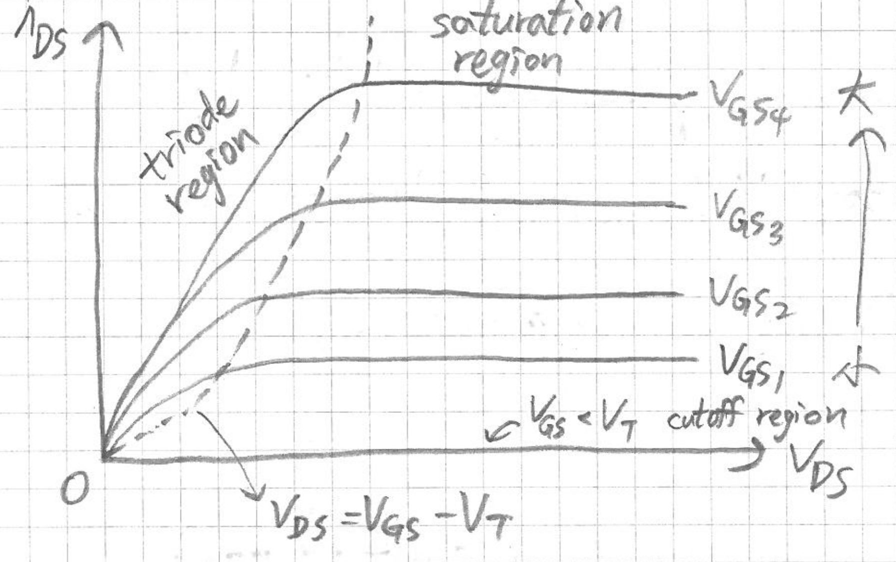
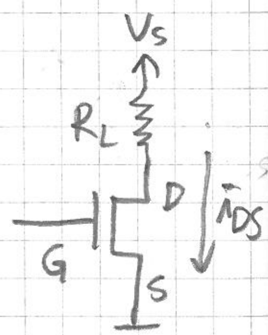
P68 ☆ The SCS model of a MOSFET

switch-current source

Compared to the S model and the SR model, the SCS model is a more accurate MOSFET model (closer to the real physical characteristic).

In the SCS model, a MOSFET can operate with three very different behaviors, and we say it has three "operational regions".

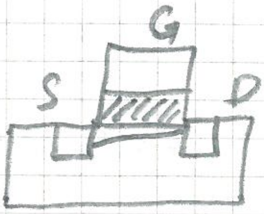




Conditions for each region:

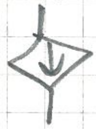
- cutoff: $V_{GS} < V_T$, i.e., $V_{GS} - V_T < 0$
- saturation: $0 \leq V_{GS} - V_T \leq V_{DS}$
- triode: $V_{DS} < V_{GS} - V_T$

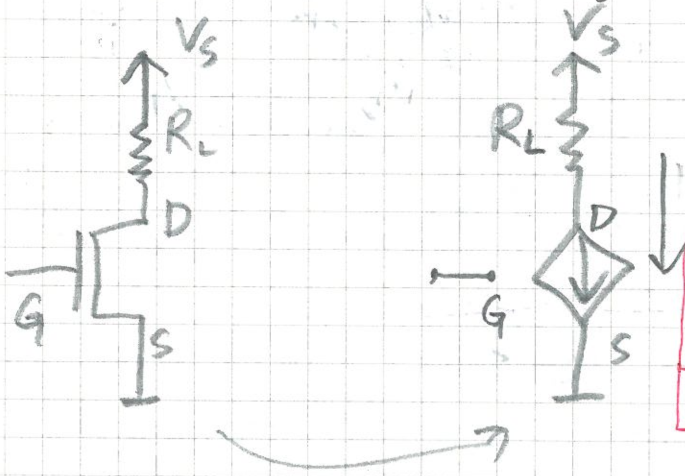
In the saturation region, current i_{DS} would stay the same as we keep increasing V_{DS}, because the channel between source and drain has become stable:



(review P_{58}, P_{66-67})

Therefore, we may consider such a behavior as if there is a "current source" (P_{13})
But i_{DS} would depend on voltage V_{GS}, and thus we say it is like a "voltage-controlled current source". symbol:





