Probabilistic Robotics Course

Particle Localization

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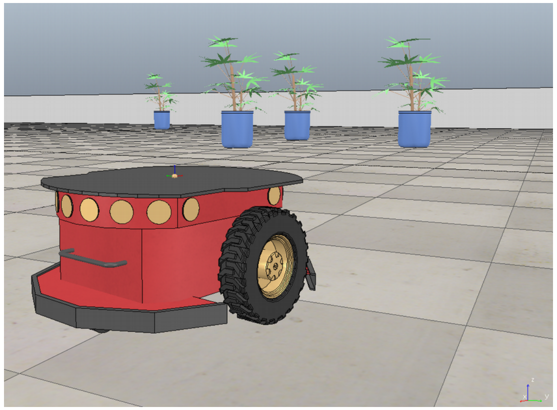
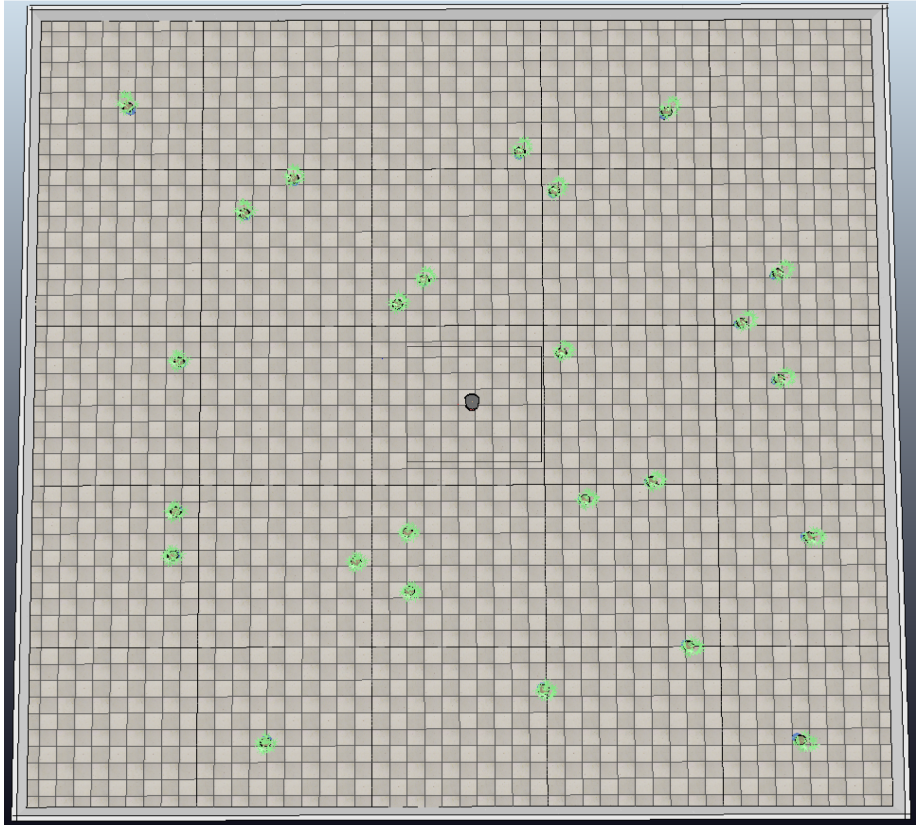
Scenario

Orazio moves on a 2D plane

- Is controlled by translational and rotational velocities
- Senses a set of uniquely distinguishable landmarks through a "2D landmark sensors"
- The location of the landmarks in the world is known





Approaching the problem

We want to develop a Particle Filter based algorithm to track the position of Orazio as it moves

