problems since the early 1990s
- In deep learning, people work with really large Neural Networks. Example: 10 layers, 1000 neurons/layer
- ResNet from 2015 had 152 layers

3: Graphical Processing Units (GPUs)

- GPUs are highly suited for the kind of number crunching, matrix/vector math involved in deep Neural Networks. GPUs have been shown to speed up training algorithms by orders of magnitude
- Their highly parallel structure makes them more effective than general-purpose CPUs for algorithms where processing of large blocks of data is done in parallel
- General-Purpose Computing on Graphics Processing Units (GPGPU) is the utilization of a graphics processing unit (GPU), which typically handles computation only for computer graphics, to perform computation in applications traditionally handled by the central processing unit (CPU)

4: Stochastic Gradient Descent SGD

- Often regular SGD is used where the gradient is calculated on a single training pattern
- "Minibatch SGD" works identically to SGD, except that more than one training example is used to make each estimate of the gradient
- Gradient clipping (to avoid huge update steps):
  if $\|\mathbf{g}\| > v$ then $\mathbf{g} \leftarrow \mathbf{g}v/\|\mathbf{g}\|$. $v > 0$ is the norm threshold ($\mathbf{g}$ is the gradient vector)
- Local optima do not appear to be a major problem: current thinking is that there are many local optima, but that they are all very good

Adaptive Learning Rates

- AdaGrad (adaptive gradient algorithm) is often used for learning rates to be adaptively altered

- Let g_j be the gradient for weight w_j accumulated over a minibatch at "time" t

  $$w_j := w_j - \frac{\eta}{\sqrt{G_{j,j}}} g_j$$

  Here

  $$G_{j,j} = \sum_{\tau=1}^{t} g_{\tau,j}^2$$

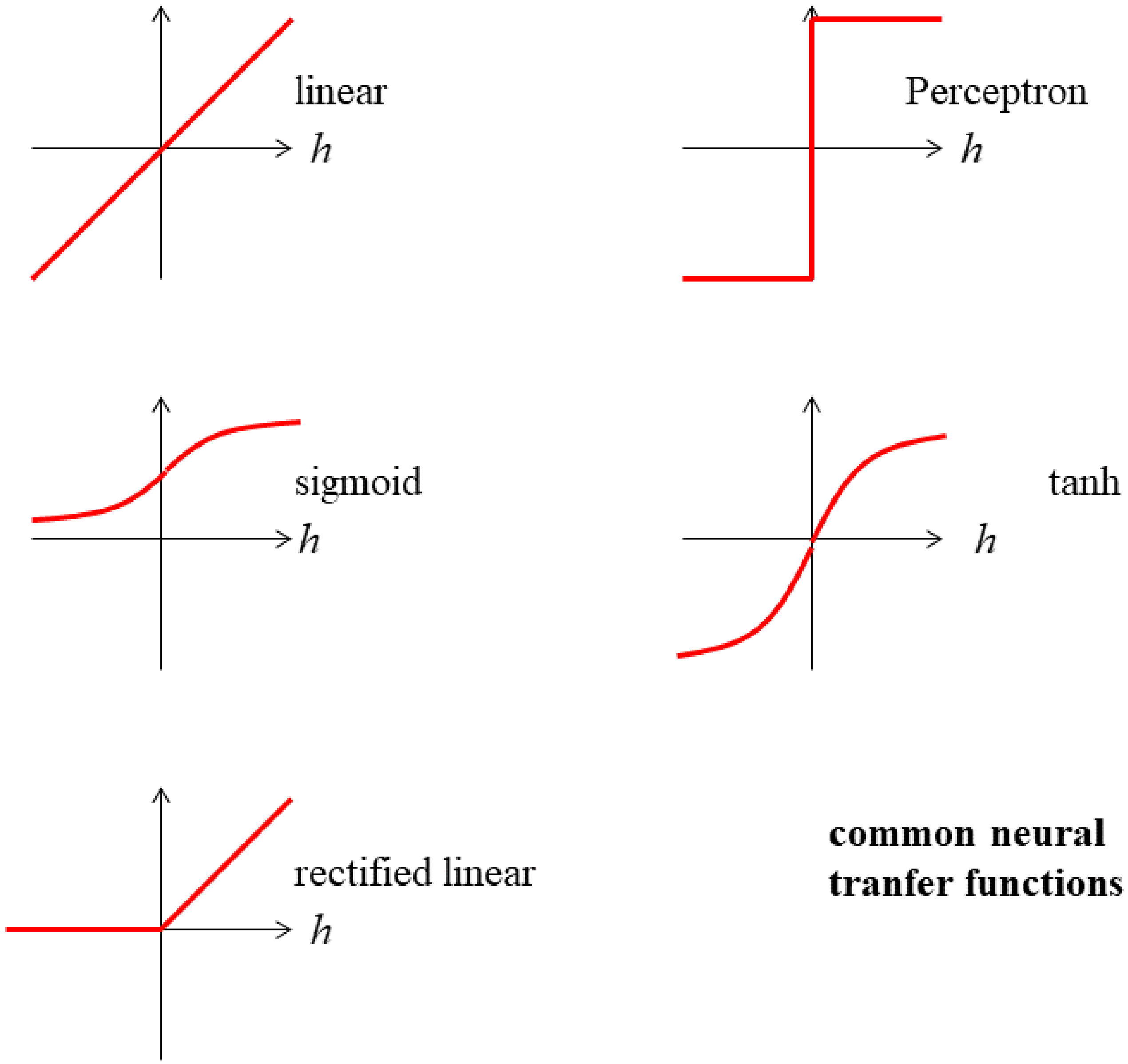
  is the accumulated squared gradient **from the start of the epoch**

- Extreme parameter updates get dampened, while parameters that get few or small updates receive higher learning rates

- The accumulation of the gradient from the start of the epoch can be problematic. Other popular variants: AdaDelta, Adam, RMSProp, Hessian-Free Optimization

5: Rectified Linear Function

- The transfer function of a Rectified Linear Unit (ReLU) is $max(0, h)$
- ReLU can be motivated in the following way: summing up the response of identical neurons (same input and output weights) where only the threshold/bias is varying. This become similar to a rectified linear neuron
- Reduces the effects of the vanishing gradient problem which can occur with sigmoid neurons! They learn much faster!
- Seems odd since some neurons become insensitive to the error, but a sufficient number stays active
- Leads to sparse gradients and to a sparse solution
- Leaky ReLU "fix" problems with "dead" neurons; GELU "fixes" problems with the discontinuity at the origin
- For training classification tasks, the output layer has sigmoidal activation functions and the cross-entropy cost function is used



common neural tranfer functions

6A: Drop-Out Regularization

- For each training instance: first remove 50% of all hidden units, randomly chosen. Only calculate error and do adaptation on the remaining network
- For testing (prediction): use all hidden units but multiply all outgoing weights by $1/2$ (gives you same expectation but no variance)
- This is like a committee machine, where each architecture is a committee member, but committee member share weights. It supposedly works like calculating the geometric mean: average the log of the predictions (and then take the exponential over the average)
- Works better than stopped training! No stopping rule required!
- Can even do drop-out in the input layer, thus different committee members see different inputs!
- Hinton: *use a large enough neural network so that it overfits on your data and then regularize using drop out*

- Goodfellow (DL): *Dropout provides an inexpensive approximation to training and evaluating a bagged ensemble of exponentially many neural networks*
- Variant: DropConnect (dropout of single connections)

6B: Weight Regularization

- Weight decay works
- But even better: for each neuron, normalize the incoming weight vector to have the same maximum length. Thus if $\|\mathbf{w}\| > \alpha$

