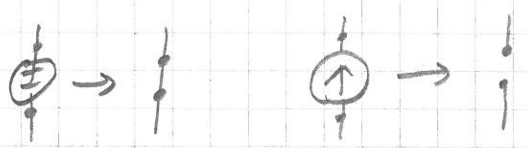
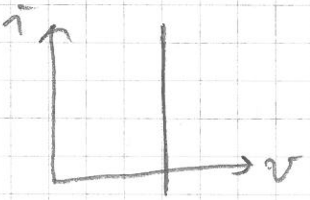
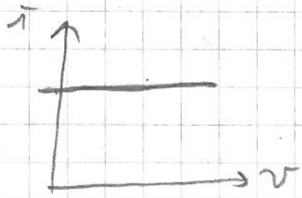
When calculating the input/output resistance, it is important that we suppose all "independent" voltage sources and current sources are turned off:



Why? Because those voltage/current sources theoretically have i-v characteristics that resemble zero/∞ resistance:



ideal voltage source



ideal current source

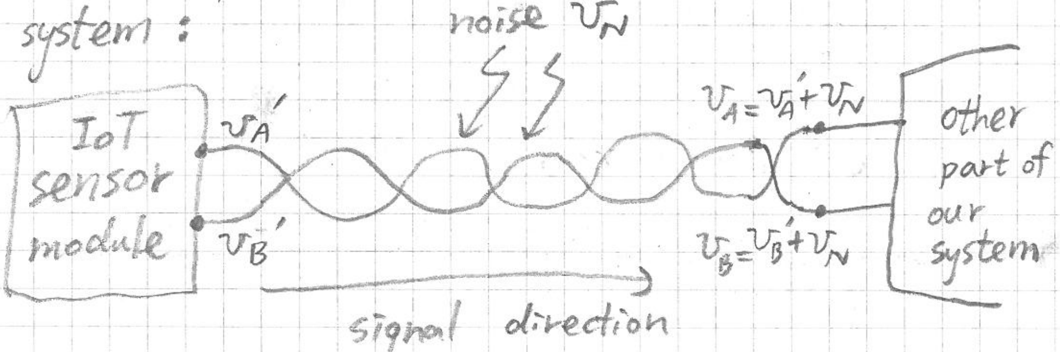
(resistance is the inverse of the slope of the curve.)

For "dependent" voltage sources and current sources, however, we do not always suppose they are turned off, because their values depend on others.

Finally, for small-signal r_i and r_o, we will still consider the corresponding DC biases because it is their existence that determines the structure of the small-signal circuit.

☆ Difference Amplifier

The physical environment in which an electronic system operates often introduces noises to the system:



It would be good if we can ① reduce the noise and ② amplify the original signal.

Difference amplifiers are designed for this purpose.

(Note that the two wires are twisted together so that the same amount of noise infects both signals, which then can be reduced by having the amplifier reduce the magnitude of the common component of the signals.)

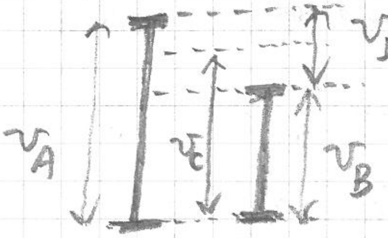
think about it as $v_O = f(v_D, v_C)$

$$v_O = \underbrace{A_d}_{\text{gain}} v_D + \underbrace{A_C}_{\text{gain}} v_C$$

where v_C is the common component shared by v_A, v_B and v_D is the difference between v_A, v_B

P96

We can visualize the setup this way:



$v_D = v_A - v_B$ is called the difference-mode component signal.

$v_C = (v_A + v_B)/2$ is called the common-mode component signal.

Therefore, we may represent v_A and v_B in terms of v_C and v_D:

$$\begin{cases} v_A = v_C + \frac{1}{2} v_D \\ v_B = v_C - \frac{1}{2} v_D \end{cases}$$

A reason to choose $v_C = (v_A + v_B)/2$ is just for convenience; as long as the quantity $v_A - v_B$ is represented by some variable (v_D here), we may choose some other linear combination of $\{v_A, v_B\}$ for v_C. It is convenient if we choose v_C such that $v_A \leq v_C \leq v_B$. Otherwise, for example, letting $v_C = v_B - \Delta v$, then we'll have

$$\begin{cases} v_A = v_C + \Delta v + v_D \\ v_B = v_C - \Delta v \end{cases}$$ which is ugly.

