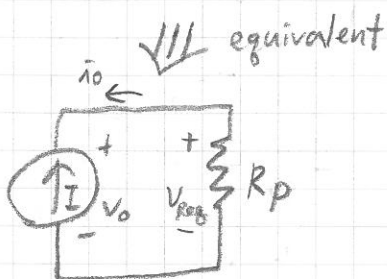
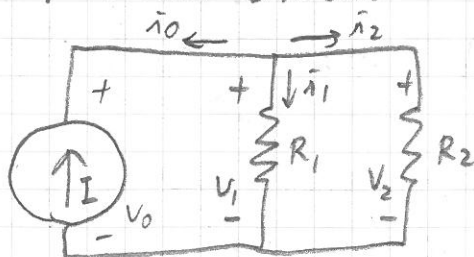


P16 * Current Divider



$$V_1 = R_1 \hat{i}_1 = \frac{R_1 R_2}{R_1 + R_2} I$$

$$V_0 = R_{eq} \cdot I$$

$$\Rightarrow R_p = \frac{R_1 R_2}{R_1 + R_2}$$

$$\Rightarrow \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

Using the "basic method"

$$\begin{cases} \hat{i}_0 = -I \\ V_1 = R_1 \hat{i}_1 \\ V_2 = R_2 \hat{i}_2 \end{cases} \quad \begin{cases} \hat{i}_0 + \hat{i}_1 + \hat{i}_2 = 0 \\ V_0 = V_1 = V_2 \end{cases}$$

$$\Rightarrow \hat{i}_1 + \hat{i}_2 = I$$

$$\hat{i}_1 + \hat{i}_2 = \frac{V_1}{R_1} + \frac{V_2}{R_2} = \frac{V_0 R_2 + V_0 R_1}{R_1 R_2}$$

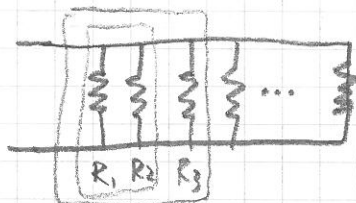
$$= \frac{R_1 + R_2}{R_1 R_2} V_0$$

$$\Rightarrow V_0 = \frac{R_1 R_2}{R_1 + R_2} \cdot I$$

$$\Rightarrow \hat{i}_1 = \frac{V_1}{R_1} = \frac{R_2}{R_1 + R_2} I$$

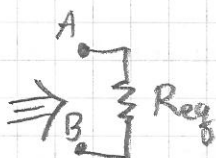
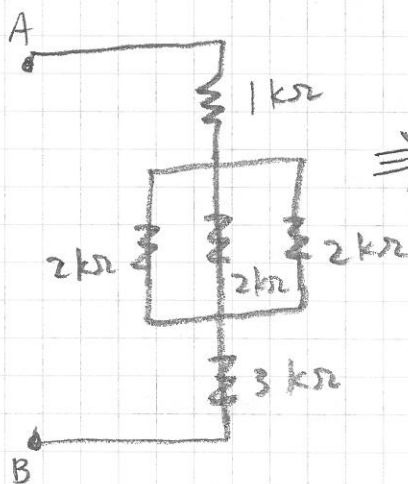
$$\hat{i}_2 = \frac{V_2}{R_2} = \frac{R_1}{R_1 + R_2} I$$

In general, for N resistors connected in parallel,



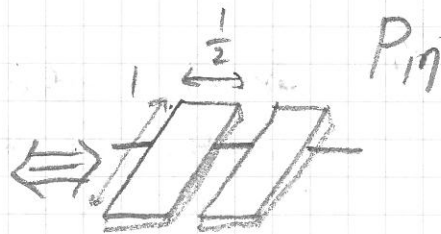
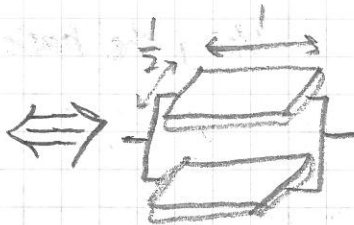
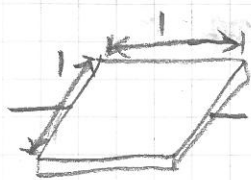
$$\frac{1}{R_p} = \sum_{n=1}^N \frac{1}{R_n}$$

this can be proved by induction.



