CMPS 102 — Fall 2018 — Homework 2

"I have read and agree to the collaboration policy." - ***Kevin Wang***

Solution to Problem 4: Ropes

Let $L = \{l_1...l_n\}$ where $l_i \in \mathbb{Z}^+$. The cost of joining two ropes, i and j, is $C = l_i + l_j$.

Algorithm 1 Joins a set of different length rope together with the minimum total cost

---

JOIN-ROPE (L , C):
if $|L| = 1$ **then**
    The rope has been fully joined. Return total cost C
else $\{|L| > 1\}$
    Let l_i and l_j be the shortest two ropes in set L
    Let L' be the new set after the ropes have been joined
    $C' = C + l_i + l_j$
    JOIN-ROPE (L' , C')
end if

Claim 1. *This algorithm is optimal as it incurs the minimum total cost.*

Proof. We observe that each set always takes exactly $n - 1$ joins in order to become a single rope. Thus, because there is a set number of additions to the total cost and due to the max cost addition being the cumulative length of the set, the most optimal method of proceeding with the costs is to keep each cost addition as small as possible. □