Going back to equation $v_0 = A_D v_D + A_C v_C$,

A_D is called the difference-mode gain, and

A_C is called the common-mode gain.

Our goal is to design a circuit so that

A_D is large and A_C is small (relatively),

to reduce noise.

In particular, people use $\frac{A_D}{A_C}$ to quantify the ability of the circuit to reject noise.

$\frac{A_D}{A_C}$ is called Common-Mode Rejection Ratio.
(CMRR)

We are in particular interested in the small-signal behavior of a difference amplifier. To be specific, represent the small-signal output as $v_o = \underbrace{A_d}_{= \frac{v_o}{v_d}} v_d + \underbrace{A_c}_{= \frac{v_o}{v_c}} v_c$, and we want a large A_d and a small A_c.

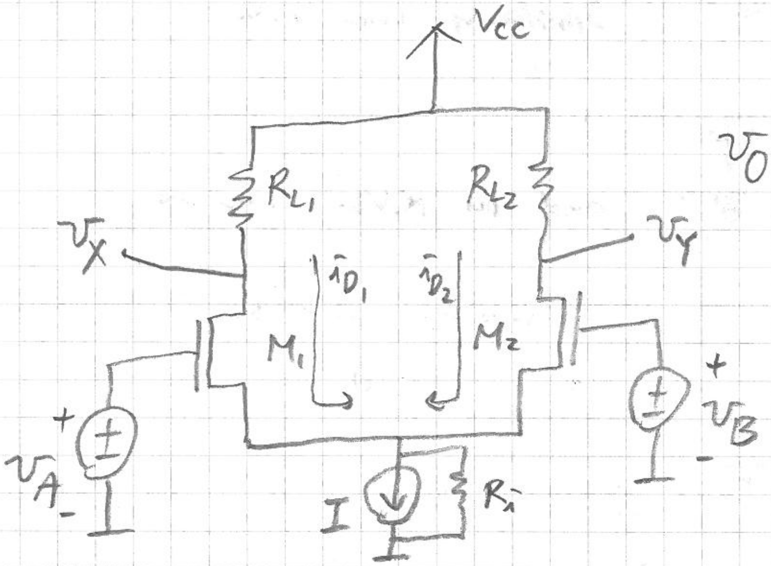
Represent the small-signal inputs as

$$\begin{cases} v_a = v_c + \frac{1}{2} v_d \\ v_b = v_c - \frac{1}{2} v_d \end{cases}$$

Conventionally, people use superposition to consider each mode separately. We'll do this in the following pages.

P98

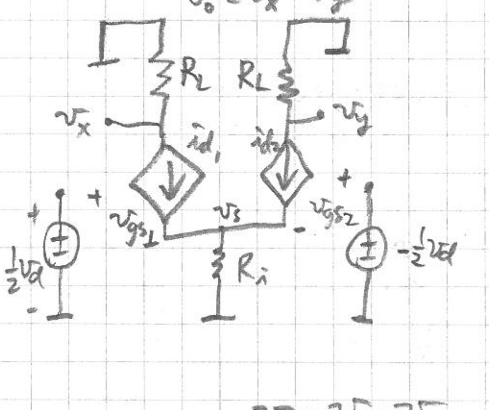
First of all, the overall structure:



$$v_O = v_X - v_Y$$

Assuming symmetry, that is, $R_{L_1} = R_{L_2}$ and MOSFETs $M_1 = M_2$. And assume R_i is very large. Also, assume that both M_1 and M_2 operates under the saturation discipline.

