ECON 2029 Macroeconomics Theory Formula

Haoling Wang

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Contents

Preface

This formula sheet is tailored for the Spring 2025 version of ECON 2029 Macroeconomics Theory, using Mankiw's Macroeconomics [1] as the main textbook. It is intended primarily for use during exam preparation to help with memorizing key formulas. You are encouraged to jot down brief notes beside each formula to help recall related concepts from the lecture slides (PPTs). If you want to learn this course in detail, try to find the textbook and previous slides.

Personally, my understanding of macroeconomics is not as deep as that of microeconomics. However, if your goal is to grasp the fundamental concepts of macroeconomics for exam purposes, I strongly recommend **carefully analyzing the dynamic movements in every graph** and **thoroughly understanding the derivation and economic interpretation of each formula**.

This course builds upon ECON 1034 Introduction to Macroeconomics, extending and modifying some of the original models. Everyone has their own preferred learning style—please feel free to stick with the one that works best for you.

Good luck with your Macroeconomics Theory study!

1 Closed Economy and National Income

1.1 Supply side

1.1.1 Assumptions

$$K = \bar{K}$$
$$L = \bar{L}$$

1.1.2 Determining GDP

$$\bar{Y} = F(\bar{K}, \bar{L})$$

1.1.3 Determining real wage

$$MPL = \frac{W}{P}$$

1.1.4 Determining rental rate

$$MPK = \frac{R}{P}$$

1.1.5 Income distribution to L and K

Total labor income

$$MPL \times \bar{L} = \frac{W}{P} \times \bar{L}$$

Total capital income

$$MPK \times \bar{K} = \frac{R}{P} \times \bar{K}$$

Constant returns to scale production function

$$\bar{Y} = MPL \times \bar{L} + MPK \times \bar{K}$$

1.1.6 Cobb-Douglas production function

Cobb-Douglas production function

$$Y = AK^{\alpha}L^{1-\alpha}$$

Total capital income

$$MPK \times \bar{K} = \frac{\partial Y}{\partial K} \times \bar{K} = A\alpha K^{\alpha-1}L^{1-\alpha} \times \bar{K} = \alpha Y$$

Total labor income

$$MPL \times \bar{L} = \frac{\partial Y}{\partial L} \times \bar{L} = A(1-\alpha)K^{\alpha}L^{-\alpha} \times \bar{L} = (1-\alpha)Y$$

capital's share of total income = α

Each factor's **marginal product** is **proportional** to its **average product**.

$$MPK = \frac{\partial Y}{\partial K} = A\alpha K^{\alpha-1}L^{1-\alpha} = \alpha\frac{Y}{K}$$

$$MPL = \frac{\partial Y}{\partial L} = A(1-\alpha)K^{\alpha}L^{-\alpha} = (1-\alpha)\frac{Y}{L}$$

1.2 Demand side

1.2.1 Components of aggregate demand

Closed economy: no NX

1. C = consumer demand for good and services
2. I = demand for investment goods
3. G = government demand for good and services

1.2.2 Consumption, C

Disposable income = total income - total taxes

$$Y - T$$

Consumption function

$$C = C(Y - T)$$

Definition 1.1 *Marginal propensity to consume (MPC)*

Change in C when disposable income increases by one dollar.

$$\Delta C = MPC \times (\Delta Y - \Delta T) = MPC \times \Delta Y - MPC \times \Delta T$$

1.2.3 Investment, I

Investment function

$$I = I(r)$$

Real interest rate, r

1. the cost of borrowing
2. the opportunity cost of using one's own funds to finance investment spending
3. I depends negatively on r

1.2.4 Government spending, G

Assumption:
Government spending and total taxes are exogenous

$$G = \bar{G}$$

$$T = \bar{T}$$

1.3 Equilibrium

1.3.1 Goods Market

Aggregate demand

$$C + I + G = C(\bar{Y} - \bar{T}) + I(r) + \bar{G}$$

Aggregate supply

$$Y = \bar{Y} = F(\bar{K}, \bar{L})$$

Equilibrium

$$\bar{Y} = C(\bar{Y} - \bar{T}) + I(r) + \bar{G}$$

*The real interest rate adjusts to equate demand with supply.

