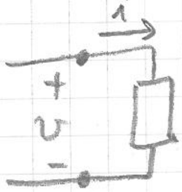
P9

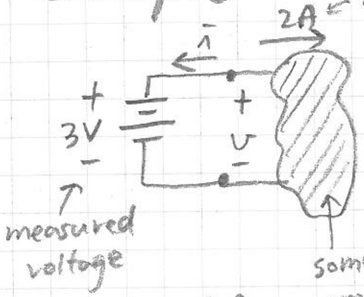
– Associated Variable Convention : (約定俗成)

For a two-terminal lumped element, define current to flow in at the element terminal assigned to be positive in voltage.



v and i are called "terminal variables."

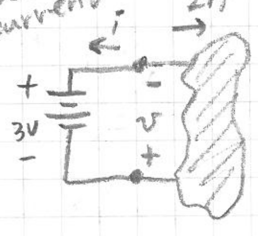
Example :



$v = 3A$

$i = -2A$

$P = v \cdot i = -6W$

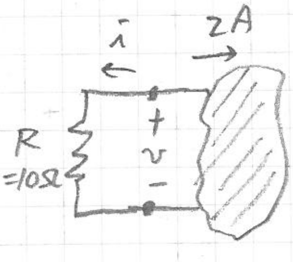


$v = -3V$

$i = -(-2) = 2A$

$P = v \cdot i = -6W$

⇒ power supplied "from" battery
by



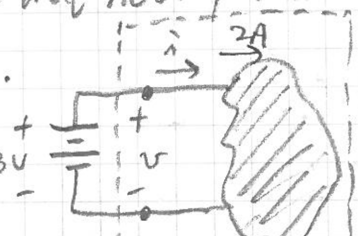
$i = -2A$

$v = iR = -20\,V$

$P = v \cdot i = 40\ W$

⇒ power supplied "to" resistor (and then is dissipated in the form of heat)

Note that the above is from the viewpoint of the battery; from the viewpoint of that complex circuit, you may try and see that now power is supplied "to" it.



P10 Section 1.8 in the textbook mentioned sinusoidal signals and the <u>root mean square</u> value. For your interest, the $\sqrt{2}$ ratio between the amplitude of a sinusoidal signal and its rms value comes from 三角函數 2 倍角轉換.

Example : let signal $i(t) = I_m \cos(\omega t)$

$$i_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t) \cdot dt}$$

$$= \sqrt{\frac{1}{T}\int_0^T (I_m^2 \cos^2 \omega t)\, dt}$$

$$= \sqrt{\frac{I_m^2}{2T}\int_0^T (1+\cos 2\omega t)\, dt}$$

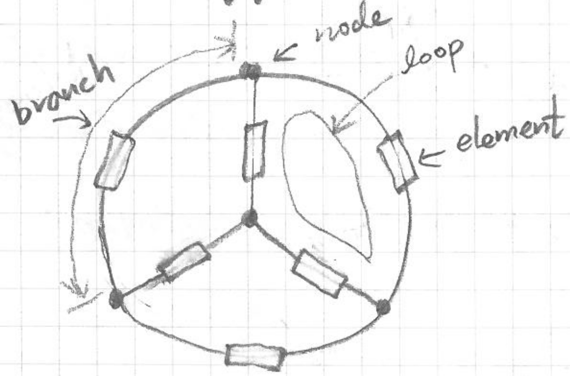
$$= \frac{I_m}{\sqrt{2}} \sqrt{\frac{1}{T}\int_0^T (1+\cos 2\omega t)\, dt}$$

