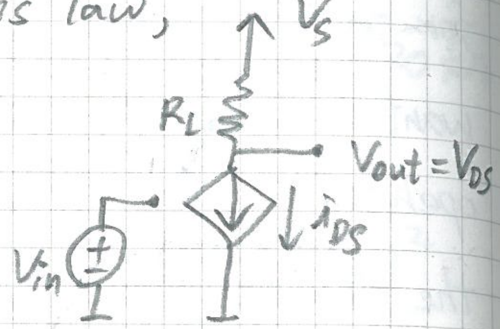
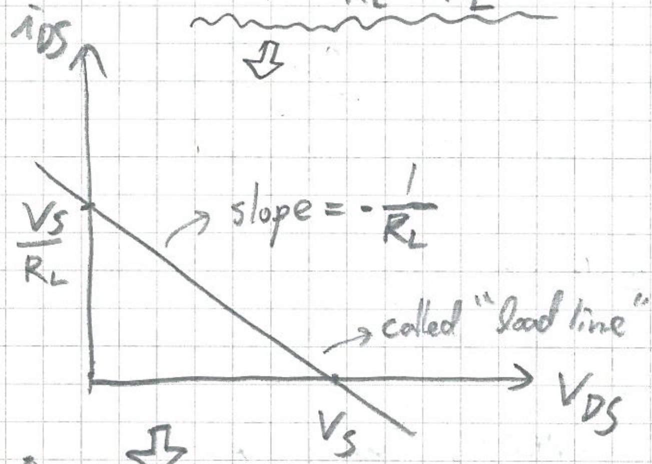
P80 Using graphically analysis, we first observe that, by Ohm's law,



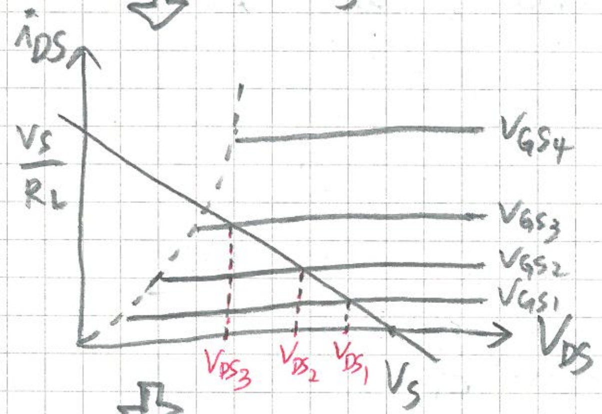

$$i_{DS} = \frac{V_S - V_{DS}}{R_L}$$

$$\Rightarrow i_{DS} = \frac{V_S}{R_L} - \frac{1}{R_L} V_{DS}$$

⇩



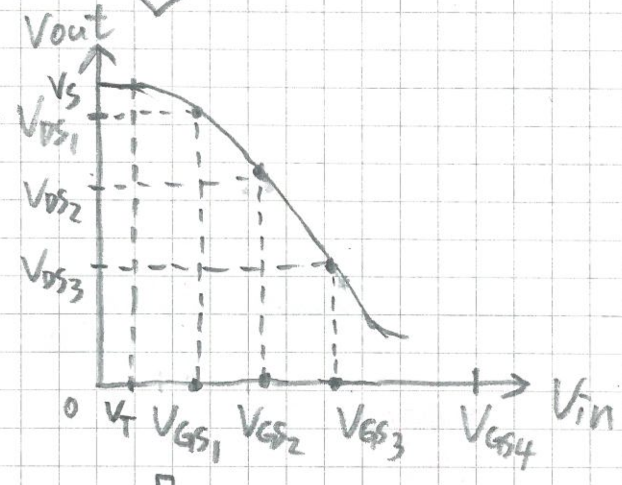

⇩




(see the plot on P68)