1.3.2 Loanable Funds Market

1. demand for funds: investment
   (a) comes from investment
   (b) depends negatively on r
   (c) investment curve shifting
      i. technological innovations
      ii. tax laws
2. supply of funds: saving
   (a) Private saving
   $$(Y - T) - C$$
   (b) Public saving
   $$T - G$$
      i. $T > G$ **budget surplus** = $T - G$
      ii. $T < G$ **budget deficit** = $G - T$
      iii. $T = G$ **balanced budget**, public saving = 0
   (c) National saving = Private saving+ Public saving
   $$S = (Y - T) - C + T - G = Y - C - G$$
   (d) saving cruve shifting
      i. public saving
      fiscal policy: changes in G or T
      ii. private saving
      preferences
      tax laws that affect saving
3. "price" of funds: real interest rate

Equilibrium

$$S = I(r)$$

1.3.3 Relationship between Goods Market and Loanable Funds Market

$$S = Y - C - G = I \quad \text{[Loanable Funds Market]} \Leftrightarrow Y = C + I + G \quad \text{[Goods Market]}$$

2 Monetary System

Money supply = currency + demand (checking account) deposits

$$M = C + D$$

Total money supply

$$\frac{1}{rr} \times \text{original deposit}$$

rr = ratio of reserves to deposits = $\frac{R}{D}$

Definition 2.1 *Reserves, R*

The portion of deposits that banks have not lent.

Leverage ratio = $\frac{assets}{capital}$ capital = assets – liabilities

2.1 A model of the money supply

Monetary base

$$B = C + R$$

Reserve-deposit ratio

$$rr = \frac{R}{D}$$

Currency-deposit ratio (depends on households' preferences)

$$cr = \frac{C}{D}$$

$$M = C + D = \frac{C+D}{B} \times B$$

Denote $m = \frac{C+D}{B}$, then $M = m \times B$. m is the money multiplier.

$$m = \frac{C+D}{B} = \frac{C+D}{C+R} = \frac{1+C/D}{1+R/D} = \frac{1+cr}{1+rr}$$

$$\Delta M = m \times \Delta B$$

3 Inflation

Velocity

$$V = \frac{T}{M} = \frac{P \times Y}{M}$$

1. T=value of all transactions (nominal GDP as a proxy for total transactions).
2. P = price of output (GDP deflator)
3. Y = quantity of output (real GDP)
4. $P \times Y$ = value of output (nominal GDP)
5. M = money supply

3.1 Quantity equation

$$M \times V = P \times Y$$

Quantity equation in growth rates

$$\frac{\Delta M}{M} + \frac{\Delta V}{V} = \frac{\Delta P}{P} + \frac{\Delta Y}{Y}$$

3.1.1 Money demand function

$$(\frac{M}{P})^d = kY$$

k = how much money people wish to hold for each dollar of income.
$\frac{M}{P}$ = real money balances, the purchasing power of the money supply.

$$k = \frac{1}{V}$$

Assumption

$$V = \bar{V}$$

$$M \times \bar{V} = P \times Y$$

$$\frac{\Delta M}{M} = \frac{\Delta P}{P} + \frac{\Delta Y}{Y}$$

Price level

$$P = \frac{nominalGDP}{realGDP}$$

Inflation rate

$$\pi = \frac{\Delta P}{P}$$

$$\pi = \frac{\Delta M}{M} - \frac{\Delta Y}{Y}$$

$$(\frac{M}{P})^d = L(i, Y) = L(r + E\pi, Y)$$

1. M: exogenous
2. r: adjusts to ensure $I = S$
3. Y: $\bar{Y} = F(\bar{K}, \bar{L})$
4. P: adjusts to ensure $\frac{M}{P} = L(i, Y)$
5. Real money demand $\frac{M}{P}$: depends
   (a) negatively on i
   (b) positively on Y