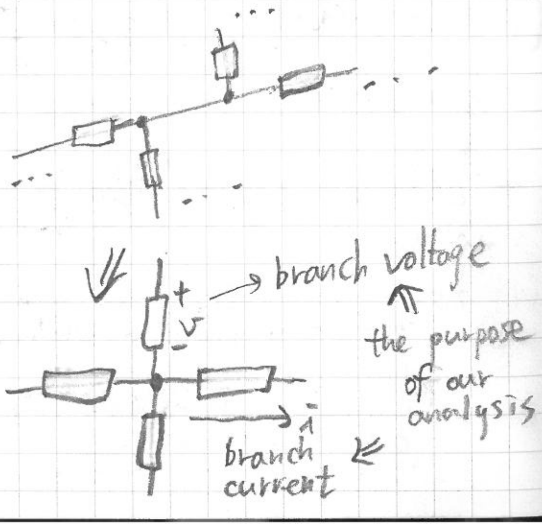
$$= \frac{I_m}{\sqrt{2}} ※$$

☆ Resistive Networks and How to Analyze Them

- Terminology

ideal wire : no resistance





- Kirchhoff's Laws $\left\{ \begin{array}{l} \underline{\text{KCL}} \\ \underline{\text{KVL}} \end{array} \right.$

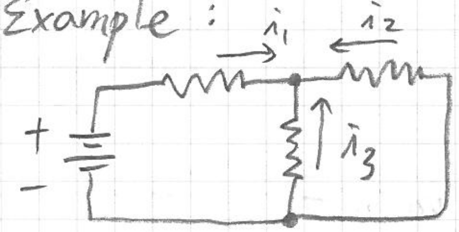
following LMD, Kirchhoff's laws are simplifications of Maxwell's Equations (see Appendix A.2 in the textbook)

→ KCL and KVL are extremely useful tools to help us analyze a circuit!

KCL : Kirchhoff's current law

The algebraic sum of all branch currents flowing into any node must be zero.

Example :



$$\sum_{n=1}^{3} i_n = 0$$

$$\sum_{n=1}^{3} (-i_n) = 0$$

In general, for integers N, M, we have

$$\sum_{n=1}^{N} i_n = 0 \Rightarrow \sum_{n=1}^{M} i_n + \sum_{n=M+1}^{N} i_n = 0$$

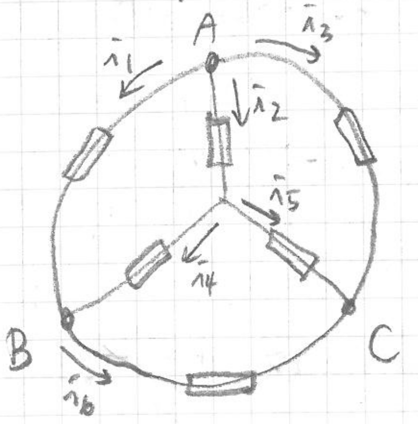
$$\Rightarrow \sum_{n=1}^{M} i_n = \sum_{n=M+1}^{N} (-i_n)$$

⇒ Sum of total current flowing into a node = sum of total current flowing out from a node.

$i_1 = i_2$ ideal wire

P12

Example:



Using KCL

node A: $0 = -i_1 - i_2 - i_3$

B: $0 = i_1 + i_4 - i_6$

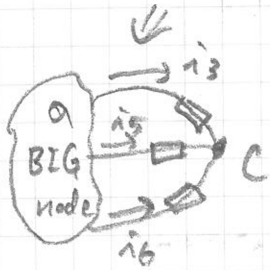
C: $0 = i_2 - i_4 - i_5$

D: $0 = i_3 + i_5 + i_6$

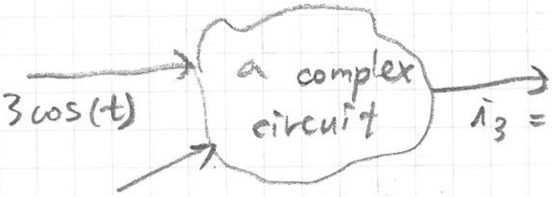
Suppose $i_1 = 1$, $i_3 = 3 \Rightarrow i_2 = -4$

In addition, suppose $i_5 = -2$

$\Rightarrow i_4 = -2$, $i_6 = -1$ #



In general, for N KCL statements, only N-1 of them are independent. Therefore, we need N-1 known values to solve the circuit.



