1 April 26th, 2019

1.1 SVD

Let $A \in \mathbb{R}^{m \times n}$ (with $m \geq= n$). Following from the previous lecture we have that the eigenvalues of AA^T are:

$$\lambda_1, \lambda_2, \ldots, \lambda_n, 0, 0, \ldots, 0$$

with corresponding eigenvectors:

$$u_1, u_2, \ldots, u_n, u_{n+1}, \ldots u_m.$$

Them:

$$AA^T = \begin{bmatrix} u_1 & u_2 & \ldots & u_n & u_{n+1} & \ldots \end{bmatrix} \begin{bmatrix} \lambda_1 \end{bmatrix} = U \Lambda U^T.$$

$$\begin{gathered}
(AA^T)(Av_1) = A(A^T A v_i) = A(\lambda_i v_i) = \lambda_i (A v_i) \\
\implies Av_i \text{ is an eigenvector of } AA^T \text{ with eigenvalue } \lambda_i \\
\implies Av_i = \sigma_i u_i, \quad i = 1, 2, \ldots, n \qquad (a). \\
(A^T A)(A^T v_1) = A^T (AA^T v_i) = A^T (\lambda_i v_i) = \lambda_i (A^T v_i) \\
\implies A^T v_i \text{ is an eigenvector of } A^T A \text{ with eigenvalue } \lambda_i \\
\implies A^T v_i = \tilde{\sigma}_i u_i, \quad i = 1, 2, \ldots, n \qquad (a).
\end{gathered}$$