3.2 Inflation and interest rates

$$r = i - \pi$$

i=Nominal interest rate
r=Real interest rate

3.2.1 Fisher effect

$$i = r + \pi$$

π = actual inflation rate
$E\pi$ = expected inflation rate

$1 - E\pi$ = the real interest rate people expect at the time they buy a bond or take out a loan
$1 - \pi$ = the real interest rate actually realized

4 Open Economy

d = domestic spending on domestic goods
f = domestic spending on foreign goods

$$C = C^d + C^f$$
$$I = I^d + I^f$$
$$G = G^d + G^f$$

IM = imports = = spending on foreign goods = $C^f + I^f + G^f$

EX = exports = foreign spending on domestic goods

NX = net exports (i.e. trade balance)= EX-IM

GDP = Expenditure on domestically produced goods and services

$$Y = C^d + I^d + G^d + EX = (C - C^f) + (I - I^f) + (G - G^f) + EX$$
$$= C + I + G + EX - (C^f + I^f + G^f) = C + I + G + EX - IM$$
$$= C + I + G + NX$$
$$\Leftrightarrow NX = Y - (C + I + G)$$

net exports = output - domestic spending

$$\Rightarrow NX = EX - IM = Y - (C + I + G)$$

1. Trade surplus
   output > spending and exports > imports
   Size of the trade surplus = NX
2. Trade deficit
   spending > output and imports > exports
   Size of the trade deficit = –NX

4.1 International capital flows

Net capital outflow

$$S - I$$

=net outflow of "loanable funds"
=net purchases of foreign assets =the country's purchases of foreign assets - foreign purchases of domestic assets

1. $S > I$ net lender
2. $S < I$ net borrower

trade balance = net capital outflow

NX = Y – (C + I + G) [international trade]
=(Y – C – G) – I
= S – I [international capital flows]

Asumptions

1. Domestic and foreign bonds are perfect substitutes
2. Perfect capital mobility: no restrictions on international trade in assets
3. Economy is small: cannot affect the **world interest rate**, denoted **r*** [exogeneous]

$$r = r*$$

4.2 Exchange rate

e = nominal exchange rate,
the relative price of domestic **currency** in terms of foreign **currency**.

ϵ = real exchange rate,
the relative price of domestic **goods** in terms of foreign **goods**.

$$\epsilon = \frac{e \times P}{P^*}$$

P: price level

$$P = \frac{nominalGDP}{realGDP}$$

ϵ is the relative price of one country's **output** in terms of the other country's **output**.

4.2.1 Net exports function

$$NX = NX(\epsilon) = \bar{S} - I(r^*)$$

Inverse relationship between NX and ϵ.

5 Unemployment and the labor market

Notation

1. L = No. of workers in labor force [exogenous]
2. E= No. of employed workers
3. U = No. of unemployed
4. $\frac{U}{L}$= unemployment rate
5. s =rate of job separations [exogenous]
   fraction of employed workers that become separated from their jobs
6. f = rate of job finding [exogenous]
   fraction of unemployed workers that find jobs

Steady state condition

No. of employed people who lose or leave their jobs = No. of unemployed people who find jobs

$$s \times E = f \times U$$

"Equilibrium" U-rate: natural rate of unemployment

$$E = L - U$$

$$f \times U = s \times E = s \times (L - U) = s \times L - s \times U$$
$$(f + s) \times U = s \times L$$
$$\frac{U}{L} = \frac{s}{s + f}$$

6 Capital accumulation and population growth

6.1 Basic Solow model build up

- Aggregate production function
$$Y = F(K, L)$$
- Individual production function
$$y = f(k)$$
- National income identity
$$Y = C + I$$
- Individual income
$$y = c + i$$
- Consumption function (individual)
$$c = (1 - s)y$$
  - s saving rate
- Individual saving = investment
$$sy = sf(k) = i$$

Capital accumulation
$$\Delta k = sf(k) - \delta k$$
Steady state
$$\Delta k = 0$$
Steady state capital stock: $k^\star$

6.2 Golden rule capital stock

Definition 6.1 *Golden rule*

The "best" steady state has the highest possible consumption per person.
$$c^\star = (1–s)f(k^\star)$$
$k^\star_{gold}$ is the Golden Rule level of capital, the steady state value of k that maximizes consumption.