$$R_{eq} = 1 + \frac{1}{\frac{1}{2} + \frac{1}{2} + \frac{1}{2}} + 3$$

$$= \frac{14}{3} \Omega$$



let $R_1 = R_0$

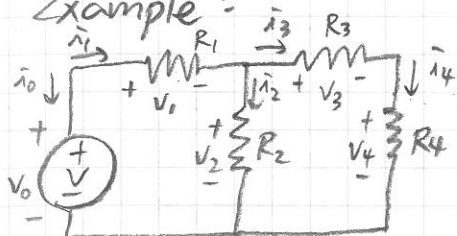
$$R_2 = \frac{2R_0 \cdot 2R_0}{2R_0 + 2R_0}$$

$$= R_0 = R_1$$

$$R_3 = \frac{1}{2}R_0 + \frac{1}{2}R_0$$

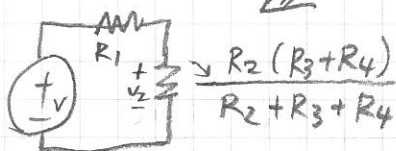
$$= R_0 = R_1$$

Example:



Determine V_1, V_2, V_3, V_4
and i_1, i_2, i_3, i_4 .

Way ②: transformation
using equivalent
resistors



$$i_1 = V / \left(R_1 + \frac{R_2(R_3+R_4)}{R_2+R_3+R_4} \right)$$

$$V_1 = i_1 R_1 = \text{---}$$

$$V_2 = \text{---} \quad (\text{voltage divider})$$

$$i_2 = V_2 / R_2, \quad i_3 = V_2 / (R_3 + R_4)$$

$$V_3 = i_3 R_3, \quad V_4 = i_3 R_4$$

Way ①: we may use the
basic method

2° element law ...

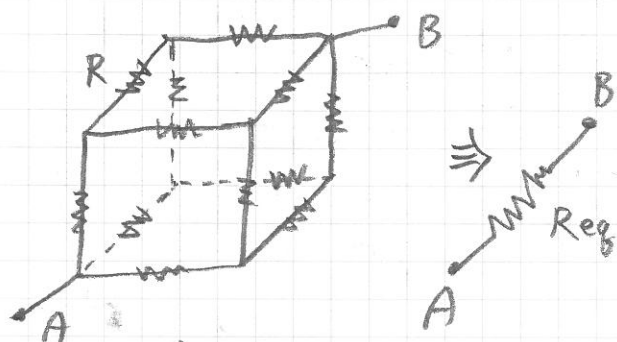
3° KCL & KVL ...

4° 解聯立方程式

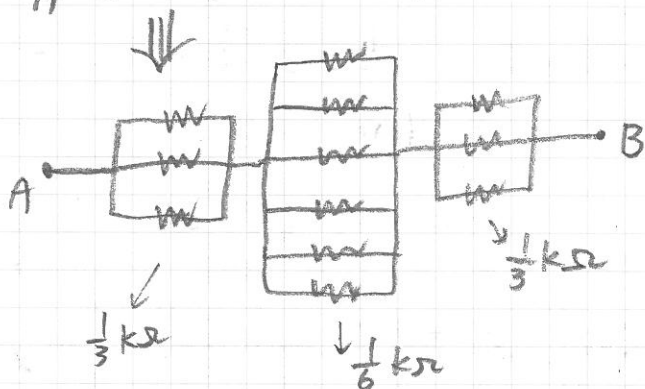
give it a try yourself here:

P18 Sometime we may leverage symmetry to greatly reduce the complexity of analysis:

Example: assuming all resistors are the same on a cube, with resistance $R = 1\text{ k}\Omega$, determine R_{eq}



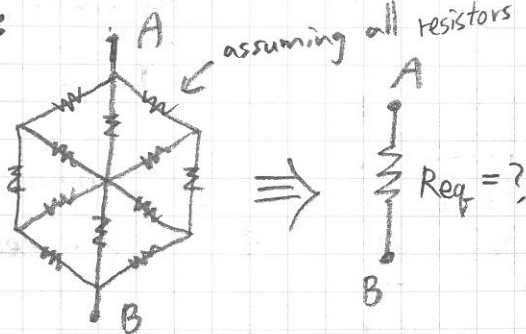
(See example 2.24 in the textbook)



$$R_{eq} = \frac{1}{3} + \frac{1}{6} + \frac{1}{3} = \frac{5}{6}\text{ k}\Omega *$$

