EE538 Digital Signal Processing I **Matlab Homework 2**

Due Date: Friday, Nov. 21, Fall 2014

Note: You should make use of the matlab codes PR4chan.m and PRRC4chan.m posted at the course web site.

Background. See relevant notes at course web site.

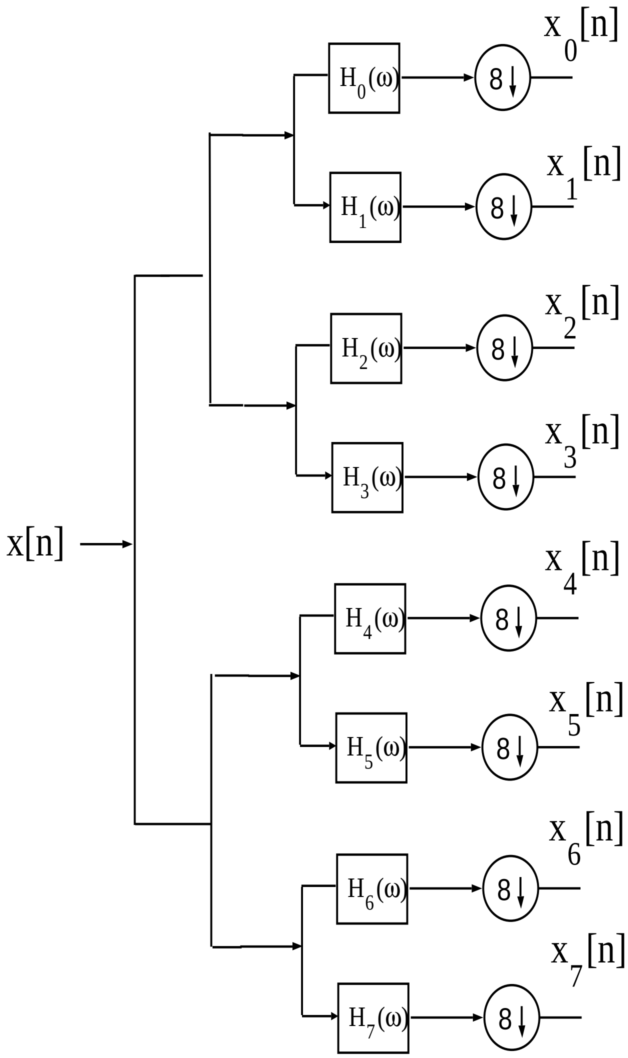

Figure 1(a). Analysis Filter Bank, $M = 8$.

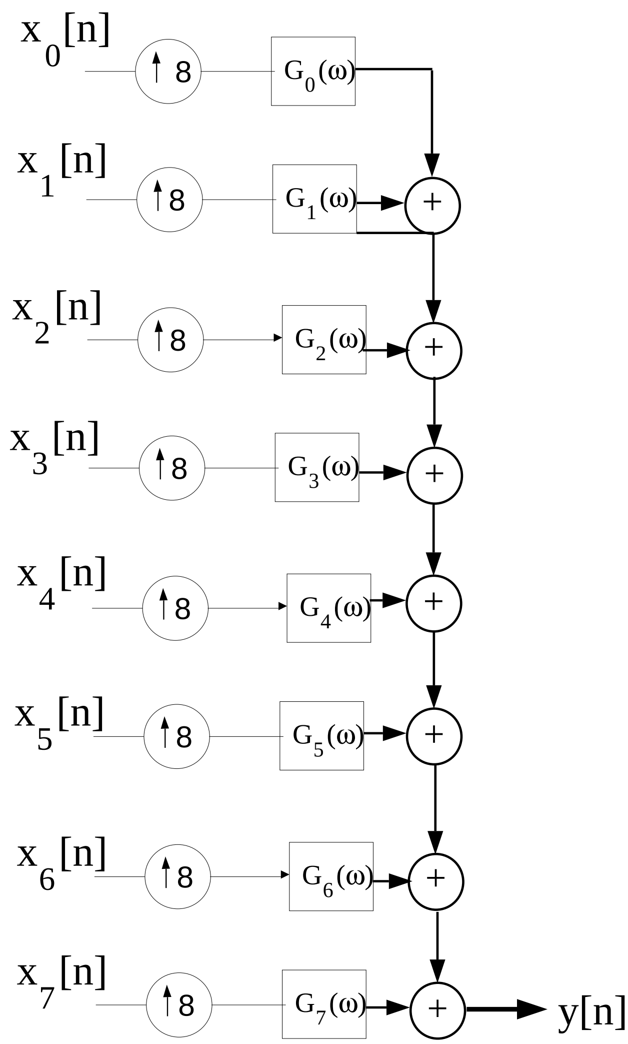

Figure 1(b). Synthesis Filter Bank, $M = 8$.

