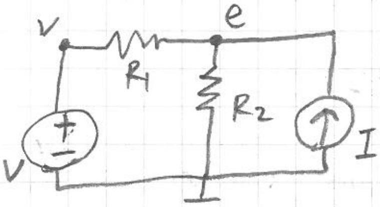
Another example:



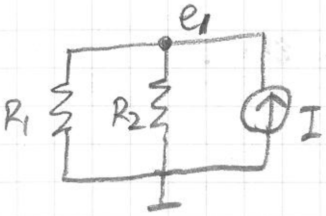
$$\text{KCL}: \frac{V-e}{R_1}+\frac{0-e}{R_2}+I=0$$

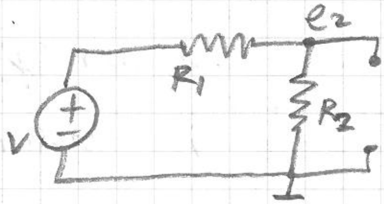
$$\Rightarrow R_2(V-e)+R_1(-e)+IR_1R_2=0$$

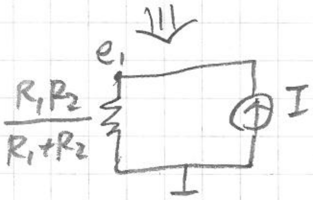
$$\Rightarrow e=\frac{1}{R_1+R_2}(R_2V+IR_1R_2)$$

$$=\frac{R_2}{R_1+R_2}V+\frac{R_1R_2}{R_1+R_2}I$$

Study the case of $V=0$ and $I=0$, respectively, we see that the original circuit can be thought of as a superposistion of one current divider and one voltage divider, where $e=e_1+e_2$:







$$e_1=I\cdot\left(\frac{R_1R_2}{R_1+R_2}\right)$$

$$e_2=\frac{R_2}{R_1+R_2}V.$$

(Note: set $V=0$ 相當於將  短路 (short circuit)
set $I=0$ 相當於將  斷路 (open circuit))

In general, for a linear circuit, we can use the concept of superposition to simply our analysis, by first considering one independent source at a time and then adding up the result.

Why does the concept of "superposition" make sense in circuit analysis?

→ Because ① each independent source contributes to the response of circuit "individually" and the contribution is independent from the contribution of any other independent source, and

② independent sources are assumed to have no resistance (see P13 of this note).

Why does the concept of "equivalence" make sense in circuit analysis?

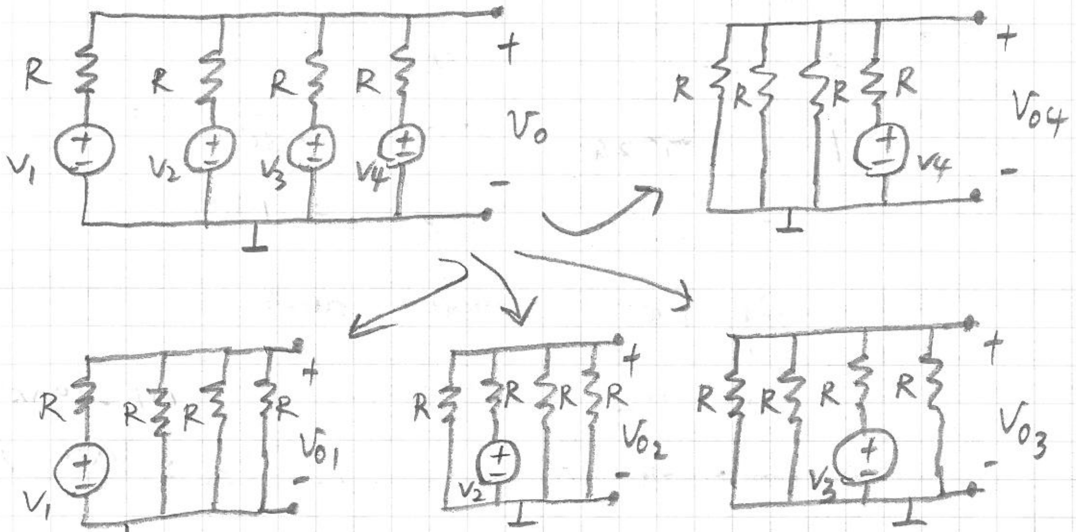
→ Because as long as the i-v characteristic are identical, from input/output viewpoint of a system, what's inside doesn't matter.

Therefore, we may replace some part of a circuit by its equivalence, solely for the purpose of simplifying our analysis. It is an extremely useful trick in engineering!

For example, we may use 訊號產生器 to feed an equivalent input to a system, emulating some physical input circuit.

