# Machine Learning

## Decision Trees

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# Goal for Today's class

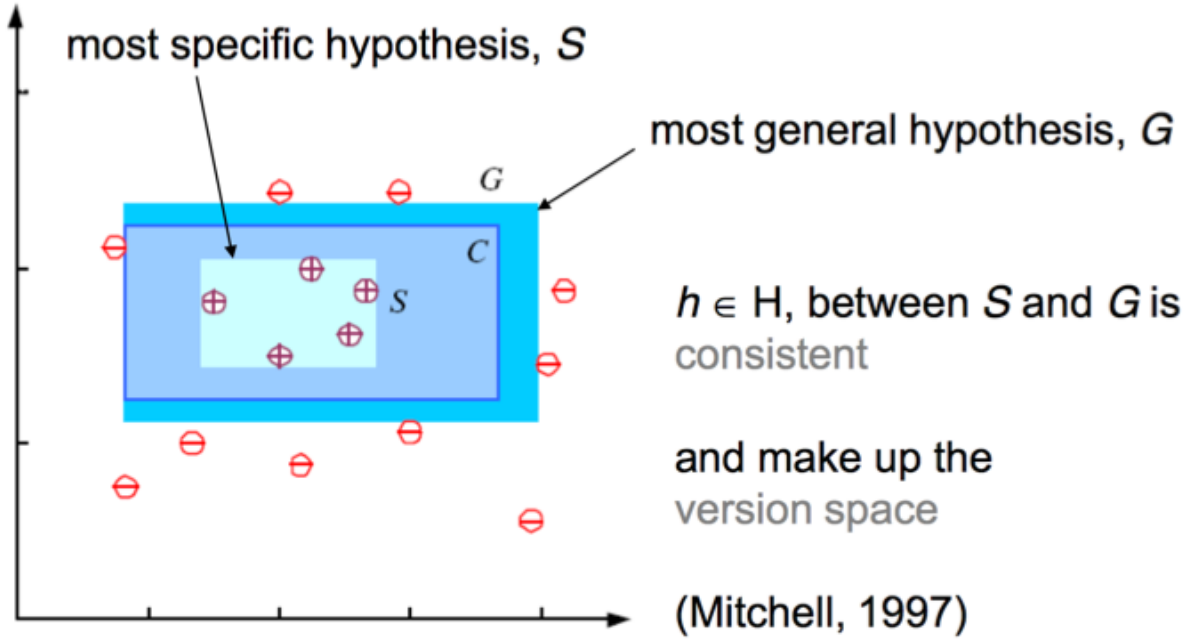
Quick Review of previous concepts

and

Decision Trees!

# Quick Review: Hypothesis space

- *Hypothesis space:* H
- *Version space*: Subset of H consistent with the training data.



# Quick Review: Hypothesis space

- **Input**: a set of examples <**x**,f(**x**)>, where f(**x**) is the unknown target function
- **Learning**: find g(x) that *approximates* f(x) well.
- ***Question***: *does the hypothesis space of f(x) and g(x) has to be similar?*

*We know the f(x) is defined over all Boolean functions but* ***it's too hard to learn over that space!***

***Learning is possible if we find the "right" hypothesis space!***

→ *Practically, this often involves tuning the representation.*

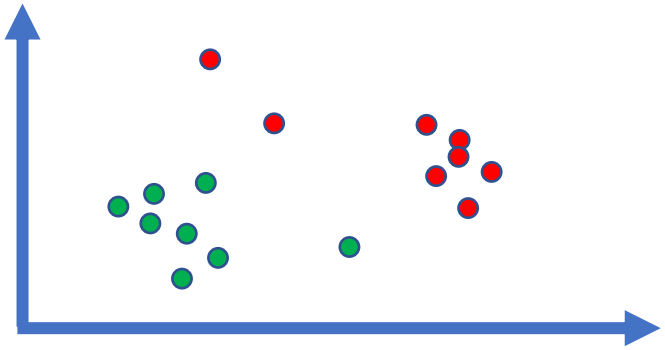
| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $y$ | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | ? | |
| 0 | 0 | 0 | 1 | ? | |
| 0 | 0 | 1 | 0 | **0** | ← |
| 0 | 0 | 1 | 1 | **1** | ← |
| 0 | 1 | 0 | 0 | **0** | ← |
| 0 | 1 | 0 | 1 | **0** | ← |
| 0 | 1 | 1 | 0 | **0** | ← |
| 0 | 1 | 1 | 1 | ? | |
| 1 | 0 | 0 | 0 | ? | |
| 1 | 0 | 0 | 1 | **1** | ← |
| 1 | 0 | 1 | 0 | ? | |
| 1 | 0 | 1 | 1 | ? | |
| 1 | 1 | 0 | 0 | **0** | ← |
| 1 | 1 | 0 | 1 | ? | |
| 1 | 1 | 1 | 0 | ? | |
| 1 | 1 | 1 | 1 | ? | |

# Quick Review: KNN




**Categorical attributes**

**Numerical Attributes:**

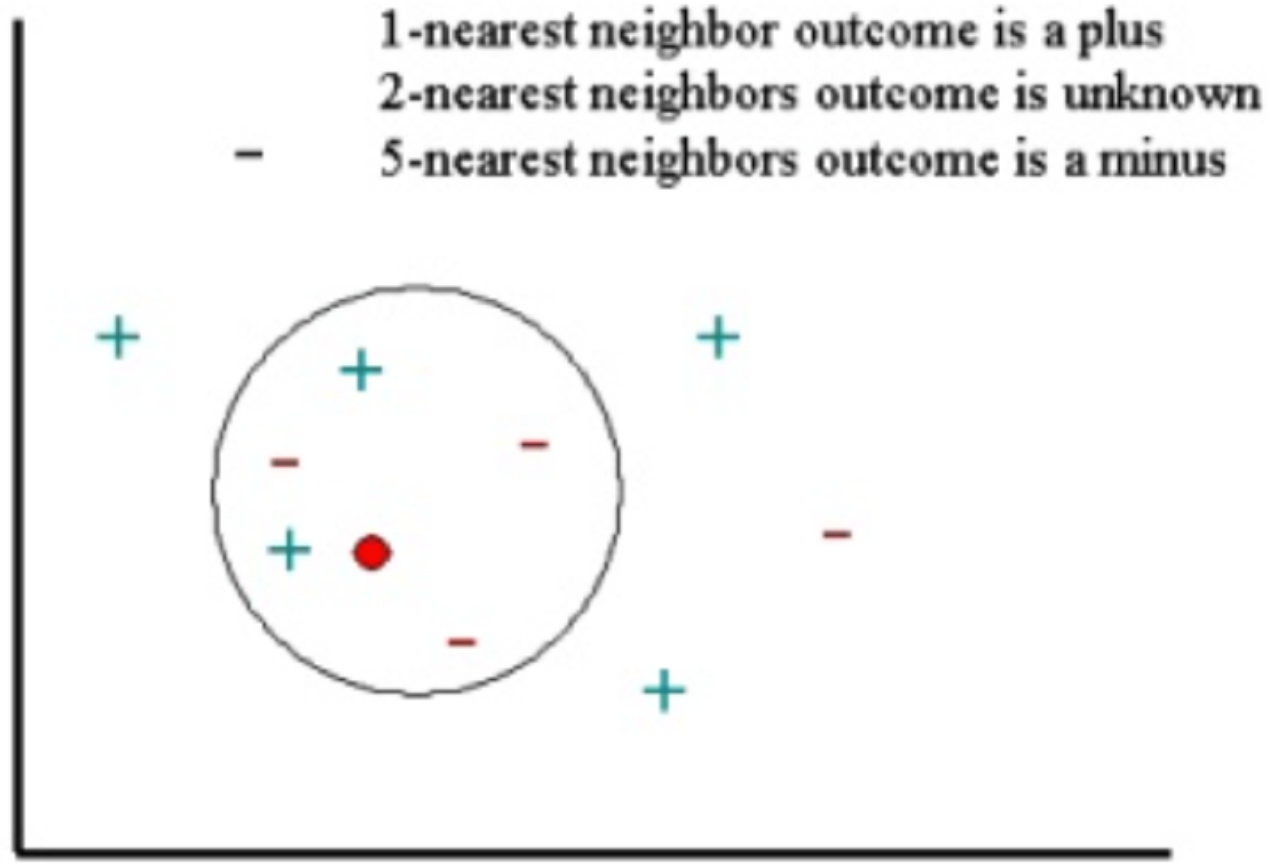


**Distance Measures**

$$d(x_1, x_2) = \sqrt[2]{(x_1 - x_2)^2}$$

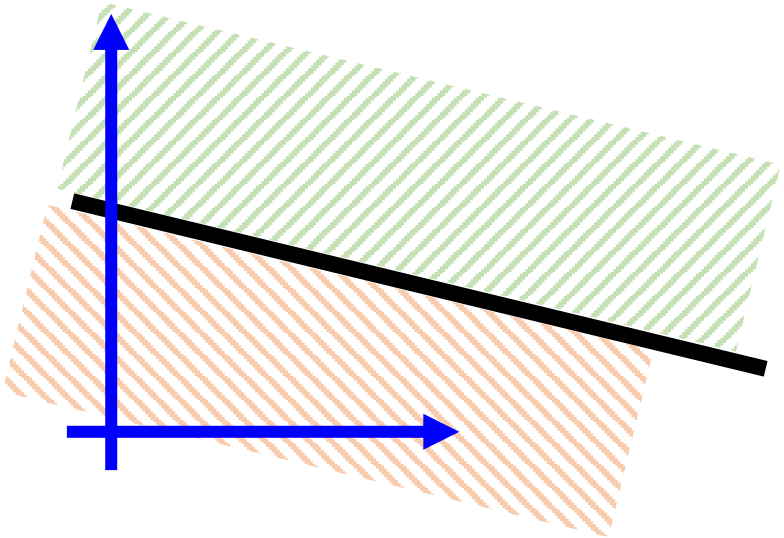
$$d(x_1, x_2) = 1 - \frac{x_1 \bigcap x_2}{x_1 \bigcup x_2}$$

