

# **Probabilistic Robotics Course**

## **EKF SLAM with unknown Data Association**

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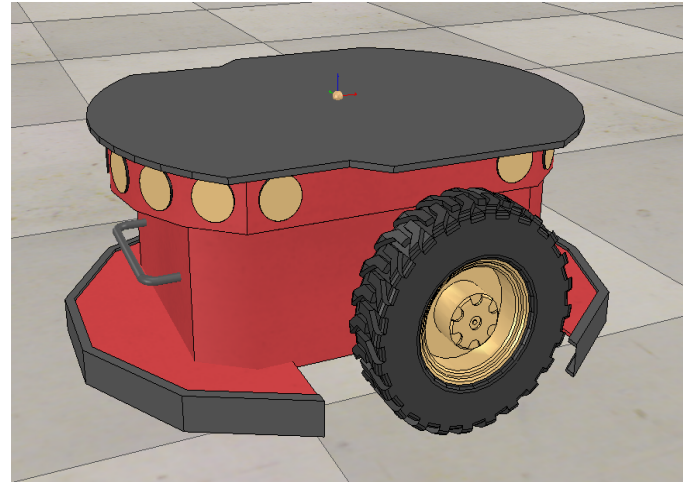
# Outline

- Scenario
- Controls
- Observations
- Non-Linear Systems and Gaussian Noise
- Data Association

# Scenario

Orazio moves on a 2D plane

- Is controlled by translational and rotational velocities
- Senses a set of **not distinguishable** landmarks through a “2D landmark sensors”
- The location of the landmarks in the world is **not known**



# Approaching the problem

We want to develop a KF based algorithm to track the position of Orazio as it moves (localization) and, at the same time, the position of the observed plant-landmarks (mapping) while performing data association.

The inputs of our algorithms will be

- velocity measurements
- landmark measurements

**We have no prior knowledge of the map.**

# Domains

Define

- state space

$$\mathbf{x}_t^r = \begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} \in \mathbb{R}^3$$

poses mapped to  
3D vectors

landmarks  
in the state

$$\mathbf{x}_t^{[n]} = \begin{pmatrix} x_t^{[n]} \\ y_t^{[n]} \end{pmatrix} \in \mathbb{R}^2$$

$n=1..N$

- space of controls (inputs)

