Practice Sheet 1

ECE 55G - Modeling and Analysis of Random 5G Networks

January 11, 2019

1. **Void Probabilities:** Assume a PPP Φ with intensity λ in $\mathbb{R}^2$.
   (a) What is the probability that there are no points lying in the annular region $\{x : a \leq ||x|| \leq b\}$?
   (b) Based on the derived formula in (a), what is the probability that there are no points in a ball of radius r centered around the origin?
   (c) Derive the cumulative density function (cdf) and the probability density function (pdf) of the nearest point of Φ from the origin.
   (d) Now let us shift our point of observation to a point of Φ itself. Given that we are now observing Φ from the perspective of a point of Φ, what is the probability that the nearest neighbor of this point is at least at a distance of r?

2. **Superposition:** Consider two homogeneous PPPs Φ_1 and Φ_2, with intensities λ_1 and λ_2 respectively, in $\mathbb{R}^2$. These points may represent two different tiers of BSs, where Φ_1 represents a macro tier, and Φ_2 represents a small cell tier.
   (a) Let us define a new PPP Φ as $\Phi_1 \cup \Phi_2$. What is the intensity of Φ?
   (b) Now assume that you are located at the origin and observing the points of Φ. What is the probability that the nearest BS from you belongs to the macro tier, i.e., Φ_1?
   (c) Can we generalize it to n tiers?

3. **Thinning:** Consider a PPP Φ with intensity λ in $\mathbb{R}^2$. To each of the points of Φ, we randomly assign a binary mark $m \in \{0, 1\}$, with $\mathbb{P}(m_i = 0) = p_0$ for an arbitrary point $\mathbf{x}_i$.
   (a) What are the intensities of the processes consisting of only the points with mark 0?
   (b) What is the probability that there lies a point with mark 0 within a circle of radius r centered around origin?
   (c) From the perspective of a point of mark 0, what is the probability that there lies a point of mark 1 within a range r from it?

4. **Transformation of a PP:** Let us assume a homogeneous PPP in $\mathbb{R}^2$ with intensity λ. These points may represent the locations of BSs of tier. We are interested in characterizing the distances of these BSs from the perspective of a typical user. Without loss of generality, let the typical user be located at the origin.
   (a) The distance of a point $\mathbf{x}_i$ from the origin is given by $||\mathbf{x}_i||$. What is the average number of BSs that have a distance less than R from the origin?
   (b) Let the distances of each point of the PPP be plotted in the number line $\mathbb{R}^+ := (0, \infty)$. This constructs a new, one-dimensional point process Ξ with its domain as $\mathbb{R}^+$ rather than $\mathbb{R}^2$. Is Ξ a PPP?
   (c) What is the intensity measure of Ξ for an arbitrary convex set $A \subset \mathbb{R}^+$? What is the intensity?
   (d) What is the pdf of the nearest point of Ξ from the origin?
   (e) Comment on the homogeneity of Ξ.