Digital Signal Processing I
Purdue University
Spring 2021

Lab 5 Report

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1 Multivariate Gaussian Distributions and Whitening

1.1 Generating Gaussian random vectors

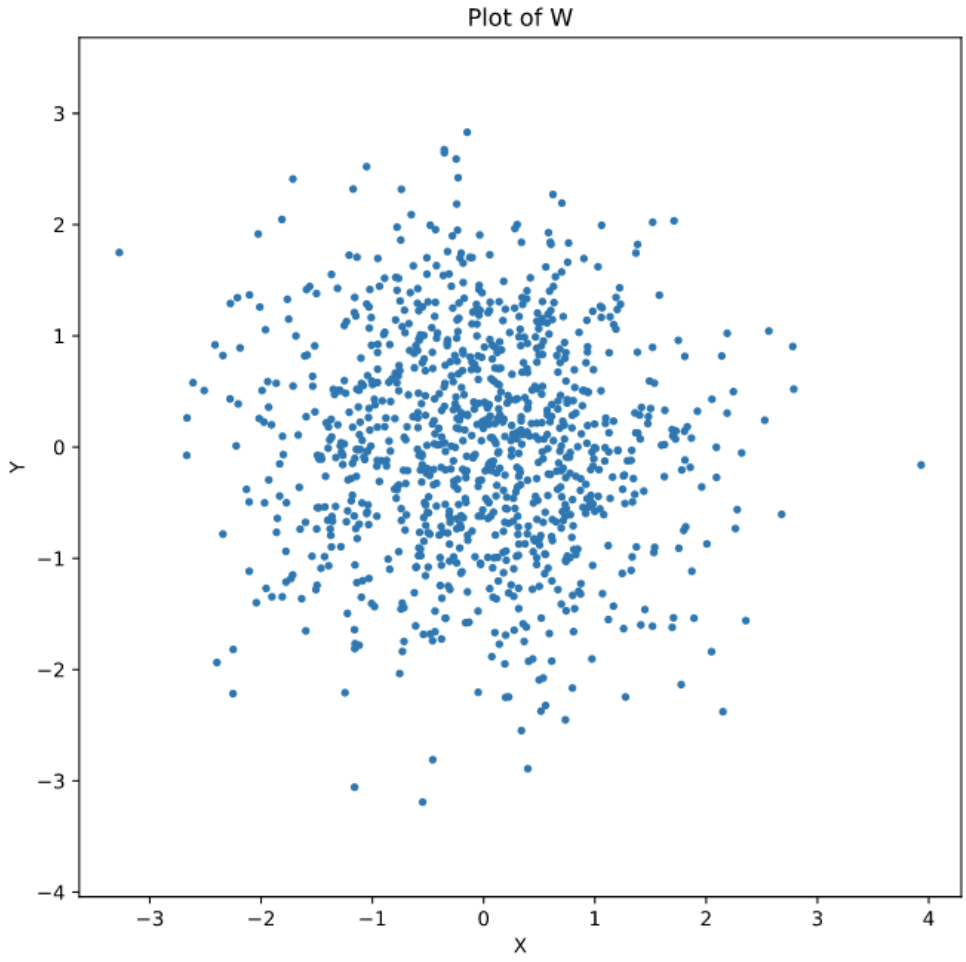

Figure 1: Plot of W

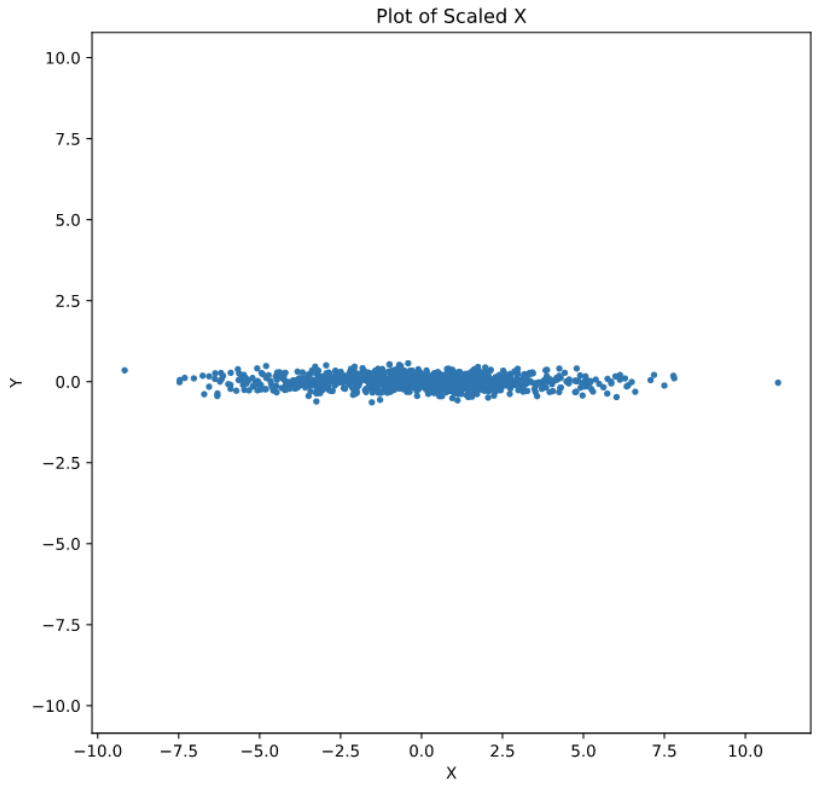

Figure 2: Plot of $\widetilde{X}$