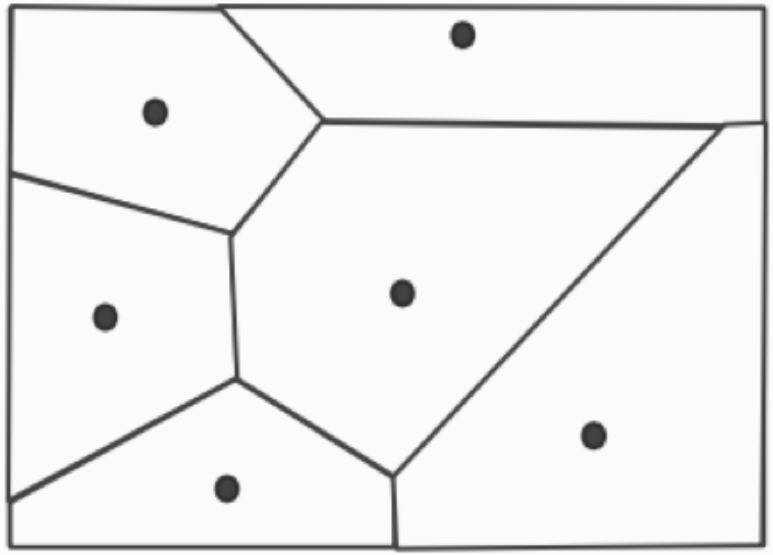
# Quick Review: KNN



# Quick Review: Expressivity

- KNN Can learn very complex decisions models
- We can try to characterize the learned function using its ***decision boundary***
  - *Visualize which elements will be classified as positive/negative*
  - *Decision Boundary is the curve separating the negative and positive regions*

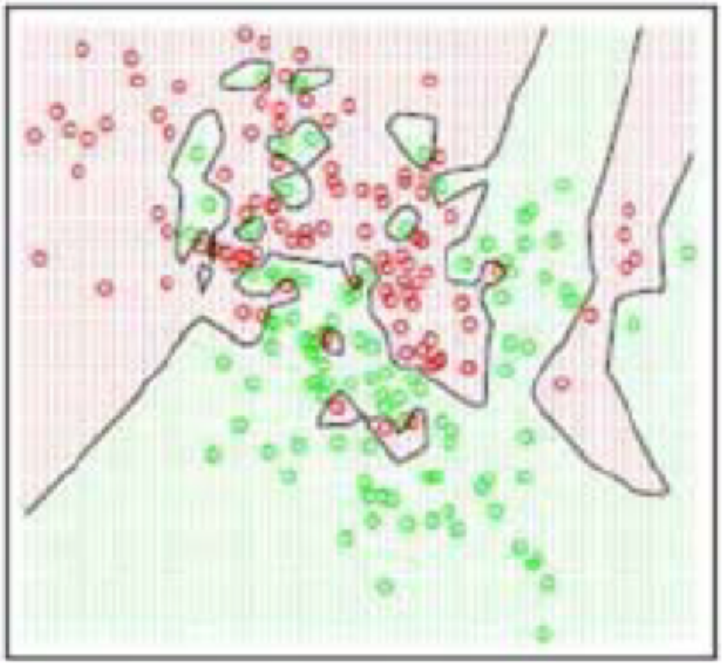


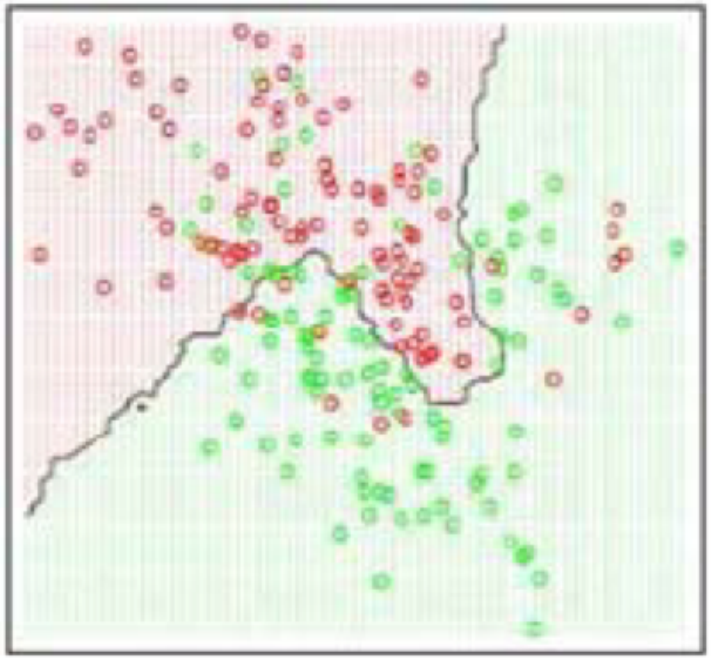


# Quick Review: Expressivity

Let's take a closer look at the learned function

→ ***High sensitivity to noise!***



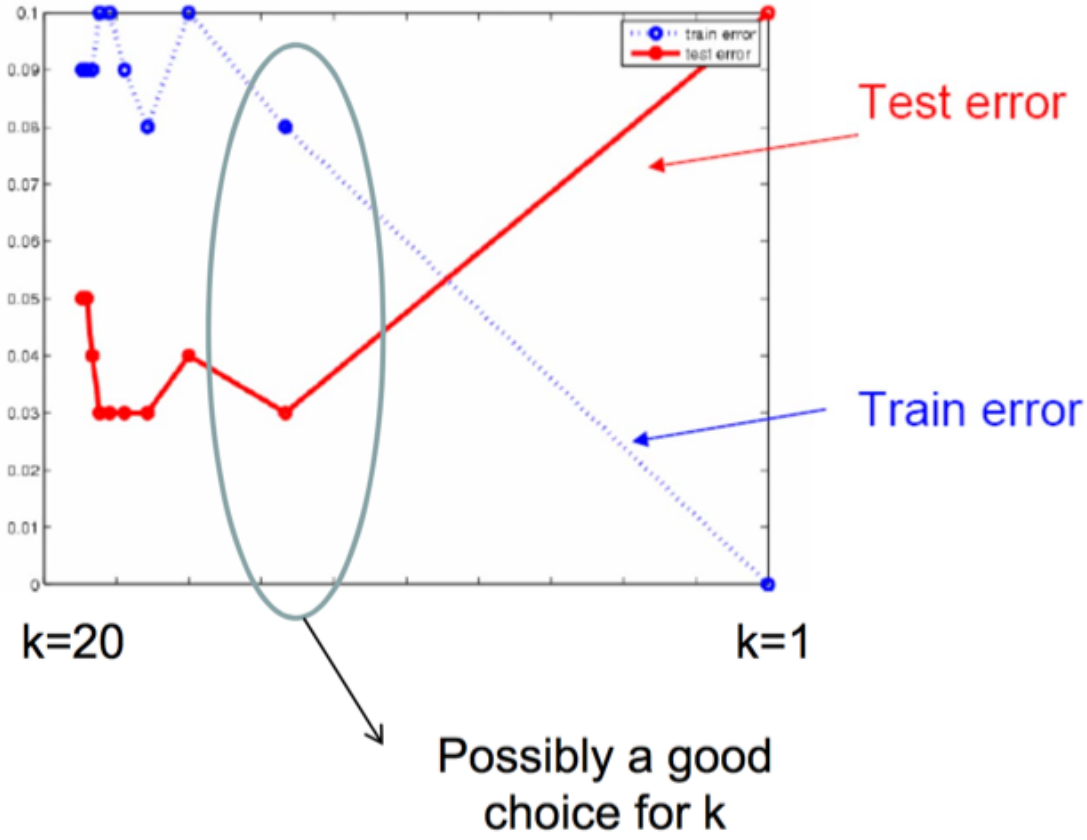


Higher k values results in smoother decision boundaries

Figures from Hastie, Tibshirani and Friedman (Elements of Statistical Learning)

# Quick Review: Model Selection

*How would the test and train error change with K?*




In general – using the training error to tune parameters will always result in a more complex hypothesis! **(why?)**

# Inductive Bias

- Unbiased learning is impossible!
- Inductive bias: a set of assumptions guiding learning **beyond the data**
- We have seen one type of bias: **Language bias**
  - **Pick the right hypothesis space**

- Today, as part of our discussion on Decision Trees we will introduce a second type – **search bias.**
  - **Similar objective**: Encode assumptions about learning, restrict the complexity of the resulting hypothesis

# **Last Time**: K Nearest Neighbors

- *Is KNN really a learning algorithm?*
  - *Yes, because…*
    - ***Well, we get a classifier out of it..***
  - *No, because…*
    - ***Well, it just maintains the data..***
- **Today we will look at another algorithm that only maintains the data**

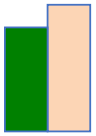
# Today's lecture: ***Learning Decision Trees***

- KNN only stored the data, Decision trees store a "*compressed*" dataset.
  - Simplified view of DT Learning : ***better Compression, with less information loss = better generalization.***

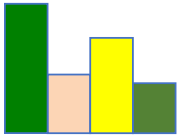
- **DT Learning overview:**
  - Decision Tree Representation
  - Algorithms for learning decision trees
  - Experimental issues
    - Controlling overfitting

# Representing Data

- Think about a large table, N attributes, and assume you want to know something about the data represented as entries in the table
  - **E.g.** own an expensive car or not;
- **Simplest way**: Histogram on the first attribute – own



- **Then**, histogram on first and second (own & gender)