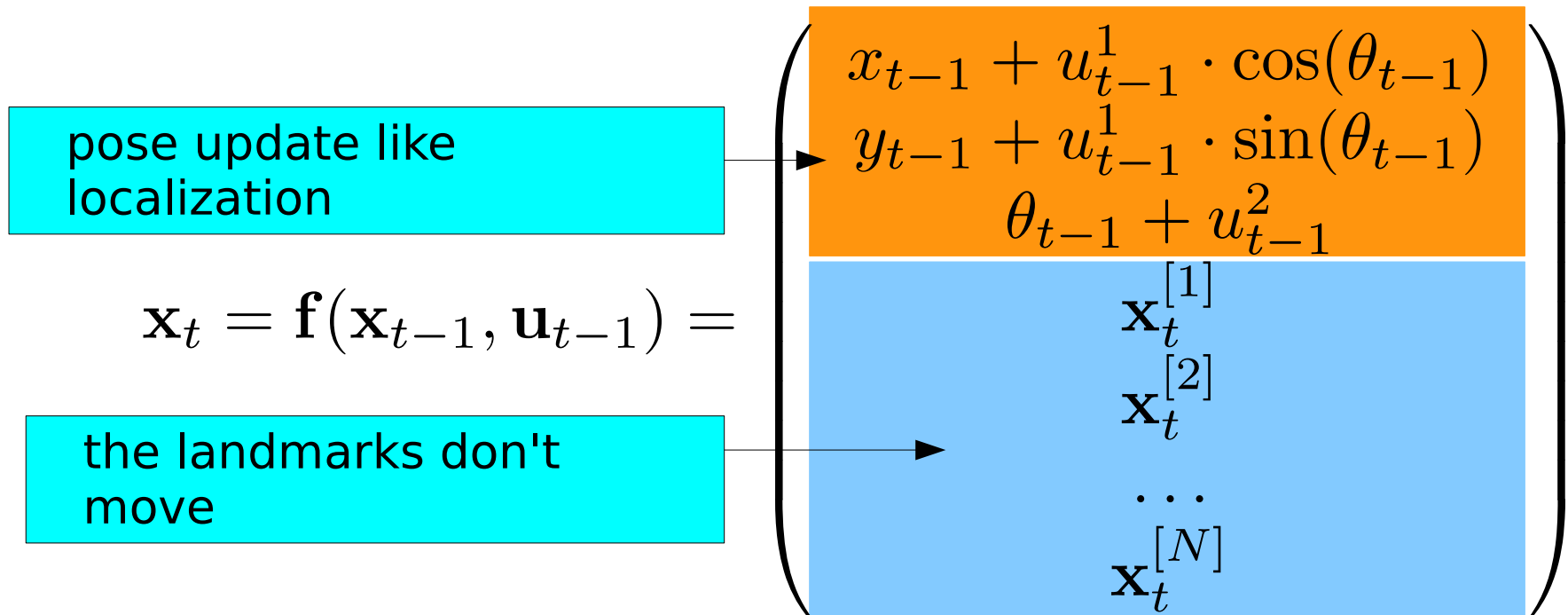
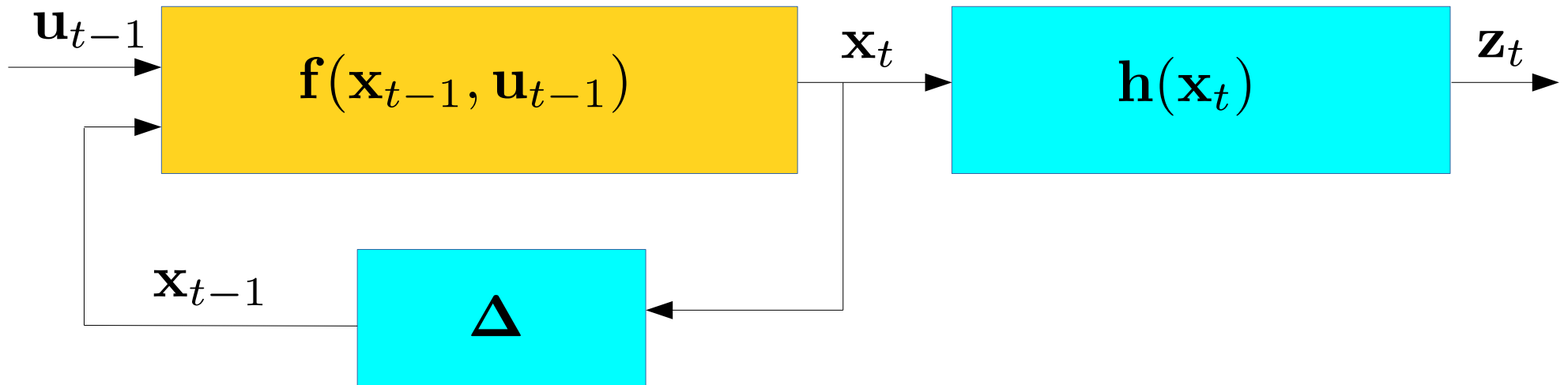
$$\mathbf{u}_t = \begin{pmatrix} u_t^1 \\ u_t^2 \end{pmatrix} \in \mathbb{R}^2$$

- space of observations (measurements)