$i_1 = 3\cos(t)$

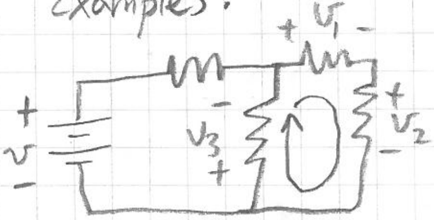
$i_2 = 6\cos(t)$

$i_3 = i_1 + i_2 = 9\cos(t)$

KVL: Kirchhoff's voltage law

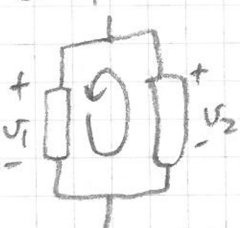
The algebraic sum of the branch voltages around any loop (i.e., closed path) in a network must be zero.

Examples:



$$\sum_{n=1}^{3} v_n = 0$$

並聯



$0 = v_1 - v_2$

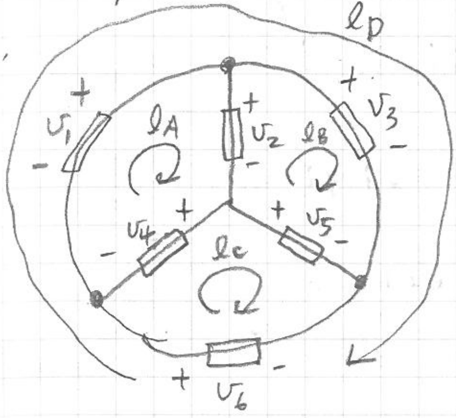
$\Rightarrow v_1 = v_2$

串聯

$0 = -v + v_1 + v_2$

$\Rightarrow v = v_1 + v_2$

Example:



Using KVL

loop l_A: $0 = -v_1 + v_2 + v_4$

l_B: $0 = -v_2 + v_3 - v_5$

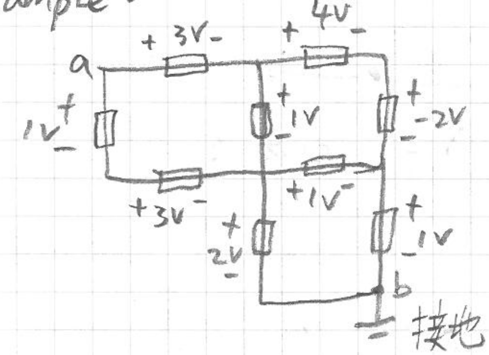
l_C: $0 = -v_4 + v_5 - v_6$

l_D: $0 = -v_1 + v_3 - v_6$

Suppose that $v_1 = 1$, $v_3 = 3$, $v_2 = 2$

then $v_4 = -1$, $v_5 = 1$, $v_6 = 2$ ☆

Example:



v_{ab} can be determined using KVL

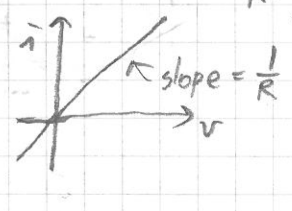
(which loop(s) would you pick?)

$v_{ab} = 6$ V.

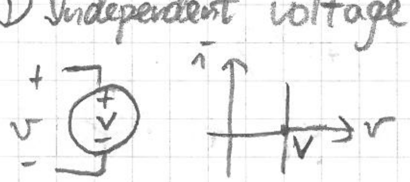
> In general, we model individual elements based on LMD assumptions
> and we analyze a circuit of elements using KCL and KVL.

☆ Two more basic elements and their i-v characteristics

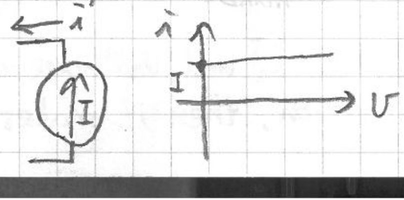
for linear resistor it obeys Ohm's law $R = \frac{v}{i}$



① Independent voltage source



② Independent current source



☆ Basic Method to Analyze A Circuit

four steps

see Page 9

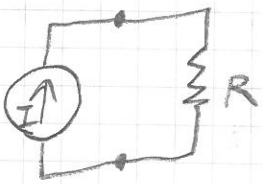
1° define branch current and voltage <u>consistently</u>

2° apply element laws for each elements (e.g., Ohm's law for linear resistors)

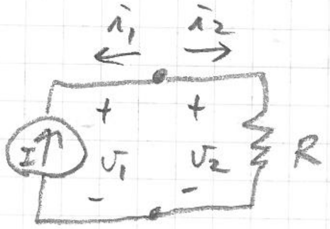
3° apply KCL and KVL

4° jointly solve the equations obtained from 2° and 3°.

