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1.1 Applications

Given the family of curves $u(x, y) = c_1$, the family of curves orthogonal to these are the solution to:

$$\frac{\partial u}{\partial x}dy = \frac{\partial u}{\partial y}dx.$$

1.1.1 2nd-Order ODE

Definition 1.1. The general form of a 2nd order differential equation is:

$$y'' = F(x, y, y').$$

Where x is the independent variable and y is the dependent variable.

We want to consider a few special cases. The first one is when the dependent variable is missing, $y'' = f(x, y')$, for example $y'' = x - y'$. In this case, you can set $v = y'$ $v' = y''$, giving us:

$$v' = f(x, v)$$

which is a first order equation. Thus we can solve the first order ODE and then integrate to get y.

Example 1.2

Consider the earlier equation $y'' = x - y'$, we have:

$$v' = x - v \implies \frac{dv}{dx} + v = x$$

$$v = e^{-x}((x-1)e^{x} + c_1) = x - 1 + c_1e^{-x} = \frac{dy}{dx}.$$

$$y = \frac{1}{2}x^2 + x + c_2e^{-x} + c_3.$$

for some constants c_2 and c_3.

Remark 1.3 — Note that for a first order ODE, there should be one arbitrary constant, but for second order, there should be 2.

The second case is where the independent variable is missing, meaning:

$$\frac{d^2y}{dx^2} = F(y, \frac{dy}{dx}) \implies v\frac{dv}{dx} = F(y, v).$$

Where v is once again $\frac{dy}{dx}$.