1 January 31st, 2020

1.1 Problem 1

Find period of motion for the equation:

$$\dot{\theta} = \sqrt{\frac{g}{L}(3 + 2\cos\theta)} \quad 0 \le \theta \le 2\pi.$$

Since the RHS only has θ, this is separable, thus:

$$\int dt = \sqrt{\frac{L}{g}} \int \frac{d\theta}{\sqrt{3 + 2\cos(\theta)}}$$

Note that the RHS gives us an elliptical equation. Since we want the period, we have:

$$T = \sqrt{\frac{L}{g}} \int_0^{2\pi} \frac{d\theta}{3 + 2\cos\theta} + C.$$

We can consider C to be the start time, and just set it to 0. This is as far as you can go analytically, so plug it into a calculator.

1.1.1 How to use in MATLAB

```
T = integral(@(theta)1./sqrCos(1,theta),2,2*pi)
tspan = [0 2.5];
y0 = 0;
data = ode45(@sqrCos,tspan,y0);

function res = sqrCos(t,theta)
   L = 2,4;
   g = 9,8;
   res = sqrt(g/L*(3+2*cos(theta)));
end(function)
```

1.2 Problem 3

Consider the equation

$$v\frac{dv}{dx} + \frac{v^2}{x + \frac{m}{\rho}} = g.$$

With the initial condition: $v_0 = v(x_0) = v(0) = 0$. To solve for $v(x)$, note that this is a Bernoulli equation:

$$\frac{dv}{dx} + \frac{1}{x + \frac{m}{\rho}} v = g^{v-1}.$$

with:

$$p(x) = \frac{1}{x + \frac{m}{\rho}} \quad Q(x) = g \quad n = -1.$$

Plugging into the formula, we have:

$$V(x) = \left(\frac{1}{\mu(x)}\left(\int (1-n)\mu(x)Q(x)dx + C\right)\right)^{\frac{1}{1-n}}.$$

Calculating the integrating factor, we have:

$$\mu(x) = e^{\int (1-n)P(x)dx} = e^{2\ln(x+\frac{m}{\rho})} = \left(x + \frac{m}{\rho}\right)^2.$$

Thus we have:

$$V(x) = \left(\frac{1}{\left(x+\frac{m}{\rho}\right)^2}\left(2\int \left(x+\frac{m}{\rho}\right)^2 g\ dx + C\right)\right)^{\frac{1}{2}}$$

$$V(x) = \left(\frac{1}{\left(x+\frac{m}{\rho}\right)^2}\left(\frac{2}{3}\left(x+\frac{m}{\rho}\right)^3 + C\right)\right)^{\frac{1}{2}} = \frac{1}{x+\frac{m}{\rho}}\sqrt{\frac{2}{3}\left(x+\frac{m}{\rho}\right)^3 g + C}.$$

Plugging in the initial condition, we get: $C = -\frac{2}{3}\frac{m^3}{\rho^3}g$, giving us:

$$v(x) = \frac{1}{x+\frac{m}{\rho}}\sqrt{\frac{2}{3}\left(x+\frac{m}{\rho}\right)^3 g - \frac{2}{3}\left(\frac{m}{\rho}\right)^3 g}.$$

The acceleration is:

$$g - \frac{v^2}{x+\frac{m}{\rho}}.$$