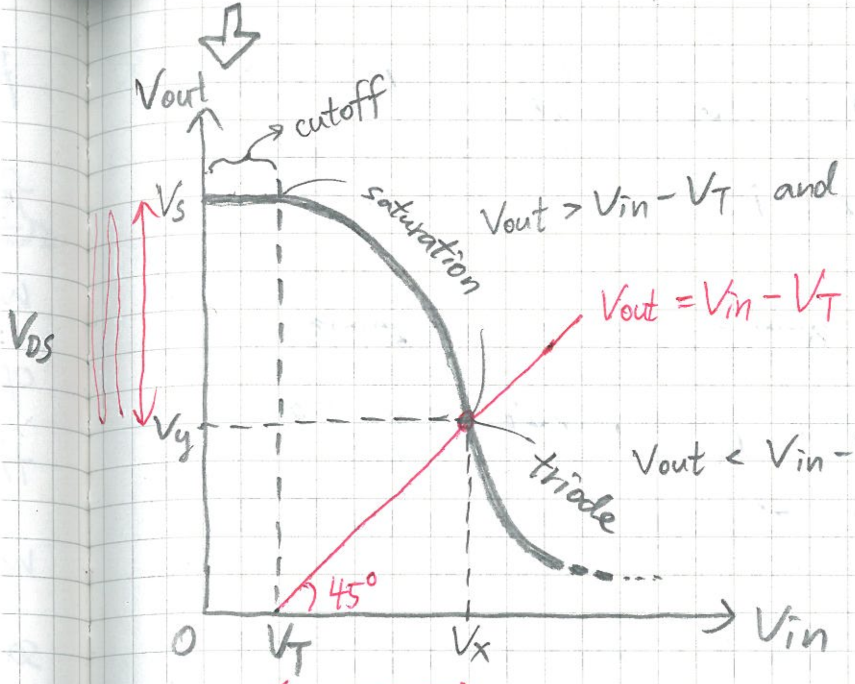
$$\begin{cases} V_{in} = V_{GS_x} & x = 1 \sim 4 \\ V_{out} = V_{DS_x} & x = 1 \sim 3 \end{cases}$$

⇩




(see the plot on P76)

⇩




To compute $V_x$, we have

$$\begin{cases} V_{out} = V_{in} - V_T \\ V_{out} = V_s - K\dfrac{(V_{in}-V_T)^2}{2}R_L \end{cases} \quad (P75)$$

$$\Rightarrow V_{in} - V_T = V_s - K\frac{(V_{in}-V_T)^2}{2}R_L$$

$$\Rightarrow \frac{KR_L}{2}(V_{in}-V_T)^2 + (V_{in}-V_T) - V_s = 0$$

$$\Rightarrow V_{in} - V_T = \frac{-1+\sqrt{1+2V_sR_LK}}{KR_L}$$

$$\Rightarrow \boxed{\begin{aligned} V_x &= \frac{-1+\sqrt{1+2V_sKR_L}}{KR_L} + V_T \\ V_y &= V_x - V_T \end{aligned}}$$

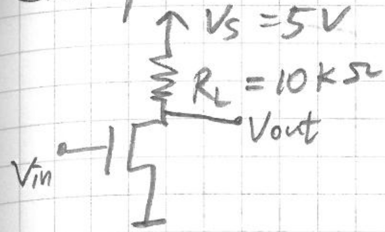
Comparing analytical analysis and graphical analysis:

- Analytical analysis is as accurate as the model (and formula) can be. For example, the use of $i_{DS} = \frac{K(V_{GS}-V_t)^2}{2}$

- Graphical analysis may be more accurate, provided that the manufacturer of the electronic device often gives "data sheet", which includes the actual measured physical values of, for example, $i_{DS}-V_{DS}$ characteristics.

- Graphical analysis also provides more insights (for example, see P37, 38).

Example:

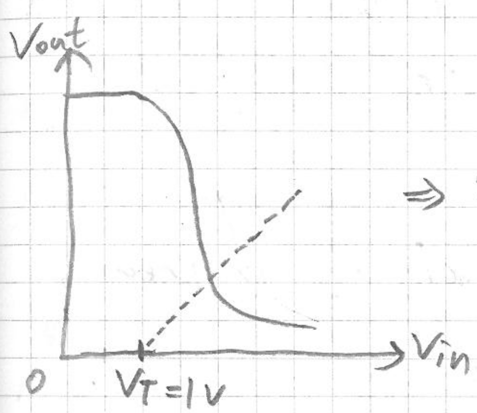



Assuming $V_T = 1V$

$K = 1\,mA/V^2$

What would be the maximum $V_{in}$ that the MOSFET can still stay in the saturation region?

