Guochun Shen, Mingjian Yu, Xin-Sheng Hu, Xiangcheng Mi, Haibao Ren, I-Fang Sun, and Keping Ma . 2009 . Species–area relationships explained by the joint effects of dispersal limitation and habitat heterogeneity. *Ecology* 90:3033–3041.

Appendix A. Main algorithms for the four spatial point models and the steps for estimating parameters used in addressing the joint effects of habitat heterogeneity and dispersal limitation on the SARs. Calculations of AIC are also detailed.

The algorithm of each model used in the present paper is described in detail in the book of Møller and Waagepetersen (2003). The steps for parameter estimation in each model are also explicitly illustrated in Waagepetersen and Guan (2007). For convenience, we summarized the main framework of the four models used in our study and the main steps of parameter estimation. For more details, please refer to the two literatures mentioned above.

POISSON PROCESS

A Poisson process X, defined in a two dimensional region $S \subset \Re^2$, with intensity measure μ and intensity function ρ, satisfies for any bounded subregion $B \subseteq S$ with $\mu(B) > 0$. Meanwhile, $N(B)$ is a Poisson variable distributed with mean $\mu(B)$. Conditional on $N(B)$, the points in $\mathbf{X}_B$ are i.i.d. with density proportional to $\rho(u)$, which has the form of

$$\rho(u) = \alpha \exp(z_{1:k}(u)\beta_{1:k}^T) \tag{A.1}$$

where $u \in B$ and $\alpha > 0$, $z_{1:k}(u)$ denotes the $1 \times k$ vector of nonconstant environmental variables; $\beta_{1:k}$ is a corresponding regression parameter.

If $\rho(u)$ is an constant λ for all $u \in S$, the Poisson process is homogeneous or stationary. This is a model for 'no interaction' and 'complete spatial randomness process. If $\rho(u)$ is not a constant, but a function of environmental variables $z_{1:k}(u)$ on location $u \in S$, we say that

this is an inhomogeneous Poisson process. It implies that there is no interaction between points, but the intensity could vary according to environmental factors.

THOMAS PROCESS

Since the independence properties of Poisson process are usually not realistic for real data, we choose two kinds of Cox processes to model the aggregation patterns. The Thomas point process X is a superposition of clusters X_c of offspring associated with mother points c in a stationary Poisson point process of intensity κ. Given c, the clusters X_c are independent Poisson processes with intensity functions

$$\rho_c(u) = \exp(z_{1:k}(u)\beta_{1:k}^T)\alpha k(u - c; \delta)$$

where $\alpha > 0$, $k(u - c; \delta)$ is a probability density depending on a parameter $\delta > 0$ determining the spread of offspring points around c. $\exp(z_{1:k}(u)\beta_{1:k}^T)$ still represents covariance between event density and environment at point u. X is a homogeneous Thomas process when $\exp(z_{1:k}(u)\beta_{1:k}^T) = 1$. Otherwise, X is a heterogeneous Thomas process

Assume that $\exp(z_{1:k}(u)\beta_{1:k}^T)$ is bounded by some constant M, A cluster X_c may then be regarded as an independent thinning of a cluster Y_c with intensity function $Mk(\cdot - c; \delta)$ where the spatially varying thinning probability is $\exp(z_{1:k}(u)\beta_{1:k}^T)/M$. Using this thinning perspective, the intensity function of Thomas process X is

$$\rho(u) = \alpha\kappa \exp(z_{1:k}(u)\beta_{1:k}^T) \tag{A.2}$$

PARAMETER ESTIMATION

For the above four processes, the intensity functions (A.1 and A.2) could also be written as:

$$\rho(u) = \exp(z(u)\beta^T), \tag{A.3}$$

where $z(u) = (1, z_{1:k}(u))$ and $\beta = (\beta_0, \beta_{1:k})$, $\beta_0 = \log(\alpha)$ for Poisson process and $\beta_0 = \log(\kappa\alpha)$ for Thomas process. Therefore, following Waagepetersen's two-step approach (Waagepetersen and Guan 2007), we could maximize the following log-likelihood function

based on the above intensity function A3 to obtain $\hat{\beta}$:

$$l(\beta) = \sum_{u \in X \cap S} z(u)\beta^T - \int_S \exp(z(u)\beta^T)du \qquad (A.4)$$

For Poisson process models, we can get all the parameters using maximum likelihood

methods based on A.4. Other parameters $\hat{\kappa}$ and $\hat{\delta}$ in Thomas process could be estimated

by minimum contrast methods:

$$m(\kappa, \delta) = \int_{rl}^{r} (\hat{K}(t)^c - K(t; \kappa, \delta)^c)^2 dt \qquad (A.5)$$

where rl, r, and c are user-specified constants, and K is the inhomogeneous K-function of X

which is defined as

$$\hat{K} = \sum_{u, \eta \in X \cap S} \frac{1[0 < \|u - \eta\| < t]}{\exp((z(u) - z(\eta))\hat{\beta}^T)} e_{u,\eta}$$

where $e_{u,\eta}$ is an edge-effect correction. Here, considering the bias of K increases with r, we

choose $rl = 0$ and $r = 100$ meters. Following Diggle's (2003) recommendation, we choose c as

$1/4$. Border edge correction method (Ripley 1988) is used for faster compute. $\hat{\mu}$, the

expected number of offspring of each mother point can be got by following $\hat{\mu} = M / \hat{\kappa}$. Finally

$\hat{\alpha} = \exp(\hat{\beta}_0) / \hat{\kappa}$ for Thomas process.

Akaike's Information criterion

Akaike's information criterion (AIC) was used to measure the goodness of fit of an estimated

statistical model. In the general case, AIC is

$$AIC = 2k - 2\ln(L)$$

where k is the number of parameters in the statistical model, and L is the maximized value of

the likelihood function for the estimated model. A problem in the application of this criterion

in our study is that estimation of our model parameters is not totally based on maximum

likelihood method. Although likelihood based parameter estimation methods have been

developed in recent years, problems of unstable and extremely time consuming restricted their

application (Moller and Waagepetersen. 2004, Guan 2006). Fortunately, we can use the

following estimation $\hat{A}$ in our current modeling framework (Webster and Mcbratney 1989):

$$\hat{A} = \{n \ln[\frac{2\pi}{n}] + n + 2\} + n \ln R + 2k$$

where n is the number of observations, k is the number of parameters estimated and R is the

sum of residual squares. The quantity in the curly brackets is constant for a given set of data

and so models can be compared by computing: $AIC = n \ln(R) + 2k$.

LITERATURE CITED

Diggle, P. J. 2003. Statistical analysis of spatial point patterns. Second edition. Academic

Press, London, UK.

Guan, Y. 2006. A composite likelihood approach in fitting spatial point process models.

Journal of the American Statistical Association 101:1502–1512.

Møller, J., and R. P. Waagepetersen. 2004. Statistical inference and simulation for spatial point

processes. Chapman and Hall CRC Press, Virginia, USA.

Ripley, B. D. 1988. Statistical inference for spatial processes. Cambridge University Press,

Cambridge, UK.

Waagepetersen, R., and Y. Guan. 2007. Two-step estimation for inhomogeneous spatial point

processes. Department of Mathematical Sciences, Aalborg University, Aalborg, Denmark.

Webster R., and A. B. Mcbratney. 1989. On the Akaike Information Criterion for choosing

models for variograms of soil properties. Journal of Soil Science 40:493–496.