P1

Note for ARQ analysis: (Problem 2.17)

Let q be the probability that a packet is correctly received and acked on a given transmission.

In the following, we show that $1/q$ is the expected number of times a packet must be transmitted for a stop-and-wait system, assuming independent errors on all frames:

let X be the number of transmissions

we have $E[X] = \sum_{k=1}^{\infty} k \cdot q(1-q)^{k-1}$

$$= \frac{q}{1-q} \sum_{k=1}^{\infty} k(1-q)^k = \frac{q}{1-q} \lim_{n\to\infty} \sum_{k=1}^{n} k(1-q)^k$$

let $z = 1-q$ and $S_n = \sum_{k=0}^{n} k \cdot z^k$

$$\Rightarrow S_n + (n+1)z^{n+1} = \sum_{k=0}^{n+1} k \cdot z^k = \sum_{k=0}^{n} (k+1) z^{k+1}$$

$$= \sum_{k=0}^{n} k \cdot z^{k+1} + \sum_{k=0}^{n} z^{k+1}$$

$$= z \cdot S_n + \frac{z - z^{n+2}}{1-z}$$

$$\Rightarrow S_n = \frac{1}{(1-z)^2}\left(z - (n+1)\cdot z^{n+1} + n \cdot z^{n+2}\right)$$

$$\Rightarrow E[X] = \frac{q}{1-q} \lim_{n\to\infty} S_n$$

(Note: by the Pinching Theorem we can prove that $\lim_{n\to\infty} n \cdot z^n = 0$.)

$$= \frac{q}{1-q}\left(\frac{z}{(1-z)^2}\right)$$

$$= \frac{q}{1-q}\left(\frac{1-q}{(1-(1-q))^2}\right)$$

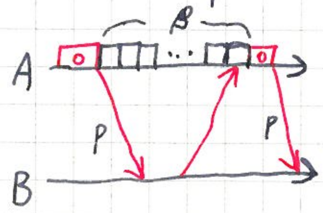
$$= \frac{1}{q} \quad \divideontimes$$

P2

ARQ analysis in terms of goodput: (page 82 and Problem 2.26)

In "go back N" and "selective repeat" protocols, let β be the expected number of transmitted frames from A to B between the transmission of a given frame and the reception of feedback about that frame.

Let p be the probability that a frame arriving at B contain errors. Let r be the expected number of transmitted frames from A to B per successfully accepted packet at B.



$$r = 1\cdot(1-p) + [1+(1+\beta)]\cdot p\cdot(1-p) + [1+2(1+\beta)]\cdot p^2(1-p) + \cdots$$

$$= (1-p)\sum_{k=1}^{\infty}\left(1+(k-1)(1+\beta)\right)\cdot p^{k-1}$$

$$= (1-p)\sum_{k=1}^{\infty} p^{k-1} + (1-p)(1+\beta)\sum_{k=1}^{\infty}(k-1)\cdot p^{k-1}$$

$$= (1-p)\cdot\frac{1}{1-p} + (1-p)(1+\beta)\sum_{k'=1}^{\infty} k' p^{k'}$$

$$= 1 + (1-p)(1+\beta)\cdot\frac{p}{(1-p)^2}$$

$$= \frac{1+p\beta}{1-p}.$$

(note that if $\beta = 0$ then you can think of it as a stop-and-wait system, and r becomes $\frac{1}{1-p}$ which is essentially the result we derived on the left side.)

The goodput $= \frac{1}{r} = \frac{1-p}{1+p\beta}$.

In "selective repeat", ideally β will decrease upon each retransmission of 0; informally, this gives goodput $\approx 1-p$.