If the $V_{out}$ - $V_{in}$ plot is accurate, we may directly estimate the maximum $V_{in}$:



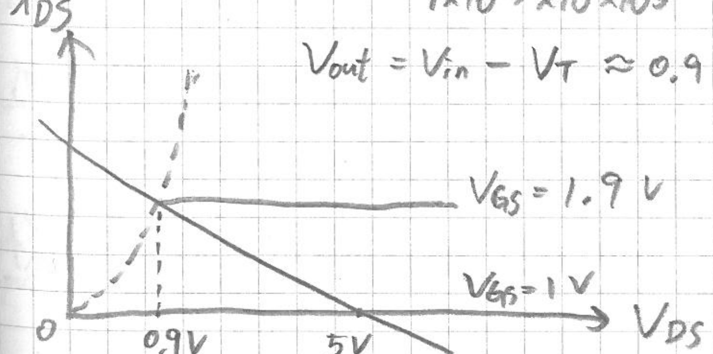

$\Rightarrow$ maximum $V_{in}$ is around 2V

Applying analytical analysis, we have

$$V_{in} = \frac{-1 + \sqrt{1 + 2V_S K R_L}}{K R_L} + V_T$$

$$= \frac{-1 + \sqrt{1 + 2 \times 5 \times 10^{-3} \times 10 \times 10^3}}{1 \times 10^{-3} \times 10 \times 10^3} + 1 \approx 1.9\,V$$

$$V_{out} = V_{in} - V_T \approx 0.9\,V$$



## ☆ The Small-Signal Model And Analysis

We have learned some small-signal analysis when we were studying diode circuits early this semester (P45-51 and P56-58; Section 4.5 in the textbook).

Small-signal model and its analysis play a critical role in both design and usage of MOSFET amplifiers because

① many real-world input signals are small signals;

② the small-signal gain $\frac{v_o}{v_i}$ is "linear", which implies less signal distortion caused by the amplifier;

③ the circuit under the small-signal model is "linear", which means we may apply existing linear circuit analysis techniques, for example Thévenin's Theorem, to help us understand the circuit's behavior and its response to input signal.

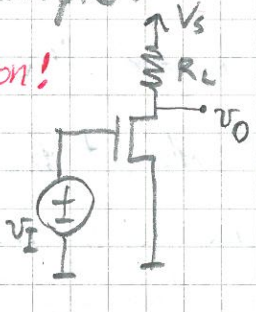
It is important to remember that small-signal model is an "approximation" of the original non-linear model. In practice, it may be needed to estimate the quality of such an approximation. That part is beyond the scope of this course, but you may study P218 in the textbook to get some idea. It is interesting to recall that the original non-linear model is itself an approximated description of how a circuit behaves in the real world.

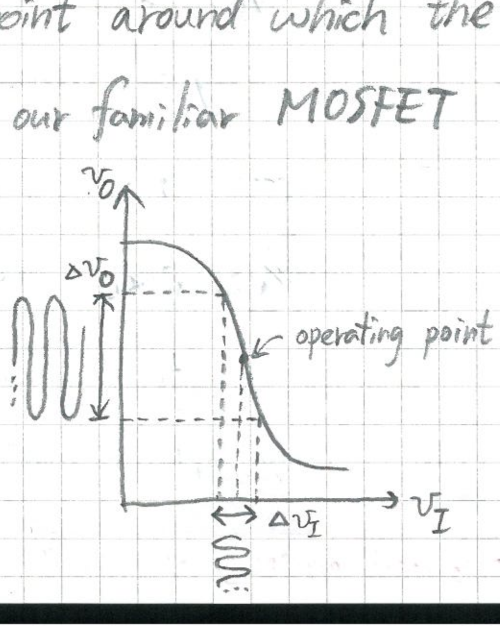
In essence, the small-signal model describes how a circuit responds to a small, time-varying signal. The large, time-invariant signal is used to determine the operating point around which the small signal oscillates. Take our familiar MOSFET amplifier, for example:

notation convention!