Difference Mode (small signal):



We have $\begin{cases} i_{d_1} = g_m \cdot v_{gs_1} \\ i_{d_2} = g_m v_{gs_2} \end{cases}$

from KVL, we may replace v_{gs_1}, v_{gs_2}

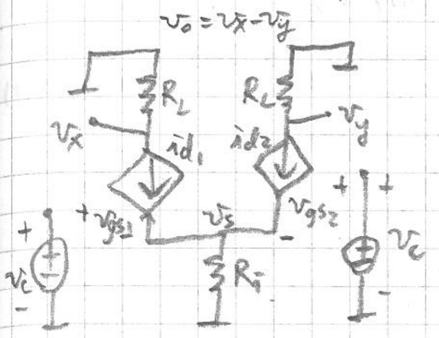
$$\Rightarrow \begin{cases} i_{d_1} = g_m\left(\frac{1}{2}v_d - v_s\right) \\ i_{d_2} = g_m\left(-\frac{1}{2}v_d - v_s\right) \end{cases}$$

$$v_o = v_x - v_y = -i_{d_1} R_L + i_{d_2} R_L$$

$$= -g_m v_d \cdot R_L$$

$$\Rightarrow \boxed{A_d = \frac{v_o}{v_d} = -g_m \cdot R_L}$$

Common Mode (small signal):



Similarly, we have

$$\begin{cases} i_{d1} = g_m \cdot v_{gs1} \\ i_{d2} = g_m \cdot v_{gs2} \end{cases}$$

and by KVL, we get

$$\begin{cases} i_{d1} = g_m (v_c - v_s) \\ i_{d2} = g_m (v_c - v_s) \end{cases}$$

$$v_o = v_x - v_y = -i_{d1} R_L - (-i_{d2} R_L) = 0$$

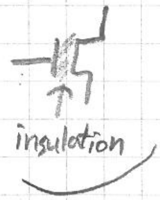
$$\Rightarrow \boxed{A_c = \frac{v_o}{v_c} = 0}$$

Therefore, with $A_d = -g_m R_L$ and $A_c = 0$, we see

$$v_o = A_d v_d + A_c v_c = -g_m R_L v_d$$

which means that, from small-signal viewpoint, the output of the difference amplifier is effectively the amplified version of the signal difference between input v_a and v_b, and with no component from the common component among v_a and v_b!

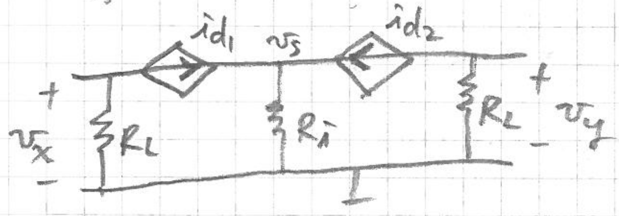
Next, let's analyze the small-signal output resistance of this difference amplifier (the input resistance is ∞, since



To do so, we in the follow page use Thévenin's Theorem to transform the circuit in each mode, and then use superposition to jointly calculate the output resistance.

Note that this is a good strategy because often Thévenin's Theorem simplifies the circuit, and, more importantly, Thévenin's Theorem will yield the same structure of the circuit, which makes it trivial to apply superposition.

Difference Mode (small signal):