# Representing Data

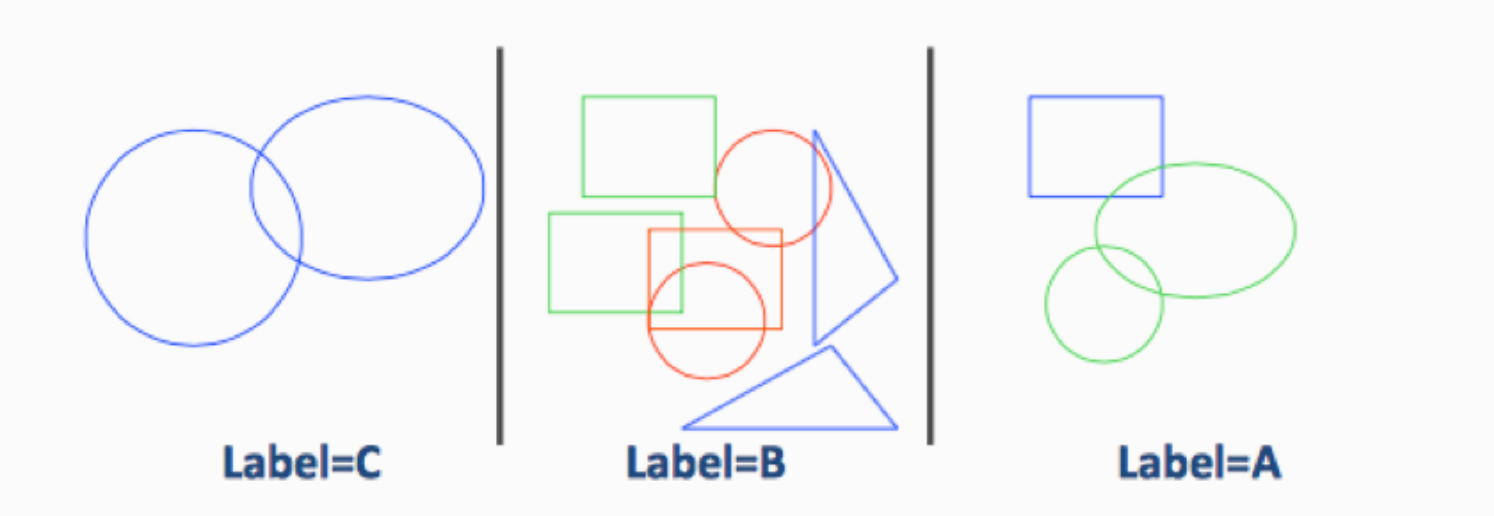
- Think about a large table, N attributes, and assume you want to know something about the data represented as entries in the table
- ***But, what if the # of attributes is larger: N=16***
  - How large are the 1-d histograms (contingency tables) ? **16 numbers**
  - How large are the 2-d histograms? **16-choose-2 = 120 numbers**
  - How many 3-d tables? **560 numbers**
  - **With 100 attributes, the 3-d tables need 161,700 numbers**
- *We need to figure out a way to represent data in a better way, and figure out what are the important attributes to look at first*.
  - we will use an information theoretic approach to represent the data better

# Decision Trees

- A hierarchical data structure (tree) that represents data by implementing a divide and conquer strategy
- **Nodes** are tests for feature values
  - There is one branch for every value that the feature can take
- **Leaves** of the tree specify the class labels

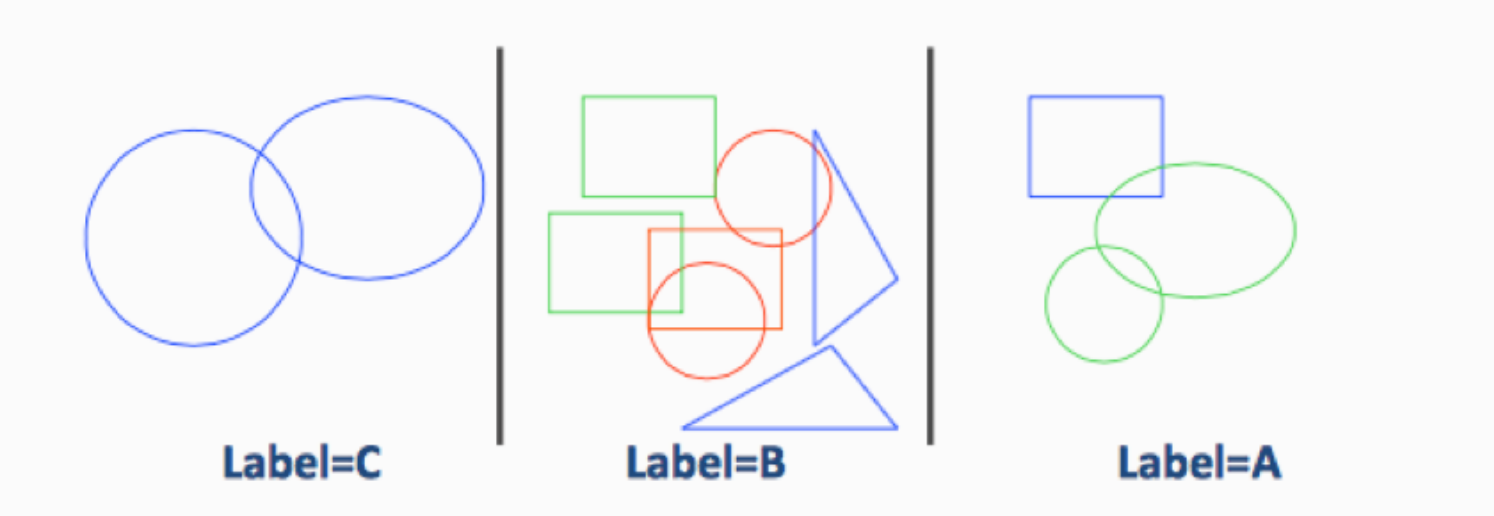
- Given a collection of examples, **learn a decision tree that represents it.**
- Use this representation to **classify new examples**
  - The tree can be used for non-parametric classification and regression

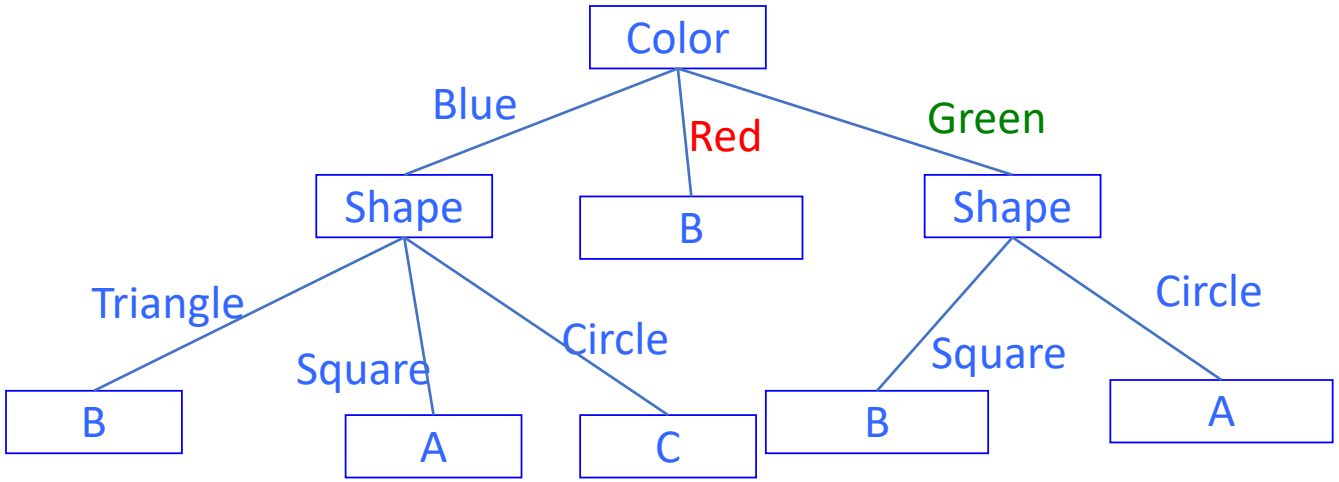
# Decision Tree Representation




- Three **output** classes: A, B, C
- Two attributes
  - **Color**: Red, Blue, Green
  - **Shape**: Circle, Triangle, Square

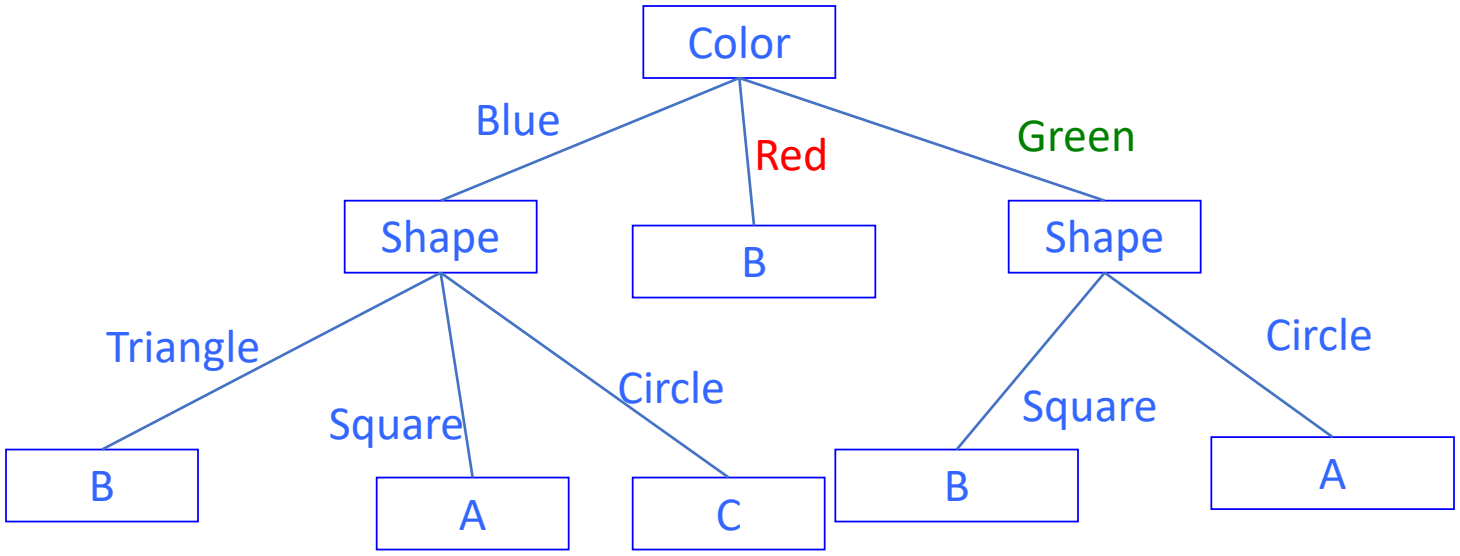
# Decision Tree Representation



# Decision Tree Representation

- **Basic Questions:**
  - *How to use Decision Trees for prediction?*
    - **follow the path from the root**
      - What is the label of *a red triangle? Green triangle?*
  - How can we learn decision trees from data?