$v_I = V_I + v_i$

total input — large signal — small signal



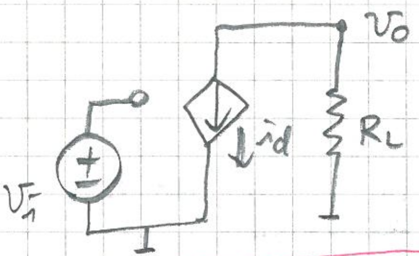



The large, time-invariant signal in this case is call <u>the DC bias</u>, or <u>the DC offset</u>.
↑ <u>直流偏壓</u>

In the small-signal model, we consider that all of the independent current sources and independent voltage sources are shut off, because their impact to the circuit does not change along with the small signal; in other word, their impact to the circuit is time-invariant.

In this sense, the voltage-controlled current source is not considered shut off for our MOSFET amplifier, because the current depends on the small-signal $v_i$ ( and depends on the large-signal $V_I$, too).

The small-signal model of our MOSFET amplifier:
on Page 85




where $\boxed{i_d = g_m \cdot v_i}$,

and $\boxed{g_m = k(V_I - V_T)}$

is called the

incremental transconductance.
( or simply, transconductance )

trans: input to output
conductance: $\frac{\text{current}}{\text{voltage}}$

★ This result applies to other circuits as well, as long as the MOSFET operates in saturation.

To obtain the small-signal gain $\frac{v_o}{v_i}$, we may apply linear circuit analysis:

$$\frac{v_o - 0}{R_L} = -i_d$$

$$\Rightarrow v_o = -i_d \cdot R_L = -(g_m \cdot v_i) \cdot R_L$$

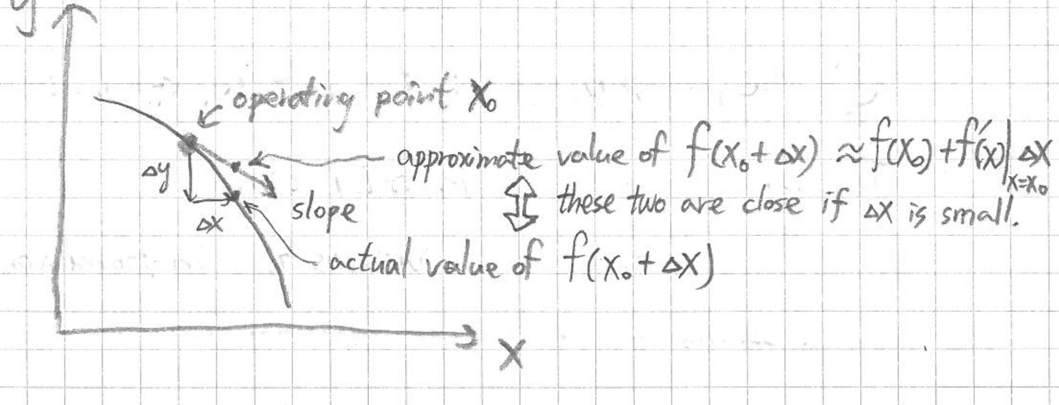
$$\Rightarrow \frac{v_o}{v_i} = -g_m \cdot R_L$$

and the magnitude of the gain is

$$\left|\frac{v_o}{v_i}\right| = g_m R_L .$$

In class, we've used the Taylor series expansion to derive that relation $i_d = K(V_I - V_T)v_i$.

The take-home message of such a derivation is that we may, in general, compute the first derivative (一次微分) of a function at the operating point to get a reasonable small-signal relation:



Study textbook Chapter 8 and Homework 6 for some examples of the small-signal analysis and the model. Finally, from a practical viewpoint, we may summarize four criteria regarding how to select an appropriate operating point, or in other word, how to set a proper DC bias:

① determine the range of input signal $v_I$ such that the MOSFET would operate in the saturation region; (P356 in textbook)

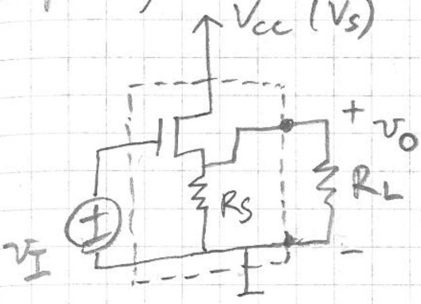
② if the large-signal is also time-varying, we may want to maximize the peak-to-peak swing of the input signal by setting the operating point at the middle of the valid range of $v_I$ for operation in the saturation region;
(review P351 and P369 in the textbook)