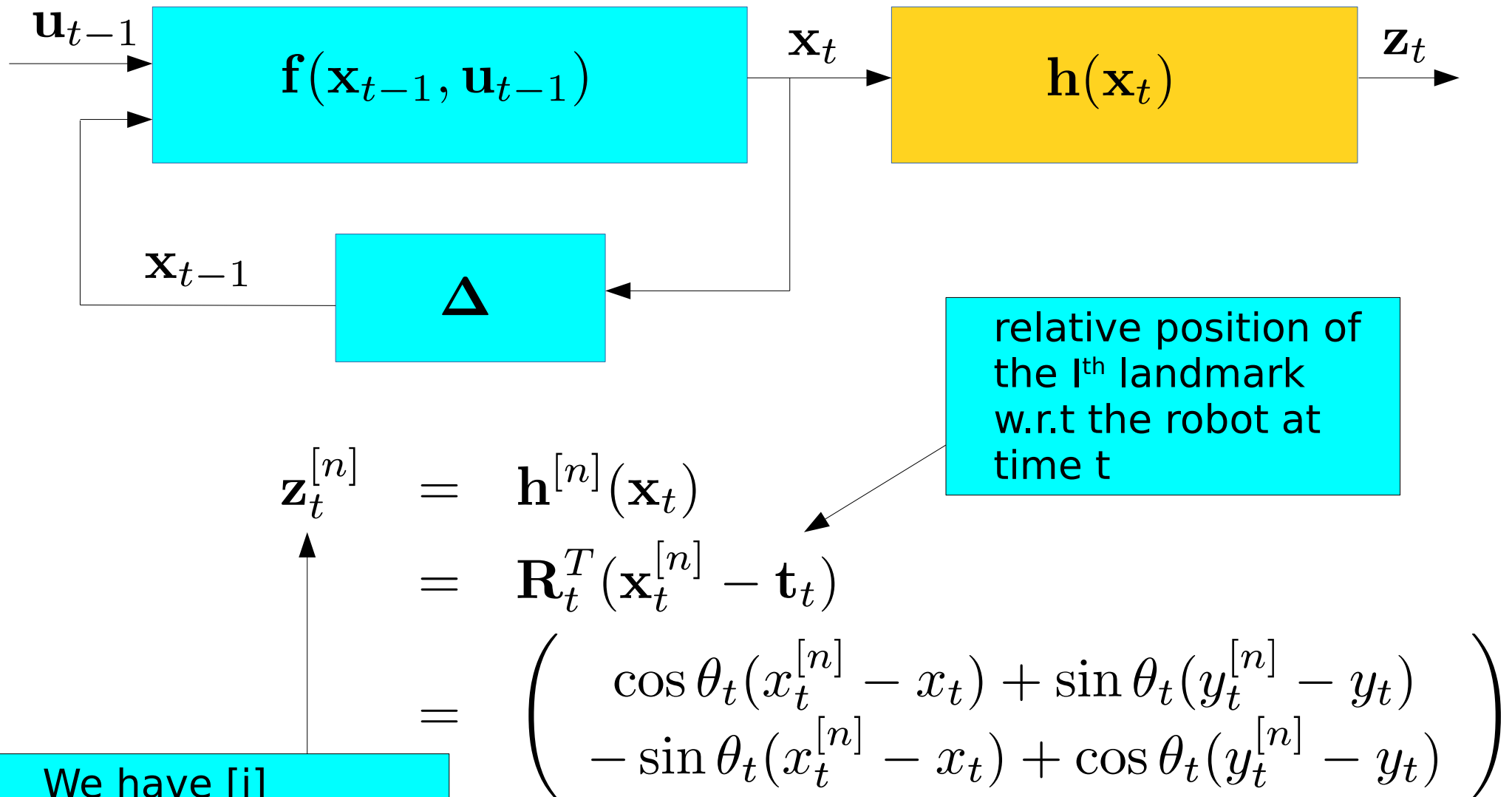
$$\mathbf{z}_t^{[m]} = \begin{pmatrix} x_t^{[m]} \\ y_t^{[m]} \end{pmatrix} \in \mathbb{R}^2$$

$m=1..M$

# Transition Function



# Measurement Function



We have  $[i]$  measurement functions, one per landmark

# Control Noise

We assume the velocity measurements are effected by a Gaussian noise resulting from the sum of two aspects

- a constant noise
- a velocity dependent term whose standard deviation grows with the speed
- translational and rotational noise are assumed independent

$$\mathbf{n}_{u,t} \sim \mathcal{N} \left( \mathbf{n}_{u,t}; \mathbf{0}, \begin{pmatrix} (u_t^1)^2 + \sigma_v^2 & 0 \\ 0 & (u_t^2)^2 + \sigma_\omega^2 \end{pmatrix} \right)$$



# Measurement Noise

We assume it is zero mean, constant

$$\mathbf{n}_z \sim \mathcal{N} \left( \mathbf{n}_z; \mathbf{0}, \begin{pmatrix} \sigma_z^2 & 0 \\ 0 & \sigma_z^2 \end{pmatrix} \right)$$

# Jacobian 1

$$\mathbf{f}(\mathbf{x}_{t-1}, \mathbf{u}_{t-1}) = \begin{pmatrix} x_{t-1} + u_{t-1}^1 \cdot \cos(\theta_{t-1}) \\ y_{t-1} + u_{t-1}^1 \cdot \sin(\theta_{t-1}) \\ \theta_{t-1} + u_{t-1}^2 \\ \mathbf{x}_t^{[1]} \\ \mathbf{x}_t^{[2]} \\ \dots \\ \mathbf{x}_t^{[N]} \end{pmatrix}$$

Our usual Jacobians:

$$\mathbf{A}_t = \frac{\partial \mathbf{f}(\cdot)}{\partial \mathbf{x}} = \begin{pmatrix} \frac{\partial \mathbf{f}(\cdot)}{\partial \mathbf{x}^r} & \frac{\partial \mathbf{f}(\cdot)}{\partial \mathbf{x}^{[1]}} & \frac{\partial \mathbf{f}(\cdot)}{\partial \mathbf{x}^{[2]}} & \dots & \frac{\partial \mathbf{f}(\cdot)}{\partial \mathbf{x}^N} \end{pmatrix}$$

$$\mathbf{B}_t = \frac{\partial \mathbf{f}(\cdot)}{\partial \mathbf{u}}$$

# Jacobian 2

Our landmark sensor perceives points, thus our measurement function will be:

$$\mathbf{h}^{[n]}(\mathbf{x}_t) = \mathbf{R}_t^T (\mathbf{x}_t^{[n]} - \mathbf{t}_t)$$