The inputs of our algorithms will be

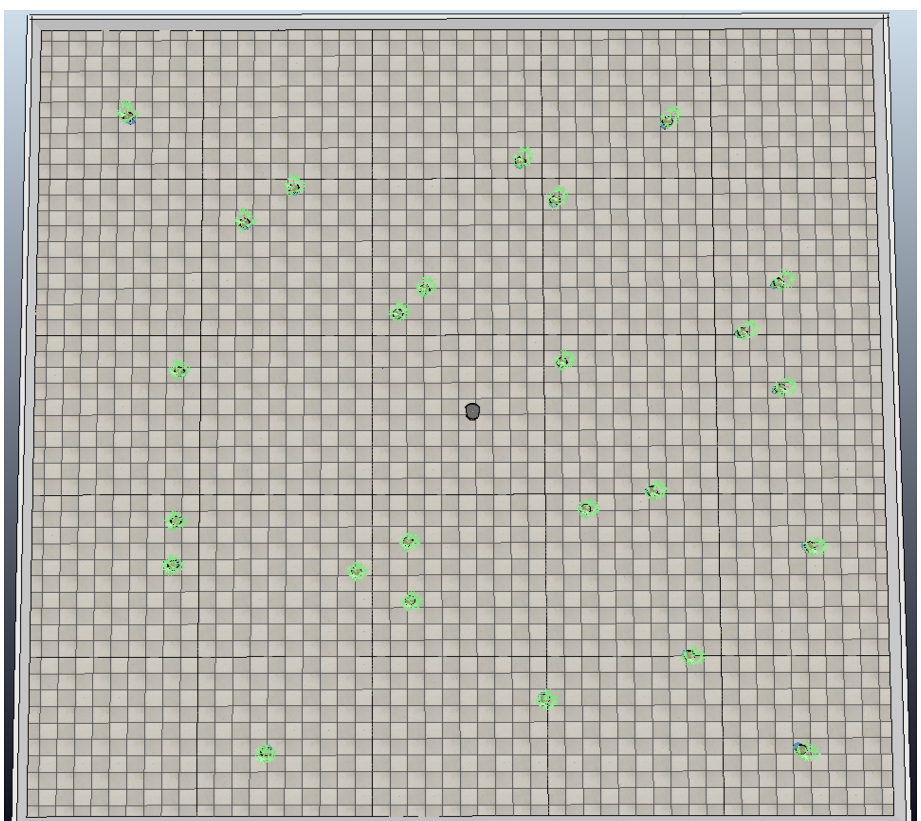
- velocity measurements
- landmark measurements

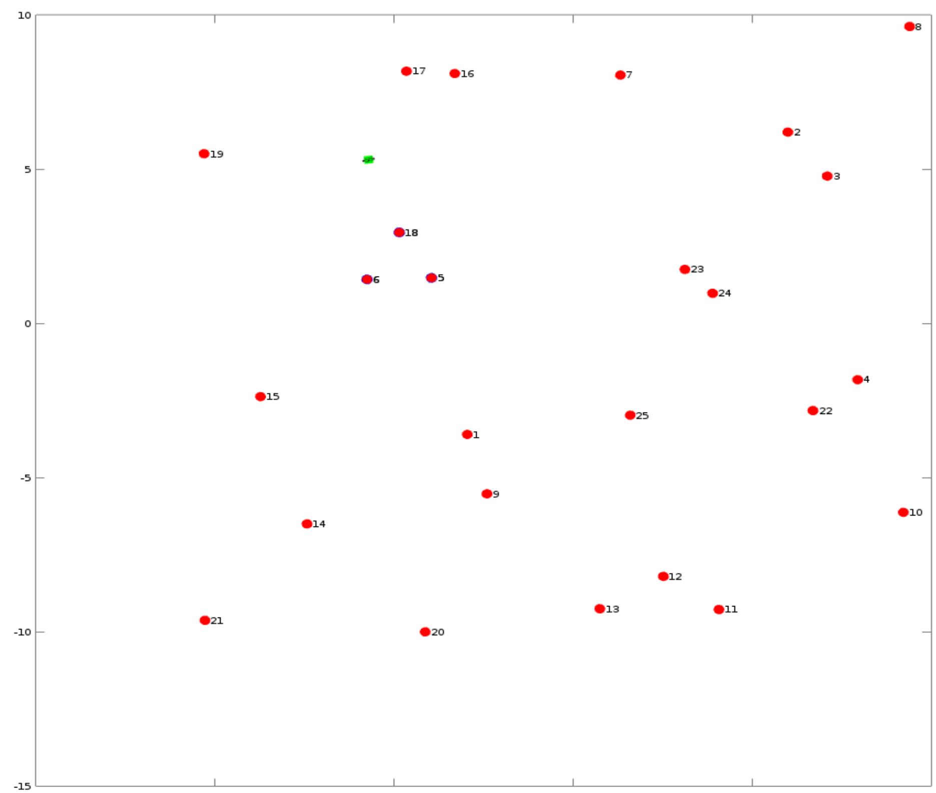
The prior knowledge about the map is represented by the location of each landmark in the world

Prior

The map is represented as a set of landmark coordinates

$$\mathbf{l}^{[i]} = \begin{pmatrix} x^{[i]} \\ y^{[i]} \end{pmatrix} \in \Re^2$$





