P84 ☆ The Small-Signal Model

We have learned some small-signal analysis when we were studying diode circuits early this semester (P43 - P48; Section 4.5 in the textbook).

Small-signal model and its analysis play a critical role in both design and usage of MOSFET amplifiers because

① many real-world input signals are small signals;

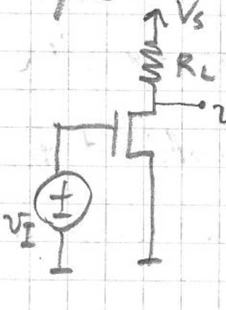
② the small-signal gain $\frac{v_o}{v_i}$ is "linear", which implies less signal distortion caused by the amplifier;

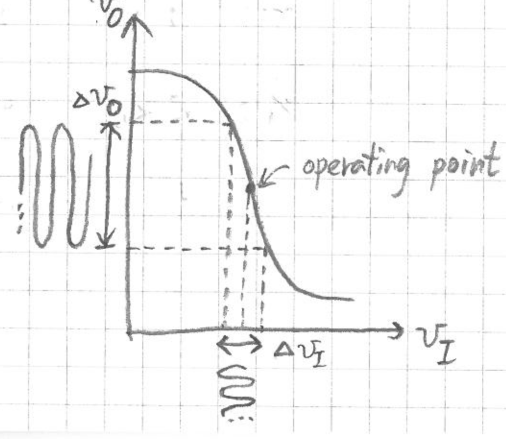
③ the circuit under the small-signal model is "linear", which means we may apply existing linear circuit analysis techniques, for example Thévenin's Theorem, to help us understand the circuit's behavior and its response to input signal.

It is important to remember that small-signal model is an "approximation" of the original non-linear model. In practice, it may be needed to estimate the quality of such an approximation. That part is beyond the scope of this course, but you may study P218 in the textbook to get some idea. It is interesting to recall that the original non-linear model is itself an approximated description of how a circuit behaves in the real world.

In essence, the small-signal model describes how a circuit responds to a small, time-varying signal. The large, time-invariant signal is used to determine the operating point around which the small signal oscillates. Take our familiar MOSFET amplifier, for example:

$$\underbrace{v_I}_{\text{total input}} = \underbrace{V_I}_{\text{large signal}} + \underbrace{v_i}_{\text{small signal}}$$





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The large, time-invariant signal in this case is call <u>the DC bias</u>, or <u>the DC offset</u>.
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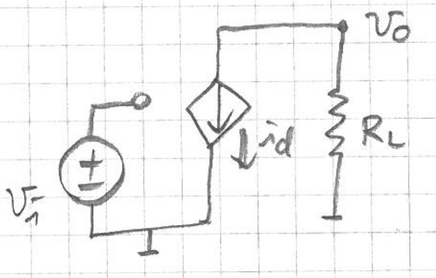
In the small-signal model, we consider that all of the independent current sources and independent voltage sources are shut off, because their impact to the circuit does not change along with the small signal; in other word, their impact to the circuit is time-invariant.
In this sense, the voltage-controlled current source is not considered shut off for our MOSFET amplifier, because the current depends on the small-signal v_i (and depends on the large-signal V_I, too).

The small-signal model of our MOSFET amplifier:



where $i_d = g_m \cdot v_i$,
and $g_m = K(V_I - V_T)$
is called the incremental transconductance.

To obtain the small-signal gain $\frac{v_o}{v_i}$, we P87
may apply linear circuit analysis :

$$\frac{v_o - 0}{R_L} = -i_d$$

$$\Rightarrow v_o = -i_d \cdot R_L = -(g_m \cdot v_i) \cdot R_L$$

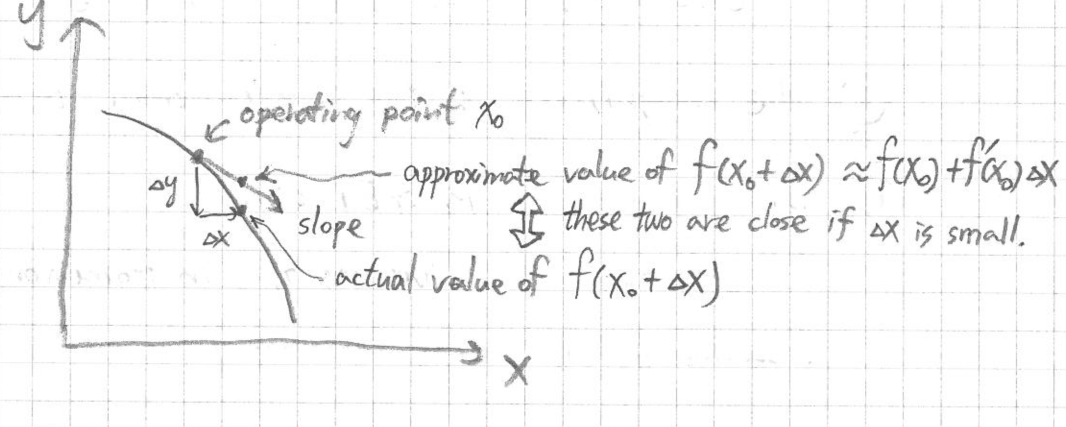
$$\Rightarrow \frac{v_o}{v_i} = -g_m \cdot R_L$$

and the magnitude of the gain is

$$\left|\frac{v_o}{v_i}\right| = g_m R_L .$$

In class, we've used the Taylor series expansion to derive that relation $i_d = K(V_I - V_T) v_i$.

The take-home message of such a derivation is that we may, in general, compute <u>the first derivative (一次微分)</u> of a function at the operating point to get a reasonable small-signal relation :



Study textbook Chapter 8 and Homework 6 for some examples of the small-signal analysis and the model. Finally, from a practical viewpoint, we may summarize four criteria regarding how to select an appropriate operating point, or in other word, how to set a proper DC bias:

① determine the range of input signal v_I such that the MOSFET would operate in the saturation region; (P356 in textbook)

② if the large-signal is also time-varying, we may want to maximize the peak-to-peak swing of the input signal by setting the operating point at the middle of the valid range of v_I for operation in the saturation region;
(review P351 and P369 in the textbook)

③ the magnitude of the small-signal gain;

④ driving another MOSFET circuit;
(P61 in this note; Questions 4, 5 in Homework 4)
(Section 8.2.3 in the textbook)

We will discuss more on the <u>difference amplifier</u> after the final exam. It will not be included in the exam.