1   March 18th, 2021

1.1   Preconditioned with Projection Methods

For one dimensional projection, we have steepest descent + preconditioning. Here we search for d_k on $\{d : \|d\|_A = \beta\}$. We obtain d_k by:

$$\min_{d_k} \|x_* - (x_k + \alpha d_k)\|_A^2 \quad \text{s.t } \|d_k\|_A = \beta$$

How ever, this is unrealistic because d_k would depend on x_*. Instead, we approximate it by searching d_k on $\{d : \|d\|_2 = \beta\}$. So, d_k is obtained by:

- Fixing $\alpha > 0$ and small

- Solving approximately (since we are consider the ellipsoid under $\|\cdot\|_2$):

$$\min_{d_k} \|x_* - (x_k + \alpha d_k)\|_2^2 \quad \text{s.t } \|d_k\|_2 = \beta$$

When the condition number of A is big, the set $\{d : \|d\|_2 = \beta\}$ is very flat in A-inner product space. To improve it, we search for d on $\S_p = \{d : \|d\|_p = \beta\}$, where p is such that S_p is rounder than S_2.