Seen from v_x $\Rightarrow$ setting $v_d = 0$, we have

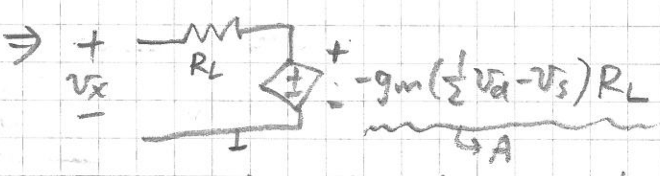
$v_{gs_1} = v_{gs_2} = 0 - v_s = -v_s$,

which implies $i_{d_1} = i_{d_2} = 0$

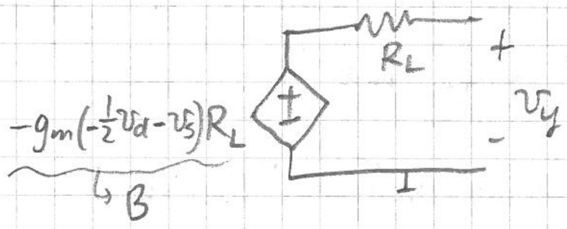
$\Rightarrow R_{TH} = R_L$

$\Rightarrow V_{TH} = -i_{d_1} \cdot R_L = -g_m(\frac{1}{2}v_d - v_s)R_L$

$\Rightarrow$

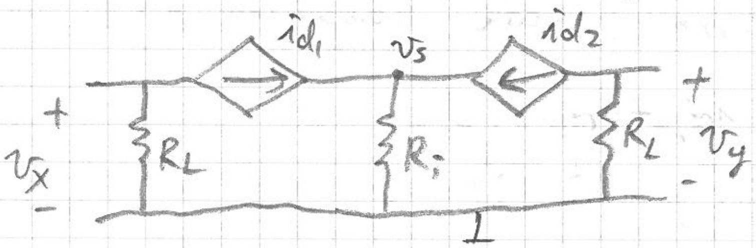


Seen from v_y, using similar derivation, we have



Common Mode (Small signal):

the following derivation will be very similar to what we did for the difference mode, since the only difference in the circuit structure is the different input voltage source:

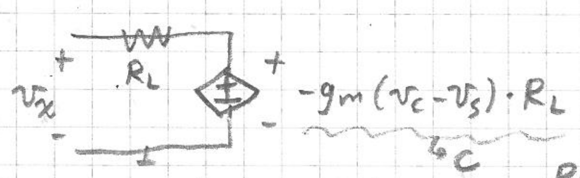


Seen from v_X ⇒ setting $v_C = 0$, we have $v_{gs_1} = v_{gs_2} = -v_S$, which again implies $i_{d_1} = i_{d_2} = 0$

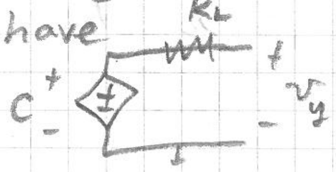
$\Rightarrow R_{TH} = R_L$

$\Rightarrow V_{TH} = -i_{d_1} \cdot R_L = -g_m(v_C - v_S) \cdot R_L$

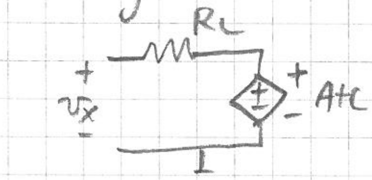
⇒



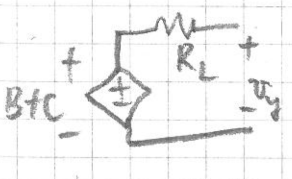
Seen from v_y, by symmetry, we have



Combining the two modes, we have



seen from v_X, and thus output resistance $r_o = R_L$



seen from v_y, and thus output resistance $r_o = R_L$ ※

You might notice that in the previous, the resulting voltage source we got is a bit different from that on P_{435} in the textbook.

Here is some explanation:

- In the common mode, applying KCL we have $i_{d_1} + i_{d_2} = \frac{v_s}{R_i}$

$$\Rightarrow 2g_m(v_c - v_s) = \frac{v_s}{R_i}$$ and since both inputs are v_c

therefore, we can say $i_{d_1} = i_{d_2} = \frac{v_s}{2R_i}$.

Further, because of the biasing current source, I, we can say that $v_s \neq 0$.

- In the difference mode, apply KCL we have $i_{d_1} + i_{d_2} = \frac{v_s}{R_i}$

$$\Rightarrow -2g_m \cdot v_s = \frac{v_s}{R_i}$$ and the textbook said

that since g_m and R_i are independent $\Rightarrow v_s = 0$

So, what we've got in the previous page is equivalent to that in the textbook.

Interestingly, in the difference mode v_s does not necessarily equal zero; it equals zero only because the inputs happened to be $\frac{1}{2}v_d$ and $-\frac{1}{2}v_d$.

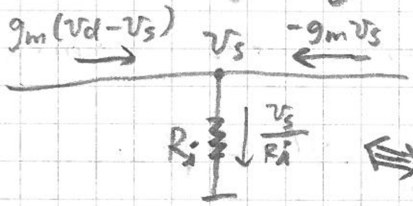
To see this, let's consider the two inputs are v_d and 0 instead:

in this case we will have, from KCL,

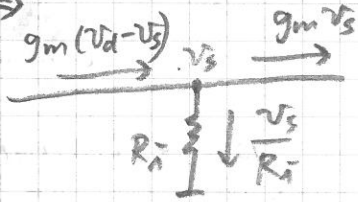
$$g_m(v_d - v_s) + g_m(0 - v_s) = \frac{v_s}{R_i}$$