Domains

Define

- state space

$$\mathbf{X}_t = [\mathbf{R}_t | \mathbf{t}_t] \in SE(2)$$

Instead of considering rotational and translational velocities, we consider the integrated motion in the interval as input

- space of controls (inputs)

$$\mathbf{u}_t = \begin{pmatrix} \Delta_t v_t \\ \Delta_t \omega_t \end{pmatrix} = \begin{pmatrix} u_t^1 \\ u_t^2 \end{pmatrix} \in \Re^2$$

- space of observations (measurements)

$$\mathbf{z}_t^{[i]} = \begin{pmatrix} x_t^{[i]} \\ y_t^{[i]} \end{pmatrix} \in \Re^2$$

Domains

Find an Euclidean parameterization of non-Euclidean spaces

$$\mathbf{X}_t = [\mathbf{R}_t | \mathbf{t}_t] \in SE(2) \quad \Rightarrow \quad \mathbf{x}_t = \begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} \in \Re^3$$

- state space

poses are not Euclidean, we map them to 3D vectors

$$\mathbf{u}_t = \begin{pmatrix} u_t^1 \\ u_t^2 \end{pmatrix} \in \Re^2$$

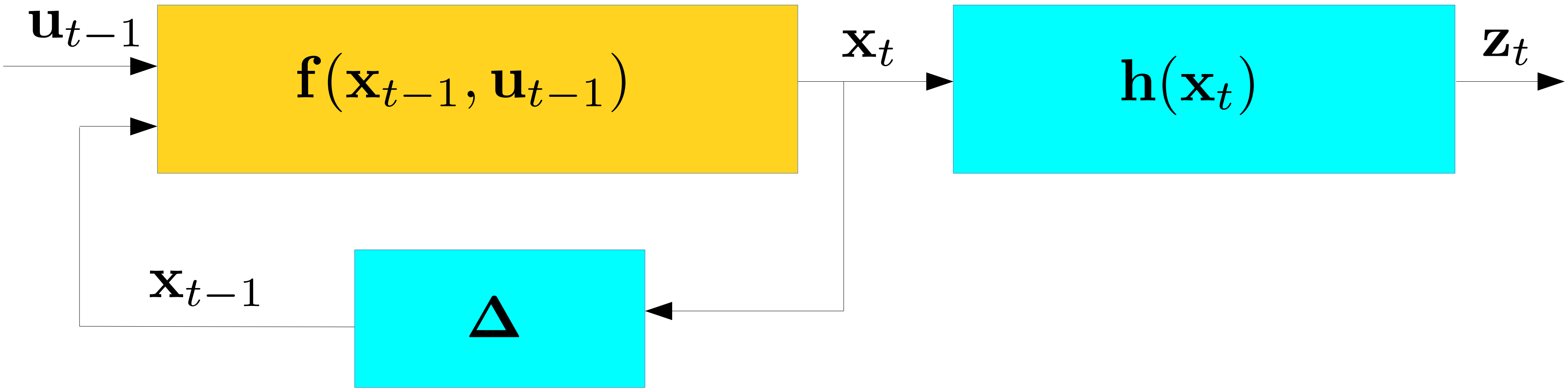
- space of controls (inputs)

$$\mathbf{z}_t = \begin{pmatrix} x_t^{[i]} \\ y_t^{[i]} \end{pmatrix} \in \Re^2$$

measurement and control, in this problem are already Euclidean

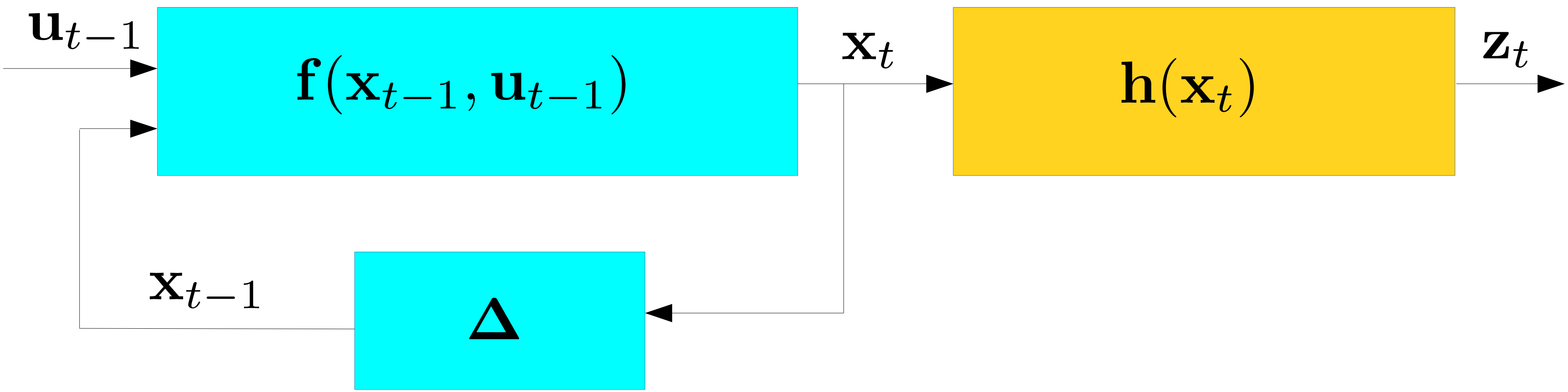
- space of observations (measurements)