Example :



1°



2° $i_1 = -I$, $v_2 = i_2 \cdot R$

3° KCL $\Rightarrow i_1 + i_2 = 0$

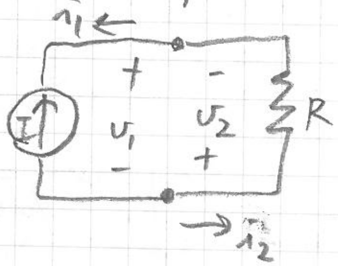
KVL $\Rightarrow -v_1 + v_2 = 0$

4° $i_2 = -i_1 = I$

$v_2 = i_2 R = IR$

$v_1 = v_2 = IR$ ※

if at step 1° we define v_2 inversely, we must



also define i_2 inversely, to be consistent. Applying the basic method you will find $i_2 = -I$ and $v_2 = -IR$. This may seem to be strange. But if we recall that both i_2 and v_2 have a reversed direction, then it makes sense.

Alternatively, we may solve a circuit by considering "Energy conservation".

Example :

power out from the source :

$$P_{out} = 2mA \times v$$

power into the resistor :

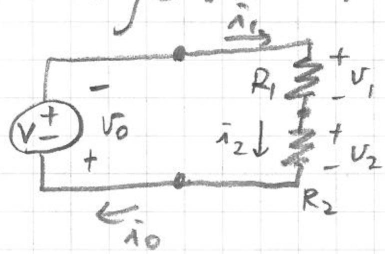
$$P_{in} = i \times v = \frac{v^2}{R} = \frac{v^2}{1k\Omega}$$

$$P_{out} = P_{in} \Rightarrow 2\times10^{-3} \cdot v = \frac{v^2}{1\times10^3}$$

$$\Rightarrow v = 2\ V \ *$$

(Example 2.14 in the textbook has a typo saying $v = 0.5\ V$)

☆ Voltage Divider



we may analyze it using the basic method :

$$\begin{cases} v_0 = -V \\ v_1 = R_1 i_1 \\ v_2 = R_2 i_2 \end{cases} \qquad \begin{cases} i_0 = i_1 \\ i_1 = i_2 \end{cases} \text{KCL} \qquad v_0 + v_1 + v_2 = 0 \quad \text{KVL}$$

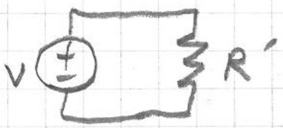
$$\Rightarrow \boxed{v_2 = \frac{R_2}{R_1 + R_2} V}$$

Further, from $i_2 = \frac{v_2}{R_2}$ we see $i = \frac{1}{R_1 + R_2} V$

in other word, $V = i \times (R_1 + R_2)$.

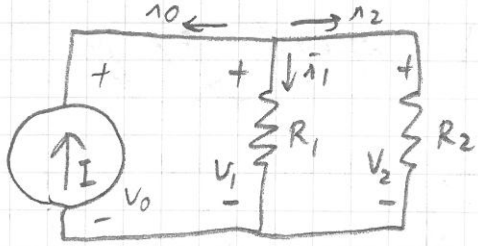
$\Rightarrow$ we may replace R_1 and R_2 by an equivalent resistor $\boxed{R' = R_1 + R_2}$

and the circuit is equivalent as

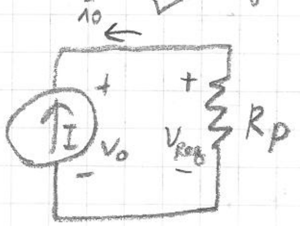


This lead to a general planar linear resistor analysis, such as that in Example 2.21 in the textbook.