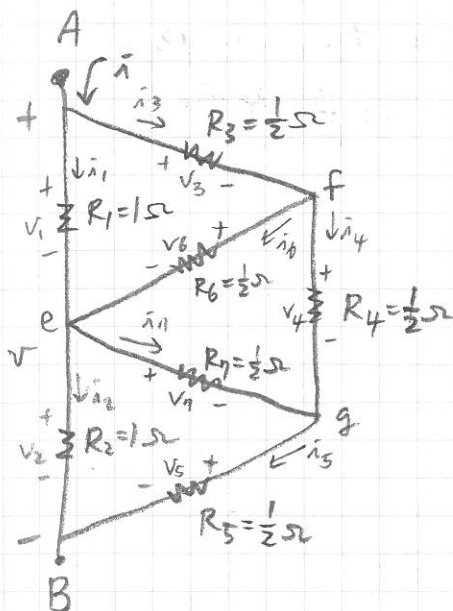
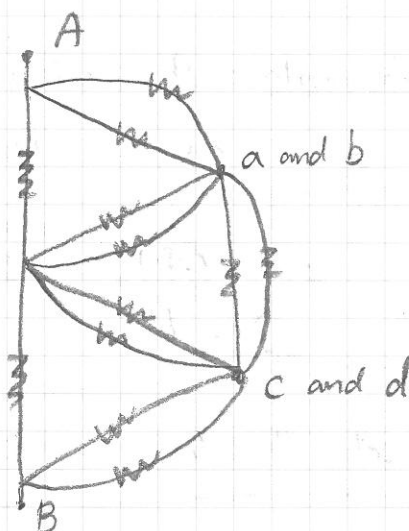
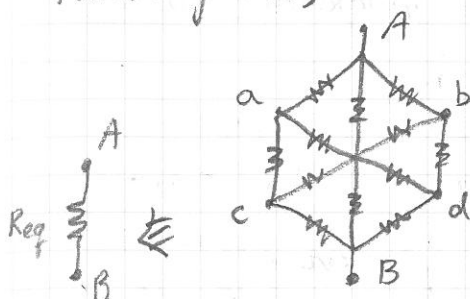
Often, we still need to apply the basic method after reducing a circuit by using equivalent resistors and symmetry !!

Example: assuming all resistors have 1Ω resistance



following P18,

P19



element laws:

$$\begin{cases} V_1 = \bar{i}_1, V_2 = \bar{i}_2, V_3 = \frac{1}{2} \bar{i}_3 \\ V_4 = \frac{1}{2} \bar{i}_4, V_5 = \frac{1}{2} \bar{i}_5, V_6 = \frac{1}{2} \bar{i}_6 \\ V_7 = \frac{1}{2} \bar{i}_7 \end{cases}$$

$$\text{KCL: } \begin{cases} \bar{i} = \bar{i}_1 + \bar{i}_3 = \bar{i}_2 + \bar{i}_5 & (\text{node A \& B}) \\ \bar{i}_1 + \bar{i}_6 = \bar{i}_2 + \bar{i}_7 & (\text{node e}) \\ \bar{i}_3 = \bar{i}_6 + \bar{i}_4 & (\text{node f}) \\ \bar{i}_5 = \bar{i}_7 + \bar{i}_4 & (\text{node g}) \end{cases}$$

$$\text{KVL: } \begin{cases} V_1 + V_2 = V_3 + V_4 + V_5 \\ V_1 = V_3 + V_6 \\ V_2 = V_7 + V_5 \end{cases}$$

$$R_{eq} = \frac{V}{\bar{i}} = \frac{V_1 + V_2}{\bar{i}_1 + \bar{i}_3} = \frac{\bar{i}_1 + \bar{i}_2}{\bar{i}_1 + \bar{i}_3}$$

therefore, 我們可將所有電壓值代換為電流值, 去解電流的聯立方程式!!

