1 April 1st, 2020

1.1 Dot Products

Recall that we defined the generic dot product between functions to be:

$$f \cdot g = \int_{\alpha}^{\beta} f(x)g(x)w(x)\ dx.$$

With $w(x)$ being a weight function. Recall that:

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta, \quad \cos\theta = \frac{\vec{A} \cdot \vec{B}}{\sqrt{\vec{A} \cdot \vec{A}}\sqrt{\vec{B} \cdot \vec{B}}}.$$

For

$$f \cdot g = \int_{\alpha}^{\beta} f(x)g(x)w(x)\ dx.$$

we have:

$$|f| = \sqrt{f \cdot f}.$$

$$\cos\theta = \frac{f \cdot g}{\sqrt{f \cdot f}\sqrt{g \cdot g}} = \frac{f \cdot g}{|f||g|}.$$

Note that f is perpendicular to g is $f \cdot g = 0$.

1.2 Recall Bessel's Equation

Consider

$$x^2 y''(x) + (a + 2b^R)xy'(x) + (c + dx^{2s} - b(1 - a - R)x^R + b^2 x^{2R})y(x) = 0.$$

Example 1.1 (Example where Bessel's equation does not work)

$$y''(x) + y'(x) + x^3 y(x) = 0.$$

Multiplying by x^2, we have:

$$x^2 y''(x) + y'(x) + x^5(x) = 0.$$

This means that $a + 2bx^R = x$, meaning that:

$$a = 0, b = \frac{1}{2}, R = 1.$$

$$c + dx^{2s} - b(1 - a - R)x^R + b^2 x^{2R} = x^5.$$

$$c + dx^{2s} + \frac{1}{4}x^2 = x^5.$$

$$c + dx^{2s} = x^{5 - \frac{1}{4}x^2}.$$

Which does not work because c and d are constants.

Example 1.2

$$y''(x) + x^5 y(x) = 0.$$

Multiplying by x^2, we have:

$$x^2 y''(x) + x^5 y(x) = 0.$$

$$\implies a + 2bx^R = 0 \implies a = 0, b = 0.$$

$$c + dx^{2s} - b(1 - a - R)x^R + b^2 x^{2R} = x^5 \implies c = 0, d = 1 > 0, s = \frac{5}{2}.$$

$$p = \left|\frac{1}{s}\right|.$$