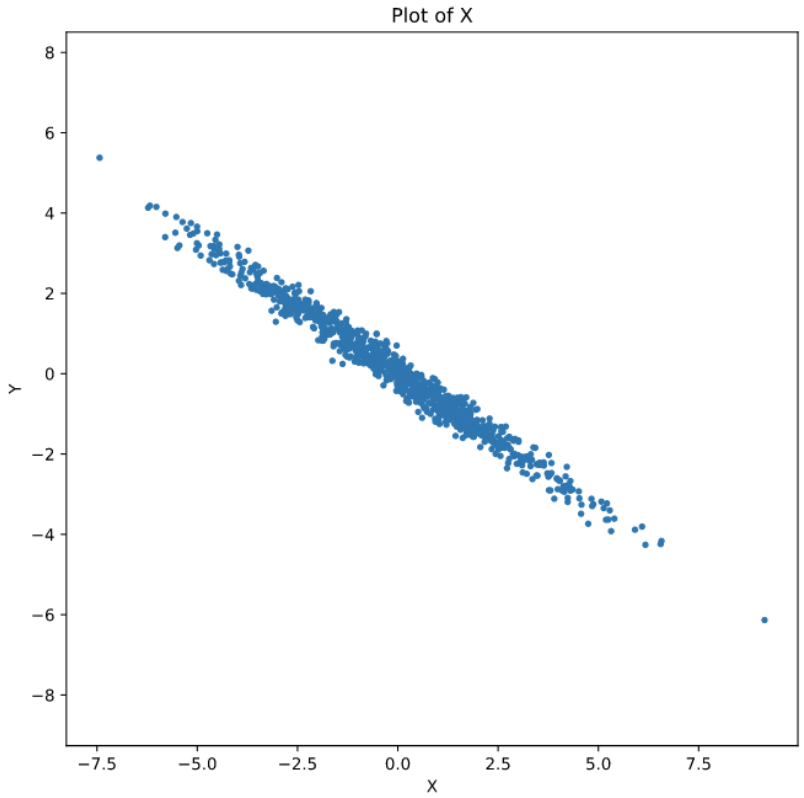

Figure 3: Plot of X

1.2 Covariance Estimation and Whitening

1. Theoretical value of the covariance matrix, R_x.

$$R_X = \begin{bmatrix} 2 & -1.2 \\ -1.2 & 1 \end{bmatrix}$$

2. Numerical listing of the covariance estimate $\hat{R}_x$

$$\hat{R}_X = \begin{bmatrix} 2.06503876 & -1.20211637 \\ -1.20211637 & 0.98290674 \end{bmatrix}$$