Consequently, the Jacobian can be computed as:

$$\mathbf{C}_t^{[n]} = \frac{\partial \mathbf{h}^{[n]}(\cdot)}{\partial \mathbf{x}_t} = \left( \begin{array}{ccc} \boxed{\frac{\partial \mathbf{h}^{[n]}}{\partial \mathbf{x}_t^r}} & \mathbf{0} & \dots & \boxed{\frac{\partial \mathbf{h}^{[n]}}{\partial \mathbf{x}_t^{[n]}}} & \dots & \mathbf{0} \end{array} \right)$$

pose block

↑

landmark block

↑

# Data Association

We do **not** observe the landmark ids.

When a new landmark appears, it's our duty to assign a unique id.

For convenience, we can keep unchanged the state-id mapping structure seen in the previous lesson

$$\begin{aligned} \text{id\_to\_state\_map} &= \begin{pmatrix} -1 & -1 & \dots & \dots & -1 \end{pmatrix} \\ \text{state\_to\_id\_map} &= \begin{pmatrix} -1 & -1 & \dots & \dots & -1 \end{pmatrix} \end{aligned}$$

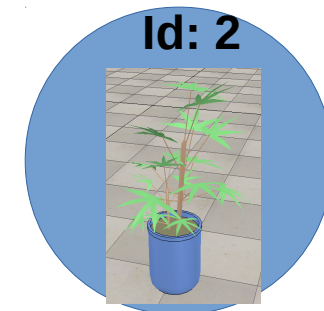
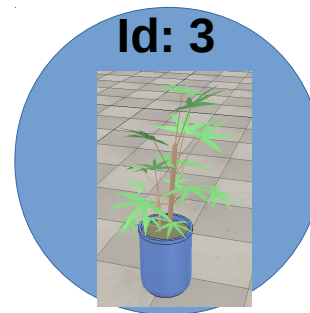
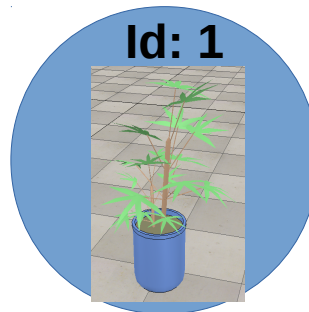
# Data Association

Time: t

current mean

$$\mu_t = \begin{pmatrix} x_t^r \\ y_t^r \\ \theta_t^r \\ l_t^{[1]} \\ l_t^{[2]} \\ l_t^{[3]} \end{pmatrix}$$

The first time an unmatched landmark is seen, results in the creation and assignment of a new id