$$\Rightarrow g_m \cdot v_d - 2 g_m \cdot v_s = \frac{v_s}{R_i}$$

which does not necessarily imply $v_s = 0$



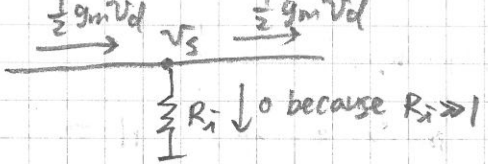
$\Leftrightarrow$



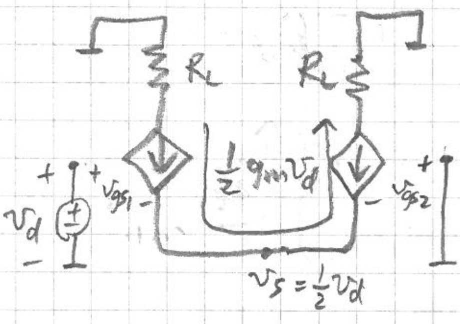
$$\Rightarrow g_m v_d = \left(2g_m + \frac{1}{R_i}\right) v_s$$

$$\Rightarrow v_s = \frac{g_m}{2g_m + \frac{1}{R_i}} \cdot v_d$$

if $R_i \gg 1$, this implies $v_s = \frac{1}{2} v_d$ and thus



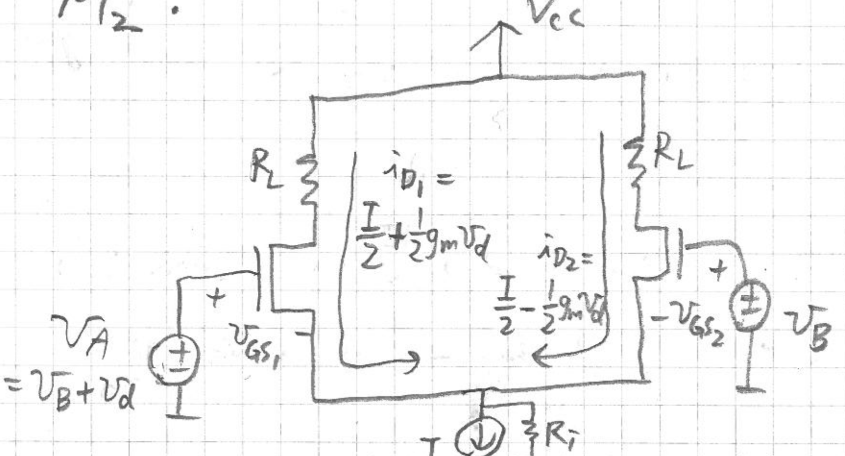
on the whole circuit it means:



notice that now $v_{gs_1} = \frac{1}{2} v_d$ and $v_{gs_2} = -\frac{1}{2} v_d$

Now, it is very interesting to ponder this from the aspect of "total signal"

P.104 ↳ If we think about the case where v_A and v_B differs only by a tiny amount, say, Δv. Then we have $v_D = v_A - v_B = \Delta v$ and $v_d = \Delta v$, $V_D = 0$. The fact that a tiny amount of current, $\frac{1}{2} g_m v_d$, flowing through MOSFETs M_1 and M_2 is equivalent as both a small amount of increase of total current flowing through M_1 and a small amount of decrease of total current flowing through M_2:



which makes sense, because we see that v_{GS_1} being slightly larger than v_{GS_2} will cause i_{D_1} being slightly larger than i_{D_2}.