✲ Current Divider



⇓ equivalent



Using the "basic method"

$$\begin{cases} i_0 = -I \\ v_1 = R_1 i_1 \\ v_2 = R_2 i_2 \end{cases} \quad \begin{cases} i_0 + i_1 + i_2 = 0 \\ v_0 = v_1 = v_2 \end{cases}$$

$$\Rightarrow i_1 + i_2 = I$$

$$i_1 + i_2 = \frac{v_1}{R_1} + \frac{v_2}{R_2} = \frac{v_0 R_2 + v_0 R_1}{R_1 R_2}$$

$$= \frac{R_1 + R_2}{R_1 R_2} v_0$$

$$\Rightarrow v_0 = \frac{R_1 R_2}{R_1 + R_2} \cdot I$$

$$\Rightarrow \boxed{\begin{aligned} i_1 &= \frac{v_1}{R_1} = \frac{R_2}{R_1 + R_2} I \\ i_2 &= \frac{v_2}{R_2} = \frac{R_1}{R_1 + R_2} I \end{aligned}}$$

$$v_1 = R_1 i_1 = \frac{R_1 R_2}{R_1 + R_2} I$$

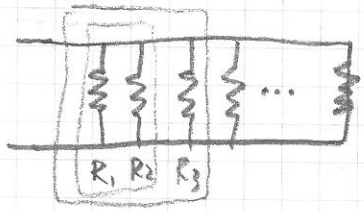
$$\parallel$$

$$v_0 = R_{eq} \cdot I$$

$$\Rightarrow \boxed{R_p = \frac{R_1 R_2}{R_1 + R_2}}$$

$$\Rightarrow \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{P_2}$$

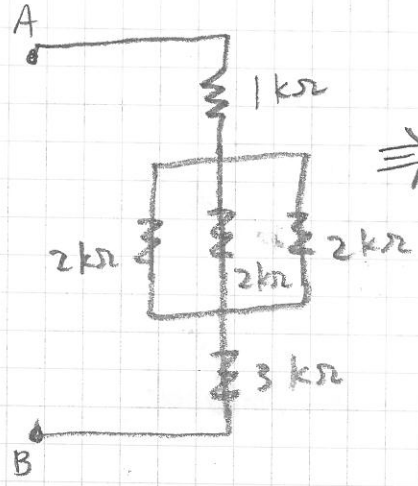
In general, for N resistors connected in parallel,



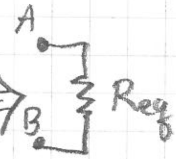
then

$$\frac{1}{R_p} = \sum_{n=1}^{N} \frac{1}{R_n}$$

this can be proved by induction.



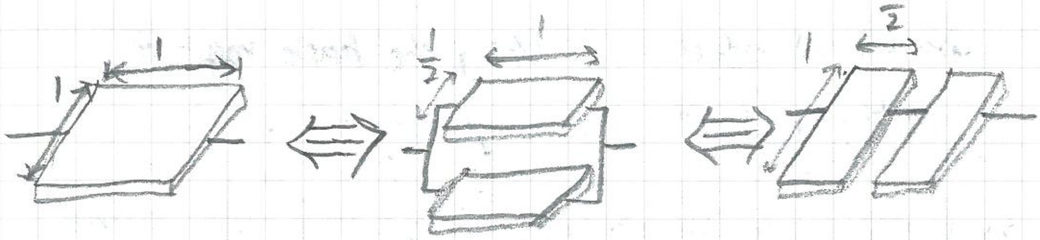
⇛



$$R_{eq} = 1 + \frac{1}{\frac{1}{2} + \frac{1}{2} + \frac{1}{2}} + 3$$

$$= \frac{14}{3} k\Omega$$

✲

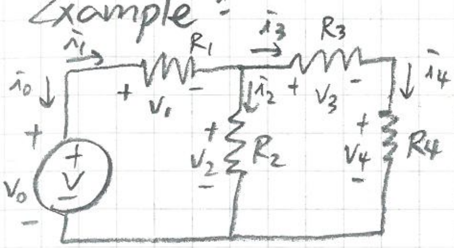


let $R_1 = R_\square$

$$R_2 = \frac{2R_\square \cdot 2R_\square}{2R_\square + 2R_\square} = R_\square = R_1$$

$$R_3 = \frac{1}{2}R_\square + \frac{1}{2}R_\square = R_\square = R_1$$

Example:



Determine v_1, v_2, v_3, v_4 and i_1, i_2, i_3, i_4.