# Decision Trees

- *Output*:
  - Boolean
  - Multiclass (discrete categories)
  - Real valued (regression tree)

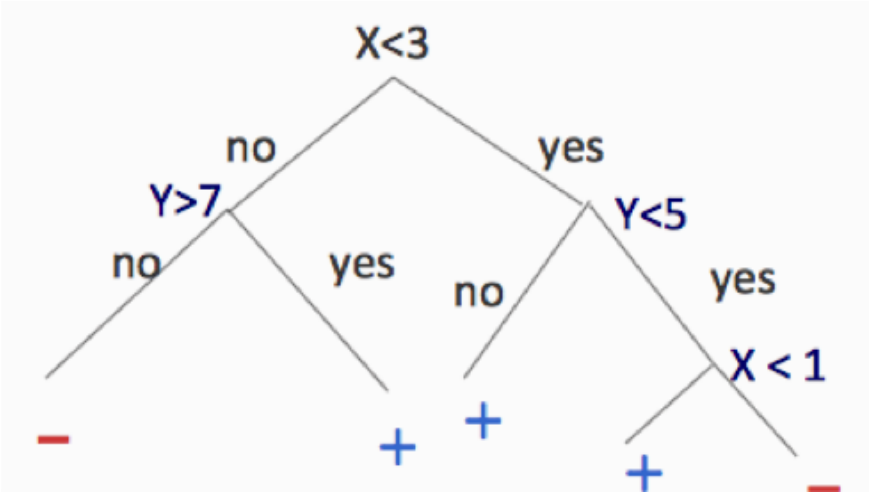
- *Concise representation of the training data*
  - Simply "organizing" the training data in tree for will severely overfit and will be very sensitive to noise (***why***?)
  - Methods for handling noisy data (noise in the label or in the features) and for handling missing attributes
  - **Pruning trees** helps with noise

# Expressivity of Decision Trees

- What kind of Boolean functions can DT represent?
  - Any Boolean function! (***why?***)
- A decision tree can be rewritten as a DNF
  - *Each path from root to leaf can be written as a rule, the tree is a disjunction of these rules.*
  - **`Green ^ square → positive`**
  - **`Blue ^ circle → positive`**
  - **`Blue ^ square → positive`**
- **Question**: *What is the size of the hypothesis space for the shape classification problem?*

# Numeric Attributes

- We have seen instances represented as attribute-value pairs (color=blue, second letter=e, etc.)
  - Values have been categorical
- **What about numeric feature values? (e.g., length)**
  - Discretize them or use thresholds on the numeric values

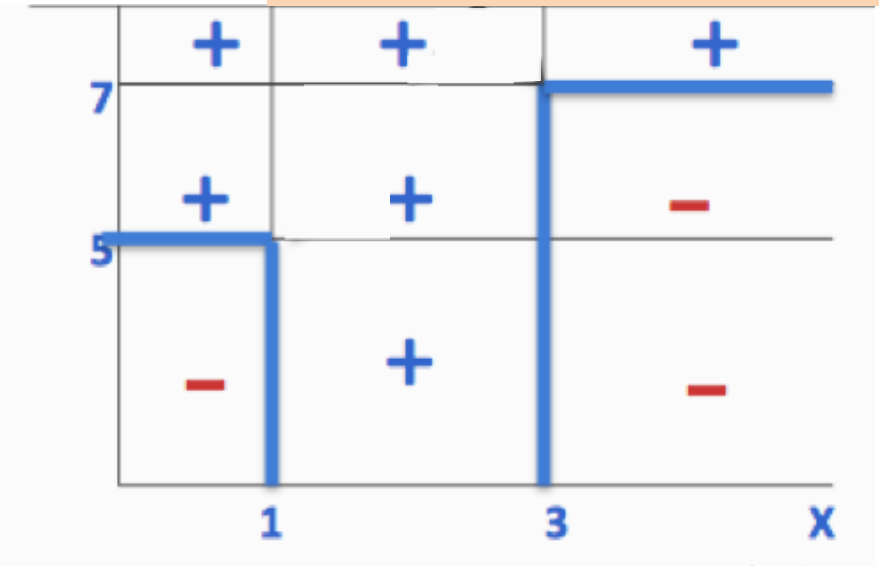


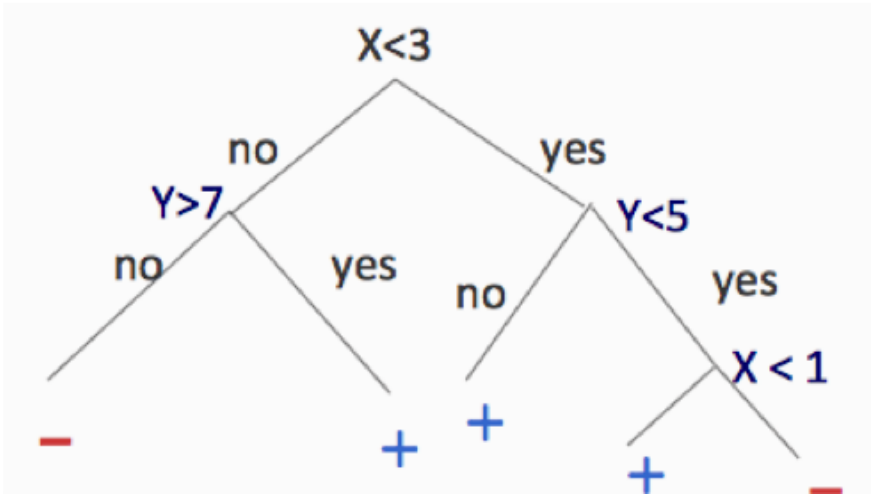
# Expressivity

- **What about numeric feature values? (e.g., length)**
  - Discretize them or use thresholds on the numeric values
  - ***This divides the feature space into axis parallel rectangles***

***This is the decision boundary for DT***

*How would you characterize the complexity of a hypothesis?*



# Summary: *Decision Tree* ***Representation***

- ***Decision tree can represent any Boolean function***
  - A way to represent lots of data
    - **Compact representation**
  - A natural representation (think 20 questions)
    - "**Explainable**" model!
- *Prediction with a decision tree is easy*
- Clearly, given a dataset, there are many decision trees that can represent it. (**why?**)
  - **How can you learn a good representation from data?**

# **Example**: *Will I play tennis today?*

- **Features**
  - *Outlook*: {Sun, Overcast, Rain}
  - *Temperature*: {Hot, Mild, Cool}
  - *Humidity*: {High, Normal, Low}
  - *Wind*: {Strong, Weak}

- **Labels**
  - *Binary classification* task: Y = {+, -}

# Basic Decision Tree Learning Algorithm

| | O | T | H | W | Play? |
|---|---|---|---|---|---|
| 1 | S | H | H | W | - |
| 2 | S | H | H | S | - |
| 3 | O | H | H | W | + |
| 4 | R | M | H | W | + |
| 5 | R | C | N | W | + |
| 6 | R | C | N | S | - |
| 7 | O | C | N | S | + |
| 8 | S | M | H | W | - |
| 9 | S | C | N | W | + |
| 10 | R | M | N | W | + |
| 11 | S | M | N | S | + |
| 12 | O | M | H | S | + |
| 13 | O | H | N | W | + |
| 14 | R | M | H | S | - |

**O**utlook: S(unny), O(vercast), R(ainy)

**T**emperature: H(ot), M(edium), C(ool)

**H**umidity: H(igh), N(ormal), L(ow)

**W**ind: S(trong), W(eak)

# Basic Decision Tree Learning Algorithm

| | O | T | H | W | Play? |
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| 8 | S | M | H | W | - |
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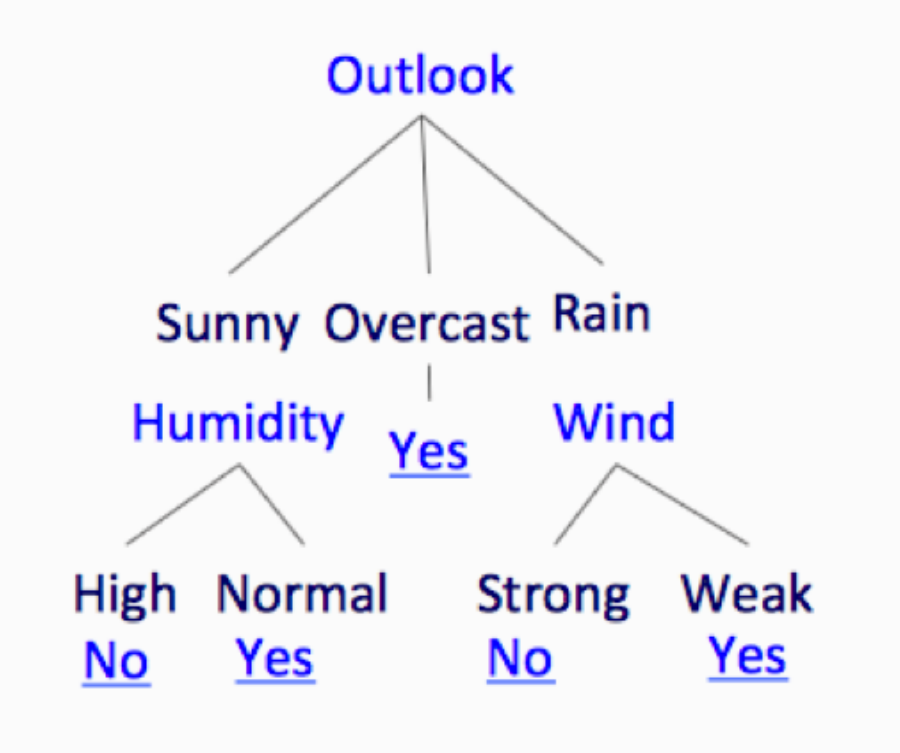
| 11 | S | M | N | S | + |
| 12 | O | M | H | S | + |
| 13 | O | H | N | W | + |
| 14 | R | M | H | S | - |

- Data is processed in Batch (i.e. all the data available)
- Recursively build a DT top down.



# *Basic Decision Tree Learning Algorithm:* ID3

**Input**:
S the set of Examples
Label is the target attribute (the prediction)
Attributes: set of measured attributes

- 1. **If** all examples are have same label:
  - *Return a single node tree with the label*
- 2. **Otherwise**
  - Create a Root node for tree
  - A = attribute in Attributes that ***best*** classifies S
  - for each possible value v of attribute A:
    - Add a new tree branch corresponding to A=v
    - Let *Sv* be the subset of examples in S with A=v
    - **if** *Sv* is empty:
      add leaf node with the common value of Label in S
    - **Else**:
      below this branch add the subtree:
      ID3(*Sv*, Attributes - {a}, Label)

Why?