③ the magnitude of the small-signal gain;

④ driving another MOSFET circuit;
(P61 in this note; Questions 4, 5 in Homework 4 in the Spring 2020 semester)
(Section 8.2.3 in the textbook)

# Example: Small-Signal Analysis of A Source Follower Circuit

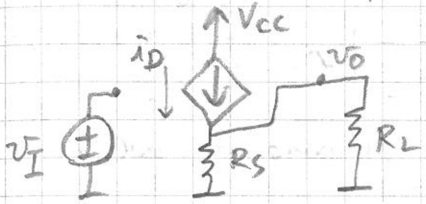
(Textbook example 8.4)




We've seen this circuit before (Example 7.8)

← consider this resistor as a load attached at the output port.

MOSFET operates in saturation



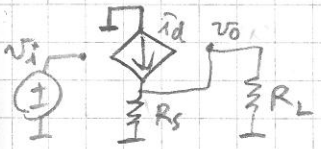

recall that $i_D = \frac{1}{2}K(v_{GS} - V_T)^2$

$= I_D + i_d$ ← interpret the result as these

$= \frac{1}{2}K(V_{GS}-V_T)^2 + K(V_{GS}-V_T)\,v_{gs}$

an approximation from Taylor expansion

small-signal model

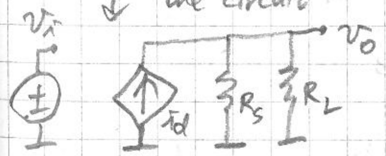



$i_d = K(V_{GS}-V_T)\cdot v_{gs} = g_m\,(v_i - v_o)$

rearrange the circuit

$\Rightarrow v_O = i_d \cdot (R_S /\!/ R_L)$ ← node analysis

$= g_m\,(v_i - v_o)(R_S /\!/ R_L)$




$\Rightarrow$ small-signal voltage gain $\frac{v_o}{v_i} = \frac{R_L R_S\, g_m}{R_L + R_S + R_L R_S\, g_m}$

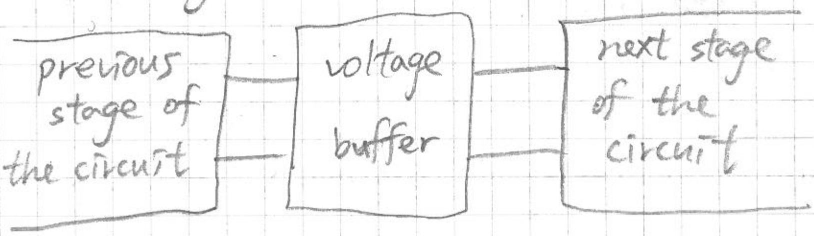
$\Rightarrow$ if $g_m$ is large, we have

$\boxed{\frac{v_o}{v_i} \approx 1}$ which is why we call this a source follower.

you may wonder why we ever want this unity-gain circuit. Read on to learn more.

Furthermore, next page

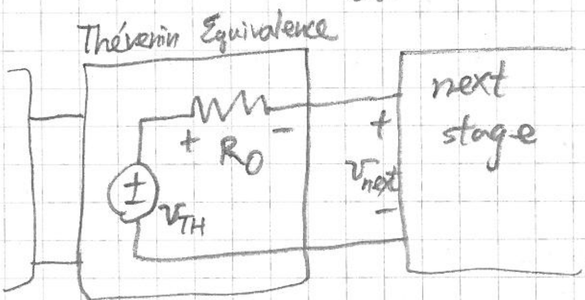
A source follower can be used as a "voltage buffer", ① to provide a unity gain of voltage to the next stage of the complex circuit, ② to provide a smaller internal resistance viewed from the next stage, and ③ to provide a larger internal resistance viewed from the previous stage.




☆ Why is the view of internal resistance important?