Golden rule capital stock
$$\max_{k^\star} c^\star = f(k^\star) - \delta k^\star$$
$$MPK = \delta$$

6.3 Solow model with population growth

Population growth
$$\frac{\Delta L}{L} = n$$
Break-even investment
$$(\delta + n)k$$
nk: to equip new workers with capital

Capital accumulation with population growth

$$\Delta k = sf(k) - (\delta + n)k$$

The Golden Rule with population growth

$$\max_{k^\star} c^\star = f(k^\star) - (\delta + n)k^\star$$

$$MPK - \delta = n$$

6.4 Solow model with technological growth

Labor efficiency growth led by technological growth

$$g = \frac{\Delta E}{E}$$

Production function

$$Y = F(K, L \times E)$$

$L \times E$ = the number of effective workers

Break-even investment

$$(\delta + n + g)k$$

gk: to provide capital for the new "effective" workers created by technological progress

Golden Rule capital stock with technological progress

$$\max_{k^\star} c^\star = f(k^\star) - (\delta + n + g)k^\star$$

$$MPK - \delta = n + g$$

6.5 Endogenous growth theory

6.5.1 Basic model

Production function

$$Y = AK$$

Equation of motion for total capital

$$\Delta K = sY - \delta K$$

Growth rate of total output

$$\frac{\Delta Y}{Y} = \frac{\Delta K}{K} = sA - \delta$$

6.5.2 A two-sector model

Manufacturing production function

$$Y = F[K, (1 - u)EL]$$

u = fraction of labor in research Research production function

$$\Delta E = g(u)E$$

Capital accumulation

$$\Delta K = sY - \delta K$$

In the steady state, manufacturing output per worker and the standard of living grow at rate

$$\frac{\Delta E}{E} = g(u)$$

7 IS-LM Model and Aggregate demand

7.1 IS-LM model in closed economy

7.1.1 Keynesian cross

Explaining IS curve.

Notation

1. I = planned investment
2. $PE = C + I + G$ = planned expenditure
3. Y = real GDP = actual expenditure
4. consumption function $C = C(Y - T)$
5. government policy variables $G = \bar{G}, T = \bar{T}$
6. planned investment (exogenous) $I = \bar{I}$
7. planned expenditure $PE = C(Y - \bar{T}) + \bar{I} + \bar{G}$

$$\text{Actual expenditure} - \text{Planned expenditure} = \text{Unplanned inventory investment}$$

Equilibrium condition

$$\text{Actual expenditure} = \text{Planned expenditure}$$

$$Y = PE$$

The government purchases multiplier

$$\frac{\Delta Y}{\Delta G} = \frac{1}{1 - MPC}$$

The tax multiplier

$$\frac{\Delta Y}{\Delta T} = \frac{-MPC}{1 - MPC}$$

7.1.2 IS curve

IS curve equation (negative sloped)
A graph of all combinations of r and Y that result in **goods market** equilibrium

$$Y = C(Y - \bar{T}) + I(r) + \bar{G}$$

7.1.3 The theory of liquidity preference

The interest rate is determined by money supply and money demand.

7.1.4 LM curve

LM curve equation (upward sloped)
The supply of real money balances is fixed

$$(\frac{M}{P})^s = \frac{\bar{M}}{\bar{P}}$$

Demand for real money balances

$$(\frac{M}{P})^d = L(r))$$

Equilibrium

$$\frac{\bar{M}}{\bar{P}} = L(r)$$

LM curve
A graph of all combinations of r and Y that equate the supply and demand for real money balances. (**money market balances**).