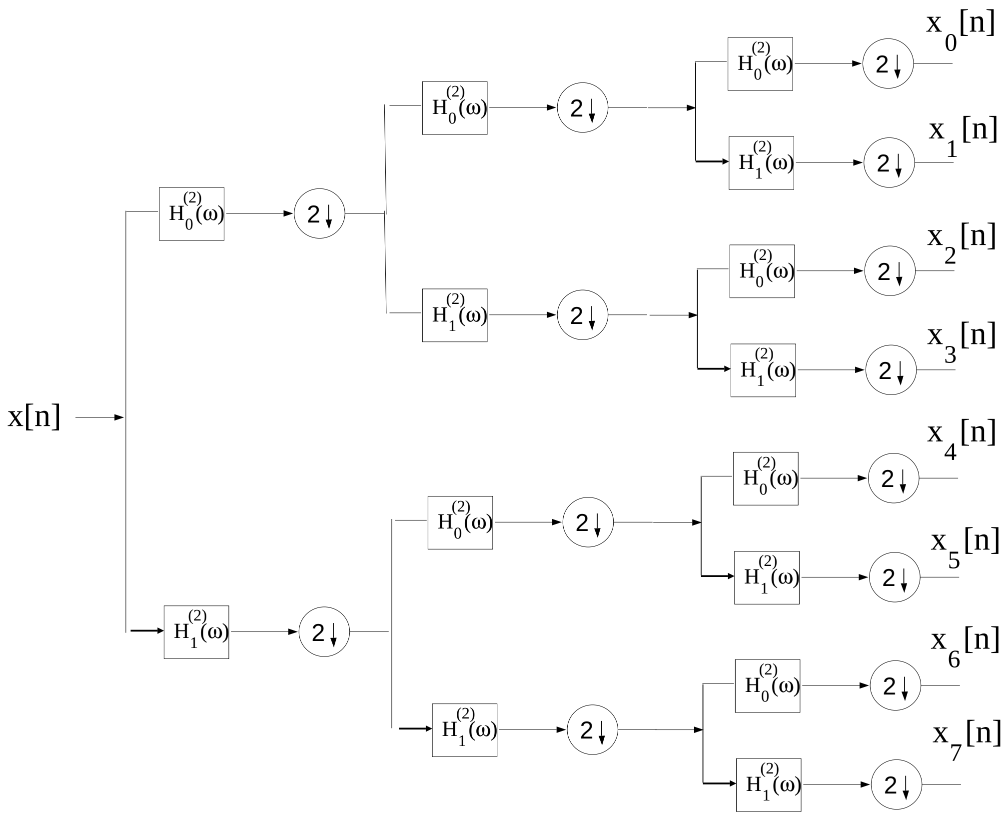

Figure 2(a). Analysis Section of Three-Stage Tree-Structured Filter Bank.