$$\mathbf{w} \to \alpha \frac{1}{\|\mathbf{w}\|} \mathbf{w}$$

- Backpropagation is performed through the normalization

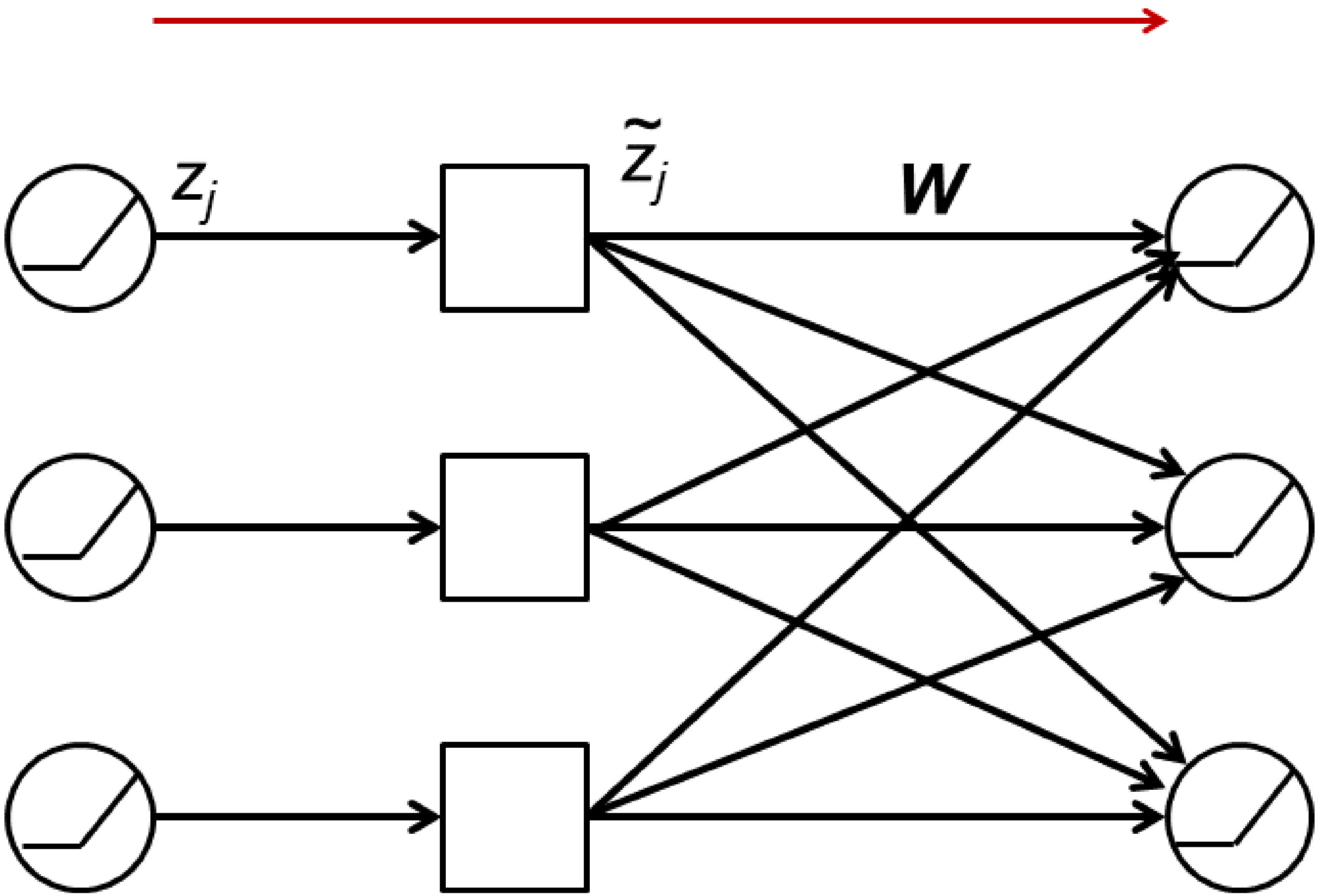
6C: Batch Normalization

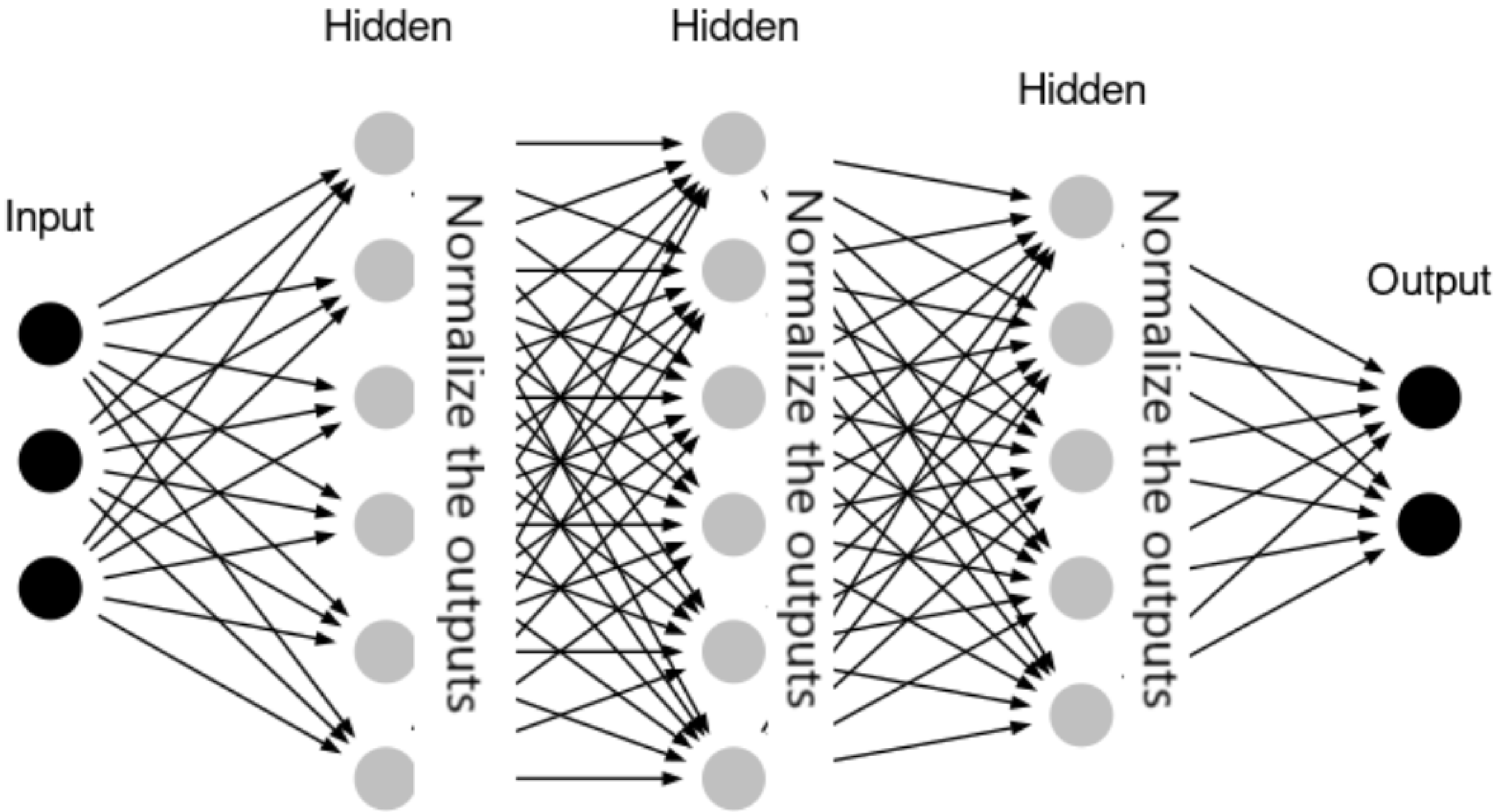
- Batch normalization: Let z_j be the output (activation) of a neuron after applying the nonlinear transfer function. Then each neuron z_j is normalized as

$$\tilde{z}_j = \frac{z_j - \mu_j}{\sqrt{\sigma_j^2 + \epsilon}}$$

- Mean μ_j and variance σ_j^2 of that neuron are calculated **over a small batch**. Back-propagation is performed through these operations. $\epsilon > 0$ is a small number, to ensure stability
- Batch normalization can greatly improve the training of deep networks! Reason: The mean and the variance of a neuron's output after the normalization don't change
- Batch normalization can be applied **after** the ReLU (as discussed here) or **before** the ReLU
- *Layer Normalization* is a variant which also works well in recurrent neural networks

Batch Normalization





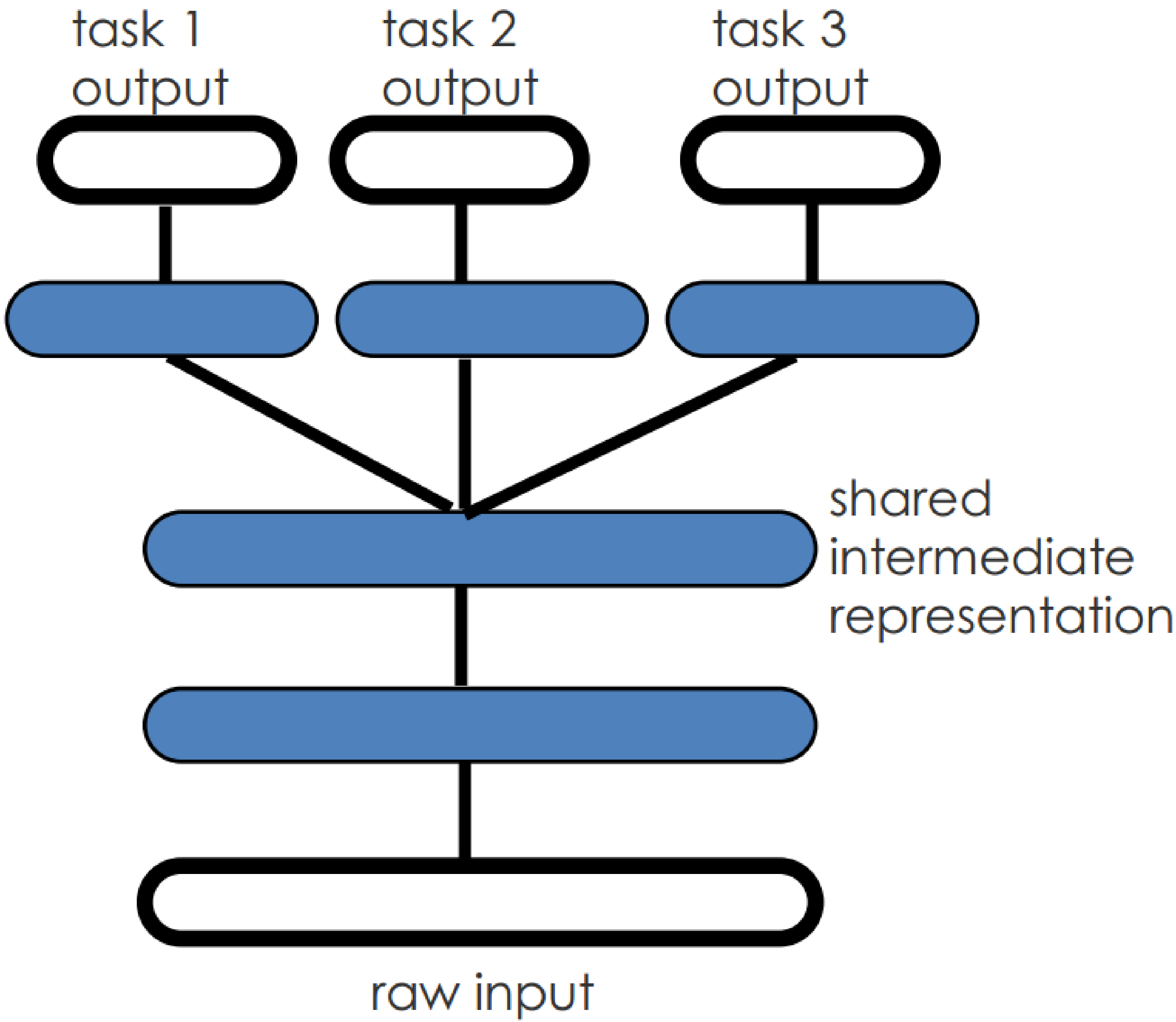
Input
Hidden
Hidden
Hidden
Output
Normalize the outputs
Normalize the outputs
Normalize the outputs

7: Initialize Weights with Unsupervised Learning

- The first layer is initialized with the encoder part of an autoencoder (separate slides)
- Alternatively: Several layers are initialized by a stacked autoencoder
- Not as popular anymore: Restricted Boltzmann Machine (RBM) for Deep Boltzmann Machines (DBMs) and Deep Belief Networks (DBNs)
- New in 2021: Pretraining is coming back with strong force in the form of "contrastive learning"!

Multitask Learning

- In the autoencoder approach, the same representation is used to train both the autoencoder and the classification task. This idea can be generalized in many ways. The idea is to learn several tasks by sharing common representations
- Another advantage is that a new task can be learned much faster!
- Current research: few-shot learning, one-shot learning, zero-shot learning


task 1 output
task 2 output
task 3 output
shared intermediate representation
raw input

Facebook's Deep Face: Face Recognition as Multi-Task Learning

- Build a deep learning NN to classify many face images from 4000 persons. Thus there are 4000 outputs, one for each person
- The next to last layer is used as a representation for any face image (also for faces and persons not in the training set)
- Note that here, the representation is close to the output layer
- Much effort is spent in the input layers to normalize the facial images
- C: convolutional layer. M: max-pooling layer. The subsequent layers (L4, L5 and L6) are locally connected, like a convolutional layer they apply a filter bank, but every location in the feature map learns a different set of filters.