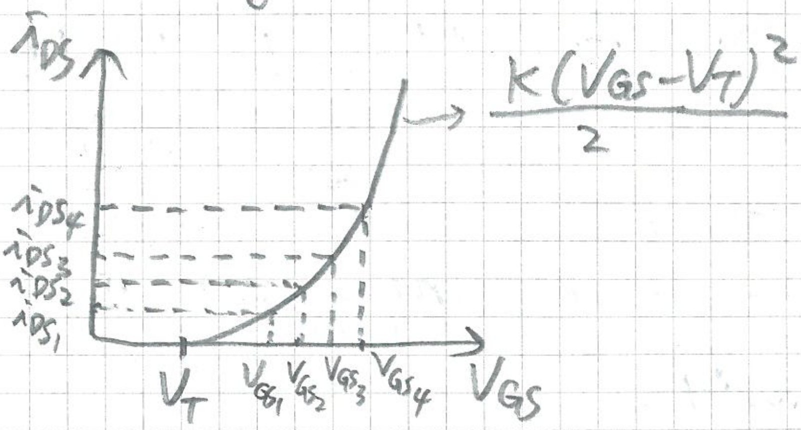
$$\boxed{i_{DS} = f(V_{GS}) = \frac{K(V_{GS}-V_T)^2}{2}}$$
(according to physics)

Note that K is a coefficient; unit: mA/V^2

if $0 \le V_{GS}-V_T \le V_{DS}$
$\rightarrow$ in saturation region

Pno

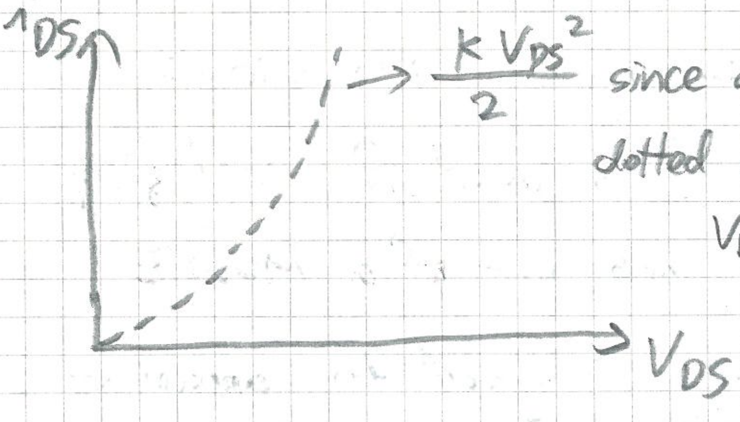
Plotting the $i_{DS}-V_{GS}$ relation, we see:



Therefore, if $|V_{GS_1}-V_{GS_2}| = |V_{GS_2}-V_{GS_3}|$
$= |V_{GS_3}-V_{GS_4}|$

we will have

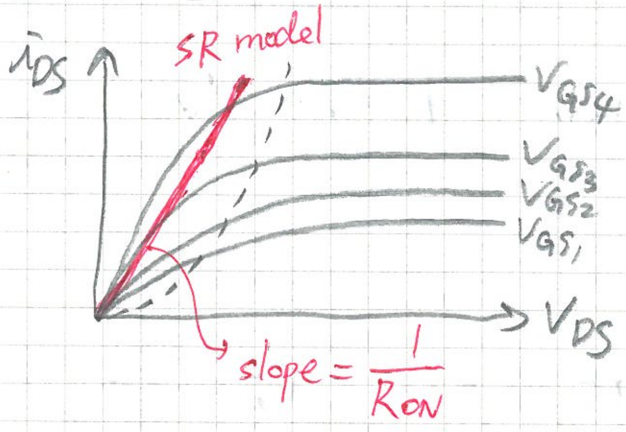
$$|i_{DS_1}-i_{DS_2}| < |i_{DS_2}-i_{DS_3}| < |i_{DS_3}-i_{DS_4}|$$



since along this dotted curve we have $V_{DS} = V_{GS} - V_T$

$$\Rightarrow \boxed{i_{DS} = \begin{cases} 0 & \text{for } V_{GS} < V_T \\ \frac{K(V_{GS}-V_T)^2}{2} & \text{for } 0 \le V_{GS}-V_T \le V_{DS} \end{cases}}$$

