

# 1 January 31st, 2020

## 1.1 Problem 1

Find period of motion for the equation:

$$\dot{\theta} = \sqrt{\frac{g}{L}(3 + 2 \cos \theta)} \quad 0 \leq \theta \leq 2\pi.$$

Since the RHS only has  $\theta$ , this is separable, thus:

$$\int dt = \sqrt{\frac{L}{g}} \int \frac{d\theta}{\sqrt{3 + 2 \cos(\theta)}}$$

Note that the RHS gives us an elliptical equation. Since we want the period, we have:

$$T = \sqrt{\frac{L}{g}} \int_0^{2\pi} \frac{d\theta}{3 + 2 \cos \theta} + C.$$

We can consider  $C$  to be the start time, and just set it to 0. This is as far as you can go analytically, so plug it into a calculator.

### 1.1.1 How to use in MATLAB

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```
T = integral(@(theta)1./sqrCos(1,theta),2,2*pi)
tspan = [0 2.5];
y0 = 0;
data = ode45(@sqrCos,tspan,y0);

function res = sqrCos(t,theta)
    L = 2,4;
    g = 9,8;
    res = sqrt(g/L*(3+2*cos(theta)));
end(function)
```

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## 1.2 Problem 3

Consider the equation

$$v \frac{dv}{dx} + \frac{v^2}{x + \frac{m}{\rho}} = g.$$

With the initial condition:  $v_0 = v(x_0) = v(0) = 0$ . To solve for  $v(x)$ , note that this is a Bernoulli equation:

$$\frac{dv}{dx} + \frac{1}{x + \frac{m}{\rho}} v = g^{v-1}.$$

with:

$$p(x) = \frac{1}{x + \frac{m}{\rho}} \quad Q(x) = g \quad n = -1.$$

Plugging into the formula, we have:

$$V(x) = \left( \frac{1}{\mu(x)} \left( \int (1-n)\mu(x)Q(x)dx + C \right) \right)^{\frac{1}{1-n}}.$$

Calculating the integrating factor, we have:

$$\mu(x) = e^{\int (1-n)P(x)dx} = e^{2\ln(x+\frac{m}{\rho})} = \left(x + \frac{m}{\rho}\right)^2.$$

Thus we have:

$$V(x) = \left( \frac{1}{\left(x + \frac{m}{\rho}\right)^2} \left( 2 \int \left(x + \frac{m}{\rho}\right)^2 g dx + C \right) \right)^{\frac{1}{2}}$$

$$V(x) = \left( \frac{1}{\left(x + \frac{m}{\rho}\right)^2} \left( \frac{2}{3} \left(x + \frac{m}{\rho}\right)^3 + C \right) \right)^{\frac{1}{2}} = \frac{1}{x + \frac{m}{\rho}} \sqrt{\frac{2}{3} \left(x + \frac{m}{\rho}\right)^3 g + C}.$$

Plugging in the initial condition, we get:  $C = -\frac{2}{3} \frac{m^3}{\rho^3} g$ , giving us:

$$v(x) = \frac{1}{x + \frac{m}{\rho}} \sqrt{\frac{2}{3} \left(x + \frac{m}{\rho}\right)^3 g - \frac{2}{3} \left(\frac{m}{\rho}\right)^3 g}.$$

The acceleration is:

$$g - \frac{v^2}{x + \frac{m}{\rho}}.$$