Example of the use of superposition:

find $V_o = ?$



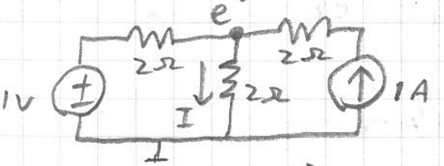
$$V_{o1} = \frac{\frac{1}{3}}{1+\frac{1}{3}} V_1 , \quad V_{o2} = \frac{\frac{1}{3}}{1+\frac{1}{3}} V_2 , \quad V_{o3} = \frac{\frac{1}{3}}{1+\frac{1}{3}} V_3 , \quad V_{o4} = \frac{\frac{1}{3}}{1+\frac{1}{3}} V_4$$

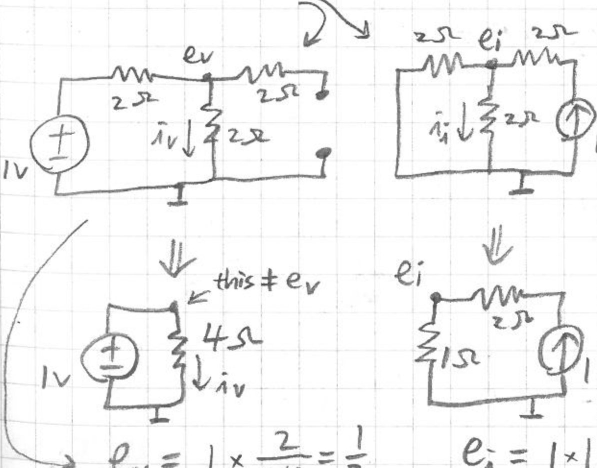
$$= \frac{1}{4} V_1 \qquad = \frac{1}{4} V_2 \qquad = \frac{1}{4} V_3 \qquad = \frac{1}{4} V_4$$

$$\Rightarrow V_o = V_{o1} + V_{o2} + V_{o3} + V_{o4} = \frac{1}{4}(V_1 + V_2 + V_3 + V_4)$$

*

Another example: find $I = ?$





$e_v = 1 \times \frac{2}{2+2} = \frac{1}{2}$ $\quad e_i = 1 \times 1 = 1$

$e = e_v + e_i = \frac{3}{2}$

$I = \frac{e - 0}{2\Omega} = 0.75\ A$

*

Alternatively, we may compute I directly:

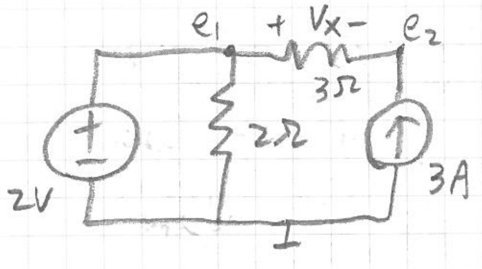
$i_v = \frac{1}{4}$ $\quad i_i = \underbrace{\frac{2}{2+2} \times 1}_{\text{current divider}} = \frac{1}{2}$

$\Rightarrow I = i_v + i_i$

$= \frac{1}{4} + \frac{1}{2} = 0.75\ A$

*

Some further example of the use of node method:



find $V_x = ?$

$e_1 = 2V$

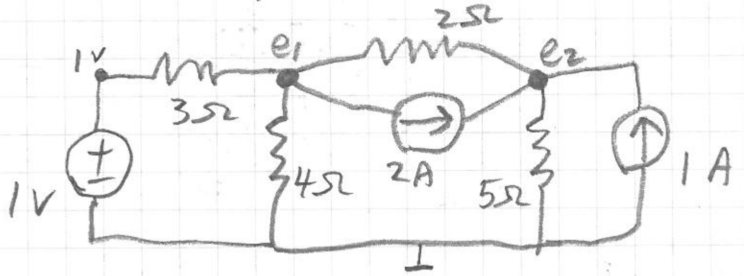
KCL: $\frac{e_1 - e_2}{3\Omega} + 3 = 0$

$\Rightarrow e_2 = 11\,V$

$\Rightarrow V_x = e_1 - e_2 = -9\,V$ ※

Compare this with the use of basic method as we did on P20 of this note! ($V_x = -9V$ there)

Another example: find e_1 and e_2



KCL on e_1: $\frac{e_1 - 1}{3} + \frac{e_1}{4} + \frac{e_1 - e_2}{2} + 2 = 0$

KCL on e_2: $-2 + \frac{e_2 - e_1}{2} + \frac{e_2}{5} - 1 = 0$

$$\Rightarrow \begin{cases} 4e_1 - 4 + 3e_1 + 6e_1 - 6e_2 + 24 = 0 \\ -20 + 5e_2 - 5e_1 + 2e_2 - 10 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} 13e_1 - 6e_2 = -20 \\ -5e_1 + 7e_2 = 30 \end{cases}$$

$\Rightarrow 13e_1 - 6\left(\frac{1}{7}(30 + 5e_1)\right) = -20$

$\Rightarrow 91e_1 - 180 - 30e_1 = -140$

$\Rightarrow 61e_1 = 40 \Rightarrow e_1 \approx 0.655$, $e_2 = \frac{1}{6}(13e_1 + 20)$

$= \frac{1}{6}(13 \times 0.655 + 20) = 4.75$ ※