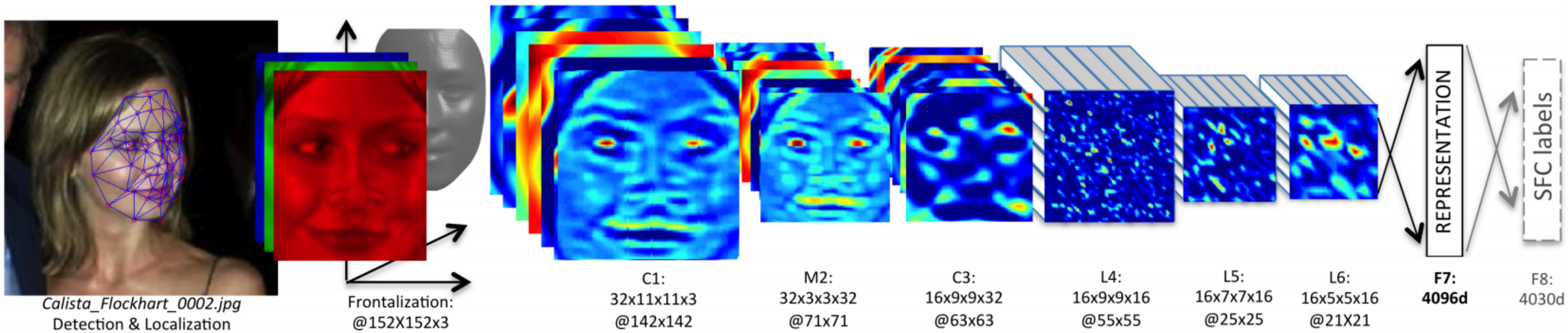


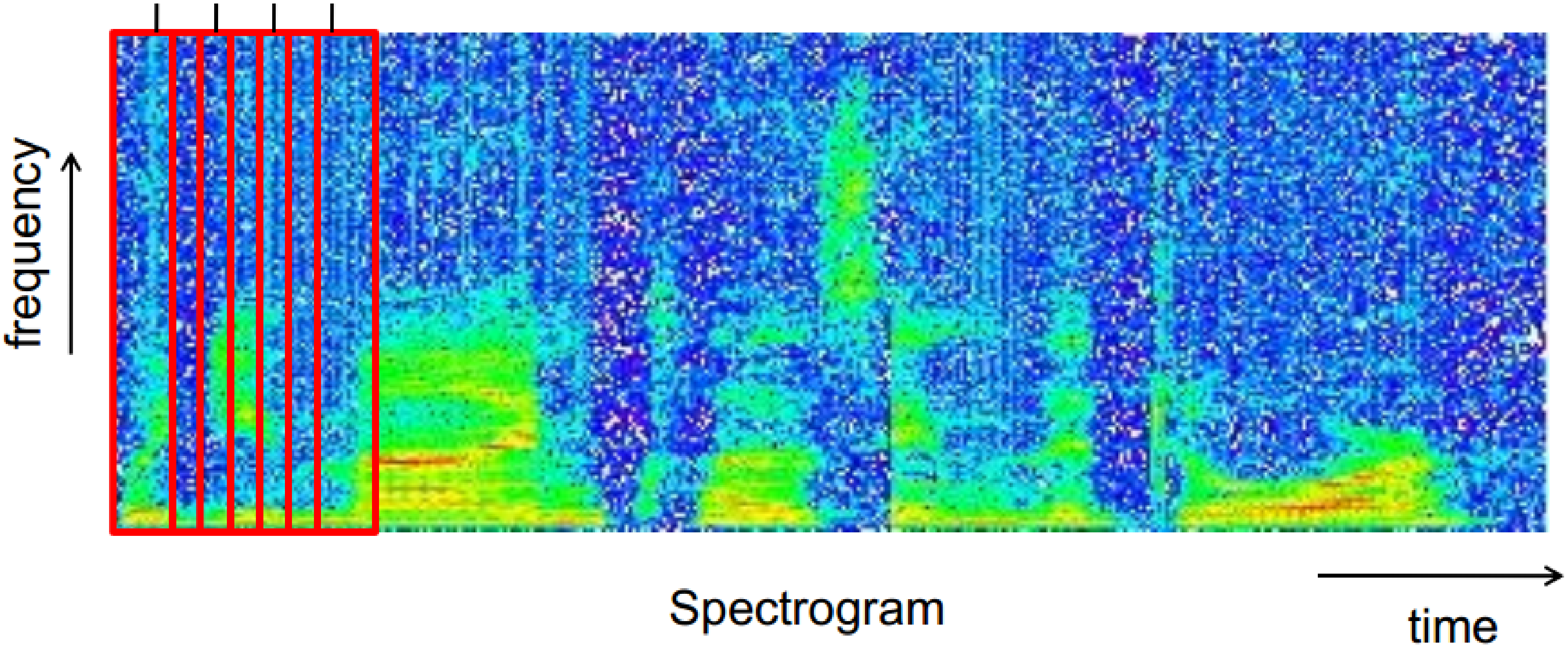
Figure 2. **Outline of the *DeepFace* architecture**. A front-end of a single convolution-pooling-convolution filtering on the rectified input, followed by three locally-connected layers and two fully-connected layers. Colors illustrate feature maps produced at each layer. The net includes more than 120 million parameters, where more than 95% come from the local and fully connected layers.

Android Server Architecture for Speech Recognition (2013)

- Part of speech recognition with Hidden Markov Models (HMMs): predict a state in the HMM (State) using a frequency representation of the acoustic signal in a time window (Frame)
- The Neural Network is trained to learn $P(State|Frame)$
- 4-10 layers, 1000-3000 nodes / layer, no pre-training
- Rectified linear activations: y=max(0,x)
- Full connectivity between layers,
- Softmax output (cross-entropy cost function) (see lecture on linear classifiers)

cont'd

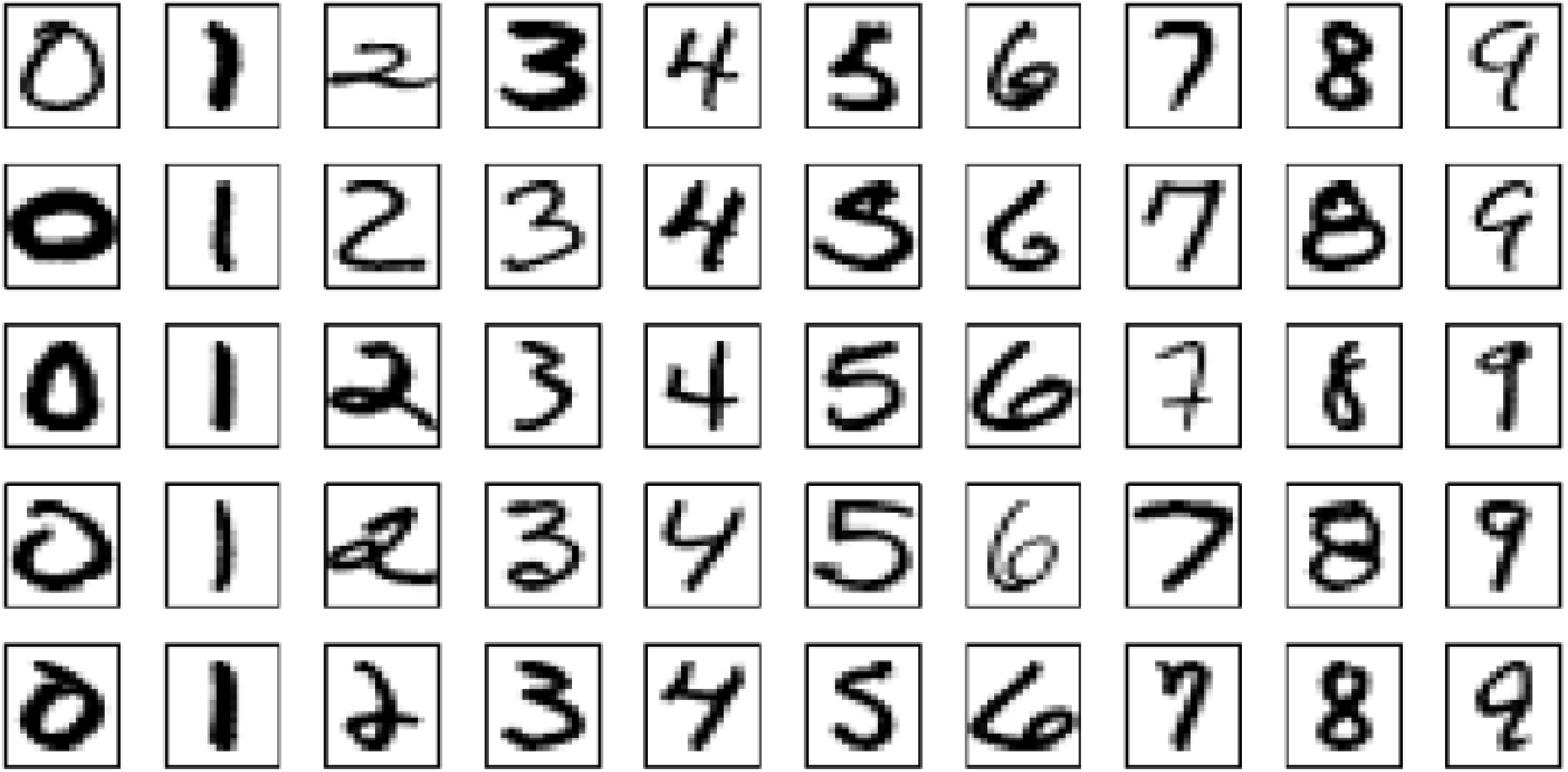
- Features:
  - 25ms window of audio, extracted every 10ms.
  - log-energy of 40 Mel-scale filterbanks, stacked for 10-30 frames.
- Training time: 2-3 weeks using GPUs!
- Online: Android uses the server solution. Offline: Small Neural Network on the Smart Phone
- Advantage: Speaker independent! Now used by Google, Microsoft, IBM, replacing Gaussian mixture models (30% reduction in error)
- Even more improvement on the task of object recognition in images (from 26% error to 16% error)) using 1.2 million training images. With convolutional neural networks.


frequency
Spectrogram
time

task	Hours of training data	Deep net+HMM	GMM+HMM same data	GMM+HMM more data
Switchboard	309	16.1	23.6	17.1 (2k hours)
English Broadcast news	50	17.5	18.8	
Bing voice search	24	30.4	36.2	
Google voice input	5870	12.3		16.0 (lots more)
Youtube	1400	47.6	52.3	

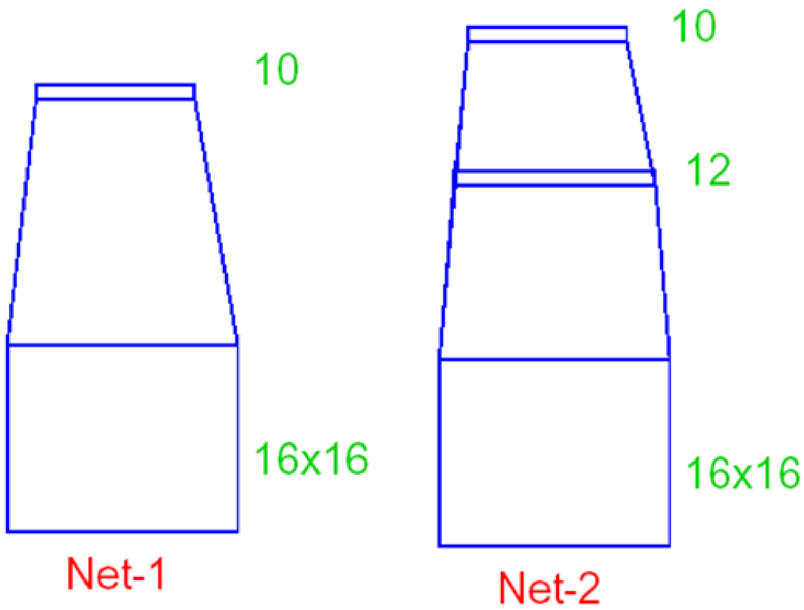
8: Convolutional Neural Networks (CNNs)

Recognition of Handwritten Digits



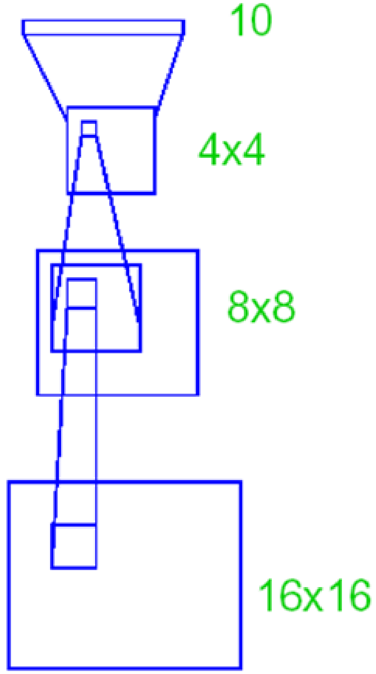
Recognition of Handwritten Digits using Neuronal Networks

- Example: 16×16 grey-valued pictures; 320 training images, 160 test images
- Net-1: No hidden layer: corresponds to 10 Perceptrons, one for each digit
- Net-2: One hidden layer with 12 nodes; fully connected ("normal MLP")


10
16x16
Net-1
10
12
16x16
Net-2

Neuronal Network with local connectivity: Net-3

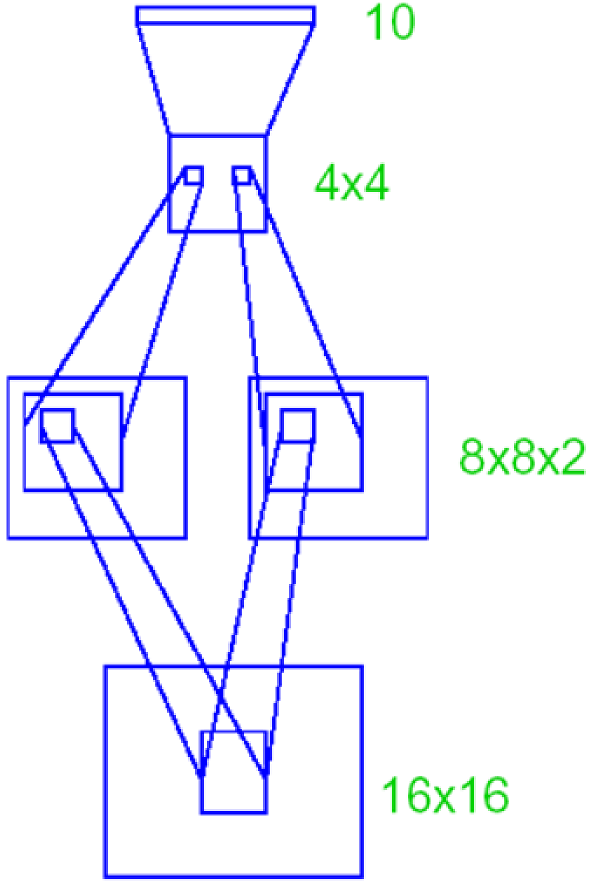
- In the following variants, the complexity was reduced
- Net-3: Two hidden layers with local connectivity (but no weight sharing yet): motivated by the local receptive fields in the brain
  - Each of the 8×8 neurons in the first hidden layer is only connected to 3×3 input neurons from a receptive field
  - In the second hidden layer, each of the 4×4 neurons is connected to 5×5 neurons in the first hidden layer
  - Net-3 has less than 50% of the weights of Net-2, but more neurons