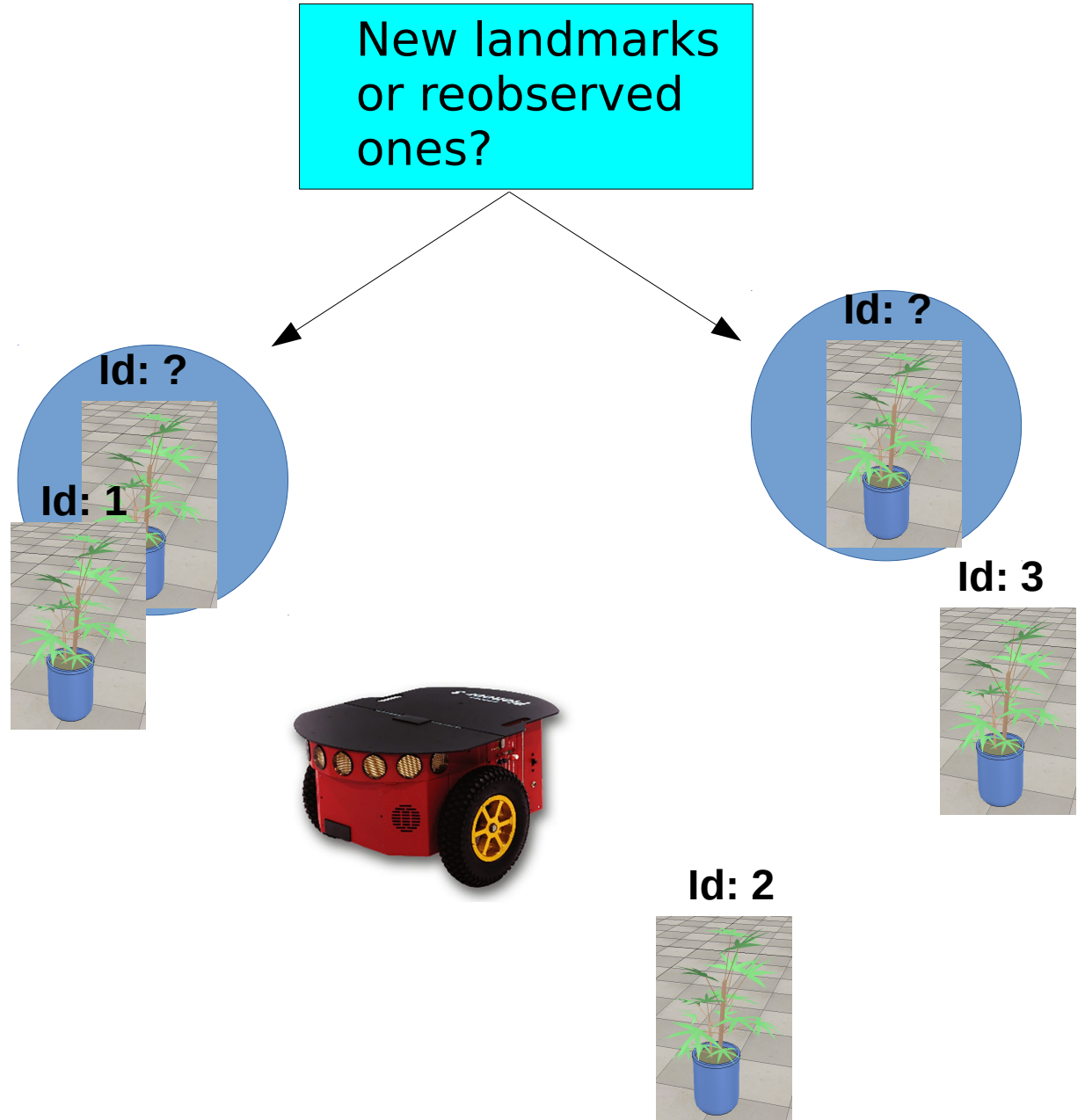


# Data Association

Time:  $t+1$

current mean

$$\mu_{t+1} = \begin{pmatrix} x_{t+1}^r \\ y_{t+1}^r \\ \theta_{t+1}^r \\ l_{t+1}^{[1]} \\ l_{t+1}^{[2]} \\ l_{t+1}^{[3]} \\ ?? \end{pmatrix}$$



# Data Association

At each time step, precompute the likelihood for each landmark/measurement pair:

$$a_{mn} = (\mathbf{z}^{[m]} - \mu_z^n)^T \Sigma_{m,m}^{-1} (\mathbf{z}^{[m]} - \mu_z^n)$$

and assemble them in a cost matrix

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots a_{1N} \\ a_{21} & a_{22} & a_{23} & \cdots a_{2N} \\ \vdots & & & \\ a_{M1} & a_{M2} & a_{M3} & \cdots a_{MN} \end{pmatrix}$$

# Gating

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1N} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2N} \\ \vdots & & & & \\ a_{M1} & a_{M2} & a_{M3} & \cdots & a_{MN} \end{pmatrix}$$

- Choose a threshold  $\tau_{accept}$
- Extract the minimum for each row  $a_{mn}$
- If  $a_{mn} < \tau_{accept}$ 
  - *then* observation  $m$  is associated with landmark  $n$
  - *otherwise*,  $m$  is a new landmark.

Multiple measurements can be assigned to the same landmark



# Best Friends

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1N} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2N} \\ \vdots & & & & \\ a_{M1} & a_{M2} & a_{M3} & \cdots & a_{MN} \end{pmatrix}$$

- Take all the accepted associations from gating and check
- If  $a_{mn} = \min_m a_{mn}$  AND  $a_{mn} = \min_n a_{mn}$ 
  - *then* keep the association
  - *otherwise* discard it

# Lonely Best Friends

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1N} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2N} \\ \vdots & & & & \\ a_{M1} & a_{M2} & a_{M3} & \cdots & a_{MN} \end{pmatrix}$$

- Define a smaller threshold  $\gamma$  and, for all surviving associations, extract the second best association for measurements  $\hat{a}_n$  and landmarks  $\hat{a}_m$  and check
- If  $a_{mn} - \hat{a}_m > \gamma$  AND  $a_{mn} - \hat{a}_n > \gamma$ 
  - *then* keep the association
  - *otherwise* discard it

**Hands On!**