Recursive call

- 4. **Return** Root node

# Picking the Root Attribute

- The goal is to have the resulting decision tree as small as possible (*Occam's Razor*)
  - But, finding the minimal decision tree consistent with the data is NP- hard
- The recursive algorithm is a greedy heuristic search for a simple tree, but **cannot guarantee optimality**
- The main decision in the algorithm is the selection of the next attribute to split on

# Picking the Root Attribute

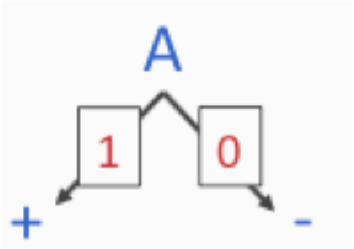
Consider data with two Boolean Attributes (A,B).

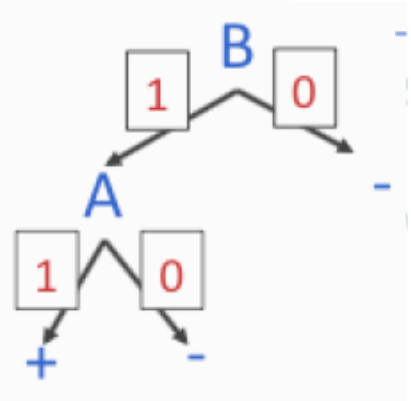
**<(A =-0, B=0), - >: 50 Examples**
**<(A =-0, B=1), - >: 50 Examples**
**<(A =-1, B=0), - >: 0 Examples**
**<(A =-1, B=1), + >: 100 Examples**

What should be the first attribute to split on?

Split on A: purely labeled nodes

Split on B: nodes are not purely labeled



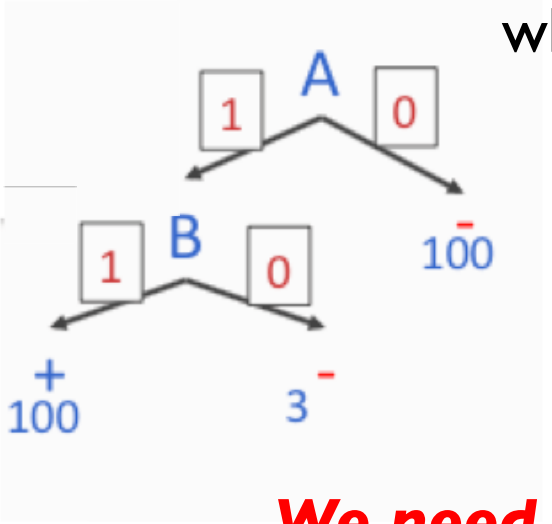

*What if we have: "<(A=1,B=0), - >: 3" examples?*

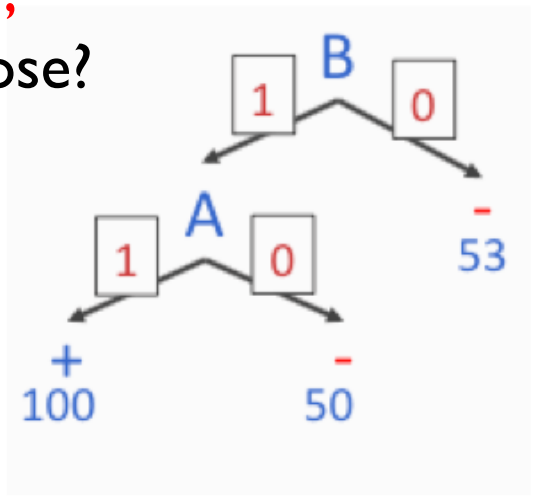
# Picking the Root Attribute

Consider data with two Boolean Attributes (A,B).

**<(A =-0, B=0), - >: 50 Examples**
**<(A =-0, B=1), - >: 50 Examples**
**<(A =-1, B=0), - >: *3 Examples***
**<(A =-1, B=1), + >: 100 Examples**

The trees look structurally similar,
which attributes should you choose?




***We need a way to quantify this preference!***

# Picking the Root Attribute

- The goal is to have the resulting decision tree as small as possible (Occam's Razor)

- The main decision in the algorithm is the selection of the next attribute to condition on

- **We want attributes that split the examples to sets that are relatively pure in one label; this way we are closer to a leaf node.**

- **The most popular heuristic is based on information gain, originated with the ID3 system of Quinlan.**

# Entropy

***Entropy*** (impurity, disorder) of a set of examples S with respect to binary classification is

$$Entropy(S) = H(S) = -p_+ \log(p_+) - p_- \log(p_-)$$

- *The proportion of positive examples is $p_+$*
- *The proportion of negative examples is $p_-$*

In general, for a discrete probability distribution with K possible values, with probabilities $\{p_1, p_2, , p_K\}$ the entropy is given by

$$H(\{p_1, p_2, \cdots, p_K\}) = -\sum_{i=1}^{K} p_i \log(p_i)$$

# Entropy

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- If all examples belong to the same category, entropy = 0
- If $p_+ = p_- = 0.5$, entropy = 1

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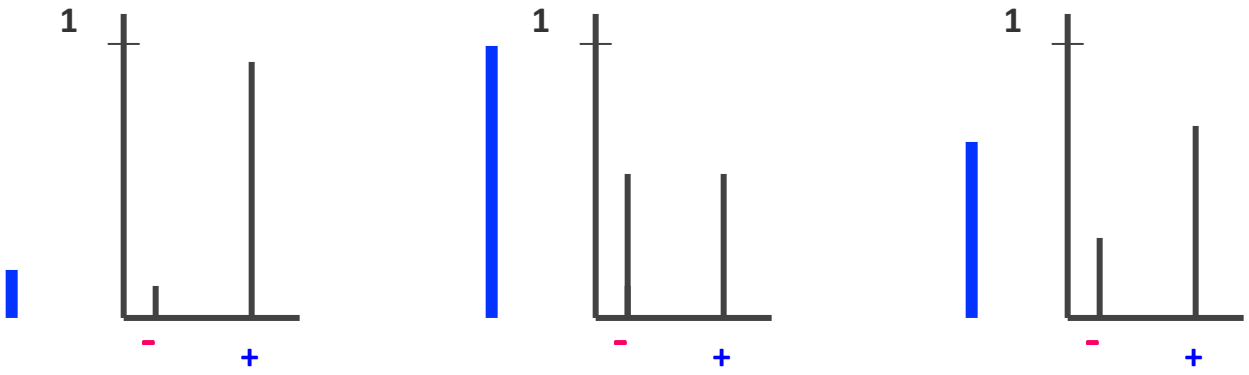


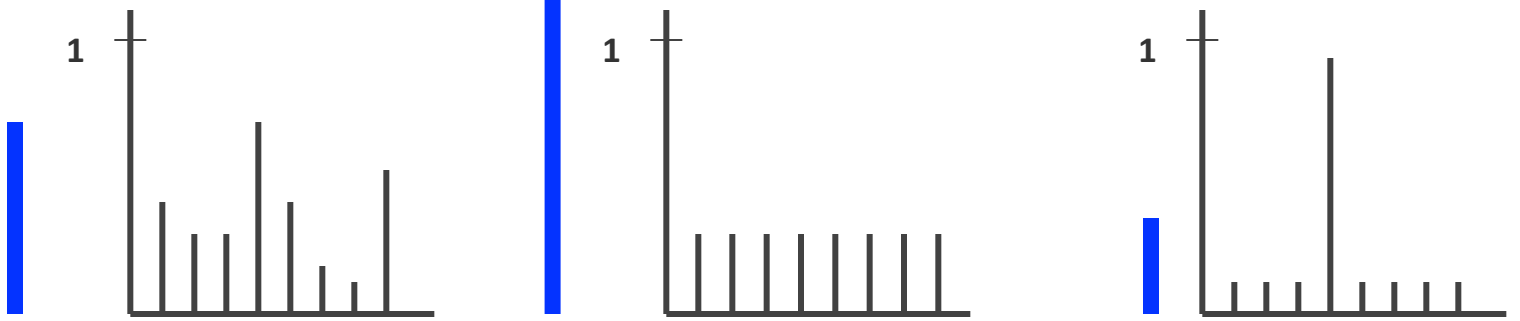
# Entropy

*Entropy* (impurity, disorder) of a set of examples S with respect to binary classification is

*The uniform distribution has the highest entropy*

**High Entropy** – High level of Uncertainty

**Low Entropy** – No Uncertainty.



# Information Gain

The *information gain* of an attribute A is the *expected reduction in entropy* caused by partitioning on this attribute

$$Gain(S, A) = Entropy(S) - \sum_{v \in \text{Values}(A)} \frac{|S_v|}{|S|} Entropy(S_v)$$

$S_v$: the subset of examples where the value of attribute A is set to value v

Entropy of partitioning the data is calculated by weighing the entropy of each partition by its size relative to the original set

- partition of low entropy (imbalanced splits) leads to high gain

*Go back to check which of the A, B splits is better!*

# Will I play tennis today?

| 1 | O | T | H | W | Play? |
|---|---|---|---|---|---|
| | S | H | H | W | -- |
| 2 | S | H | H | S | -- |
| 3 | O | H | H | W | + |
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| 3 | O | H | H | W | + |
| 4 | R | M | H | W | + |
| 5 | R | C | N | W | + |
| 6 | R | C | N | S | -- |
| 7 | O | C | N | S | + |
| 8 | S | M | H | W | -- |
| 9 | S | C | N | W | + |
| 10 | R | M | N | W | + |
| 11 | S | M | N | S | + |
| 12 | O | M | H | S | + |
| 13 | O | H | N | W | + |
| 14 | R | M | H | S | -- |

**Current entropy:**

$p = 9/14$

$n = 5/14$

$H(Y) =$

$-(9/14)\log_2(9/14) \quad -(5/14)\log_2(5/14)$

$= 0.94$

# InformaFon Gain: Outlook

| | O | T | H | W | Play? |
|---|---|---|---|---|---|
| 1 | S | H | H | W | -- |
| 2 | S | H | H | S | -- |
| 3 | O | H | H | W | + |
| 4 | R | M | H | W | + |
| 5 | R | C | N | W | + |
| 6 | R | C | N | S | -- |
| 7 | O | C | N | S | + |
| 8 | S | M | H | W | -- |
| 9 | S | C | N | W | + |
| 10 | R | M | N | W | + |
| 11 | S | M | N | S | + |
| 12 | O | M | H | S | + |
| 13 | O | H | N | W | + |
| 14 | R | M | H | S | -- |