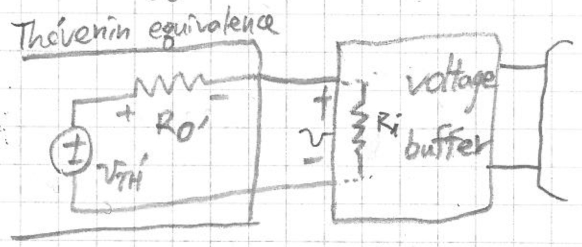
Reason 1. From the view of the next stage, having a smaller internal resistance makes less energy waste:

$R_O$ : resistance viewed from the output




From the concept of voltage divider, we see that $R_O \downarrow$ then $v_{next}$ will be closer to $v_{TH}$.
(Think about internal resistance as Thévenin resistance)

Reason 2. From the view of the previous stage, having a larger internal resistance could also make less energy waste :




$R_i$ : resistance viewed from the input.

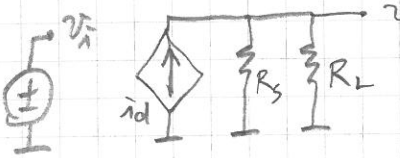
Also, from the concept of voltage divider, we see that $v$ will be closer to $v_{TH}'$ if $R_i \uparrow$

Now, let's see what is $R_O$ and $R_i$ of the source follower circuit!

Note that from now on, we'll call $R_o$ the output resistance, $r_o$ the small-signal output resistance, $R_i$ the input resistance, and $r_i$ the small-signal input resistance.

Since the need for this input/output resistance notion arises from the practical concern, we may attach a testing voltage $v_{test}$ at the input/output port and get its current response, say $i_{test}$, and the resistance can be calculated by $R = \dfrac{v_{test}}{i_{test}}$

The small-signal source follower circuit is



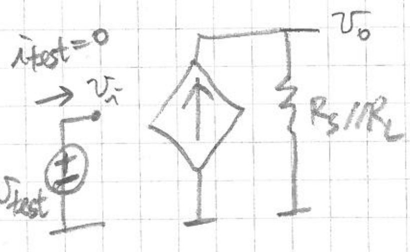

and now we're going to find $r_i$ and $r_o$.

$r_i$ and $r_o$ are important parameters, not only because of energy efficiency, but also because small signals themselves are represented by voltage variations. With a high $r_i$ and a low $r_o$ we may better preserve the signal. In this regard, energy waste can be thought of as signal distortion.

Now, to find $r_i$, we first note that physically there's an insulating layer between the gate and the rest of the MOSFET


Therefore, there's no current flowing through the gate.

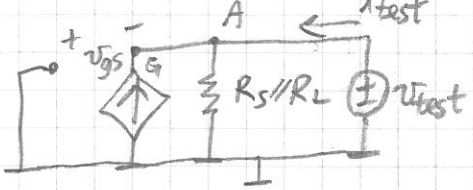
Thus, if we attach $v_{test}$ to the gate-ground (i.e., the input port) we see $i_{test} = 0$.




This implies $r_i = \frac{v_{test}}{i_{test}} = \infty$ ※

Similarly, $R_i = \infty$

Now, to find $r_o$, we attach $v_{test}$ at the output port of the source follower :




Applying the node analysis at A, we have

$$(i_d + i_{test})(R_S // R_L) = v_{test}$$

$$\Rightarrow (g_m v_{gs} + i_{test})(R_S // R_L) = v_{test}$$

$$\Rightarrow (g_m \cdot (-v_{test}) + i_{test})(R_S // R_L) = v_{test}$$

$$\Rightarrow i_{test} (R_S // R_L) = v_{test} (1 + g_m (R_S // R_L))$$

$$\Rightarrow r_o = \frac{v_{test}}{i_{test}} = \frac{R_S // R_L}{1 + g_m (R_S // R_L)} \approx \frac{1}{g_m}$$ ※

If $g_m$ is large, then $r_o$ is small.
We can increase $g_m$ by increasing the DC bias voltage and/or by widening the channel between the drain and the source, because $g_m = K (V_{GS} - V_T)$

$$= K' \left(\frac{W}{L}\right) (V_{GS} - V_T)$$

Thus, we now see that
the source follower circuit is good for voltage buffering because it has large $r_i$ and may have small $r_o$ and have an unity voltage gain. (To be continued ↓)