From the aspect of the SCS model, the R_{ON} in the SR model can be thought of as a piecewise approximation of the $i_{DS}-V_{DS}$ relation in the triode region in (P39) (Section 4.4 in textbook) the SCS model:



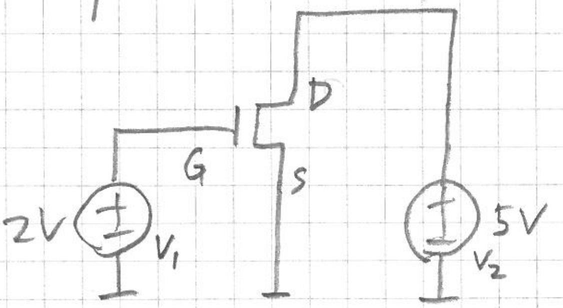
We may summarize in this way:

The S model is good for analyzing the ON/OFF behavior of a MOSFET.

The SR model is good for approximating the MOSFET's behavior in the triode region.

P72 Let's work on two examples to get ourselves familiar with the SCS model and its analysis:

Example 1.



Assuming
$V_T = 1\,V$
$K = 1\,mA/V^2$

① $i_{DS} = ?$ since $V_{GS} = 2V > V_T$
and $V_{DS} = 5V > V_{GS} - V_T = 1$,
we see the MOSFET is operating in the saturation region $\Rightarrow i_{DS} = \frac{K(V_{GS}-V_T)^2}{2} = 0.5\,mA$

② If we keep V_1 and decrease V_2, at what condition of V_2 will the MOSFET enter the triode region?

→ As long as $V_{DS} \geq V_{GS} - V_T$ the MOSFET will stay in the saturation region

→ if $V_2 = V_{DS} < V_{GS} - V_T = 1\,V$, the MOSFET will enter the triode region.

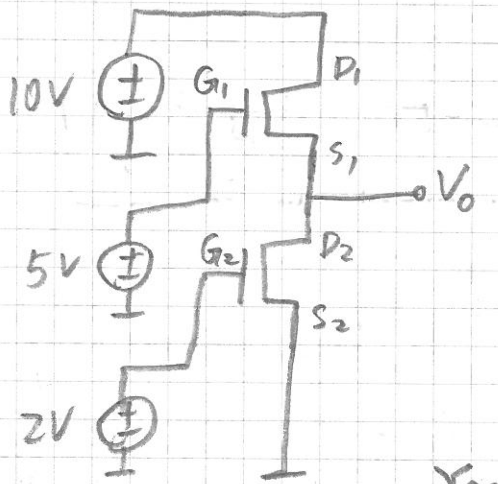
③ If we keep V_2, what would be the range of V_1 for the MOSFET to stay in the saturation region?

$\rightarrow \quad V_{DS} \geq V_{GS} - V_T$

$\rightarrow \quad 5 \geq V_{GS} - 1 \rightarrow V_{GS} \leq 6$

$\rightarrow \quad V_1 = V_{GS} \leq 6$ ※

Example 2.



Assuming $V_T = 1\ V$

$k = 4\ mA/V^2$

If $V_0 = 3$, are both MOSFETs operating in the saturation region?

Yes, since $V_{G_1S_1} - V_T$

$= (5-3) - 1 = 1 > 0$

and $\underbrace{V_{G_1S_1} - V_T}_{=1} \leq \underbrace{V_{D_1S_1} = 10 - 3 = 7}$

And $V_{G_2S_2} - V_T = 2 - 1 = 1 > 0$

and $\underbrace{V_{G_2S_2} - V_T}_{= 2-1} \leq \underbrace{V_{D_2S_2}}_{=3}$

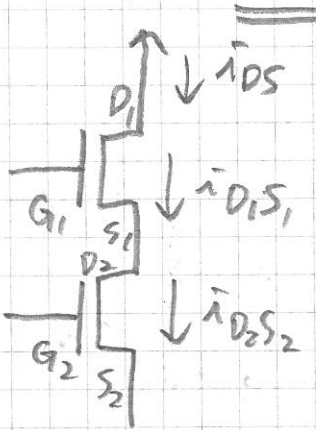
And $V_{G_1S_1} = V_{G_2S_2}$ correctly implies $i_{D_1S_1} = i_{D_2S_2}$ ※