Transition Function



$$\mathbf{x}_t = \mathbf{f}(\mathbf{x}_{t-1}, \mathbf{u}_{t-1}) = \left(\begin{array}{c} x_{t-1} + u^1_{t-1} \cdot \cos(\theta_{t-1}) \\ y_{t-1} + u^1_{t-1} \cdot \sin(\theta_{t-1}) \\ \theta_{t-1} + u^2_{t-1} \end{array} \right)$$

Measurement Function



We have [i] measurement functions, one per landmark

$$
\begin{aligned}
\mathbf{z}_t^{[i]} &= \mathbf{h}^{[i]}(\mathbf{x}_t) \\
&= \mathbf{R}_t^T(\mathbf{l}^{[i]} - \mathbf{t}_t) \\
&= \begin{pmatrix} \cos\theta_t(x^{[i]} - x_t) + \sin\theta_t(y^{[i]} - y_t) \\ -\sin\theta_t(x^{[i]} - x_t) + \cos\theta_t(y^{[i]} - y_t) \end{pmatrix}
\end{aligned}
$$

relative position of the Ith landmark w.r.t the robot at time t

$$
\mathbf{R}_t = \begin{pmatrix} \cos\theta_t & -\sin\theta_t \\ \sin\theta_t & \cos\theta_t \end{pmatrix}
$$

rotation matrix of theta

Measurement Function

At each point in time, our robot will sense only a subset of *K* landmarks in the map

The measurement is thus consisting of a stack of measurements

$$\mathbf{z}_t = \begin{pmatrix} \mathbf{z}^{[i_1]} \\ \mathbf{z}^{[i_2]} \\ \dots \\ \mathbf{z}^{[i_K]} \end{pmatrix} = \mathbf{h}(\mathbf{x}_t) = \begin{pmatrix} \mathbf{h}^{[i_1]}(\mathbf{x}_t) \\ \mathbf{h}^{[i_2]}(\mathbf{x}_t) \\ \dots \\ \mathbf{h}^{[i_K]}(\mathbf{x}_t) \end{pmatrix}$$

index of the landmark
generating the measurement

Control Noise

We assume the velocity measurements are effected by a uniform noise resulting from the sum of two aspects

- a constant noise
- a velocity dependent term whose amplitude grows with the speed
- translational and rotational noise are assumed independent

$$\mathbf{n}_{u,t} \sim \begin{pmatrix} a_1 ||u_t^1|| U(-0.5, 0.5) \\ a_2 ||u_t^2|| U(-0.5, 0.5) \end{pmatrix}$$

Measurement Noise and Observation model

We assume it is zero mean, Gaussian and constant

$$\mathbf{n}_z \sim \mathcal{N}\left(\mathbf{n}_z; \mathbf{0}, \begin{pmatrix} \sigma_z^2 & 0 \\ 0 & \sigma_z^2 \end{pmatrix}\right)$$

With this noise the observation model becomes

$$p(\mathbf{z}_t^{[i]} \mid \mathbf{x}_t) \propto \exp\left(-(\mathbf{h}^{[i]}(\mathbf{x}_t) - \mathbf{l}^{[i]})^T \Sigma_z^{-1} (\mathbf{h}^{[i]}(\mathbf{x}_t) - \mathbf{l}^{[i]})\right)$$

$$p(\mathbf{z}_t \mid \mathbf{x}_t) = \prod_i p(\mathbf{z}_t^{[i]} \mid \mathbf{x}_t);$$