3. Scatter plots for $\hat{X}_i$ and W_i

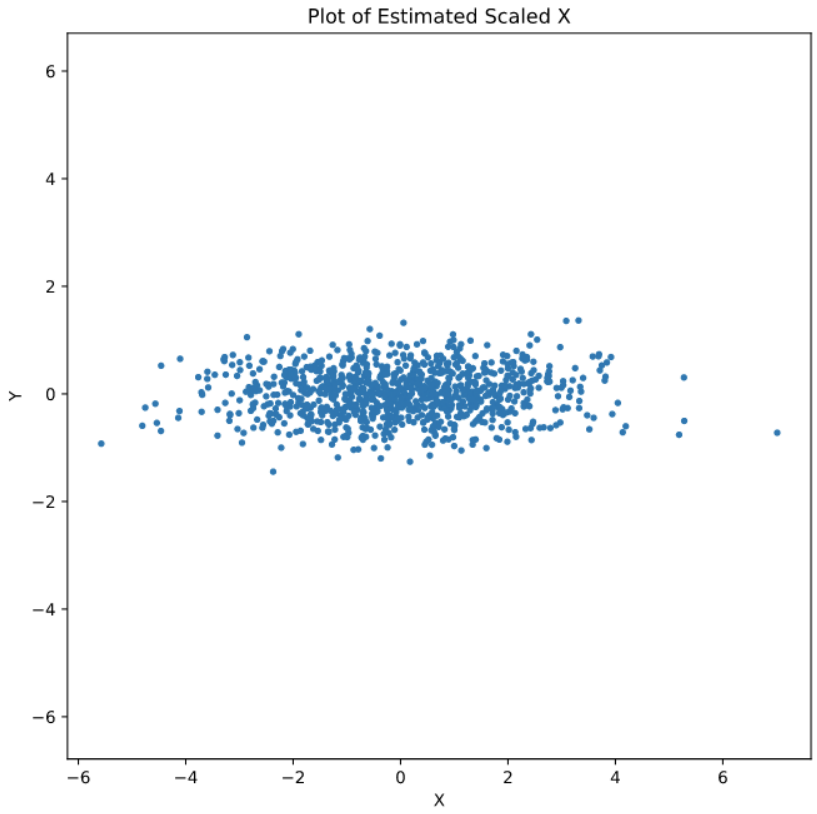

Figure 4: Plot of estimation of $\widetilde{X}$

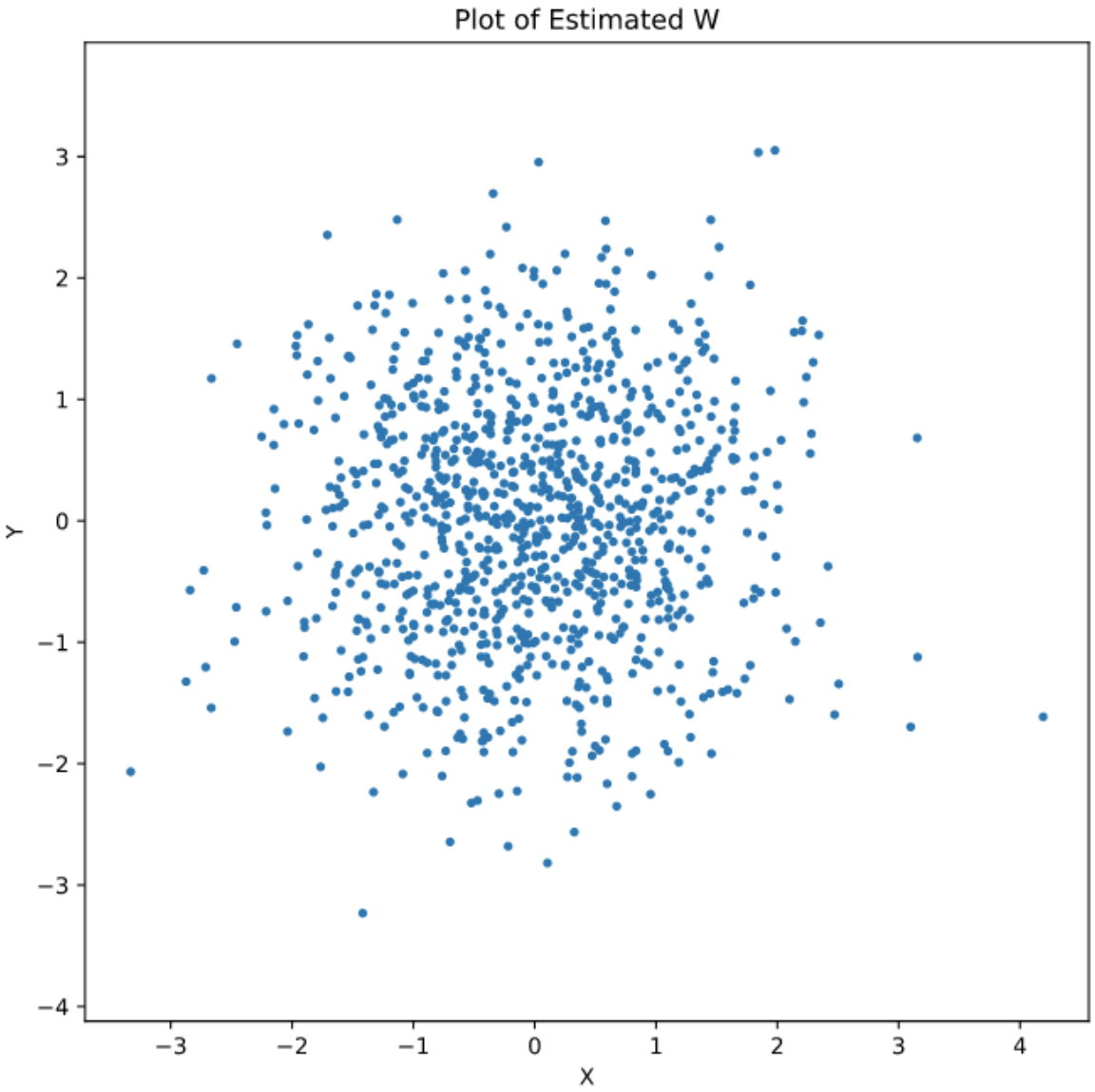

Figure 5: Plot of estimation of W

4. Numerical listing of the covariance estimate $\hat{R}_W$.

$$\hat{R}_W = \begin{bmatrix} 1.01490034 & 0.04956774 \\ 0.04956774 & 1.0311228 \end{bmatrix}$$