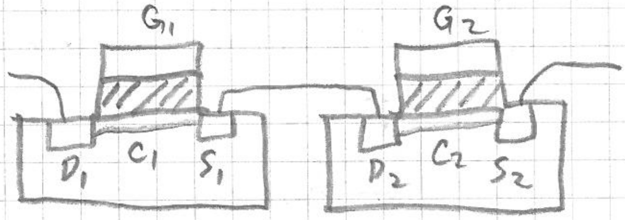
P74 (Note on 2020/6/1 3PM:

I apologize for the confusion regarding whether $V_0 = 2$ in Example 2 makes sense or not.

After some more thoughts, I think what I said this morning is wrong.

It is <u>wrong</u> that $i_{DS} = i_{D_1S_1} + i_{D_2S_2}$





In physics, $i_{D_1S_1}$ is constrained by channel C_1, and $i_{D_2S_2}$ is constrained by channel C_2. It should be that

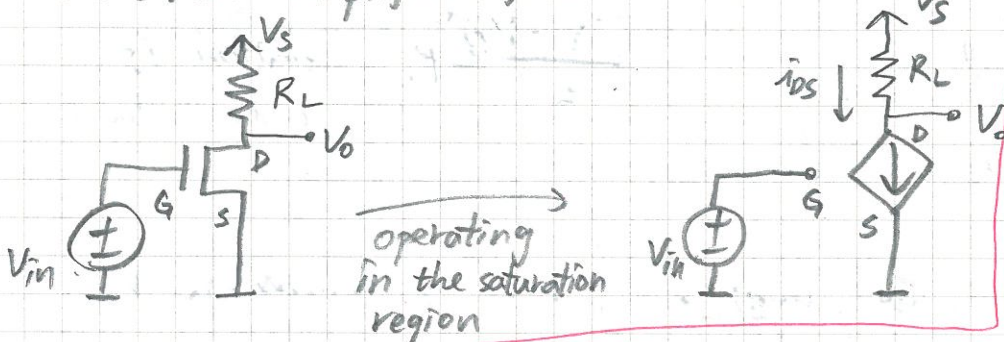
$$i_{DS} = i_{D_1S_1} = i_{D_2S_2}.$$

And therefore $V_0 = 2$ does NOT make sense.)

Now let's see how we may leverage a MOSFET operating in the saturation region for some useful purposes.

It turns out that in the saturation region a MOSFET may be used to amplify a signal. Amplifiers are used in many real-world applications and systems. Headphones and speakers are two examples.

- MOSFET Amplifier, Version 1



The voltage-controlled current source gives

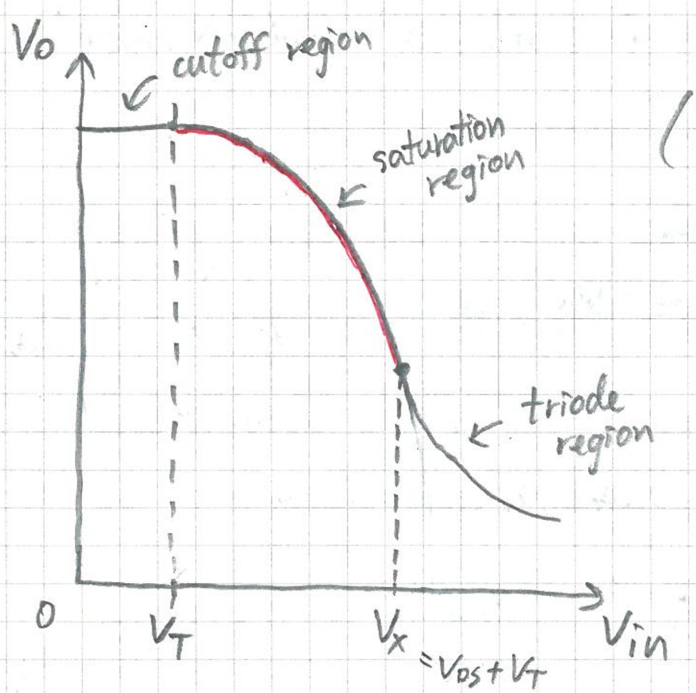
$$i_{DS} = \frac{K(V_{GS}-V_T)^2}{2}$$ (see P69)

and in addition, by Ohm's law $i_{DS} = \dfrac{V_S - V_O}{R_L}$

$$\Rightarrow \frac{K(V_{GS}-V_T)^2}{2} = \frac{V_S - V_O}{R_L} \Rightarrow \boxed{V_O = V_S - K\cdot\frac{(V_{in}-V_T)^2}{2}\cdot R_L}$$

and that $V_{GS} = V_{in}$

P76 following the equation on P75, we have



(compare this to the plots on P57 and P65)