Net-3
Local Connectivity

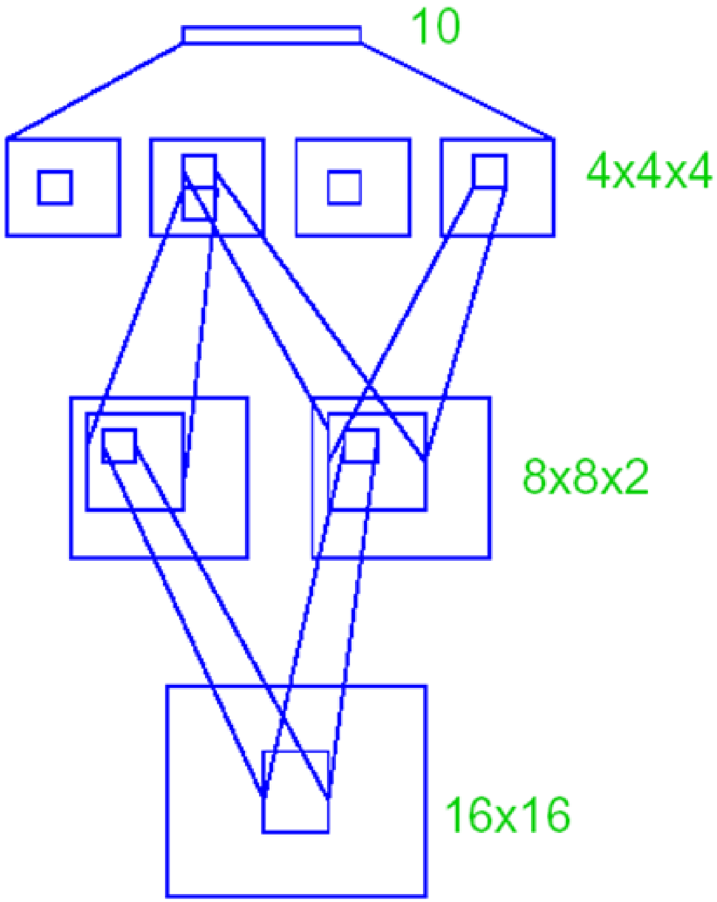
Neuronal Networks with Weight-Sharing (Net-4)

- Net-4: Two hidden layers with local connectivity and *weight-sharing*
- All receptive fields in the left 8×8 block have the same weights; the same is true for all neurons in the right 8×8 block
- The 4×4 block in the second hidden layer, as before


10
4x4
8x8x2
16x16

Neural Networks with Weight Sharing (Net-5)

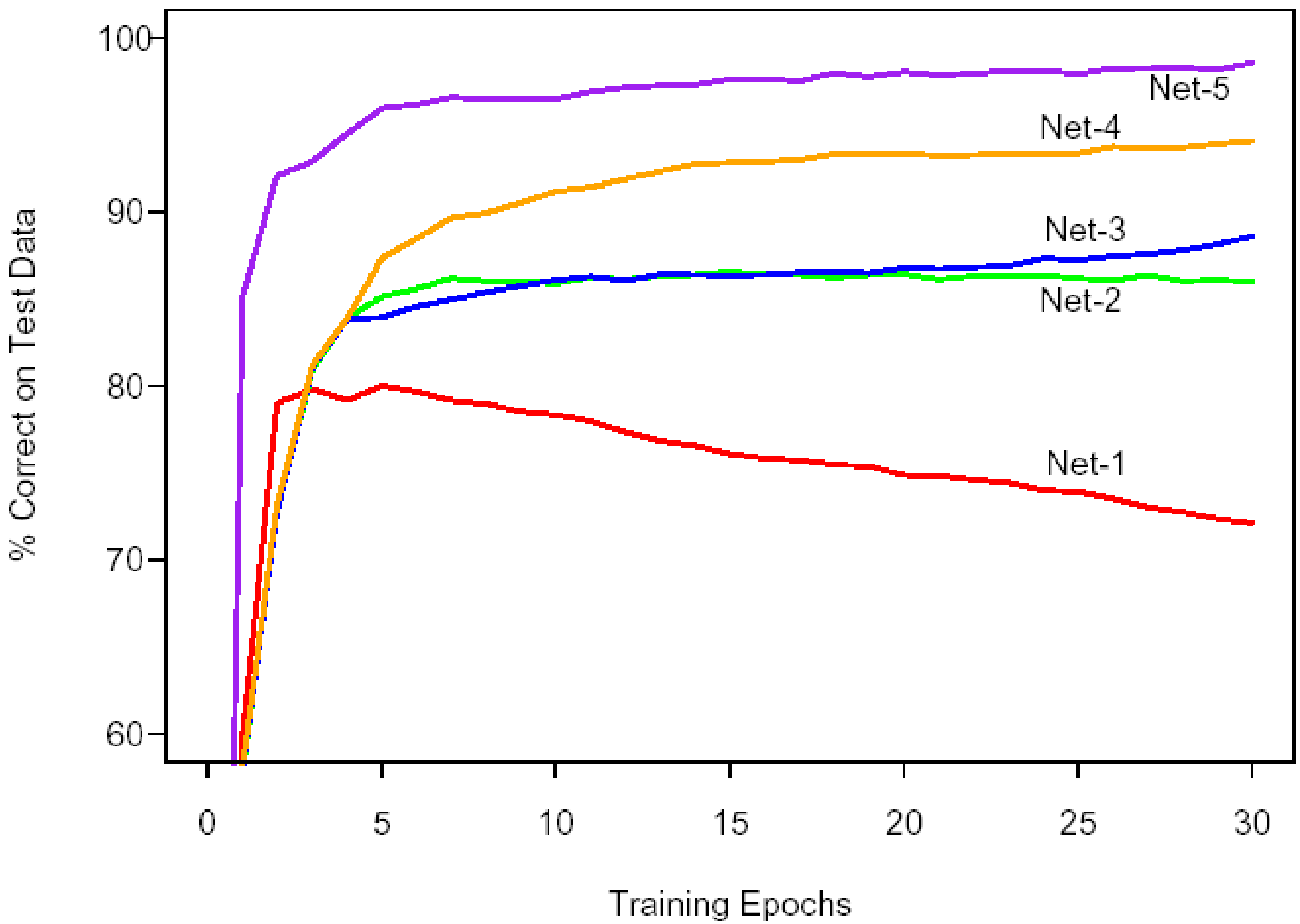
- Net-5: Two hidden layers with local connectivity and two layers of *weight-sharing*



Net-5

Learning Curves

- One training epoch is one pass through all data
- The following figure shows the performance on the test set
- Net-1: One sees overfitting with increasing epochs
- Net-5: Shows best results without overfitting


100
90
80
70
60
% Correct on Test Data
Net-5
Net-4
Net-3
Net-2
Net-1
0
5
10
15
20
25
30
Training Epochs

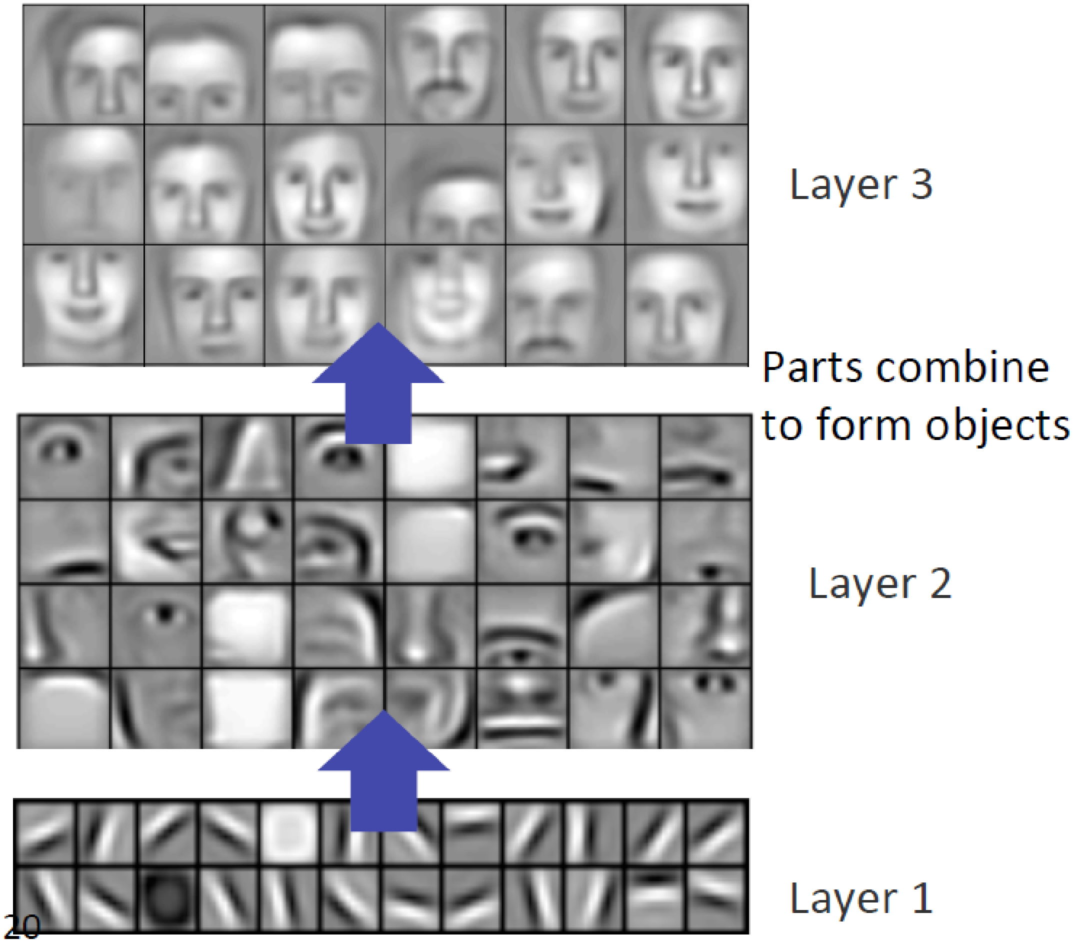
Statistics

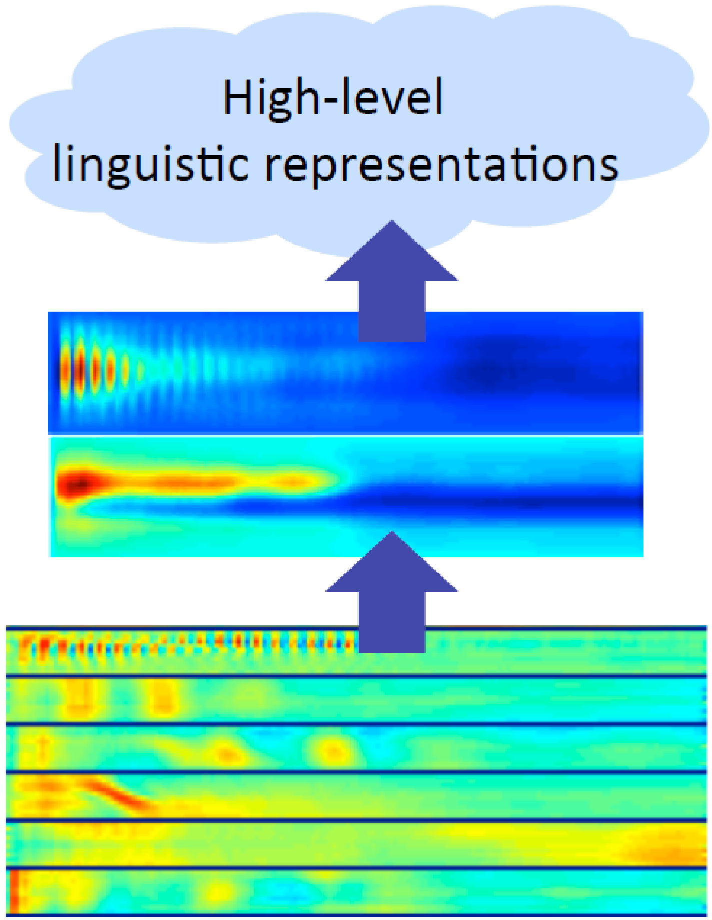
- Net-5 has best performance. The number of free parameters (1060) is much smaller than the total number of parameters (5194)