We may double check this by observing

that $v_{GS_1} = v_{GS_2} + v_d$, and thus

$$i_{D_1} = \frac{1}{2} K (v_{GS_2} + v_d - V_T)^2$$

$$i_{D_2} = \frac{1}{2} K (v_{GS_2} - V_T)^2$$

$$
\begin{aligned}
i_{D_1} - i_{D_2} &= \frac{1}{2} K \left((v_{GS_2} - V_T + v_d)^2 - (v_{GS_2} - V_T)^2 \right) \\
&= \frac{1}{2} K (2 v_{GS_2} - 2V_T + v_d) \cdot v_d \qquad \left(\begin{matrix} a^2 - b^2 \\ = (a+b)(a-b) \end{matrix} \right) \\
&\approx \frac{1}{2} K (2 v_{GS_2} - 2V_T) \cdot v_d \qquad (\because v_d \ll v_{GS_2}) \\
&= K (v_{GS_2} - V_T) \cdot v_d \\
&= g_m \cdot v_d
\end{aligned}
$$

which is equal to

$$i_{D_1} - i_{D_2} = \left(\frac{I}{2} + \frac{1}{2} g_m v_d \right) - \left(\frac{I}{2} - \frac{1}{2} g_m v_d \right) = g_m \cdot v_d$$

Amazing.

- Now, back to P98 and P101 we saw that $A_d = -g_m R_L$ and the output resistance is R_L.

This gives us a dilemma:

If we want to have a larger voltage gain we will have a larger output resistance should we choose to increase R_L; and if we want to have a smaller output resistance we will have to decrease the voltage gain.

Facing trade-offs is common to engineers. Often, a good system design involves finding a good balance that suits a particular application requirement.

Fortunately, in our case here we may have a way to increase the voltage gain while not sacrificing the output resistance.

In Homework 7, Question 2, we see that

$$A_d = -R_L \sqrt{K} \sqrt{I}.$$

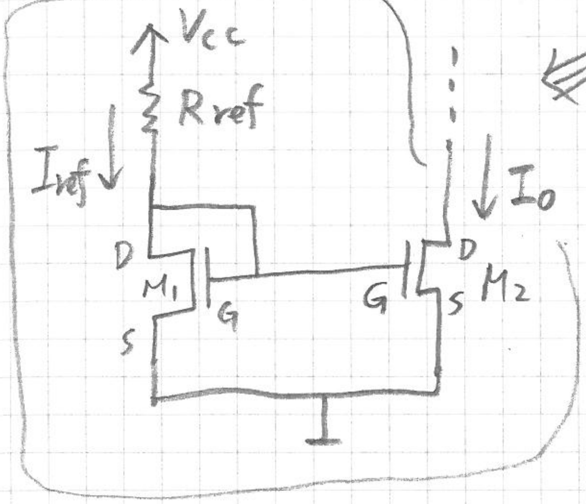
This means that we may increase the voltage gain by increasing I :)

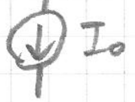
(Perhaps that's also a reason why we use such a biasing current source)

To conclude our discussion of MOSFET, we look at two interesting applications:

① MOSFET as a small-signal resistor (see Example 8.1 in the textbook)

② MOSFET as a current source (next page)



$\Longleftarrow$ 

In this configuration, we'll have $\boxed{I_o = I_{ref}}$ if both MOSFETs are

① identical, and
② operating under the saturation discipline.

The left MOSFET ($\to M_1$) is in saturation because

$V_{GS} = V_{DS}$ (i.e., $V_{GS} - V_T < V_{DS}$)

Then $I_{ref} = \frac{1}{2}\mathbb{K}_1(v_{GS_1} - V_T)^2$

$I_O = \frac{1}{2}\mathbb{K}_2(v_{GS_2} - V_T)^2.$

Since M_1, M_2 have both gates and sources short-circuited,

$v_{GS_1} = v_{GS_2}$ and $\boxed{\dfrac{I_o}{I_{ref}} = \dfrac{\mathbb{K}_2}{\mathbb{K}_1}}$

Considering that $\begin{cases} \mathbb{K}_2 = \mathbb{K}'\sqrt{\frac{W_2}{L_2}} \\ \mathbb{K}_1 = \mathbb{K}'\sqrt{\frac{W_1}{L_1}} \end{cases}$

$\Rightarrow$ We may change the ratio between I_o and I_{ref} by changing the geometric structure of the MOSFET :)

Finally, observe that I_{ref} can be determined by

$$\begin{cases} I_{ref} = \frac{1}{2}\mathbb{K}_1(v_{GS_1} - V_T)^2 \\ V_{CC} - v_{DS_1} = I_{ref} \cdot R_L \\ v_{DS_1} = v_{GS_1} \end{cases}$$

※