$$\begin{bmatrix} 1 & -1 & 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & -1 \\ 2 & 2 & -1 & -1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \bar{i}_1 \\ \bar{i}_2 \\ \bar{i}_3 \\ \bar{i}_4 \\ \bar{i}_5 \\ \bar{i}_6 \\ \bar{i}_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

note: equation ④ is dependent,

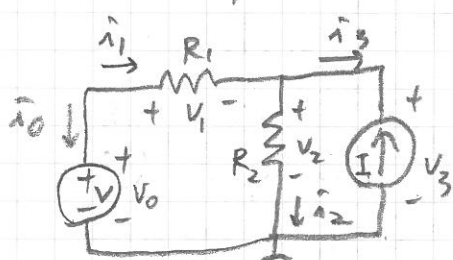
使用高斯消去法 可得 $\begin{cases} \bar{i}_1 = 2\bar{i}_7 \\ \bar{i}_2 = 2\bar{i}_7 \end{cases}$

or www.mathtools.com Matrix Calculator

$$\therefore R_{eq} = \frac{\bar{i}_1 + \bar{i}_2}{\bar{i}_1 + \bar{i}_3} = \frac{2+2}{2+3} = \frac{4}{5} \Omega$$

Calculation is a necessary part in engineering!

P₂₀ Example: A circuit with two independent sources



Determine \hat{i}_2 .

element law

$$\begin{cases} V_0 = V \\ V_1 = R_1 \hat{i}_1 \\ V_2 = R_2 \hat{i}_2 \\ \hat{i}_3 = -I \end{cases}$$

KCL

$$\begin{cases} \hat{i}_0 = -\hat{i}_1 \\ \hat{i}_1 = \hat{i}_2 + \hat{i}_3 \end{cases}$$

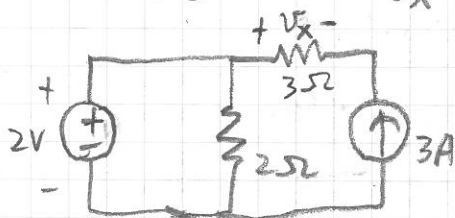
KVL

$$\begin{cases} V_0 = V_1 + V_2 \\ V_2 = V_3 \end{cases}$$

then solve these linear equations (see textbook Pg5).

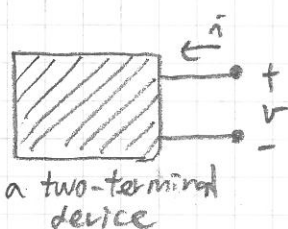
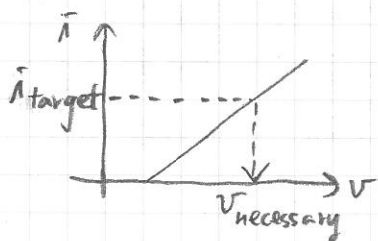
$$\hat{i}_2 = \frac{R_1}{R_1 + R_2} I + \frac{1}{R_1 + R_2} V$$

Exercise: $V_X = ?$



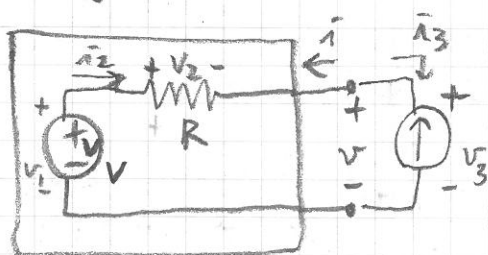
(ans: $V_X = -9V$)

★★ The I-V characteristic of a circuit



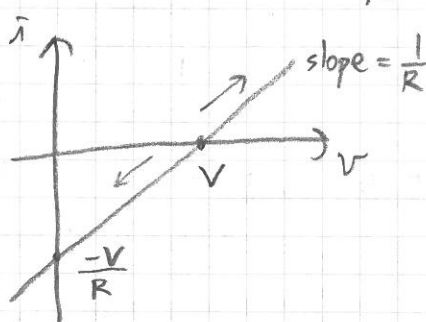
我們可藉由給定 v 量測 \hat{i}
(or vice versa) 來繪製
I-V relation.

If we know the device's internals,
we may also determine its I-V relation:



using the basic method, we have

$$V = V + \hat{i}R \Rightarrow \hat{i} = \frac{1}{R}V - \frac{V}{R}$$



example usage:
預測電流流向

$$\begin{cases} v \geq V \rightarrow \hat{i} \geq 0 \\ v < V \rightarrow \hat{i} < 0 \end{cases}$$