TABLE 11.1. *Test set performance of five different neural networks on a handwritten digit classification example (Le Cun, 1989).*

	Network Architecture	Links	Weights	% Correct
Net-1:	Single layer network	2570	2570	80.0%
Net-2:	Two layer network	3214	3214	87.0%
Net-3:	Locally connected	1226	1226	88.5%
Net-4:	Constrained network 1	2266	1132	94.0%
Net-5:	Constrained network 2	5194	1060	98.4%

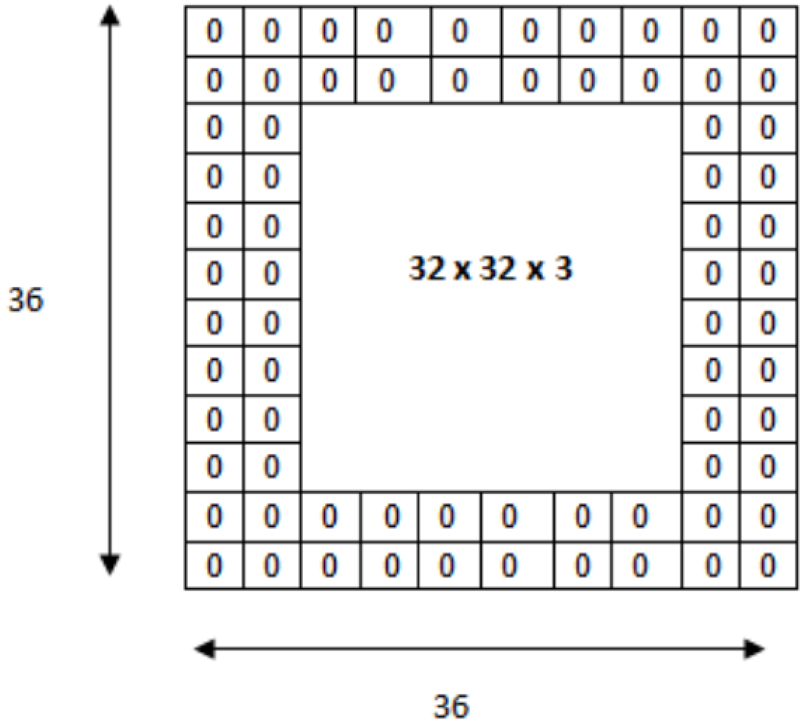
Successive model layers learn deeper intermediate representations





General Case

- The current layer might consist of R^{in} sub-layers of size $H^{in} \times H^{in}$
- The next might consist of R^{out} sub-layers of size $H^{out} \times H^{out}$
- This implies that one has different R^{out} filter kernels (one for each output sub-layer); each filter kernel is a tensor of dimension (*size*) $h \times h \times R^{in}$, where typically $h << H^{in}$
- The *stride* is H^{in}/H^{out}; often the stride is 1; by using a larger stride, I can down-sample the image
- Note that I might need to pad the image with zeros

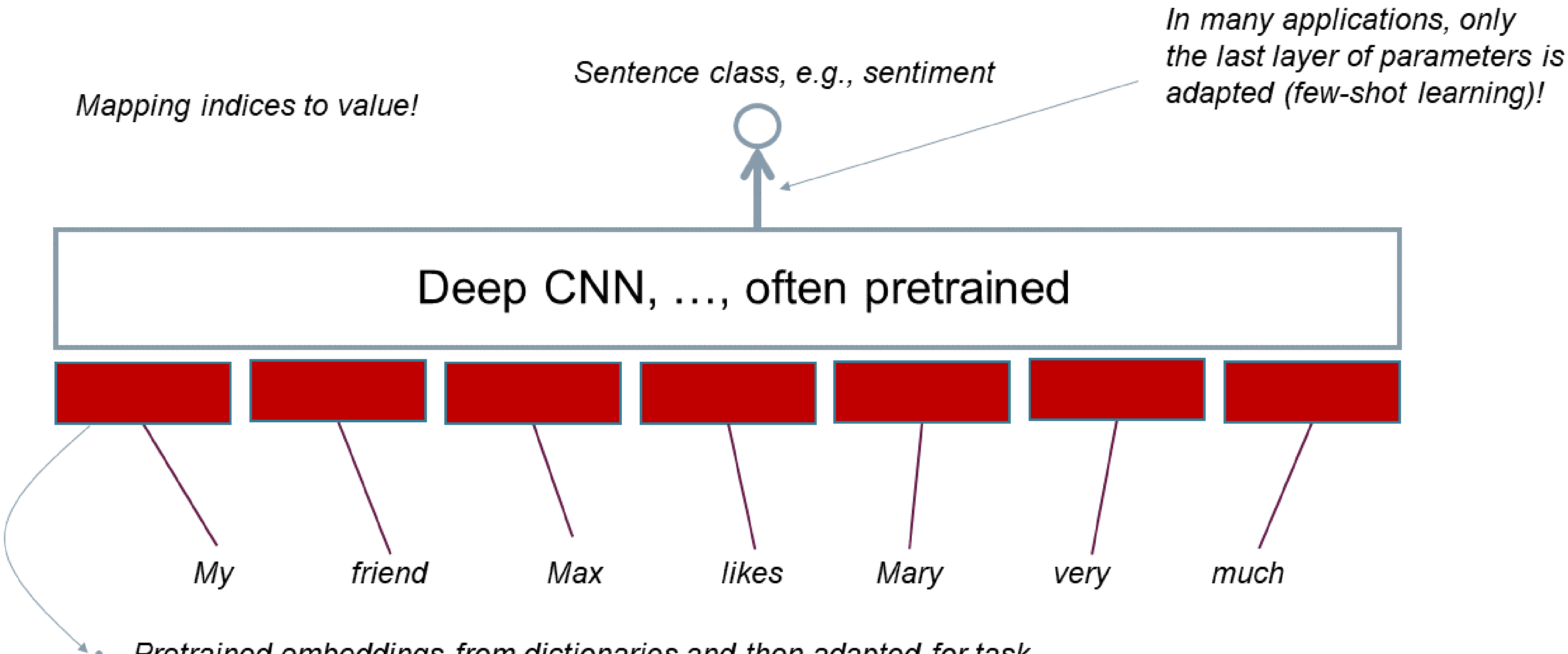


The input volume is 32 x 32 x 3. If we imagine two borders of zeros around the volume, this gives us a 36 x 36 x 3 volume. Then, when we apply our conv layer with our three 5 x 5 x 3 filters and a stride of 1, then we will also get a 32 x 32 x3 output volume.

Representation Learning and Convolutional Neural Networks in Sentence Classification

- Representation Learning and Convolutional Neural Networks in Sentence Classification

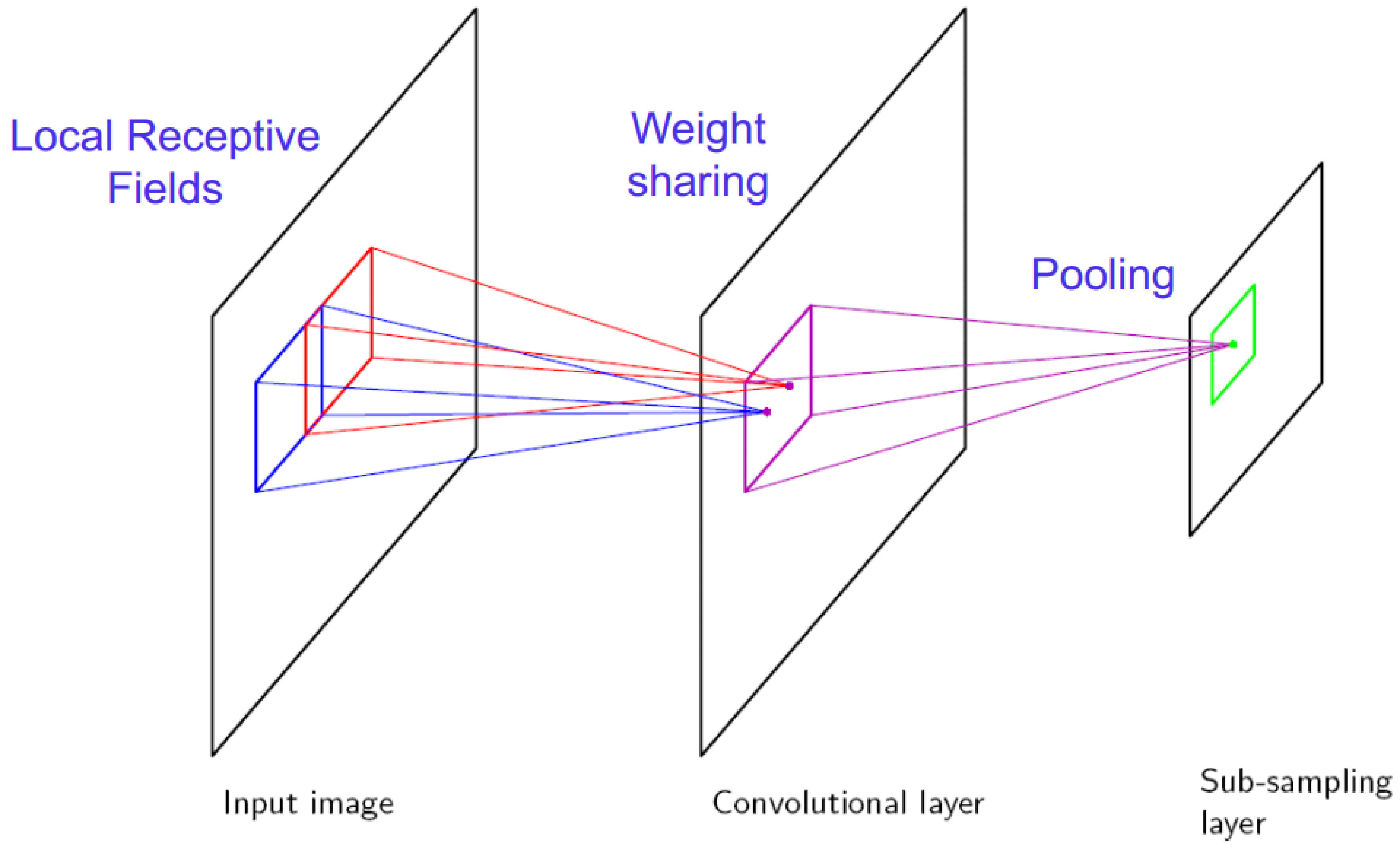
Nonlinear Mappings:
Tensor Modelling as Representation Learning in NLP



- *Pretrained embeddings from dictionaries and then adapted for task*
- *No feature engineering: NLP for everyone!*

Pooling

- For example, one could compute the mean (or max) value of a **particular feature** over a **region of the image**. These summary statistics are much lower in dimension (compared to using all of the extracted features) and can also improve results (less over-fitting). We aggregation operation is called this operation pooling, or sometimes **mean pooling** or **max pooling** (depending on the pooling operation applied).
- Max-pooling is useful in vision for two reasons: (1) it reduces the computational complexity for upper layers and (2) it provides a form of translation invariance
- Since it provides additional robustness to position, max-pooling is thus a "smart" way of reducing the dimensionality of intermediate representations.
- Mean pooling is related to a convolutional layer with a rectangular (uniform) kernel and a large stride