Way ①: we may use the basic method

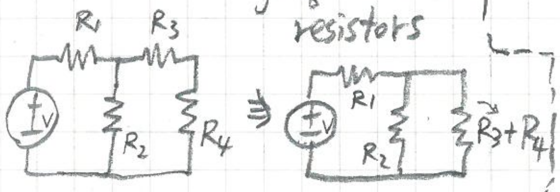
2° element law ...

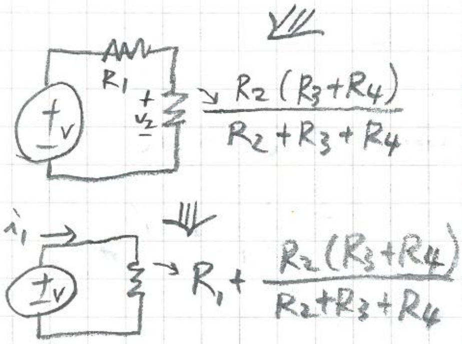
3° KCL & KVL ...

4° 解聯立方程式

give it a try yourself here:

Way ②: transformation using equivalent resistors





$$i_1 = V \Big/ \left(R_1 + \frac{R_2(R_3+R_4)}{R_2+R_3+R_4}\right)$$

$v_1 = i_1 R_1 = $ ___

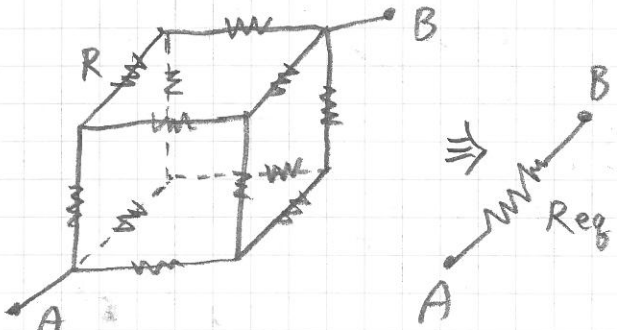
$v_2 = $ ___ (voltage divider)

$i_2 = v_2 / R_2$, $i_3 = v_2 / (R_3 + R_4)$

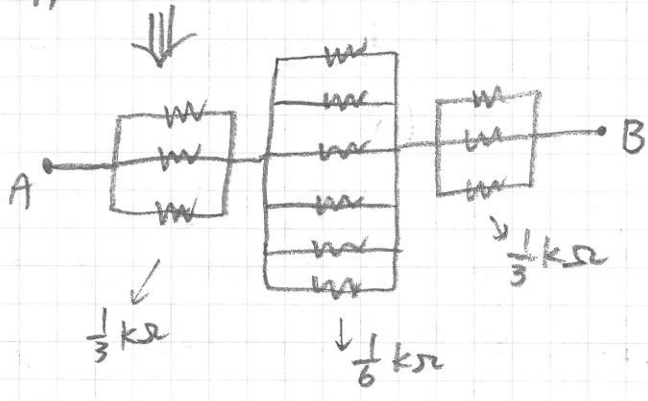
$v_3 = i_3 R_3$, $v_4 = i_3 R_4$

P18 Sometimes we may leverage symmetry to greatly reduce the complexity of analysis:

Example: assuming all resistors are the same on a cube, with resistance $R = 1\,k\Omega$, determine R_{eq}



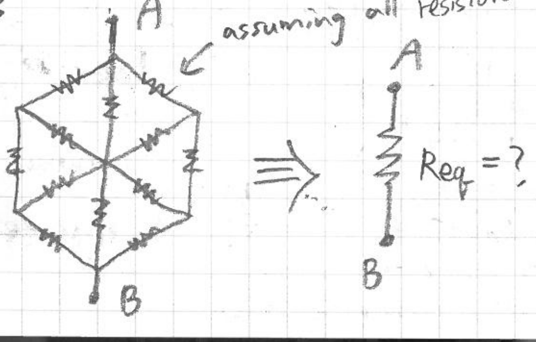
(see example 2.24 in the textbook)