Predict

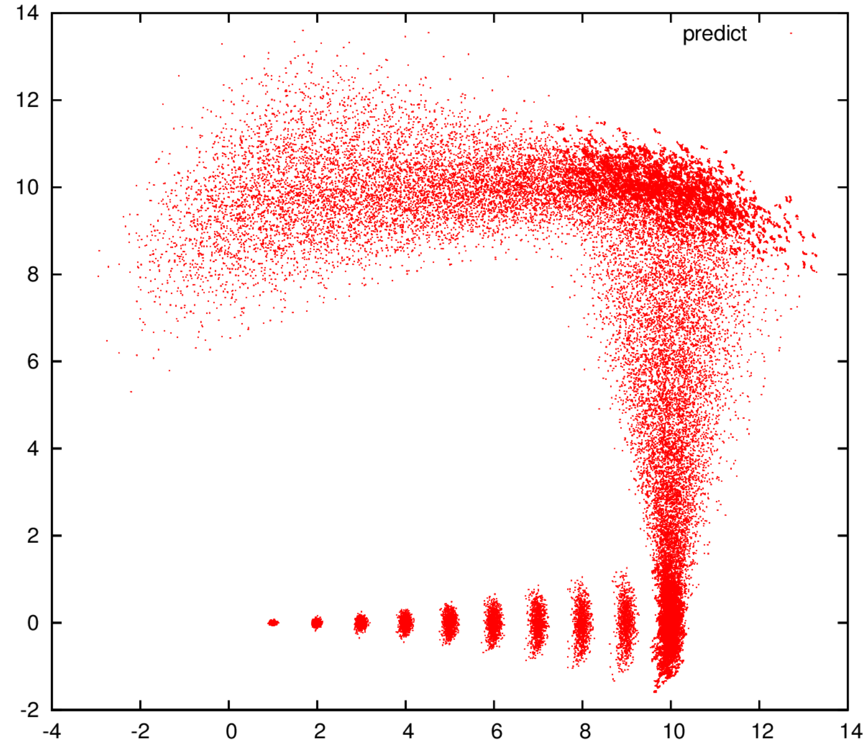
When a new control u_{t-1} becomes available, we compute the location by

- generating an independent noise sample for each particle
- computing the location of the new sample through the transition function, evaluated at the particle, at the control and at the noise sample

$$\mathbf{n}_u^{(i)} \sim p(\mathbf{n}_u)$$

$$\mathbf{x}_{t|t-1}^{(i)} = \mathbf{f}(\mathbf{x}_{t-1|t-1}^{(i)}, \mathbf{u}_{t-1} + \mathbf{n}_u^{(i)})$$

$$w_{t|t-1}^{(i)} = w_{t-1|t-1}^{(i)}$$



evolution of the particles when the robot moves on a square

Prediction: code

```
function samples = prediction(samples, transition)

 dim_particles = size(samples,2);

 u = transition.v;
 %it returns u = [ux, uy, utheta]. simply not consider uy
 u_x = u(1);
 u_theta = u(3);

 a1 = abs(u_x);
 a2 = abs(u_theta);
 %apply transition to every particle
 for i=1:dim_particles
  % sample noise from uniform between
  % [-0.5; 0.5]
  noise_x = (rand() - 0.5)*a1;
  noise_theta = (rand() - 0.5)*a2;

  samples(:,i) = %TODO;
 end
endfunction
```

Apply motion model to each particle

$$\mathbf{x}_{t|t-1}^{(i)} = \mathbf{f}(\mathbf{x}_{t-1|t-1}^{(i)}, \mathbf{u}_{t-1} + \mathbf{n}_u^{(i)})$$

Update

To compute the update we need to address the data association

We solve it in a greedy manner, for each particle

- given a particle, we evaluate the likelihood of all combinations landmarks/measurements

sample

$$\mathbf{x}_t^{(i)}$$

negative log likelihood of measurement m w.r.t landmark n seem by particle i

$$l_{mn}^{(i)} = (\mathbf{h}^{[m]}(\mathbf{x}^{(i)}) - \mathbf{z}^{[m]})^T \boldsymbol{\Sigma}_z^{-1} (\mathbf{h}^{[n]}(\mathbf{x}^{(i)}) - \mathbf{z}_t^{[m]})$$

$$l^{(i)[m]} = \min_m l_{mn}^{(i)}$$

negative log likelihood is the one of the “closest” landmark

$$p(\mathbf{z}_t \mid \mathbf{x}_t^{(i)}) \propto \exp\left(-k^{(i)} l_{\text{miss}} - \sum_m l_m^{(i)}\right)$$