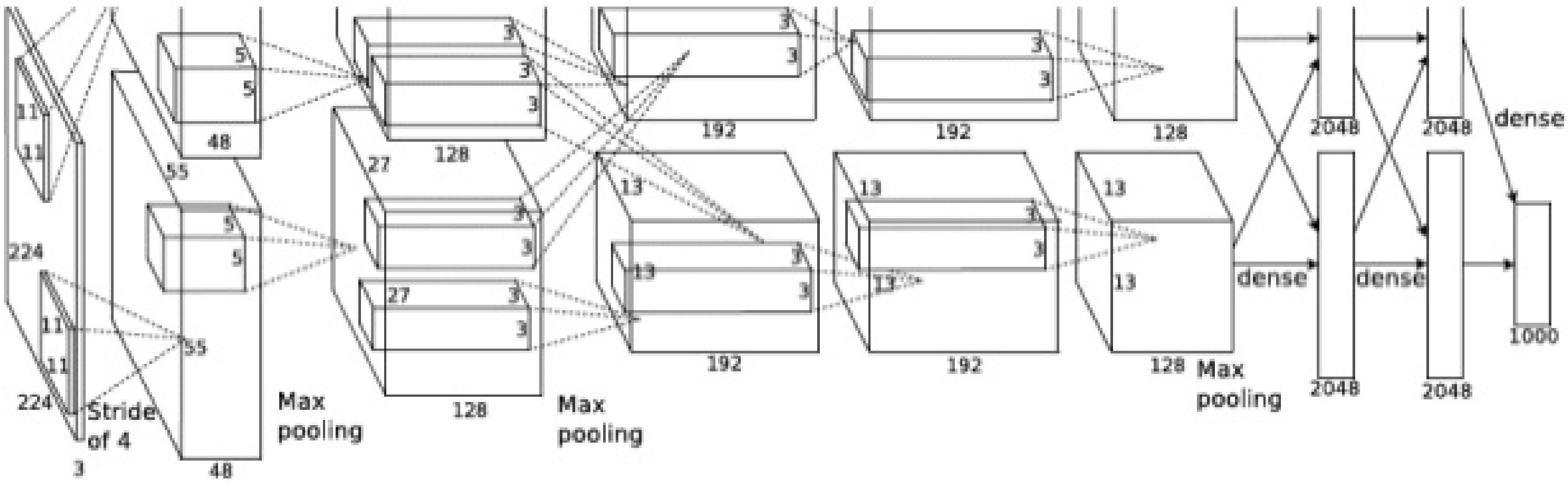

Local Receptive
Fields
Weight
sharing
Pooling
Input image
Convolutional layer
Sub-sampling
layer

Architectures

- The next slide shows AlexNet
- Alternatives: Inception (Google), Visual Geometry Group (VGG) (Oxford)

AlexNet

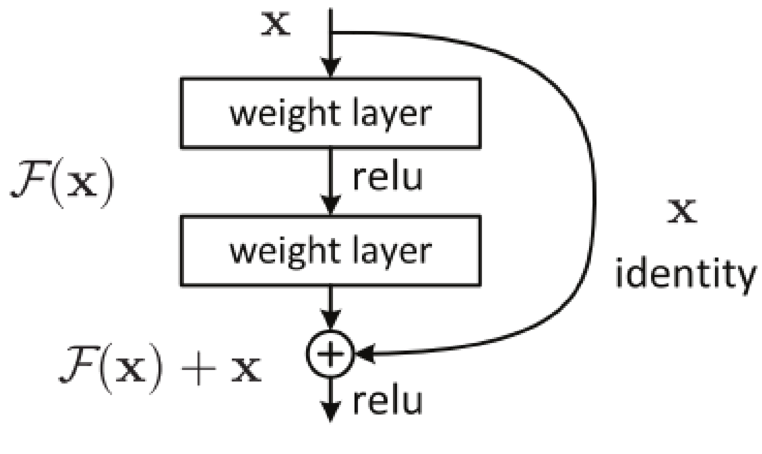
- Similar framework to LeCun'98 but:
  - Bigger model (7 hidden layers, 650,000 units, 60,000,000 params)
  - More data (10^6 vs. 10^3 images)
  - GPU implementation (50x speedup over CPU)
    - Trained on two GPUs for a week



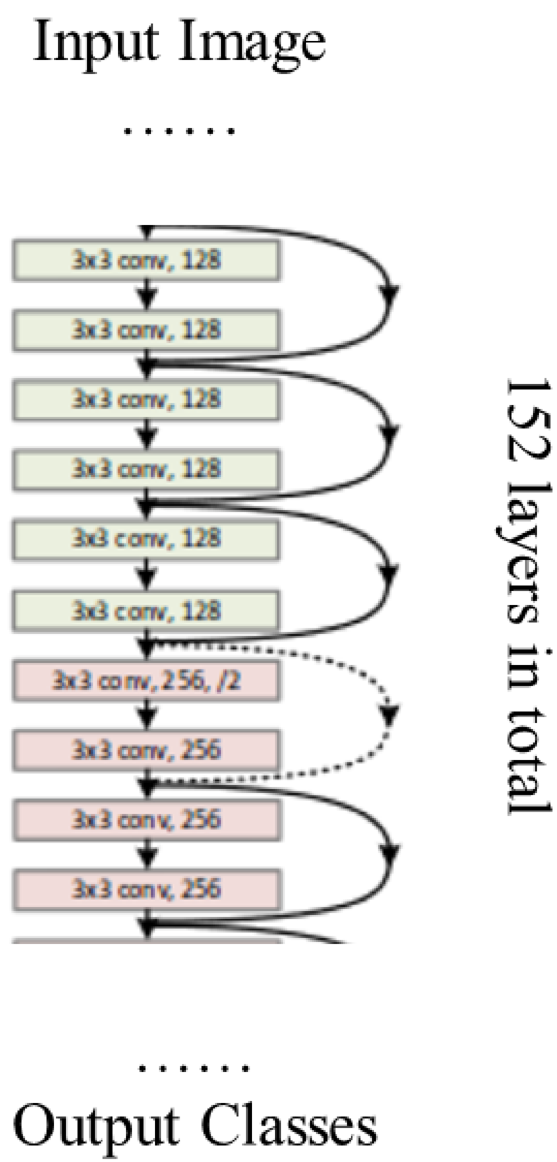
A. Krizhevsky, I. Sutskever, and G. Hinton,
ImageNet Classification with Deep Convolutional Neural Networks, NIPS 2012

Deep Residual Network

- The next figure shows a Deep Residual Network (ResNet) (2015)
- A ResNet of 152 layers became world champion in the ImageNet data base
- Other special architectures used in image classification: AlexNet (5 convolutional layers) (2012), VGG network (19 convolutional layers) (2014), GoogleNet (Inception) (22 convolutional layers) (2015), and many variants

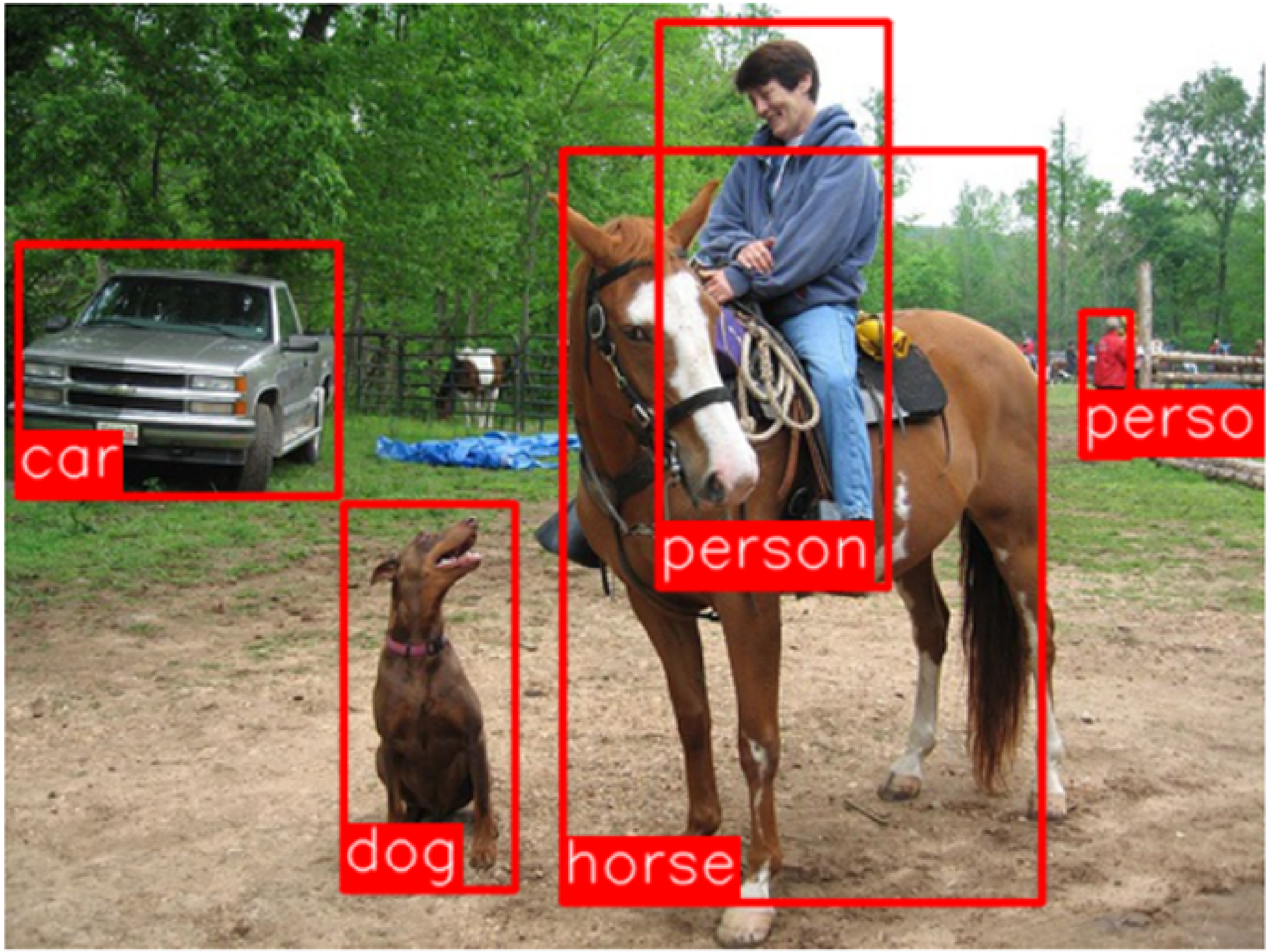


a residual block



Regional CNN

- Task: Finding bounding boxes in images (object detection, object segmentation)
- R-CNN, Fast R-CNN, Faster R-CNN, Mask R-CNN



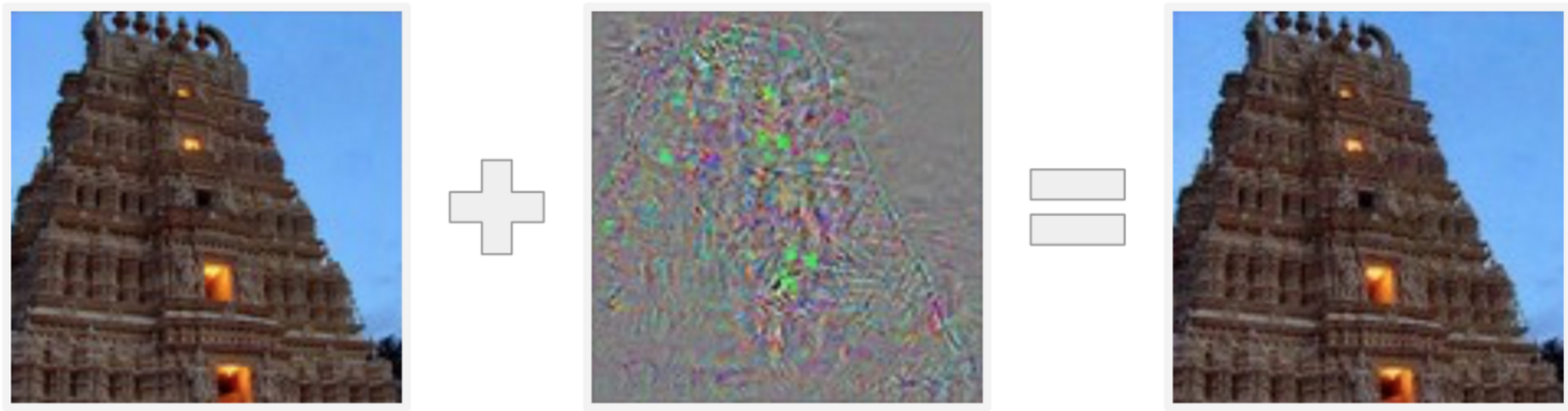
Object detection algorithms such as R-CNN take in an image and identify the locations and classifications of the main objects in the image. Source: https://arxiv.org/abs/1311.2524.

Adversarial Examples

- Deep high-performing DNNs can be fooled by examples intentionally constructed by using an optimization procedure to search for an input that is very close to a real data point and produces a very different label

- A good explanation might be that training data lies on a manifold and for new data on the same manifold, performance is very good: *the adversarial examples are away from the manifold and, there, the model behaves rather unpredictable* (see lecture on manifolds)

- Adversarial training are attempts to make DNNs less prone to adversarial examples (active research area)



Original image Temple (97%)

Perturbations

Adversarial example Ostrich (98%)

Where from here?