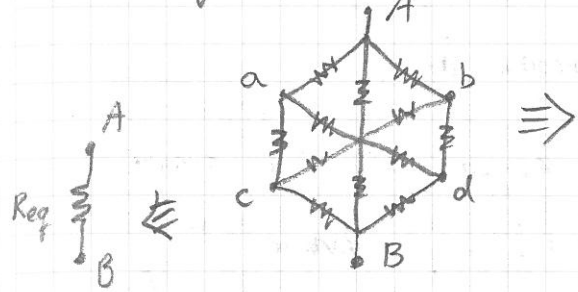
$$R_{eq} = \frac{1}{3} + \frac{1}{6} + \frac{1}{3}$$
$$= \frac{5}{6}\,k\Omega \quad *$$

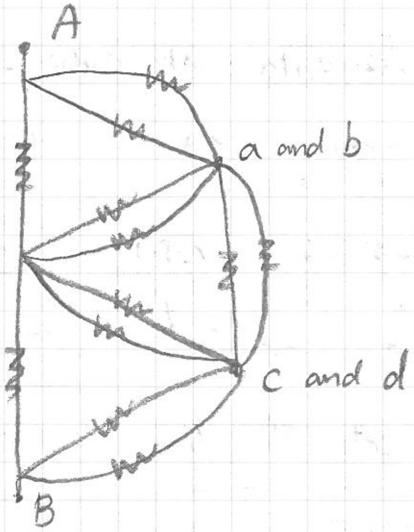
Often, we still need to apply the basic method after reducing a circuit by using equivalent resistors and symmetry !!

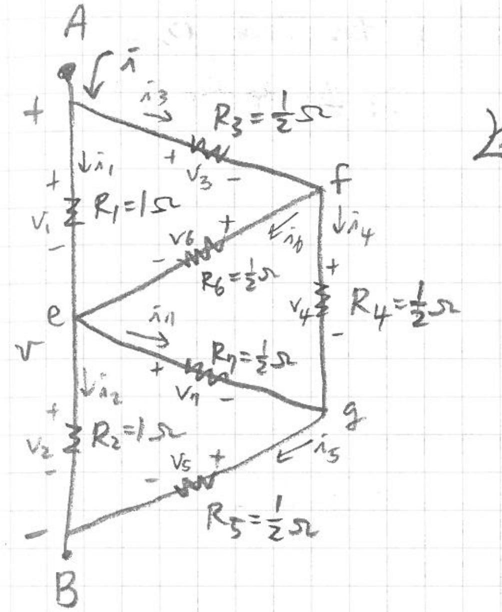
Example: assuming all resistors have $1\,\Omega$ resistance



following P_{18},







element laws:

$$\begin{cases} v_1 = i_1,\ v_2 = i_2,\ v_3 = \frac{1}{2} i_3 \\ v_4 = \frac{1}{2} i_4,\ v_5 = \frac{1}{2} i_5,\ v_6 = \frac{1}{2} i_6 \\ v_7 = \frac{1}{2} i_7 \end{cases}$$

KCL:

$$\begin{cases} ① & i = i_1 + i_3 = i_2 + i_5 & \text{(node A \& B)} \\ ② & i_1 + i_6 = i_2 + i_7 & \text{(node e)} \\ ③ & i_3 = i_6 + i_4 & \text{(node f)} \\ ④ & i_5 = i_7 + i_4 & \text{(node g)} \end{cases}$$

KVL:

$$\begin{cases} ⑤ & v_1 + v_2 = v_3 + v_4 + v_5 \\ ⑥ & v_1 = v_3 + v_6 \\ ⑦ & v_2 = v_7 + v_5 \end{cases}$$

$$Req = \frac{v}{i} = \frac{v_1 + v_2}{i_1 + i_3} = \frac{i_1 + i_2}{i_1 + i_3}$$

therefore, 我們可將所有電壓值代換為電流值，去解電流的聯立方程式!!