**Outlook = sunny:** 5 of 14 examples

*p = 2/5* *n = 3/5* $\mathbf{H_S = 0.971}$

**Outlook = overcast:** 4 of 14 examples

*p = 4/4* *n = 0* $\mathbf{H_o = 0}$

**Outlook = rainy:** 5 of 14 examples

*p = 3/5* *n = 2/5* $\mathbf{H_R = 0.971}$

**Expected entropy:**

(5/14)×0.971 + (4/14)×0
+ (5/14)×0.971 = **0.694**

**Information gain:**

0.940 – 0.694 **= 0.246**

# InformaFon Gain: Humidity

| 1 | O | T | H | W | Play? |
|---|---|---|---|---|---|
| | S | H | H | W | -- |
| 2 | S | H | H | S | -- |
| 3 | O | H | H | W | + |
| 4 | R | M | H | W | + |
| 5 | R | C | N | W | + |
| 6 | R | C | N | S | -- |
| 7 | O | C | N | S | + |
| 8 | S | M | H | W | -- |
| 9 | S | C | N | W | + |
| 10 | R | M | N | W | + |
| 11 | S | M | N | S | + |
| 12 | O | M | H | S | + |
| 13 | O | H | N | W | + |
| 14 | R | M | H | S | -- |

**Humidity = high:**

*p = 3/7* *n = 4/7* $\mathbf{H_h = 0.985}$

**Humidity = Normal:**

*p = 6/7* *n = 1/7* $\mathbf{H_o = 0.592}$

**Expected entropy:**

(7/14)×0.985 + (7/14)×0.592= **0.7885**

**Information gain:**

0.940 – 0.7885 **= 0.1515**

# Which feature to split on?

| 1 | O | T | H | W | Play? |
|---|---|---|---|---|---|
| | S | H | H | W | -- |
| 2 | S | H | H | S | -- |
| 3 | O | H | H | W | + |
| 4 | R | M | H | W | + |
| 5 | R | C | N | W | + |
| 6 | R | C | N | S | -- |
| 7 | O | C | N | S | + |
| 8 | S | M | H | W | -- |
| 9 | S | C | N | W | + |
| 10 | R | M | N | W | + |
| 11 | S | M | N | S | + |
| 12 | O | M | H | S | + |
| 13 | O | H | N | W | + |
| 14 | R | M | H | S | -- |

**Information gain:**

*Outlook*: 0.246

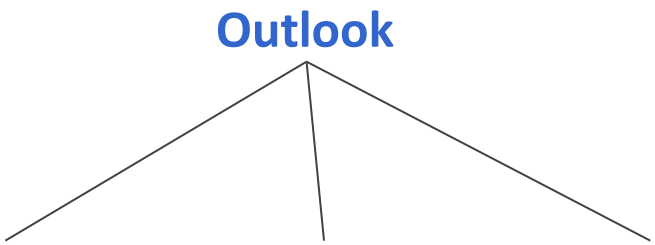
*Humidity*: 0.151

*Wind*: 0.048

*Temperature*: 0.029

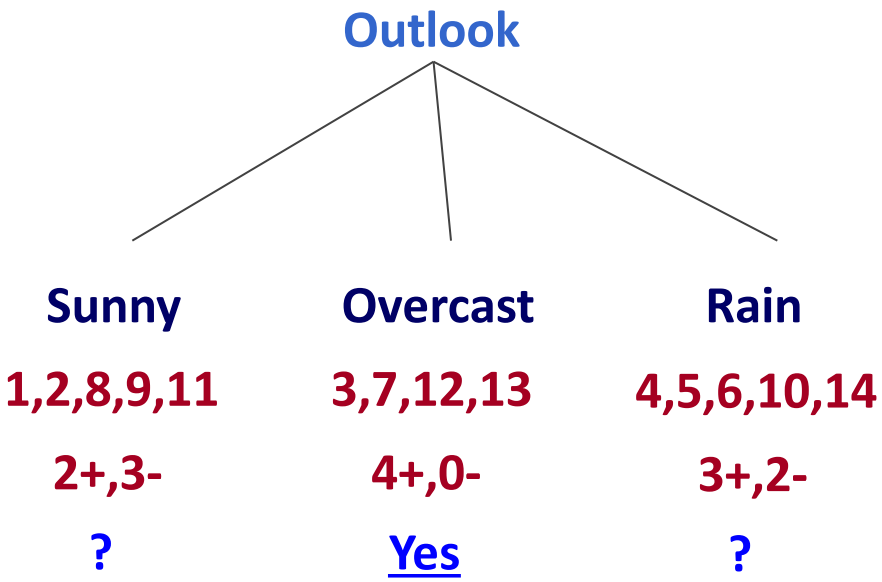
→ ***Split on Outlook***

# An Illustrative Example



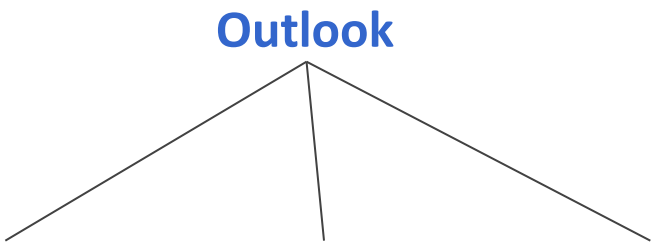

Gain(S,Humidity)=0.151
Gain(S,Wind) = 0.048
Gain(S,Temperature) = 0.029
Gain(S,Outlook) = 0.246

# An Illustrative Example




| 1 | O | T | H | W | Play? |
|---|---|---|---|---|---|
| | S | H | H | W | -- |
| 2 | S | H | H | S | -- |
| 3 | O | H | H | W | + |
| 4 | R | M | H | W | + |
| 5 | R | C | N | W | + |
| 6 | R | C | N | S | -- |
| 7 | O | C | N | S | + |
| 8 | S | M | H | W | -- |
| 9 | S | C | N | W | + |
| 10 | R | M | N | W | + |
| 11 | S | M | N | S | + |
| 12 | O | M | H | S | + |
| 13 | O | H | N | W | + |
| 14 | R | M | H | S | -- |

# An Illustrative Example



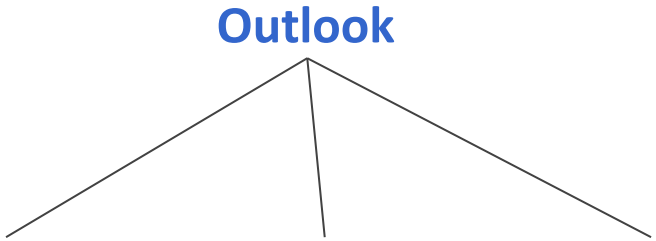

| Sunny | Overcast | Rain |
|---|---|---|
| 1,2,8,9,11 | 3,7,12,13 | 4,5,6,10,14 |
| 2+,3- | 4+,0- | 3+,2- |
| ? | Yes | ? |

| | O | T | H | W | Play? |
|---|---|---|---|---|---|
| | S | H | H | W | -- |
| 2 | S | H | H | S | -- |
| 3 | O | H | H | W | + |
| 4 | R | M | H | W | + |
| 5 | R | C | N | W | + |
| 6 | R | C | N | S | -- |
| 7 | O | C | N | S | + |
| 8 | S | M | H | W | -- |
| 9 | S | C | N | W | + |
| 10 | R | M | N | W | + |
| 11 | S | M | N | S | + |
| 12 | O | M | H | S | + |
| 13 | O | H | N | W | + |
| 14 | R | M | H | S | -- |

**Continue until:**

- Every attribute is included in path, or,
- All examples in the leaf have same label

# An Illustrative Example




| Sunny | Overcast | Rain |
|---|---|---|
| 1,2,8,9,11 | 3,7,12,13 | 4,5,6,10,14 |
| 2+,3- | 4+,0- | 3+,2- |
| ? | Yes | ? |

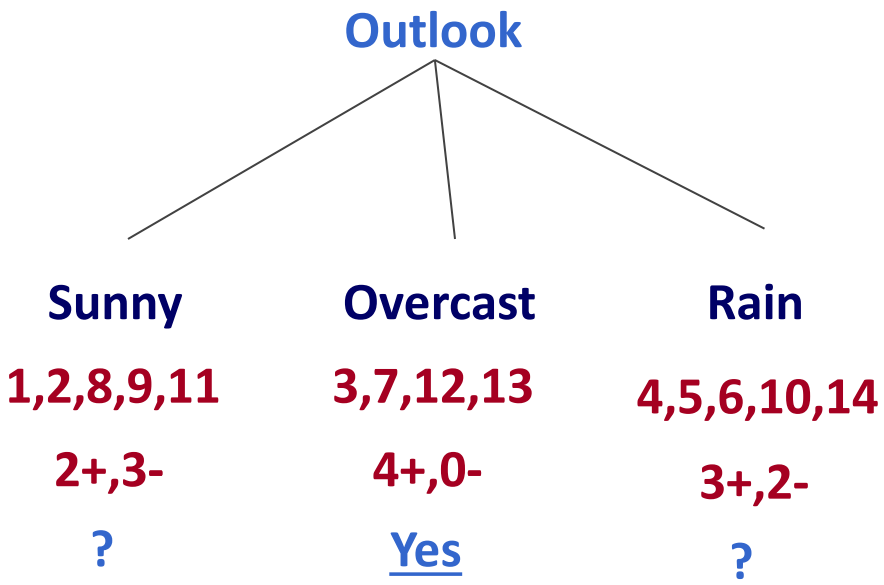
$Gain(S_{sunny}, Humidity)$ .97-(3/5) 0-(2/5) 0= .97