- There will never be enough labelled data to learn it all
- The Google cat recognizer sees more cat images than any child and is not as good
- If one assumes that cat features are not encoded genetically, then **unsupervised learning**, i.e., understanding the world's statistics might do the job! First attempts: RBM, all sorts of Clustering, autoencoders, ...
- LeCun advertises: self-supervised leaning

Tools

- **Torch7** is used at facebook, Deep Mind and several groups at Google (based on LuaJIT which is similar to Python)
- **PyTorch** is an open-source machine learning library for **Python**, based on Torch (initial release 2016)
- **GP-GPU-CUDA:** Facebook, NYU, and Google/Deep Mind all have custom CUDA back-ends for fast/parallel convolutional network training (CUDA (Compute Unified Device Architecture) is a parallel computing platform and programming model implemented by the graphics processing units (GPUs) that they produce. CUDA gives program developers direct access to the virtual instruction set and memory of the parallel computational elements in CUDA GPUs)
- **Theano:** Python library. Popular in the deep learning community. Theano family:
  - **Blocks + Fuel:** Blocks and Fuel are machine learning frameworks for Python developed by the Montreal Institute of Learning Algorithms (MILA) at the University of Montreal. Blocks is built upon Theano (also by MILA) and allows for rapid

prototyping of neural network models. Fuel serves as a data processing pipeline and data interface for Blocks.

  - **Keras:** Keras is a minimalist, highly modular neural networks library, written in Python and capable of running on top of either TensorFlow or Theano.
  - **Lasagne:** Lasagne is a lightweight library to build and train neural networks in Theano.
  - **PyLearn2:** Pylearn2 is a machine learning library.

- **TensorFlow:** TensorFlow is an open source software library for machine learning in various kinds of perceptual and language understanding tasks. Under development: **Tensor Processing Unit (TPU) custom chip**

- **Deeplearning4j** is an open source deep learning library for Java and the Java Virtual Machine

- **Caffe** is a deep learning framework made with expression, speed, and modularity in mind. It is developed by the Berkeley Vision and Learning Center (BVLC) and by community contributors. Yangqing Jia created the project during his PhD at UC Berkeley.

Successes

- Microsoft Speech (2012): Chief Research Officer Rick Rashid demonstrates a speech recognition breakthrough via machine translation that converts his spoken English words into computer-generated Chinese language.
- Google: Android Speech Recognition: Maps; Image+ (cats etc.); improve Google translate (ongoing project); Google used the (deep learning) program to update its Google+ photo-search software in May 2013; Apple: SIRI (the iPhone's voice-activated digital assistant, Siri, relies on deep learning.)
- In September 2016, a research team at Google announced the development of the Google Neural Machine Translation system (GNMT) and by November Google Translate began using neural machine translation (NMT)
- Facebook: DeepFace (2014), of the steps detect-align-represent-classify, the representation step is done by a DNNs. Asked whether two unfamiliar photos of faces show the same person, a human being will get it right 97.53 percent of the time. New software developed by researchers at Facebook can score 97.25 percent on the same

challenge, regardless of variations in lighting or whether the person in the picture is directly facing the camera.

- AlexNet is the name of a convolutional neural network, originally written with CUDA to run with GPU support, which competed in the ImageNet Large Scale Visual Recognition Challenge in 2012. The network achieved a top-5 error of 15.3%, more than 10.8 percentage points ahead of the runner up. AlexNet was designed by the SuperVision group, consisting of Alex Krizhevsky, Geoffrey Hinton, and Ilya Sutskever

- ResNet (2015)

- GANs have been used to produce samples of photorealistic images for the purposes of visualizing new interior/industrial design, shoes, bags and clothing items or items for computer games' scenes (2014)

- Spectacular success of AlphaGo (2015) and AlphaZero (2017) in Go, Chess, and other games

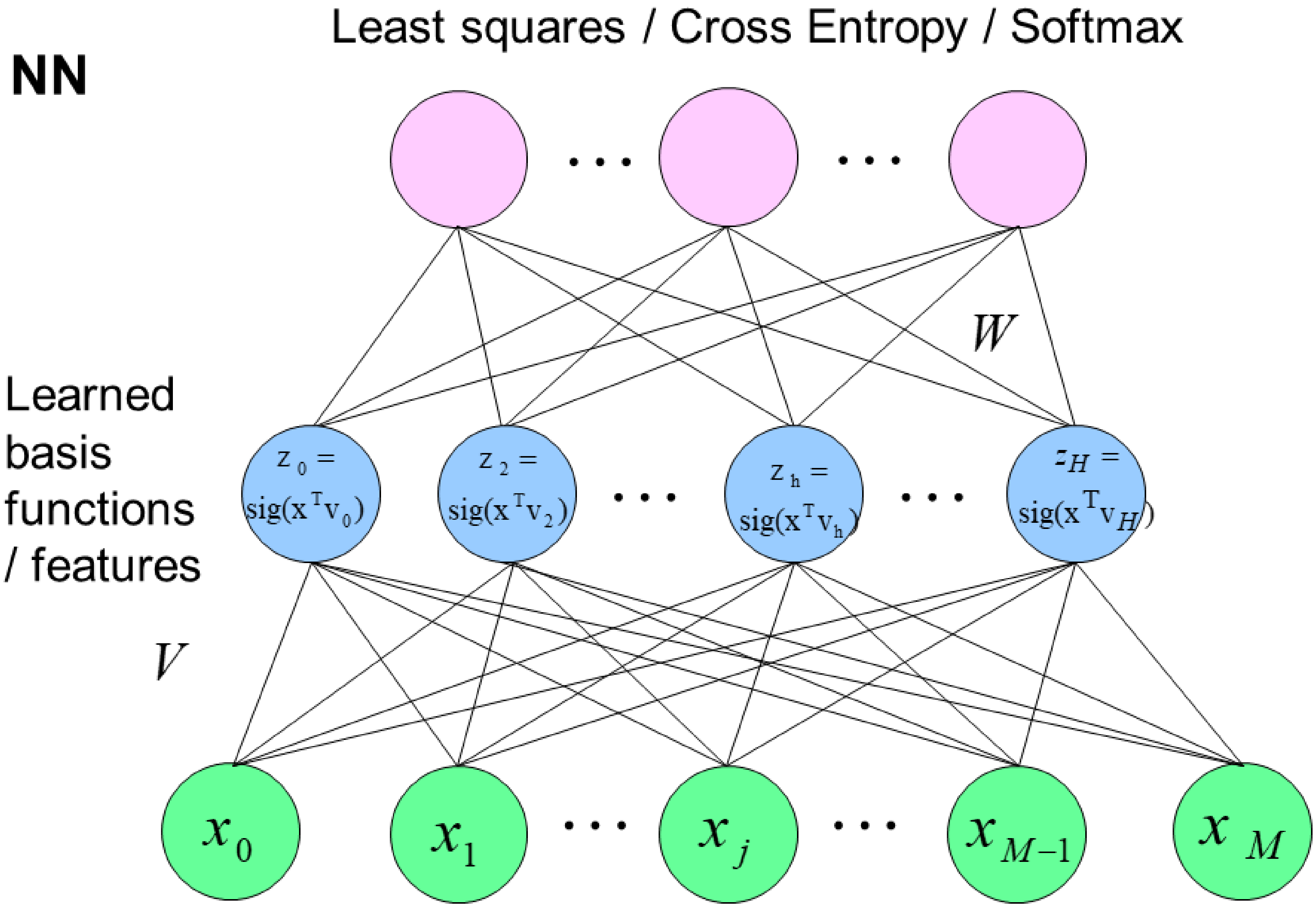
- AutoML, Automatic Statistician, ...

- Self-driving cars

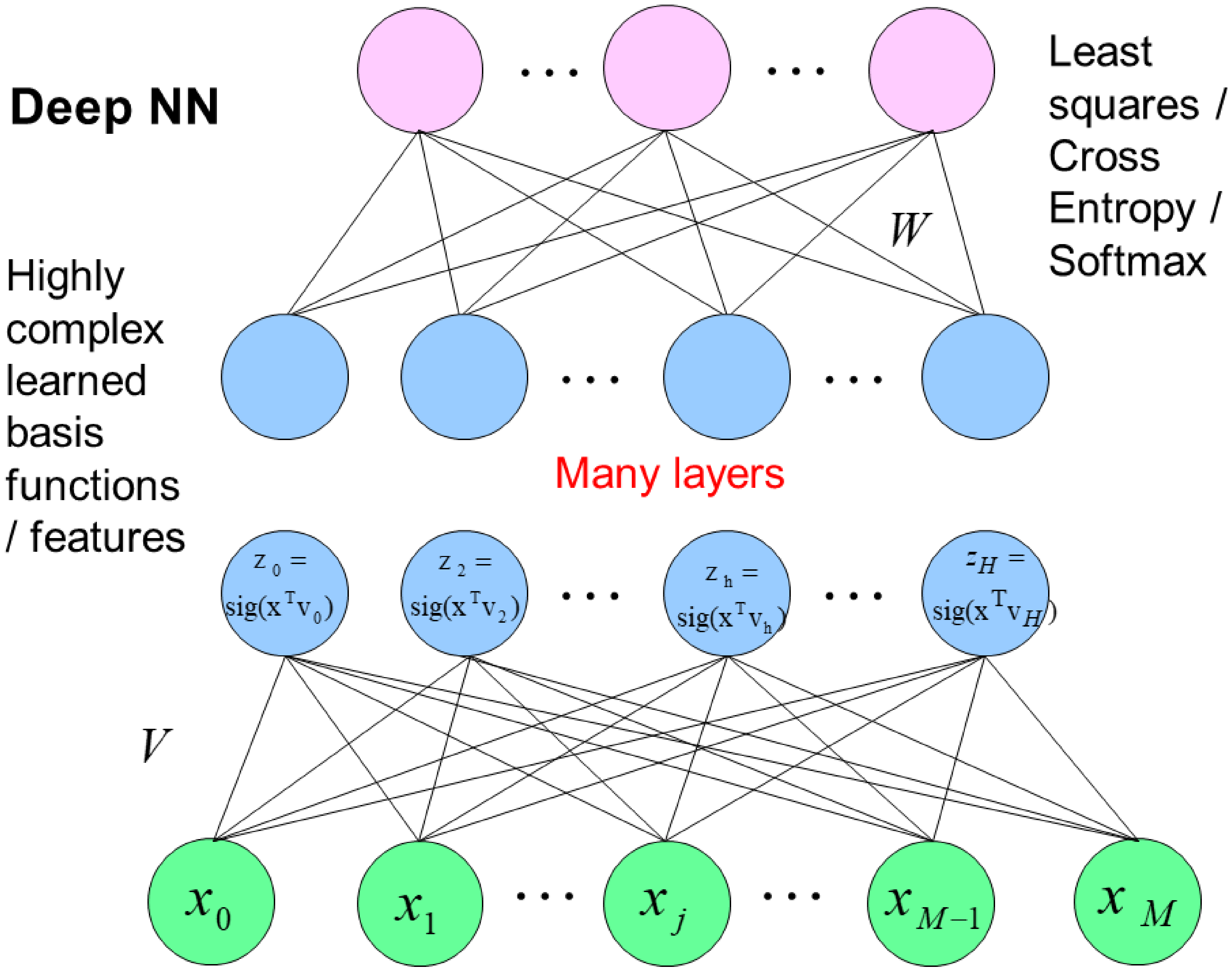
Deep Learning also Learns Fancy Basis Functions / Features

- Similar ro our analysis of a neural network, also a deep neural network learns basis functions
- The difference is that, due to the many hidden layers, these basis functions now can be highly complex

NN



Least squares / Cross Entropy / Softmax
W
Learned basis functions / features
$z_0 = \mathrm{sig}(x^T v_0)$
$z_2 = \mathrm{sig}(x^T v_2)$
$z_h = \mathrm{sig}(x^T v_h)$
$z_H = \mathrm{sig}(x^T v_H)$
V
x_0
x_1
x_j
x_{M-1}
x_M



Deep NN
Least squares / Cross Entropy / Softmax
W
Highly complex learned basis functions / features
Many layers
$z_0 = \mathrm{sig}(x^T v_0)$
$z_2 = \mathrm{sig}(x^T v_2)$
$z_h = \mathrm{sig}(x^T v_h)$
$z_H = \mathrm{sig}(x^T v_H)$
V
x_0
x_1
x_j
x_{M-1}
x_M

Why does Deep Learning Work so Well?