The $V_o = V_s - \underbrace{\frac{K(V_{in}-V_T)^2}{2} R_L}_{\text{a parabola}}$ relation is only valid when $\underline{V_T \leq V_{in} \leq V_x}$.

Later we will study how to determine V_x. (P81)

The slope $\frac{V_o}{V_{in}}$ is also the ratio between V_{in} and V_o. We see that in some part of the saturation region the magnitude of the slope is greater than one. Therefore we may amplify V_{in} by $\frac{V_o}{V_{in}}$ times and output the result as V_o.

The "gain" of the amplifier is defined to be $\frac{V_o}{V_{in}}$.

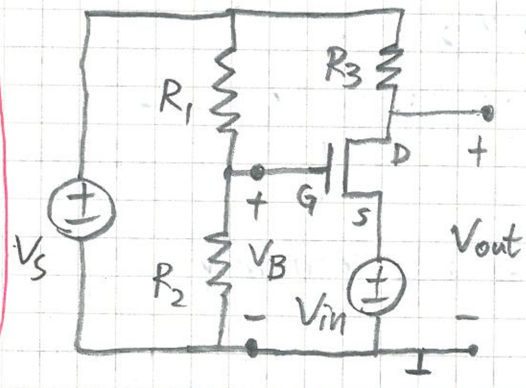
Using this MOSFET amplifier, however, the output will be inverted, which might not be what we want:

$$\begin{cases} V_{in} \downarrow \Rightarrow V_o \uparrow \\ V_{in} \uparrow \Rightarrow V_o \downarrow \end{cases}$$

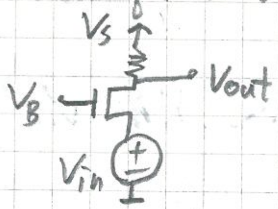
(can be verified by the sign of the slope in the V_o - V_{in} plot.)

Let's study an alternative design ↓

- MOSFET Amplifier, Version 2



In this configuration, we use one voltage source to provide voltages to the gate and the drain. This is equivalent to:



$$V_{GS} = V_B - V_{in}$$
$$= \left(V_s \times \frac{R_2}{R_1+R_2}\right) - V_{in}$$

In saturation region, we have

$$i_{DS} = \frac{K(V_{GS}-V_T)^2}{2}$$

and by Ohm's law

$$i_{DS} = \frac{V_s - V_{out}}{R_3}$$

Therefore, we have

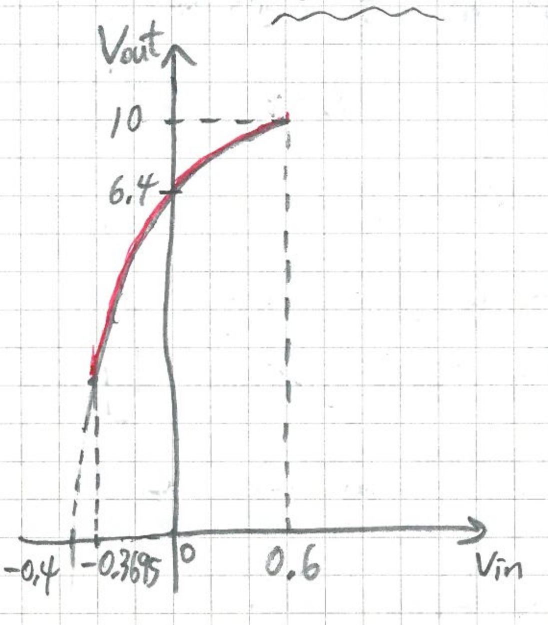
$$\frac{K(V_{GS}-V_T)^2}{2} = \frac{V_s - V_{out}}{R_3}$$

$$\Rightarrow \frac{K\left(V_s \times \frac{R_2}{R_1+R_2} - V_{in} - V_T\right)^2}{2} = \frac{V_s - V_{out}}{R_3}$$