$$\begin{array}{c} ① \\ ② \\ ③ \\ ④ \\ ⑤ \\ ⑥ \\ ⑦ \end{array} \begin{bmatrix} 1 & -1 & 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & -1 \\ 2 & 2 & -1 & -1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

note: equation ④ is dependent,

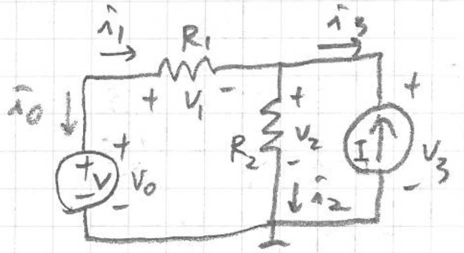
使用高斯消去法 可得 $\begin{cases} i_1 = 2 i_7 \quad i_3 = 3 i_7 \\ i_2 = 2 i_7 \end{cases}$

or www.mathtools.com Matrix Calculator

$$\therefore Req = \frac{i_1 + i_2}{i_1 + i_3} = \frac{2+2}{2+3} = \frac{4}{5}\Omega \ ※$$

Calculation is a necessary part in engineering!

P20 Example: A circuit with two independent sources



Determine i_2.

element law $\begin{cases} v_0 = V \\ v_1 = R_1 i_1 \\ v_2 = R_2 i_2 \\ i_3 = -I \end{cases}$

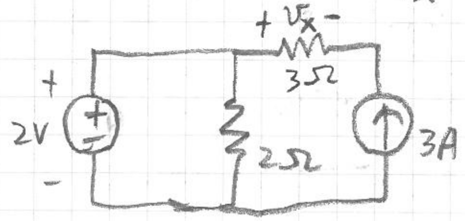
KCL $\begin{cases} i_0 = -i_1 \\ i_1 = i_2 + i_3 \end{cases}$

KVL $\begin{cases} v_0 = v_1 + v_2 \\ v_2 = v_3 \end{cases}$

→ then solve these linear equations (see textbook P95).

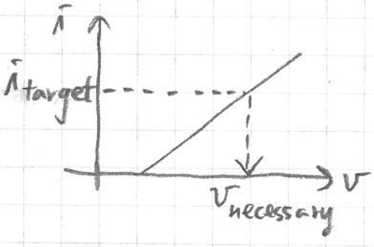
$$i_2 = \frac{R_1}{R_1+R_2} I + \frac{1}{R_1+R_2} V \quad ※$$

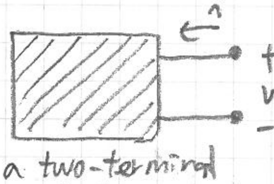
Exercise: $v_x = ?$



(ans: $v_x = -9V$)

☆ The I-V characteristic of a circuit

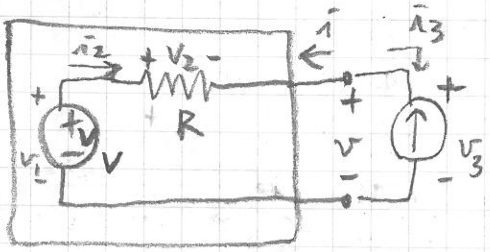




a two-terminal device

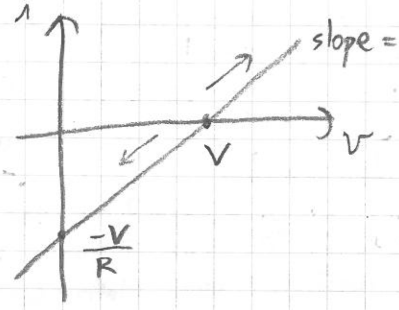
我們可藉由給定v量測i (or vice versa) 來繪製 I-V relation.

If we know the device's internals,
we may also determine its I-V relation:



using the basic method, we have

$$v = V + iR \Rightarrow i = \frac{1}{R} v - \frac{V}{R}$$



example usage:

預測电流流向

$\begin{cases} v \geq V \rightarrow i \geq 0 \\ v < V \rightarrow i < 0 \end{cases}$ ⇐