2 Eigenimages, PCA, and Data Reduction

1. First 12 eigenimages

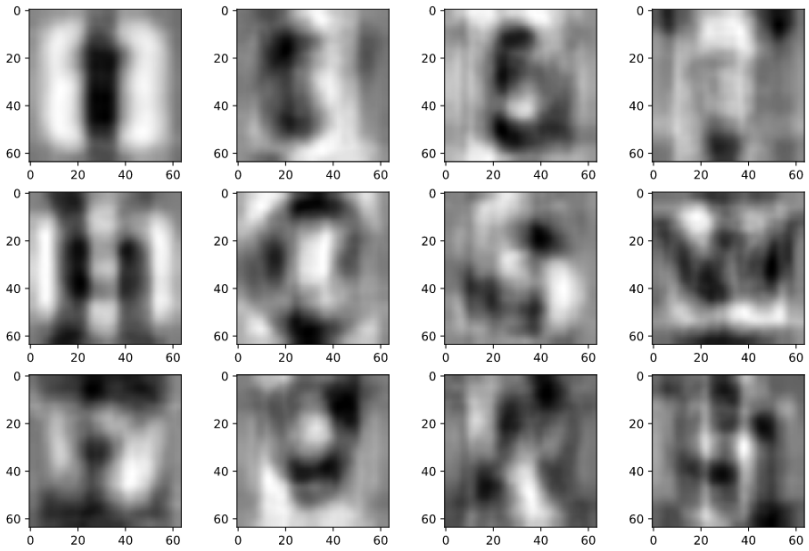

Figure 6: First 12 Eigenimages

2. Plots of projection coefficient vs eigenvector number

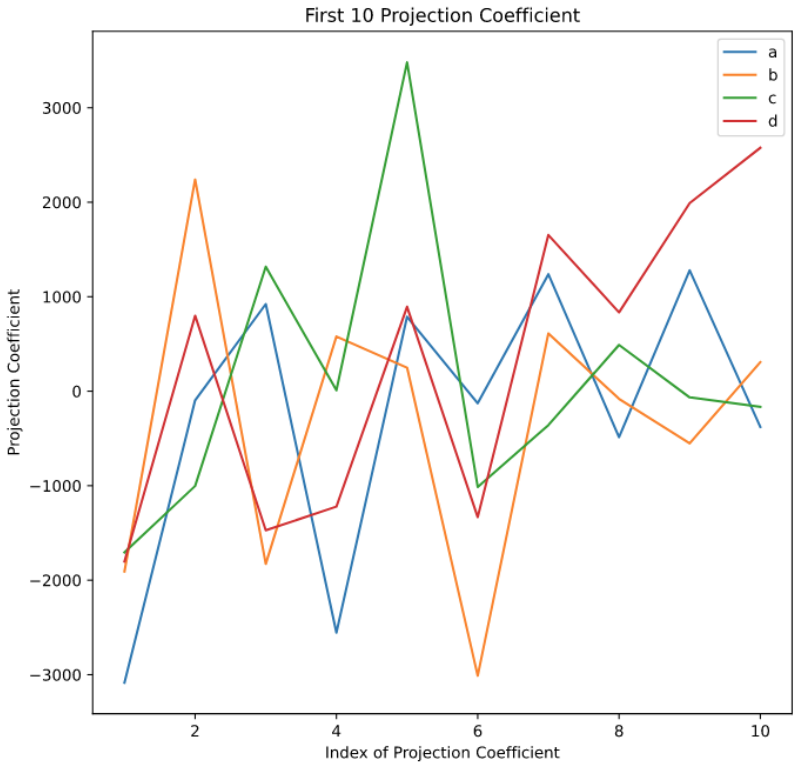

Figure 7: Plot of projection coefficient for letter: a,b,c,d

3. original image and 6 resynthesized versions

Figure 8: Original Image

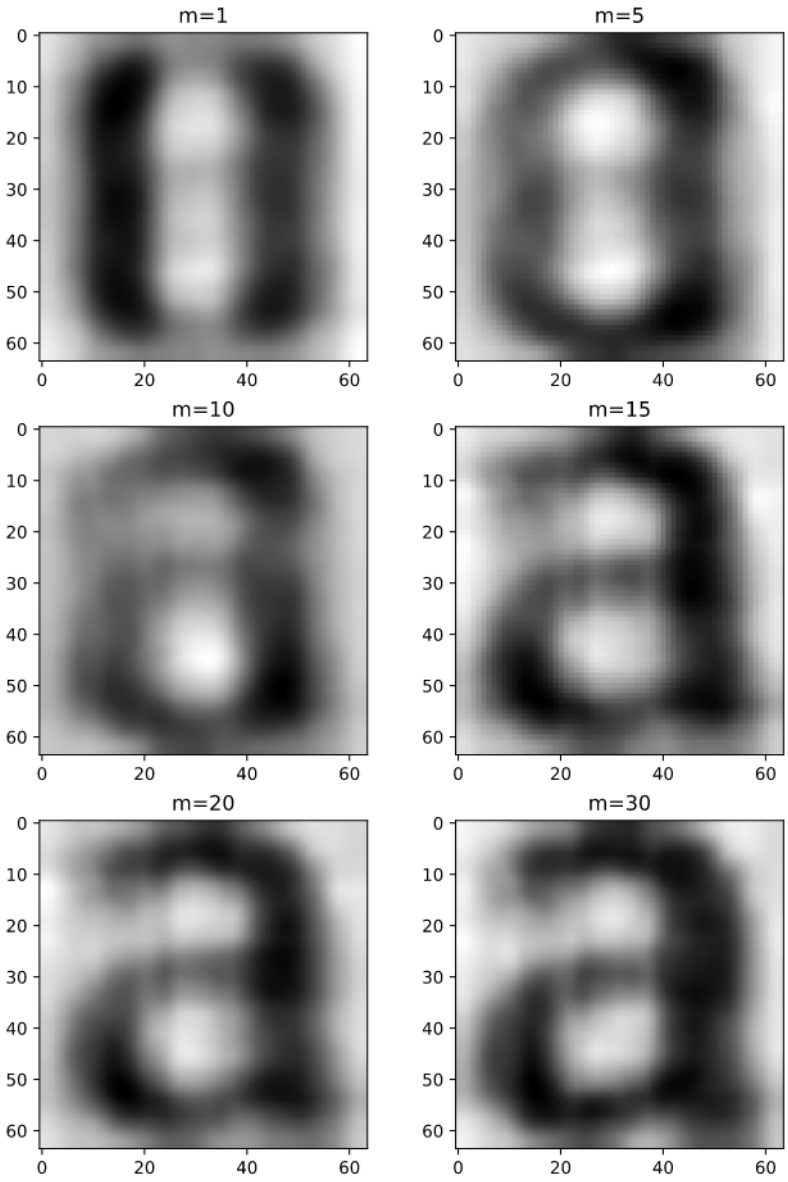

Figure 9: Resynthesized images with $m = [1, 5, 10, 15, 20, 30]$

3 Image Classification

3.1 Image Classification using R_k

Input Character	Classifier Output
a	s
b	g
d	a
l	i
n	v
q	g
u	v
y	v

3.2 Image Classification using other Covariance Matrix

Input Character	Classifier Output
n	o
u	v
y	v

Table 1: List of Misidentified letters for $B_k = \Lambda_k$

Input Character	Classifier Output
f	l
g	q
y	v

Table 2: List of Misidentified letters for $B_k = R_{wc}$

Input Character	Classifier Output
f	t
y	v

Table 3: List of Misidentified letters for $B_k = \Lambda$

Input Character	Classifier Output
f	t
g	q
y	v

Table 4: List of Misidentified letters for $B_k = I$

1. Which of the above classifier worked the best in this experiment?

   The first three options worked the best in this experiments which $B_k = \Lambda_k$, R_{wc} , or Λ. All three options has an accuracy of 92.3%

2. In constraining the covariance, what is the trade off between the accuracy of the data model and the accuracy of the estimates?

   From observation, as the covariance function is more constrained, the estimation perform better, but the performance on the training data decreases because the covariance function is no longer specific the class. The trade off is between the accuracy of the training data and accuracy of the testing data. The constrained covariance allows more generalization of the image classification and smaller classification model.