P78 following the equation on P77, suppose that

$$\begin{cases} V_S = 10\ \mathrm{V} \\ R_1 = 84\ \mathrm{k\Omega},\ R_2 = 16\ \mathrm{k\Omega},\ R_3 = 20\ \mathrm{k\Omega} \\ V_T = 1\ \mathrm{V},\ K = 1\ \mathrm{mA/V^2} \end{cases}$$

then $V_B = 1.6$ and $V_{out} = 10 - 10(0.6 - V_{in})^2$



To operate in the saturation region, both of the following conditions must be satisified:

$$\begin{cases} V_{GS} \geq V_T & \text{— ①} \\ V_{GS} - V_T \leq V_{DS} & \text{— ②} \end{cases}$$

from ①, $1.6 - V_{in} \geq 1 \Rightarrow V_{in} \leq 0.6\ \mathrm{V}$

from ②, $(1.6 - V_{in}) - 1 \leq V_{out} - V_{in}$

$\Rightarrow V_{out} \geq 0.6\ \mathrm{V}$

$\Rightarrow 10 - 10(0.6 - V_{in})^2 \geq 0.6$

$\Rightarrow -0.3695\ \mathrm{V} \leq V_{in} \leq 1.5695\ \mathrm{V}$

$-0.3695\ \mathrm{V} \leq V_{in} \leq 0.6\ \mathrm{V}$

Thus we see that it is an amplifier $\left(\frac{V_{out}}{V_{in}} > 0\right)$

and $\begin{cases} V_{in} \uparrow \Rightarrow V_{out} \uparrow \\ V_{in} \downarrow \Rightarrow V_{out} \downarrow \end{cases}$ ※

For the purpose of signal amplification, and for the SCS model in general, we would want to make sure that the MOSFET operates in the saturation region. Given the circuit's parameters, in order to determine whether the MOSFET will operate in the saturation region (or to determine the valid range of values of parameters), we may use the conditions listed on P68 or use graphical analysis.

For example, for the MOSFET amplifier on P75, suppose $V_S = 5V$, $R_L = 1\,k\Omega$, $V_T = 0.8\,V$, $V_{in} = 2.5\,V$ is the MOSFET in the saturation region?

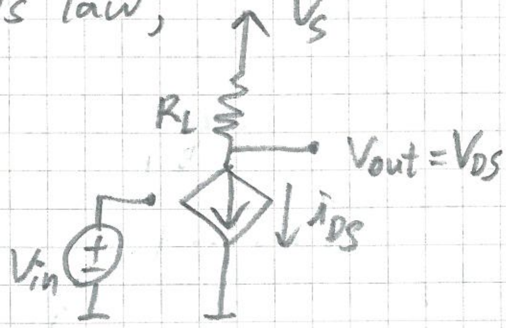
Answer: $V_{GS} = V_{in} = 2.5\,V$

$$V_{DS} = V_O = V_S - K\frac{(V_{in} - V_T)^2}{2} R_L = 4.28\,V$$

and we see that both $\begin{cases} V_{GS} \geq V_T \\ V_{DS} \geq V_{GS} - V_T \end{cases}$

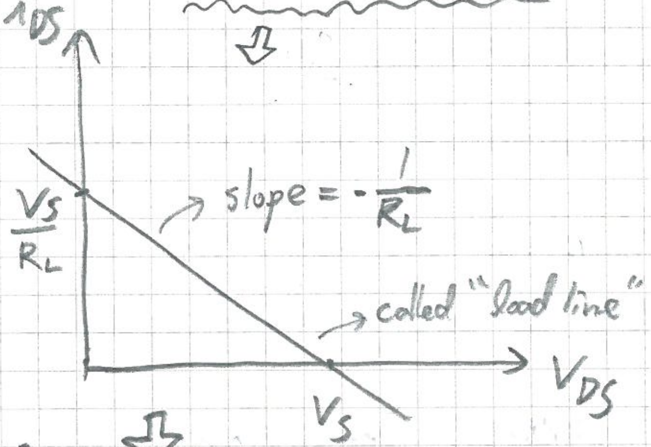
Therefore, it is indeed in the saturation region. #