- A number of different theories are being developed
- We follow Tomaso Poggio, who addresses approximation theory, optimization, and learning theory
- Youtube: DALI 2018 - Tomaso Poggio: Deep Networks: Three Theory Questions
- Mhaskar, Hrushikesh, Qianli Liao, and Tomaso Poggio. "When and why are deep networks better than shallow ones?" Thirty-First AAAI Conference on Artificial Intelligence. 2017.

Approximation theory and Case Ic (compositional functions). (When and why are deep networks better than shallow networks?)

- Given any function out of a target function class: what is the best fit a DNN (with some architectural constraints) can obtain
- Both shallow and deep networks can approximate a function equally well/badly, in case we cannot make any particular assumptions on the target function class, except for smoothness m. This is our Case I (curse)
- Both suffer from the curse of dimensionality; the **number of required parameters** grows as $\mathcal{O}\left(\frac{1}{\epsilon^{M/m}}\right)$ (see previous lecture on approximation theory)
- ϵ is the average approximation error

Compositional Functions

- If we go from generic functions $f(\cdot)$ to a compositional function (functions of functions of functions, ...) and each of those functions only depends on a small number of arguments, then the number of required parameters for a DNN is

$$\mathcal{O}\left(\frac{1}{\epsilon^2}\frac{M}{m^2}\right)$$

($m = 1/K$, again, is a smoothness parameter where K is the Lipschitz constant for the target function class; the smaller K, the smoother the function)

- This result can, e.g., be found in: “Why and When Can Deep-but Not Shallow-networks Avoid the Curse of Dimensionality: A Review” Tomaso Poggio et al., International Journal of Automation and Computing, 2017. Theorem 4.

Compositional Functions (cont'd)

- Example of a compositional formula

$$f(x_1, \cdots, x_8) =$$

$$h3(h21(h_{11}(x_1, x_2), h_{12}(x_3, x_4)), h_{22}(h_{13}(x_5, x_6), h_{14}(x_7, x_8)))$$

- So we can consider compositional function as a new Case Ic (compositional) of target functions

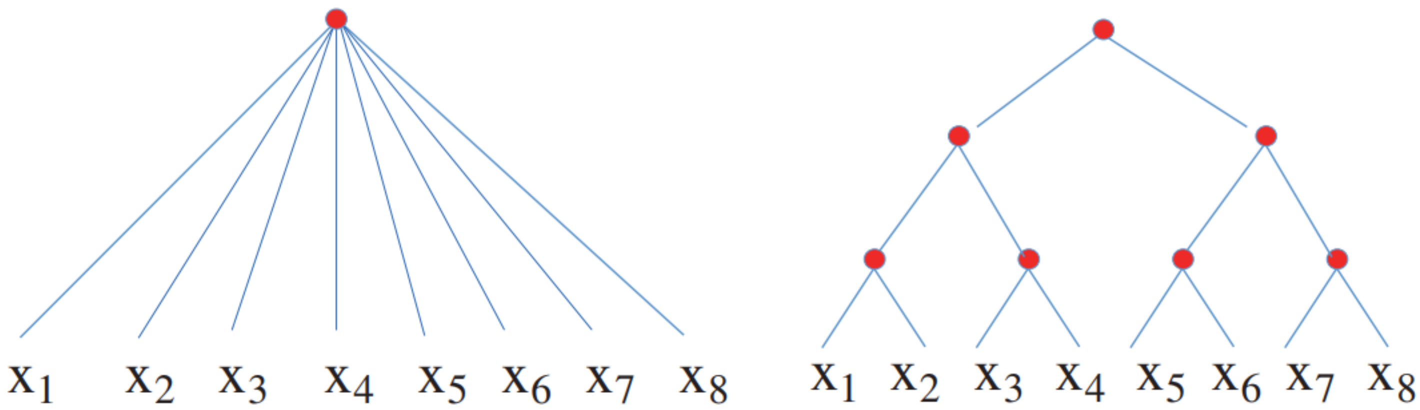


Figure 1: On the left a shallow universal network in 8 variables and N units which can approximate a generic function $f(x_1, \cdots, x_8)$. On the right, a binary tree hierarchical network in $n = 8$ variables, which approximates well functions of the form $f(x_1, \cdots, x_8) = h_3(h_{21}(h_{11}(x_1, x_2), h_{12}(x_3, x_4)), h_{22}(h_{13}(x_5, x_6), h_{14}(x_7, x_8)))$. Each of the $n - 1$ nodes consists of Q smoothed ReLU units with $Q(n-1) = N$ and computes the ridge function (Pinkus 1999) $\sum_{i=1}^{Q} a_i(\langle \mathbf{v}_i, \mathbf{x} \rangle + t_i)_+$, with $\mathbf{v}_i, \mathbf{x} \in \mathbb{R}^2$, $a_i, t_i \in \mathbb{R}$. Each term, that is each unit in the node, corresponds to a "channel". In a binary tree with n inputs, there are $log_2 n$ levels and a total of $n - 1$ nodes. Similar to the shallow network, a hierarchical network can approximate any continuous function; the text proves how it approximates a compositional functions better than a shallow network. No invariance is assumed here.

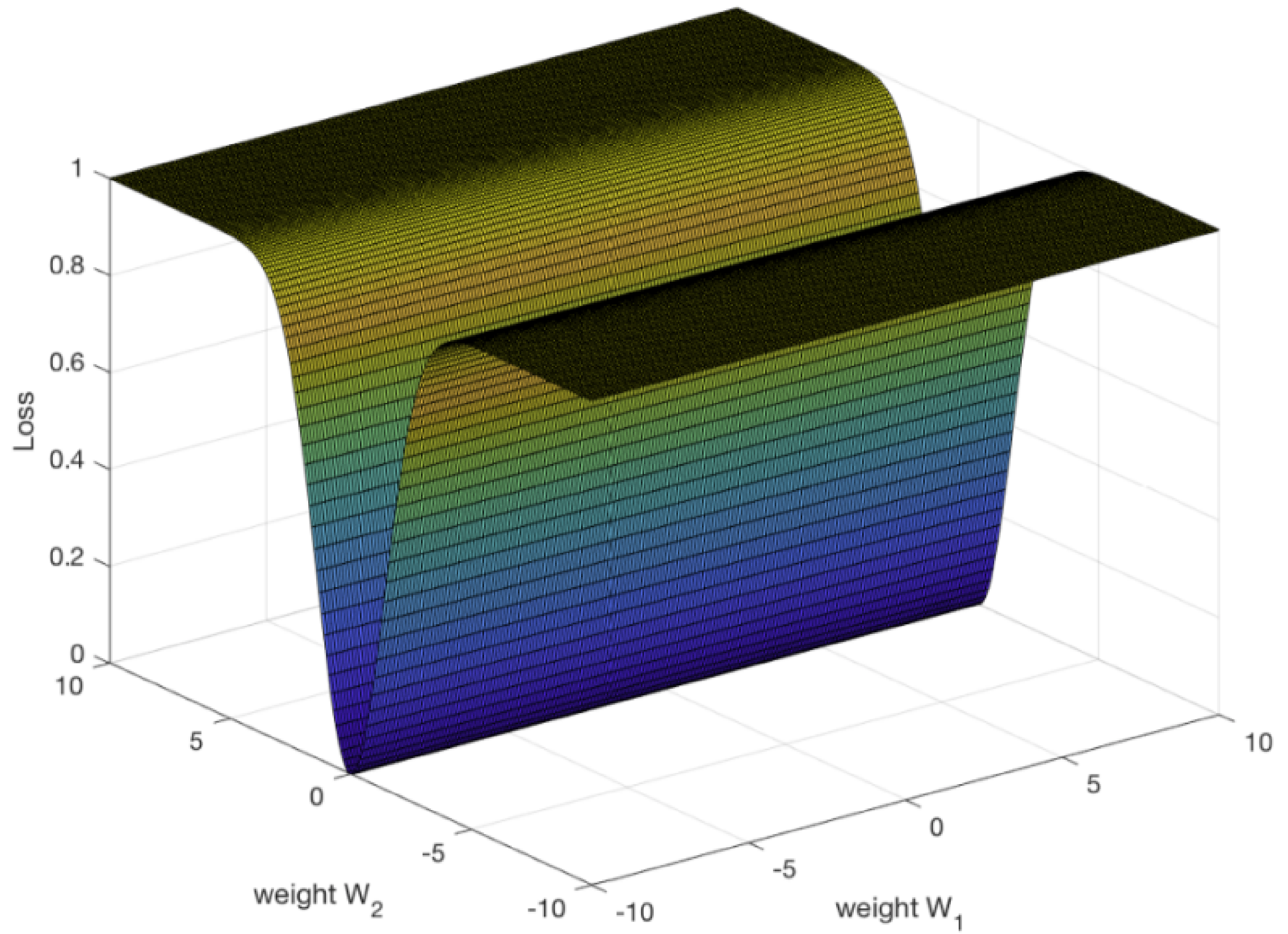
Comparison

Target Function	Model Function	Parameters	$\epsilon = 0.1$	Reference
Smoothness	Fixed Basis Functions	$\mathcal{O}\left(\frac{1}{\epsilon^{M/m}}\right)$	$\mathcal{O}\left(10^{M/m}\right)$	e.g., Poggio
C_f	Neural Network	$\mathcal{O}\left(\frac{4}{\epsilon}\frac{M}{m^2}\right)$	$\mathcal{O}\left(40\frac{M}{m^2}\right)$	Barron
Compositional	Deep Neural Network	$\mathcal{O}\left(\frac{1}{\epsilon^2}\frac{M}{m^2}\right)$	$\mathcal{O}\left(100\frac{M}{m^2}\right)$	Poggio

- Barron: We had, $H = \mathcal{O}\left(\frac{4}{\epsilon m^2}\right)$, and with $M_p \approx MH$ (number of inputs times number of hidden units) we get $M_p = \mathcal{O}\left(\frac{4M}{\epsilon m^2}\right)$
- Note that the smoothness m is defined differently in the three cases

Optimization: What is the landscape of the empirical risk?

- With over-parametrization ($M_p >> N$), the global optima of the cost functions are degenerate and thus take on more volume in parameter space and are easier to find by SGD
- So, as confirmed empirically, over-parameterized DNNs do not have major problems with local optima
- Another "blessing of dimensionality"
- Poggio: "Over-parametrized deep networks have many global minimizers that are generically degenerate; other critical points of the gradient are generically isolated."
- Poggio: "SGDL (a variant of SGD) finds with very high probability large volume, zero minimizers; empirically SGD behaves in a similar way."


1
0.8
0.6
0.4
0.2
0
Loss
10
5
0
-5
-10
weight W2
-10
-5
0
5
10
weight W1

Learning Theory and Overfitting (Learning Theory: How can deep learning generalize so well and not overfit?)

- How many training data points N are required to obtain a good model?
- When the optimum is degenerate, the number of parameters well defined by the data $M_{\textit{eff}}$ is much smaller than the number of parameters in the DNN, M_p
- Terms estimating the difference between generalization error and training error (i.e., the overfitting) contain expressions like M_p/N: we get overfitting with many parameters M_p and few data points N
- Thus, if we can substitute $M_p \rightarrow M_{\textit{eff}}$, overfitting is largely reduced!
- Thus, also for a good generalization performance, over-parameterizations ($M_p >> N$) does not hurt, as long as $M_{\textit{eff}}$ is small

cont'd

- One estimates for the required number of data points (sample size),

$$\frac{N_{shallow}}{N_{deep}} \approx \epsilon^{-M}$$

With $\epsilon = 0.1$,

$$N_{shallow} \approx 10^M N_{deep}$$

- Poggio: "Theorem: Much used variants of SGD - Batch Normalization and Weight Normalization - perform minimization with unit norm constraint, which is equivalent to maximize margin under norm constraint" (thus the DNN is regularized)

- Poggio: "Theorem: Standard gradient descent implicitly performs minimization with unit norm constraint"

- Poggio: "Together the theorems explain why the training of overparametrized deep networks satisfy the classification bounds leading to generalization despite overparametrization"

Conclusions

- Why is this a lecture on Machine Learning and not Deep Learning?
- "If you only know deep learning, you're pretty shallow" (VT)