$$\frac{\bar{M}}{\bar{P}} = L(r, Y)$$

7.1.5 Short run equilibrium

The short-run equilibrium is the combination of r and Y that **simultaneously** satisfies the **equilibrium** conditions in the **goods** & **money markets.**

$$Y = C(Y - \bar{T}) + I(r) + G$$
$$\frac{\bar{M}}{\bar{P}} = L(r)$$

7.2 IS-LM model in small open economy

7.2.1 Mundell-Fleming Model

Key assumption

$$r = r^{\star}$$

$IS^{\star}$ **curve** Goods market equilibrium

$$Y = C(Y - T) + I(r^{\star}) + G + NX(e)$$

e = nominal exchange rate
= foreign currency per unit domestic currency

$LM^{\star}$ **curve** Money market equilibrium

$$\frac{M}{P} = L(r^{\star}, Y)$$

Interest-rate differentials

$$r = r^{\star} + \theta$$
$$Y = C(Y - T) + I(r^{\star} + \theta) + G + NX(e)$$
$$\frac{M}{P} = L(r^{\star} + \theta, Y)$$

Mundell-Fleming and the AD curve

$$Y = C(Y - T) + I(r^{\star}) + G + NX(\varepsilon)$$

$$\frac{M}{P} = L(r^{\star}, Y)$$

8 Aggregate Supply

8.1 Sticky-price model

SRAS

$$Y = \bar{Y} + \alpha(P - EP)$$

8.2 Phillips curve for inflation and unemployment

Phillips curve

$$\pi = E\pi - \beta(u - u^n) + v$$

Adaptive expectations

$$E\pi = \pi_{-1}$$

So Phillips curve equilibrium becomes

$$\pi = \pi_{-1} - \beta(u - u^n) + v$$

Sacrifice ratio = $\frac{\text{lost GDP}}{\text{total inflation}}$ = 5

9 Dynamic AD-AS model

Output

$$Y_t = \bar{Y}_t - \alpha(r_t - \rho) + \varepsilon_t$$

- Y_t output
- $\bar{Y}_t$ natural level of output
- r_t real interest rate
- ρ "Natural rate of interest." In absence of demand shocks, $Y_t = \bar{Y}_t$ and $r_t = \rho$
- ε_t demand shock, random and zero on average

Fisher equation

$$r_t = i_t - E_t\pi_{t+1}$$

- r_t ex ante (i.e. expected) real interest rate
- i_t nominal interest rate
- $E_t\pi_{t+1}$ expected inflation rate

Inflation: The Phillips Curve

$$\pi_t = E_{t-1}\pi_t + \phi(Y_t - \bar{Y}_t) + v_t$$

- π_t current inflation
- $E_{t-1}\pi_t$ previously expected inflation
- ϕ indicates how much inflation responds when output fluctuates around its natural level
- v_t supply shock, random and zero on average

Expected Inflation: Adaptive Expectations

$$E_t\pi_{t+1} = \pi_t$$

The Nominal Interest Rate: The Monetary-Policy Rule

$$i_t = \pi_t\rho + \theta_\pi(\pi_t - \pi_t^\star) + \theta_Y(Y_t - \bar{Y}_t)$$

- i_t nominal interest rate, set each period by the central bank
- ρ natural rate of interest
- $\pi_t^\star$ central bank's inflation target
- θ_π measures how much the central bank adjusts the interest rate when inflation deviates from its target
- θ_Y measures how much the central bank adjusts the interest rate when output deviates from its natural rate

9.1 Long run equilibrium

No shocks

$$\varepsilon_t = v_t = 0$$

Constant inflation

$$\pi_{t-1} = \pi_t$$
$$Y_t = \bar{Y}_t$$
$$r_t = \rho$$
$$\pi_t = \pi_t^\star$$
$$E_t\pi_{t+1} = \pi_t^\star$$
$$i_t = \rho + \pi_t^\star$$

9.2 Short run

Dynamic Aggregate Supply Curve (DAS)

$$\pi_t = \pi_{t-1} + \phi(Y_t - \bar{Y}_t) + v_t$$

Dynamic Aggregate Supply Curve (DAD)

$$Y_t = \bar{Y}_t - \frac{\alpha\theta_\pi}{1+\alpha\theta_Y}(\pi_t - \pi_t^\star) + \frac{1}{1+\alpha\theta_Y}\varepsilon_t$$

References

[1] N. Mankiw. *Macroeconomics*. Macroeconomics. Worth Publishers, 2000.