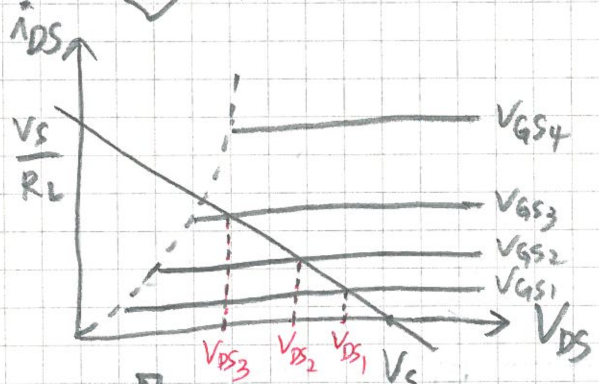
P80 Using graphically analysis, we first observe that, by Ohm's law,



$$i_{DS} = \frac{V_S - V_{DS}}{R_L}$$

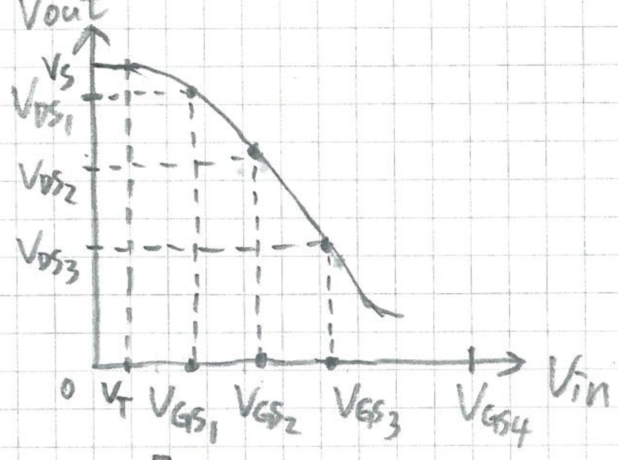
$$\Rightarrow i_{DS} = \frac{V_S}{R_L} - \frac{1}{R_L} V_{DS}$$