$Gain(S_{sunny}, Temp)$ = .97- 0-(2/5) 1 = .57

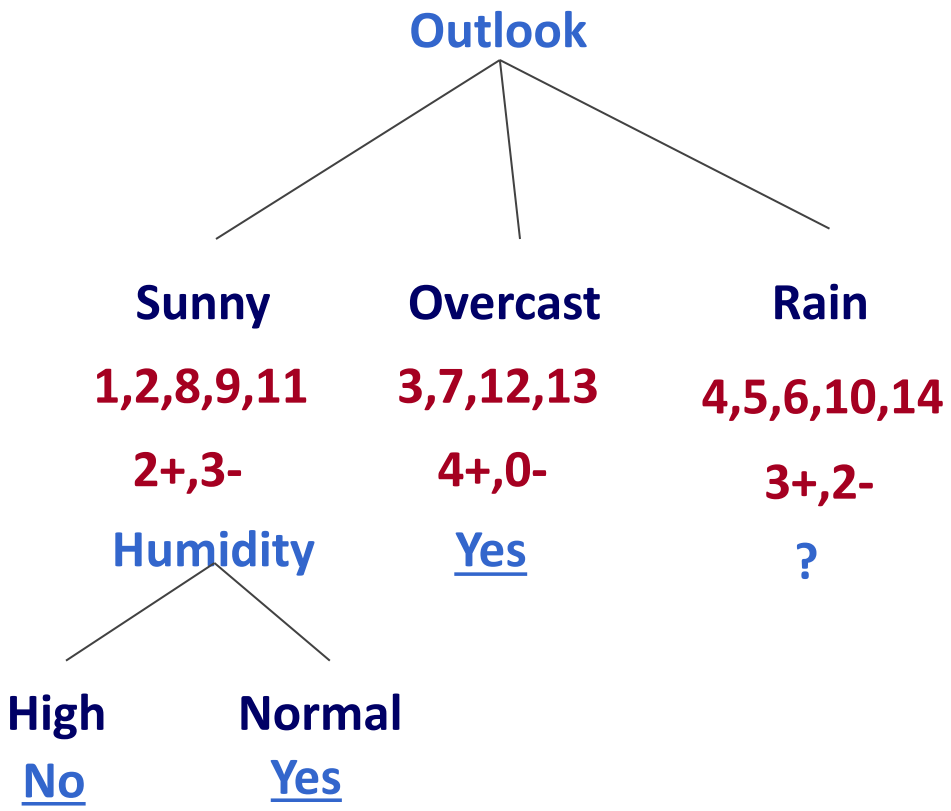
$Gain(S_{sunny}, wind)$ = .97-(2/5) 1 - (3/5) .92 = .02

| Day | Outlook | Temperature | Humidity | Wind | PlayTennis |
|---|---|---|---|---|---|
| 1 | Sunny | Hot | High | Weak | No |
| 2 | Sunny | Hot | High | Strong | No |
| 8 | Sunny | Mild | High | Weak | No |
| 9 | Sunny | Cool | Normal | Weak | Yes |
| 11 | Sunny | Mild | Normal | Strong | Yes |

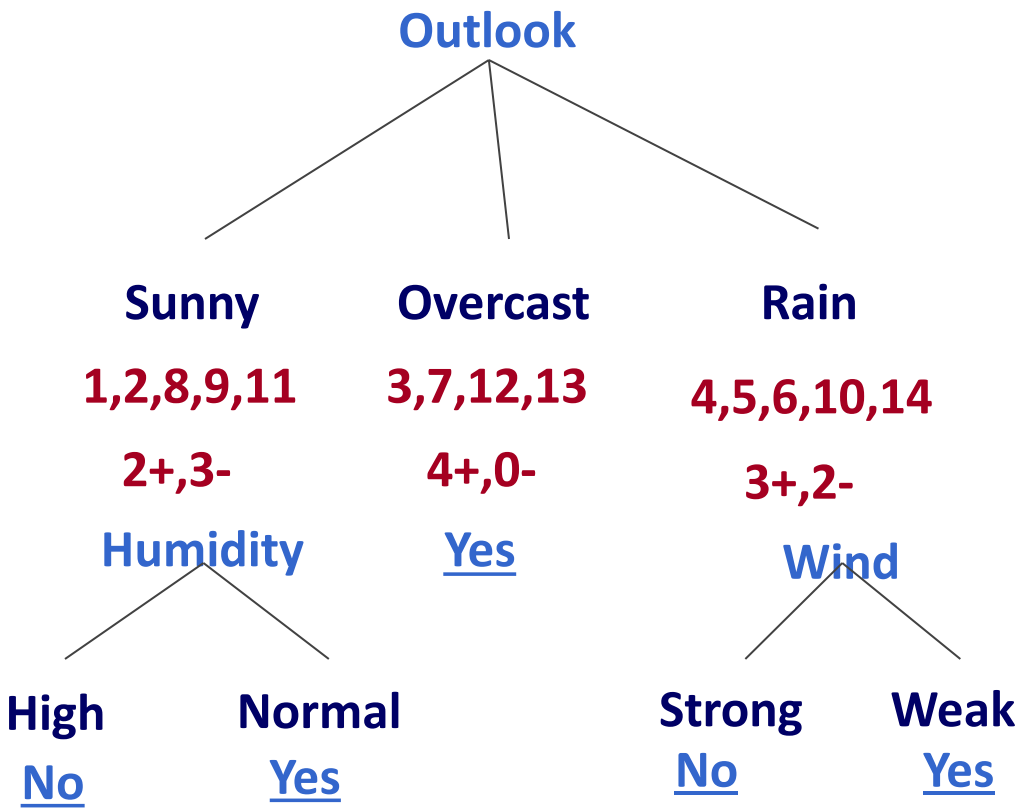
# An Illustrative Example



# An Illustrative Example



# An Illustrative Example



# induceDecisionTree(S)

- 1. Does S uniquely define a class?

  **if** all s ∈ S have the same label y: **return** S;

- 2. Find the feature with the most information gain:

  i = argmax $_i$ *Gain*(S, $X_i$)

- 3. Add children to S:

  **for** *k* **in** Values($X_i$):

  $S_k$ = {s ∈ S | $x_i$ = *k*}

  addChild(S, $S_k$)

  induceDecisionTree($S_k$)

  **return** S;

# Variants of Information Gain

- Information gain is defined using entropy to measure the disorder/ impurity of the labels.

- **Other ways to measure disorde**r, e.g., *MajorityError*, which computes:
  - *"Suppose the tree was not grown below this node and the majority label were chosen, what would be the error?"*
  - Suppose at some node, there are 15 + and 5 − examples.
  - What is the *MajorityError*?
- Answer: ¼

- *Similar idea to entropy*

# Missing feature values

Suppose an example is missing the value of an attribute. What can we?

| Day | Outlook | Temperature | Humidity | Wind | PlayTennis |
|---|---|---|---|---|---|
| 1 | Sunny | Hot | High | Weak | No |
| 2 | Sunny | Hot | High | Strong | No |
| 8 | Sunny | Mild | ??? | Weak | No |
| 9 | Sunny | Cool | High | Weak | Yes |
| 11 | Sunny | Mild | Normal | Strong | Yes |

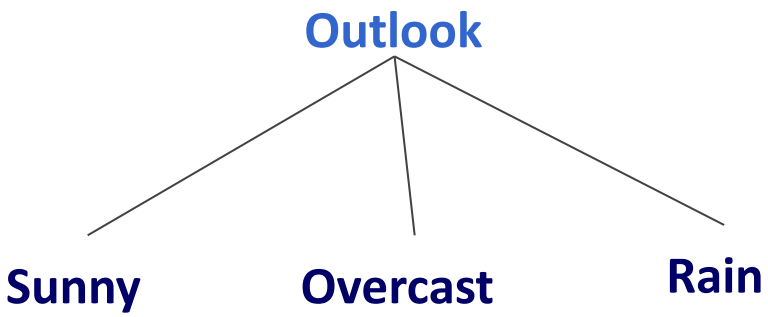
# Missing feature values

- Suppose an example is missing the value of an attribute . What can we do?

- "Complete the example" by
  - Use the most common value of that attribute in data
  - Use the most common value of that attribute in data, with the same class label (at train time)
  - Assign fractional count instead of majority vote on value

# Non Boolean Features

- If the features can take multiple values
  - We have seen one edge per value



# Non Boolean Features

- If the features can take multiple values
  - We have seen one edge per value (i.e a multiway split)
  - Another option: Make the attributes Boolean by testing for each value

Outlook=Sunny . . [Outlook:Sunny=True,
Outlook:Overcast=False,
Outlook:Rain=False]

# Non Boolean Features

What can you do with numeric features?

Use threshold *or* ranges to get Boolean tests

***How should you determine the thresholds?***

**Problem:**

You should consider all split points (c) to define node test $X_j > c$

**Is there an easier way?**

# Bias in Decision Tree Induction

- Conduct a search of the space of decision trees
  - Can represent all possible functions
- **We prefer short trees!**
  - In DT learning this is implemented as a search bias
  - We bias the search to prefer shorter trees
- Other alternatives?
  - How would you implement language bias on DT?
- Search bias is implemented using greedy heuristics
  - Hill-climbing without backtracking
  - **Overfitting can still be an issue..**

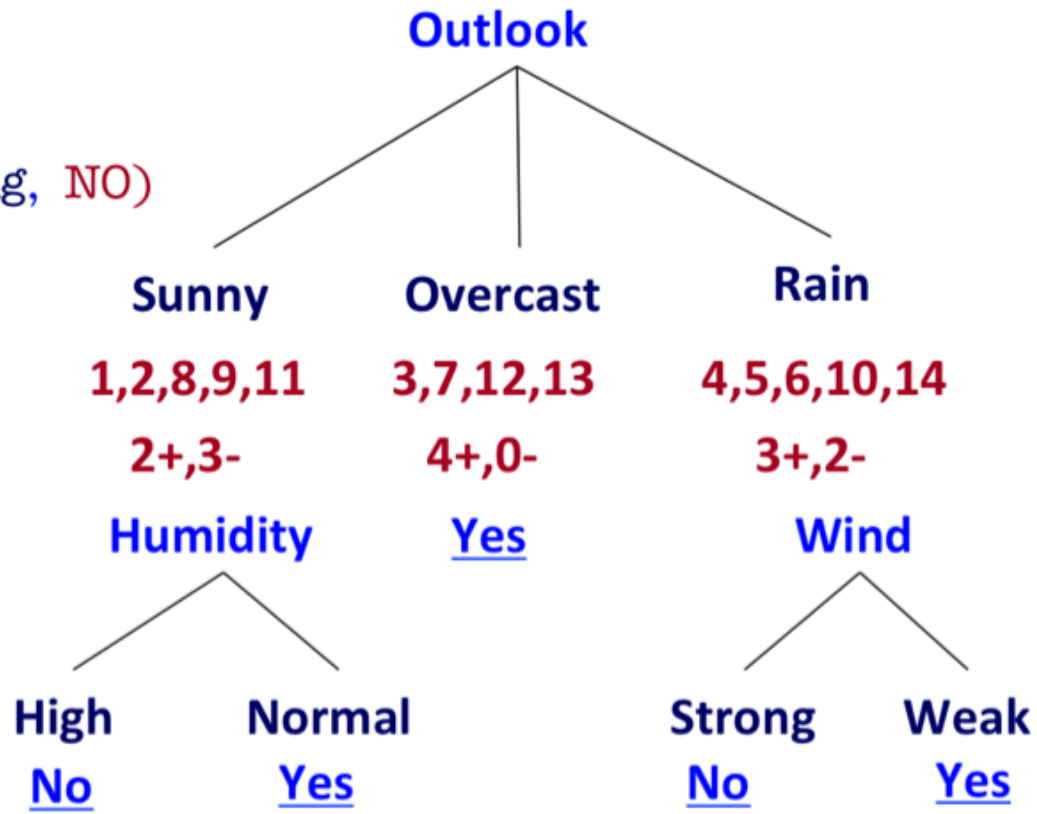
# Overfitting

- Learning a tree classifying the training data perfectly may not have the best generalization
  - Algorithm fits tree to noise in training data
  - Sparse data set

**A hypothesis h is said to overfit the training data** if there is another hypothesis h', such that h has a smaller error than h' on the training data but h has larger error on the test data.