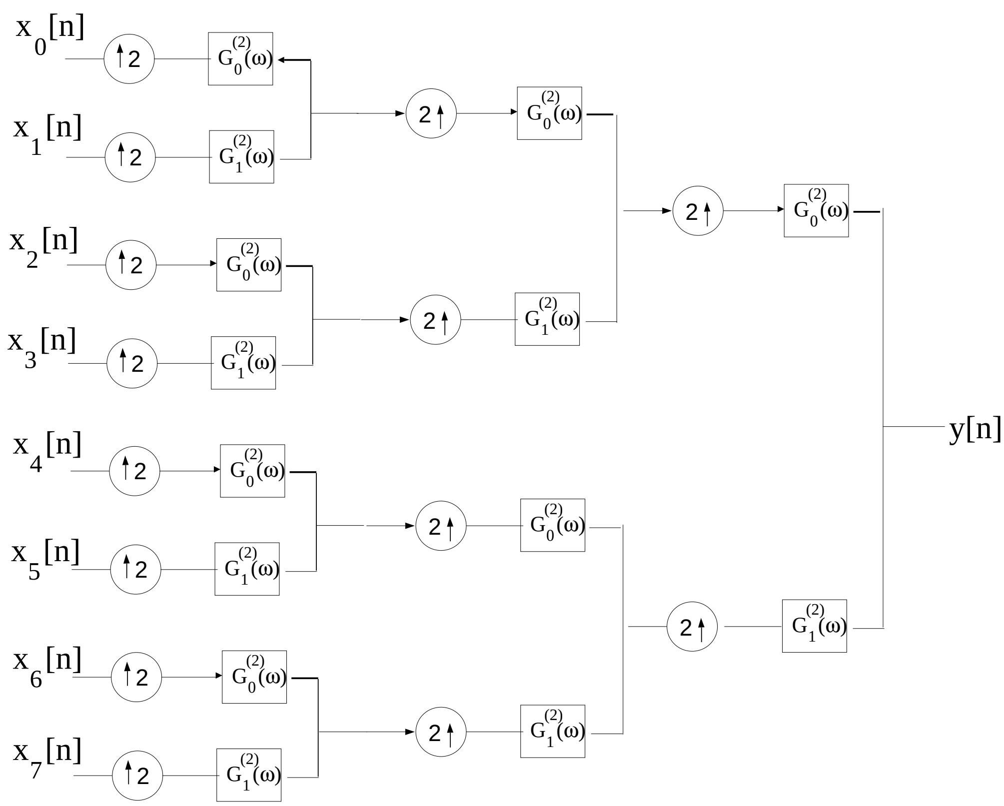

Figure 2(b). Synthesis Section of Three-Stage Tree-Structured Filter Bank.

Synthesizing M=8 Channel Perfect Reconstruction Filter Banks from Tree-Structured Filter Banks. This Matlab assignment is centered on synthesizing an M=8 channel uniform PR filter bank from a three stage tree-structured PR filter bank. As discussed in class, if the number of channels, M, is a power of two, an M-channel uniform PR filter bank may be synthesized via an equivalent (in terms of I/O relationship) tree-structured PR filter bank having $\log_2(M)$ stages with each stage formed from a two-channel QMF filter bank as depicted in Figure 2. That is, the combination of the analysis filter pair, $\{H_0^{(2)}(\omega), H_1^{(2)}(\omega)\}$, and synthesis filter pair $\{G_0^{(2)}(\omega), G_1^{(2)}(\omega)\}$, form a two-channel PR filter bank.

Part I. Deriving the Uniform Filter Bank Equivalent to Tree-Structured Filter Bank. Using Noble's First Identity to express each analysis filter, $H_m(\omega)$, $m = 0, 1, ..., 7$, in terms of $H_0^{(2)}(\omega)$ and $H_1^{(2)}(\omega)$. In each case, express the corresponding impulse response $h_m[n]$, $m = 0, 1, ..., 7$, in terms of $h_0^{(2)}[n]$ and $h_1^{(2)}[n]$.

Next, use Noble's Second Identity to express each synthesis filter, $G_m(\omega)$, $m = 0, 1, ..., 7$, in terms of $G_0^{(2)}(\omega)$ and $G_1^{(2)}(\omega)$. In each case, express the corresponding impulse response $h_m[n]$, $m = 0, 1, ..., 7$, in terms of $g_0^{(2)}[n]$ and $g_1^{(2)}[n]$. Note $g_0^{(2)}[n] = h_0^{(2)}[n]$ and $g_1^{(2)}[n] = -h_1^{(2)}[n]$; this is assumed throughout.

Part II. Matlab Calculations. For **each pair** of $h_0^{(2)}[n]$ and $h_1^{(2)}[n]$ specified below, use Matlab and the results derived above to compute the numerical values of the analysis filters $h_m[n]$, $n = 0, 1, ..., N-1$, for $m = 0, 1, ..., 7$. Plot all of the corresponding DTFT's $H_m(\omega)$, $m = 0, 1, ..., 7$, superimposed on a single graph using (at least) a 1024 pt. FFT of each $h_m[n]$, $m = 0, 1, ..., 7$. Next, compute the numerical values of the synthesis filters $g_m[n]$, $n = 0, 1, ..., N-1$, for $m = 0, 1, ..., 7$. Plot all of the corresponding DTFT's $G_m(\omega)$, $m = 0, 1, ..., 7$, superimposed on a single graph using (at least) a 1024 pt. FFT of each $g_m[n]$, $m = 0, 1, ..., 7$. Also, using Matlab, for each case below place each impulse response $h_m[n]$, $n = 0, 1, ..., 7$, as the row of a matrix called $\mathbf{H}$ and compute $\mathbf{H}\mathbf{H}^H$. Put the elements of the resulting 8×8 matrix in a Table. Finally, generate a sample function of Gaussian random process with zero mean and unity power of length 128 as the input signal $x[n]$. Plot the magnitude of the DTFT of the $x[n]$ using at least a 1024 pt. FFT. In addition, plot the magnitude of the DTFT of the corresponding output of the M=8 channel uniform PR filter bank $y[n]$, using at least a 1024 pt. FFT.

(A) $h_0^{(2)}[n] = \{1, 1\}$ and $h_1^{(2)}[n] = \{1, -1\}$. How does the resulting $\mathbf{H}$ compare with a Hadamard matrix of dimension 8?