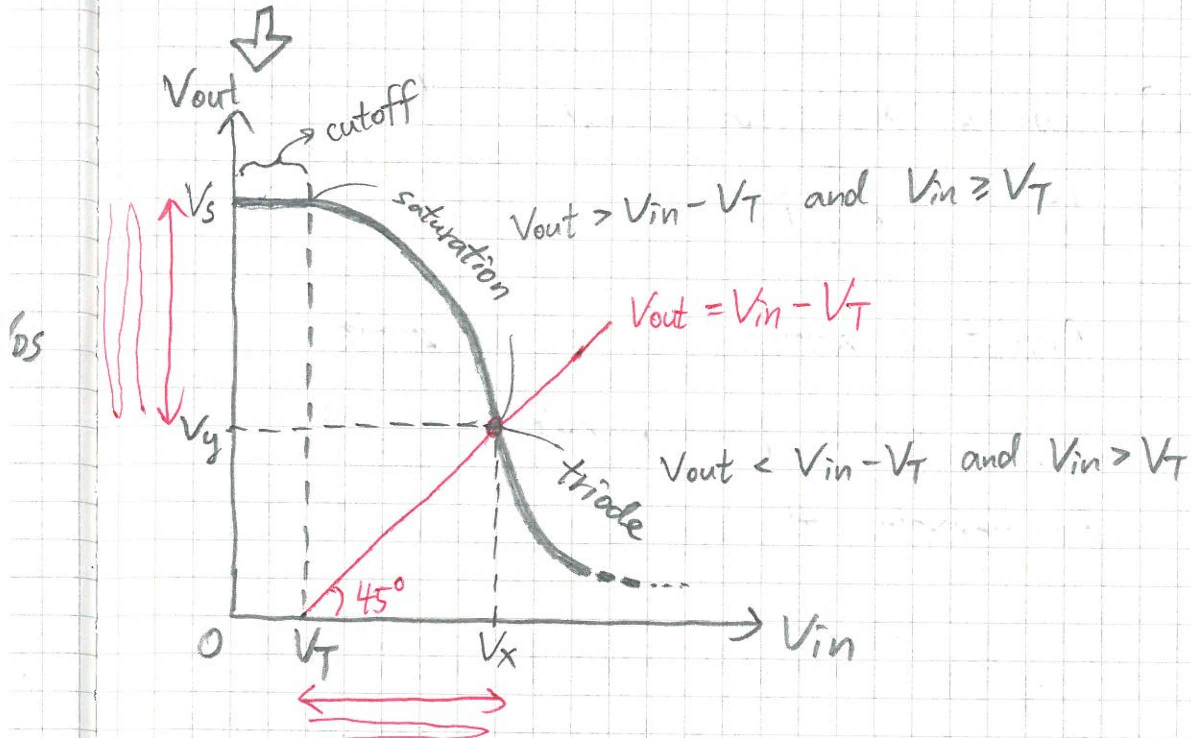


(see the plot on P68)

$$\begin{cases} V_{in} = V_{GS_x} & x = 1 \sim 4 \\ V_{out} = V_{DS_x} & x = 1 \sim 3 \end{cases}$$



(see the plot on P76)



To compute V_x, we have

$$\begin{cases} V_{out} = V_{in} - V_T \\ V_{out} = V_S - K\dfrac{(V_{in}-V_T)^2}{2}R_L \quad (P75) \end{cases}$$

$$\Rightarrow V_{in} - V_T = V_S - K\frac{(V_{in}-V_T)^2}{2}R_L$$

$$\Rightarrow \frac{KR_L}{2}(V_{in}-V_T)^2 + (V_{in}-V_T) - V_S = 0$$

$$\Rightarrow V_{in} - V_T = \frac{-1+\sqrt{1+2V_S R_L K}}{KR_L}$$

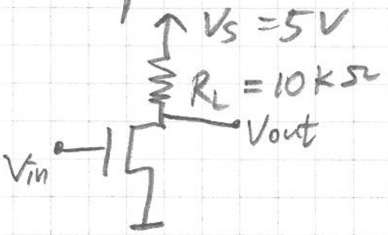
$$\Rightarrow \boxed{\begin{aligned} V_x &= \frac{-1+\sqrt{1+2V_S K R_L}}{KR_L} + V_T \\ V_y &= V_x - V_T \end{aligned}}$$

Comparing analytical analysis and graphical analysis:

- Analytical analysis is as accurate as the model (and formula) can be. For example, the use of $i_{DS} = \frac{K(V_{GS}-V_T)^2}{2}$

- Graphical analysis may be more accurate, provided that the manufacturer of the electronic device often gives "data sheet", which includes the actual measured physical values of, for example, $i_{DS}-V_{DS}$ characteristics.
- Graphical analysis also provides more insights (for example, see P37, 38).

Example:

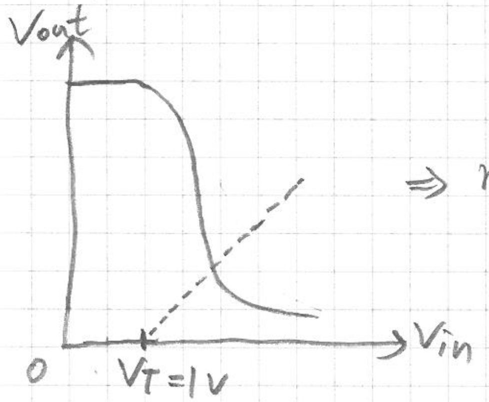


Assuming $V_T = 1V$

$K = 1\,mA/V^2$

What would be the maximum V_{in} that the MOSFET can still stay in the saturation region?

If the $V_{out} - V_{in}$ plot is accurate, we may directly estimate the maximum V_{in}:



$\Rightarrow$ maximum V_{in} is around 2V

Applying analytical analysis, we have

$$V_{in} = \frac{-1 + \sqrt{1 + 2V_S K R_L}}{K R_L} + V_T$$

$$= \frac{-1 + \sqrt{1 + 2 \times 5 \times 10^{-3} \times 10 \times 10^3}}{1 \times 10^{-3} \times 10 \times 10^3} + 1 \approx 1.9\,V$$

$$V_{out} = V_{in} - V_T \approx 0.9\,V$$