CMPS 102 — Fall 2018 — Homework 4

"I have read and agree to the collaboration policy." - ***Kevin Wang***

Collaborators: *None*

Solution to Problem 1: Coffee Shops

Given n minutes to study, Charlie has two sequences $V = \{v_1, v_2, \cdots, v_i, \cdots, v_n\}$ and $R = \{r_1, r_2, \cdots, r_i, \cdots, r_n\}$ which represents the work he can do at the i-th minute at Valve and Ruru, respectively. It costs Charlie 10 minutes to switch coffee shops.

Let $W(n)$ be the maximum total work Charlie can do.

Sub-Problems

$W_V[i]$ is the max total work done by time i when currently at Valve. At time i while at Valve, Charlie could have: (1) been studying there already or (2) just arrived from Ruru.

Base Case: Charlie has done no work before 9AM. $\longrightarrow W_V[i \leq 0] = 0$
Case 1: Charlie has been studying there already.

$$\begin{aligned} W_V[i] &= \text{max work done a minute ago} + \text{work done at the i-th minute at Valve} \\ &= W_V[i-1] + v_i \end{aligned}$$

Case 2: Charlie just arrived from Ruru.

$$\begin{aligned} W_V[i] &= \text{max work done when at Ruru} + \text{work done at the i-th minute at Valve} \\ &= W_R[i-10] + v_i \end{aligned}$$

Therefore, the max total work done by time i when currently at Valve is the max of cases 1 and 2.

$$\longrightarrow W_V[i] = \max(\ W_V[i-1] + v_i\ ,\ W_R[i-10] + v_i\ )$$

Similarly, **$W_R[i]$ is the max total work done by time i when currently at Ruru.**

Base Case: Charlie has done no work before 9AM. $\longrightarrow W_R[i \leq 0] = 0$

Case 1: Charlie has been studying there already. $\longrightarrow W_R[i] = W_R[i-1] + r_i$

Case 2: Charlie just arrived from Valve. $\longrightarrow W_R[i] = W_V[i-10] + r_i$

$$\longrightarrow W_R[i] = \max(\ W_R[i-1] + r_i\ ,\ W_V[i-10] + r_i\ )$$

Therefore, the maximum work Charlie can complete in n minutes past 9AM is:

$$\text{W}(n) = \begin{cases} 0 & \text{if } n \leq 0 \\ \max(W_V[n], W_R[n]) & \text{otherwise} \end{cases}$$

Algorithm 1 Finds the max work Charlie can do in n minutes

MAX-WORK (n):
Let $W_V[i \leq 0] = 0$ and $W_R[i \leq 0] = 0$
for $i = 1$ to n **do**
    $W_V[i] = \max(\ W_V[i-1] + v_i\ ,\ W_R[i-10] + v_i\ )$
    $W_R[i] = \max(\ W_R[i-1] + r_i\ ,\ W_V[i-10] + r_i\ )$
end for
$\text{W}(n) = \max(W_V[n], W_R[n])$

**** To find out when Charlie should move coffee shops, just flag when V $\leftrightarrow$ R. ****

Time Complexity: $O(n)$
The for-loops iterates over 2 comparisons, n times, finishing with 1 last comparison. Total time used is $O(2n+1)$.

Space Complexity:
The 2 arrays V and R are each of size n. The 2 sequences, $v_1, \cdots, v_n$ and $r_1, \cdots, r_n$, also each have size n. Total space used is $O(4n)$.