- (i) Label the plot of all of the corresponding DTFT's $H_m(\omega)$, $m = 0, 1, ..., 7$ *superimposed* as Figure 1(a).
- (ii) Label the plot of all of the corresponding DTFT's $G_m(\omega)$, $m = 0, 1, ..., 7$ *superimposed* as Figure 1(b).
- (iii) Label the table containing the values of the 8×8 matrix $\mathbf{H}\mathbf{H}^H$ as Table 1.
- (iv) Label the plot of the magnitude of the DTFT of the Gaussian random process input signal as Figure 1(c).
- (v) Label the plot of the magnitude of the DTFT of the corresponding output of the M=8 channel uniform PR filter bank as Figure 1(d).

(B) $h_0^{(2)}[n] = h_{sr}[n-16]$, $n = 0, 1, ..., 31$, $h_1^{(2)}[n] = (-1)^n h_0^{(2)}[n]$, and $\beta = 0.35$ where

$$h_{sr}[n] = \frac{2\beta\cos[(1+\beta)\pi(n+.5)/2]}{\pi[1-4\beta^2(n+.5)^2]} + \frac{\sin[(1-\beta)\pi(n+.5)/2]}{\pi[(n+.5)-4\beta^2(n+.5)^3]}, n = -16, ..., 1, ..., 15.$$

(i) Label the plot of all of the corresponding DTFT's $H_m(\omega)$, $m = 0, 1, ..., 7$ *superimposed* as Figure 2(a).

(ii) Label the plot of all of the corresponding DTFT's $G_m(\omega)$, $m = 0, 1, ..., 7$ *superimposed* as Figure 2(b).

(iii) Label the table containing the values of the 8×8 matrix $\mathbf{HH}^H$ as Table 1.

(iv) Label the plot of the magnitude of the DTFT of the Gaussian random process input signal as Figure 2(c).

(v) Label the plot of the magnitude of the DTFT of the corresponding output of the M=8 channel uniform PR filter bank as Figure 2(d).

(C) $h_0^{(2)}[n] = h_{sr}[n-24]$, $n = 0, 1, ..., 47$, $h_1^{(2)}[n] = (-1)^n h_0^{(2)}[n]$, and $\beta = 0.1$ where

$$h_{sr}[n] = \frac{2\beta\cos[(1+\beta)\pi(n+.5)/2]}{\pi[1-4\beta^2(n+.5)^2]} + \frac{\sin[(1-\beta)\pi(n+.5)/2]}{\pi[(n+.5)-4\beta^2(n+.5)^3]}, n = -24, ..., 1, ..., 23.$$

(i) Label the plot of all of the corresponding DTFT's $H_m(\omega)$, $m = 0, 1, ..., 7$ *superimposed* as Figure 3(a).

(ii) Label the plot of all of the corresponding DTFT's $G_m(\omega)$, $m = 0, 1, ..., 7$ *superimposed* as Figure 3(b).

(iii) Label the table containing the values of the 8×8 matrix $\mathbf{HH}^H$ as Table 1.

(iv) Label the plot of the magnitude of the DTFT of the Gaussian random process input signal as Figure 3(c).

(v) Label the plot of the magnitude of the DTFT of the corresponding output of the M=8 channel uniform PR filter bank as Figure 3(d).

(D) $h_0^{(2)}[n] = \{1, j\}$ and $h_1^{(2)}[n] = \{1, -j\}$.

(i) Label the plot of all of the corresponding DTFT's $H_m(\omega)$, $m = 0, 1, ..., 7$ *superimposed* as Figure 4(a).

(ii) Label the plot of all of the corresponding DTFT's $G_m(\omega)$, $m = 0, 1, ..., 7$ *superimposed* as Figure 4(b).

(iii) Label the table containing the values of the 8×8 matrix $\mathbf{HH}^H$ as Table 1.

(iv) Label the plot of the magnitude of the DTFT of the Gaussian random process input signal as Figure 4(c).

(v) Label the plot of the magnitude of the DTFT of the corresponding output of the M=8 channel uniform PR filter bank as Figure 4(d).

General Information.

Deliverables for this project include:

- the derivation required in Part I
- 16 plots and 4 tables
- a paragraph summarizing your observations and any conclusions you can draw from this set of computer experiments.
- your source code appended to the report

The collection of plots and accompanying explanation should be put together in a cohesive manner in the form of a brief report. You may use any Matlab command you like in solving these problems. Each student is expected to do his/her own work and each must turn in his/her own report. Again, your write-up for this homework should be in the form of a brief report. Handwriting is acceptable but please be sure it is legible.