Sample likelihood

number of dropped measurements

Update(1): code

```
function weights = update(samples, weights, landmarks, observations)

 % init useful stuff
 [...]

 sigma_z_noise = 10;
 Sigma_z_noise = [sigma_z_noise^2 0;
     0 sigma_z_noise^2];
 Omega_z_noise = inv(Sigma_z_noise);

 l_miss = 30; %this is a threshold

 for i=1:num_particles
  %init association matrix
  l_mn = zeros(num_landmarks_seen, num_landmarks);
  particle = samples(:,i); %current particle

  for n=1:num_landmarks
    landmark = landmarks(n); %current landmark
    land_x = landmark.x_pose;
    land_y = landmark.y_pose;
    [curr_h, _] = measurement_function(particle, [land_x; land_y]);

    for m=1:num_landmarks_seen
      %current measurement
      measurement = observations.observation(m);
      delta = %TODO;
      l_mn(m,n) = %TODO;
    endfor
  endfor
```

We have access to measurement.x_pose and .y_pose

Update

Being able to evaluate the likelihood of each predicted sample we can perform the conditioning

$$w_{t|t}^{(i)} = w_{t|t-1}^{(i)} p(\mathbf{z}_t \mid \mathbf{x}_{t|t-1}^{(i)})$$

$$\mathbf{x}_{t|t}^{(i)} = \mathbf{x}_{t|t-1}^{(i)}$$

After conditioning we NEED to resample

$$\{j^{(i)}\} = \text{uniformSample}(\{w_{t|t}^{(i)}\})$$

this will be the distribution after resampling

$$\boxed{\tilde{\mathbf{x}}_{t|t}^{(i)}} = \mathbf{x}_{t|t}^{(j(i))}$$

$$\boxed{\tilde{w}_{t|t}^{(i)}} = \frac{1}{\#\text{samples}}$$

Update(2): code

```
for i=1:num_particles
 %still in the particle cycle
  [...]
 %once the association matrix of
 %the current particle is filled

        negative_log_likelihood = min(l_mn');
        dropped_measurements = 0;
        for k=1:num_landmarks_seen
                if(negative_log_likelihood(k) > l_miss)
                        negative_log_likelihood(k) = 0;
                        dropped_measurements++;
                endif;
        endfor

        sum_of_negative_log_likelihood = sum(
    negative_log_likelihood);
        weights(i) *= %TODO;

endfor
endfunction
```

We can update the weights as:

$$p(\mathbf{z}_t \mid \mathbf{x}_t^{(i)}) \propto \exp\left(-k^{(i)} l_{\text{miss}} - \sum_m l_m^{(i)}\right)$$

$$w_{t|t}^{(i)} = w_{t|t-1}^{(i)} p(\mathbf{z}_t \mid \mathbf{x}_{t|t-1}^{(i)})$$

UniformSample: code

```
function sampled_indices = uniformSample(weights, num_desired_samples)

dim_weights = size(weights,1);
%normalize the weights (if not normalized)
normalizer = 1./sum(weights);
%resize the indices
step = 1./num_desired_samples;

y0 = rand()*step;   %sample between 0 and 1/num_desired_sample;
yi = y0;    %value of the sample in the y space
cumulative = 0;   %this is our running cumulative distribution

for weight_index=1:dim_weights
 cumulative += normalizer*weights(weight_index); %update cumulative
 % fill with current_weight_index
 % until the cumulative does not become larger than yi
 while(cumulative > yi)
  sampled_indices(end+1,1) = weight_index;
  yi += step;
 endwhile

endfor

endfunction
```

Resampling: code

```
function [new_samples, new_weights] = resample(samples, weights,
    dim_samples)

 indices = uniformSample(weights', dim_samples);

 new_samples = samples;
 new_weights = ones(1,dim_samples)/dim_samples;

 for i=1:dim_samples
  new_samples(:,i) = %TODO;
 endfor

endfunction
```

Call uniformSample

$$\{j^{(i)}\} = \mathrm{uniformSample}(\{w_{t|t}^{(i)}\})$$

$$\tilde{\mathbf{x}}_{t|t}^{(i)} = \mathbf{x}_{t|t}^{(j(i))}$$

Reset weights

$$\tilde{w}_{t|t}^{(i)} = \frac{1}{\#\mathrm{samples}}$$