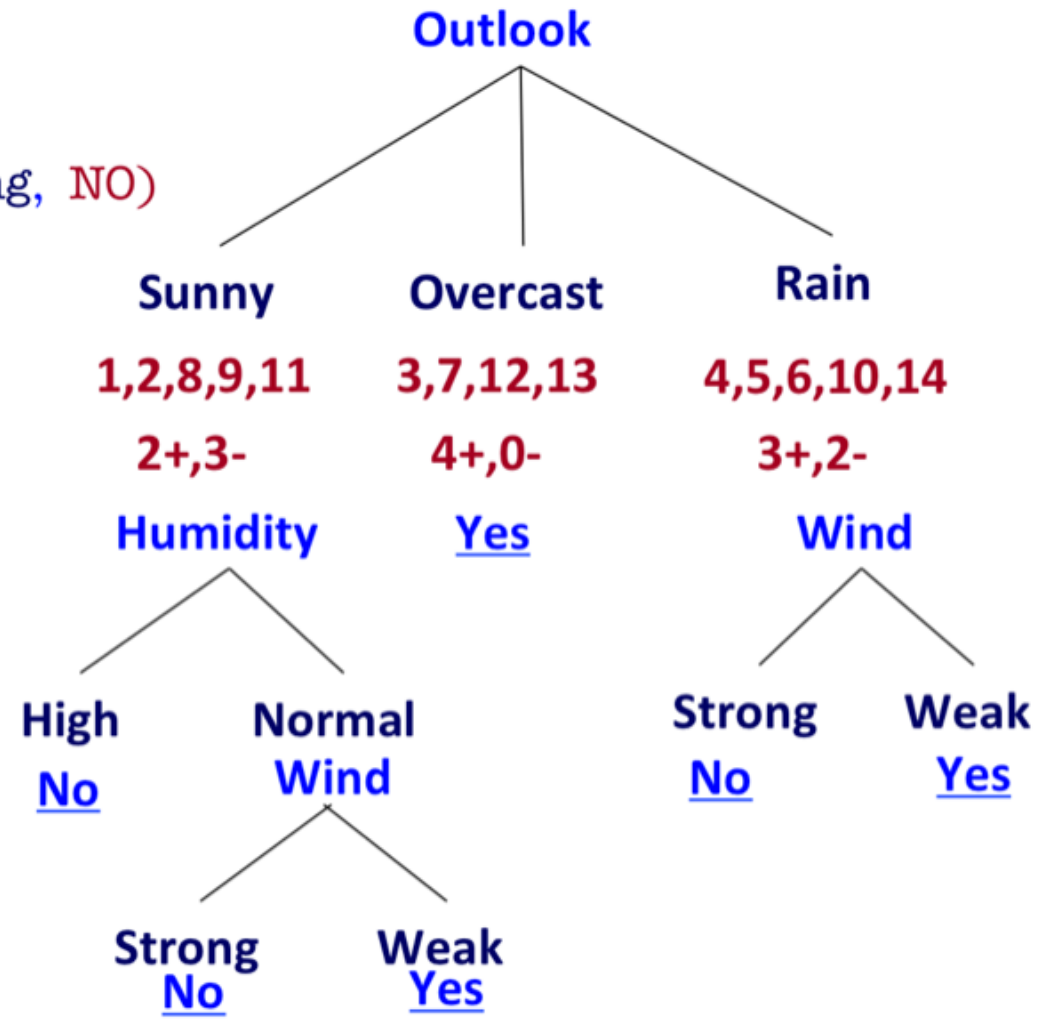
- Noisy example:

(Outlook = Sunny, Temp = Hot,
Humidity = Normal, Wind = Strong, NO)



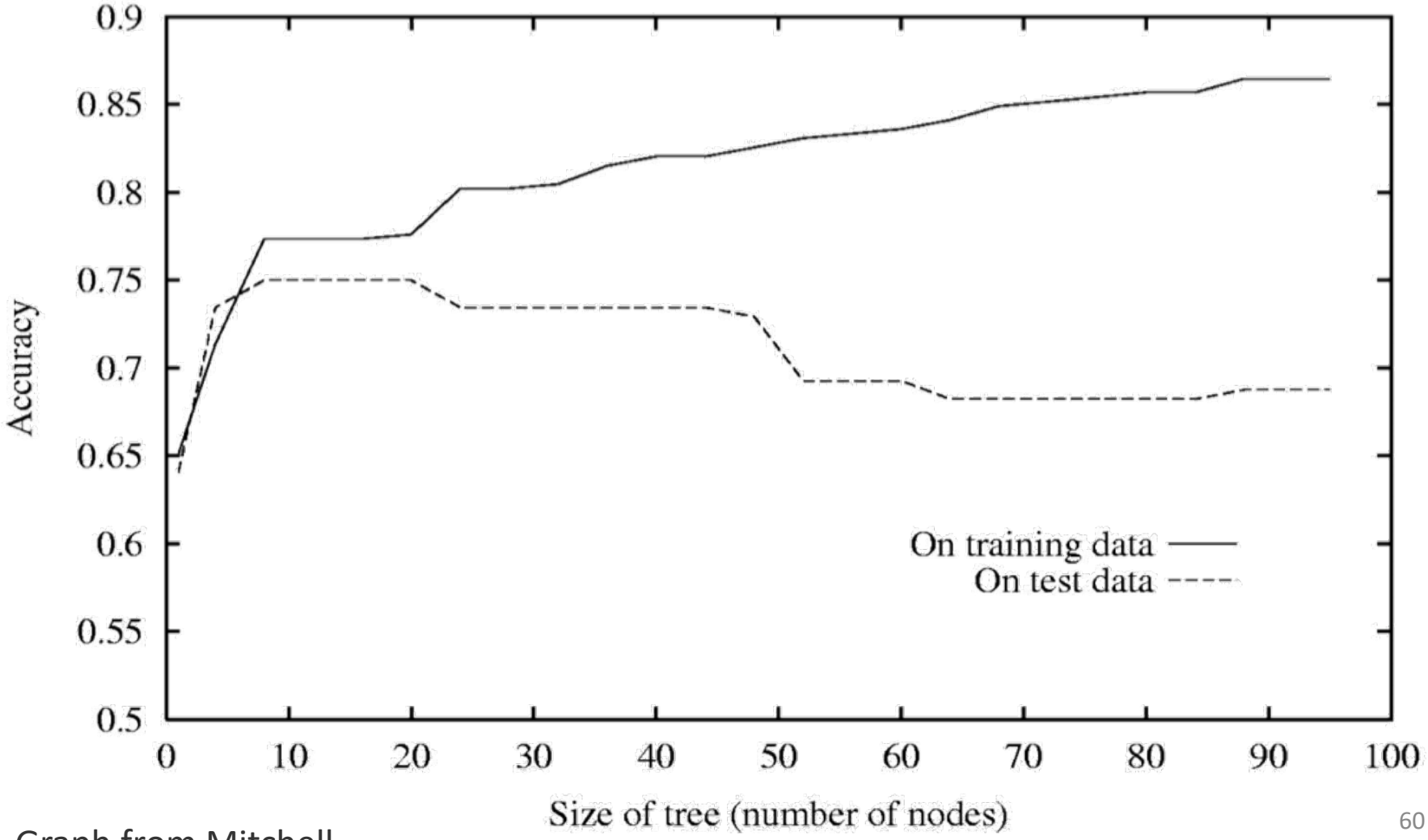
- Noisy example:

(Outlook = Sunny, Temp = Hot,
Humidity = Normal, Wind = Strong, NO)




*may fit noise or other coincidental regularities*

# Decision trees *will* overfit




Graph from Mitchell

# Avoiding overfitting in decision trees

- **Occam's Razor**
  - Favor simpler (in this case, shorter) hypotheses
  - Fewer shorter trees, less likely to fit better by coincidence
- **Static**: Fix the depth of the tree
  - Only allow trees of size K
    - Tune K using held-out validation set
    - ***Decision stump*** = a decision tree with only one level
- **Dynamic**: optimize while growing the tree
  - Grow tree on training data
  - Check performance on held-out data after adding a new node

# Avoiding overfitting in decision trees

- **Occam's Razor**
  - Favor simpler (in this case, shorter) hypotheses
  - Fewer shorter trees, less likely to fit better by coincidence

- **Post Pruning:**
- While accuracy on validation set decreases. Bottom up:
  - For each non leaf node:
    - Replace sub-tree under node by a majority vote
    - Test accuracy on validation set

# Decision Trees as Features

- When learning over a large number of features, learning decision trees is difficult and the resulting tree may be very large
- **Instead of pruning you can try:**
  - learn small decision trees, with limited depth.
  - Then, learn another function over these trees
- For example, Linear combination of decision stumps

# Decision Tree summary

- **Very popular tool**
  - Prediction is easy (and cheap!)
  - *Expressive and easy to interpret*
  - "debugging" the model is easy!
- **Learning**: greedy heuristic for representing the data
  - ID3 based on information gain
- Prone to **overfitting**!
  - Several ways to deal with it!

# Further reading

**Machine Learning. Tom Mitchell.**

Chapter 3

**A Course in Machine Learning. Hal Daumé III.**

Chapter 1

*(available on line: http://ciml.info/dl/v0_9/ciml-v0_9-ch01.pdf)*

# Questions

- What is *inductive bias*? Why is it important?
- What is the difference between **Language** bias and **search** bias?
- *Which hypothesis space is more expressive?*
  - Boolean functions , Decision trees, linear functions
- How does tree size effect generalization?
- What is the main decision when learning DT?
- How can you deal with noise? Missing attributes? Continuous values?
- *why are decision trees popular?*
  - *When should you use them?*
- *Do you think you can implement a decision tree?*