Solution

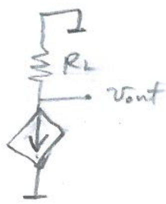
P1

CSU0007 Basic Electronics, Homework 6 (updated on Dec. 27th 18:30PM)

- 4 questions, 100 points total. Submit your work via Moodle before **9PM, Jan 5th, next year** (i.e., 2021).
- Textbook coverage: Section 7.7 and Chapter 8.
- Here is a summary or related textbook examples for the three circuits that we've discussed:
  - **Inverting amplifier** (version 1 as we called it):
    - Example 7.7, and as a driving example throughout Chapter 8
  - **Non-inverting amplifier** (version 2 as we called it):
    - Examples 7.12 and 8.5
  - **Source follower** (also known as a voltage buffer):
    - Examples 7.8, 7.10, 7.11, 8.4 and Problem 7.5

1. (40 points) In the following we review some important concepts regarding electronic circuits, in particular for those using MOSFETs. P352 ← in textbook → P406
   1. (20 points) State the *saturation discipline* and the *small-signal discipline*.
   2. (10 points) State the conceptual meaning of *input resistance* and *output resistance* (Note: in general, we should have called it *input impedance* and *output impedance*, since the resistance is just a part of the impedance. The concept of impedance is, however, beyond the scope of this course, and therefore we chose not to talk about it).
   3. (10 points) Following Question 1.2, explain why in practice we would like to have a circuit with large input resistance and small output resistance.

see my lecture note P90-91



2. (20 points) Now, analyze the following circuit (it helps to first review Sections 8.2.2 and 8.2.3), assuming that $V_{CC} = 10$ V, $K = 1$ mA/V^2, $R_L = 10$ kΩ, and $V_T = 1$ V:

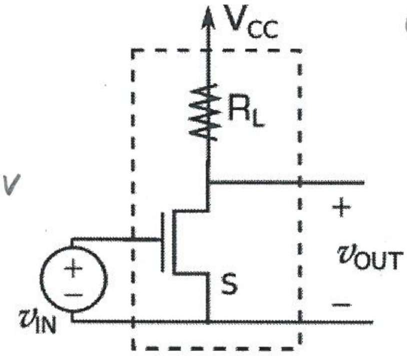
(2.1) $V_{OUT} = V_{CC} - R_L \times I_D$

$= V_{CC} - R_L \times \frac{1}{2} K (V_{GS} - V_T)^2$

$= V_{CC} - R_L \times \frac{1}{2} K (V_{IN} - V_T)^2$

$= 10 - 10 \times \frac{1}{2} \times 1 \times (1.5-1)^2 = 8.75$ V

$\frac{V_{OUT}}{V_{IN}} = \frac{8.75}{1.5} \approx 5.83$ ※



(2.2) $v_{out} = -i_d \cdot R_L$

$= -g_m v_{gs} \cdot R_L = -g_m v_{in} \cdot R_L$

$\Rightarrow \frac{v_{out}}{v_{in}} = -g_m R_L$

$= -K(V_{GS} - V_T) \cdot R_L$

$= -K(V_{IN} - V_T) \cdot R_L$

$= -1 \times (1.5-1) \times 10$

$\Rightarrow \frac{v_{out}}{v_{in}} = -5$ ※

   1. (5 points) Suppose V_{IN}= 1.5 V. Compute $\frac{V_{OUT}}{V_{IN}}$ (that is, the *large-signal* voltage gain).
   2. (10 points) Following Question 2.2, now compute $\frac{v_{out}}{v_{in}}$ (that is, the *small-signal* voltage gain). You see that the small-signal voltage gain may be different from the large-signal voltage gain.
   3. (5 points) Now, by changing V_I, is it possible to make the magnitude of a small-signal voltage gain larger than 13 while still enforcing the saturation discipline? If your answer is yes, determine the minimum value of V_I to achieve so; if your answer is no, determine the maximum achievable magnitude of the small-signal voltage gain while still enforcing the saturation discipline.

(2.3) $g_m R_L = K(V_I - V_T) \cdot R_L$

$= 10(V_I - V_T)$

$< 10 \cdot V_{DS} = 10(V_{CC} - I_D R_L)$

$= 10(10 - 5(V_I - 1)^2)$

$10 - 5(V_I - 1)^2 > 1.3$

$(V_I - 1)^2 < 1.74$

$\Rightarrow V_I < 2.319$

($V_I \neq -0.319$ otherwise, $V_I - V_T < 0$)

$K(V_I - V_T) \cdot R_L > 13$

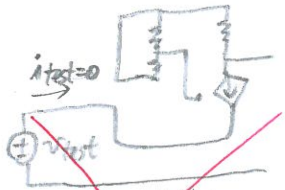
$\Rightarrow V_I - 1 > 1.3$

$\Rightarrow V_I > 2.3$ V ※

P2

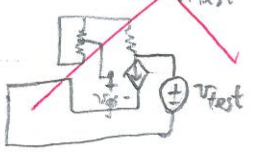
3. (15 points) Compute both the small-signal input resistance and the small-signal output resistance of the following non-inverting amplifier, under the saturation discipline:

③ r_i

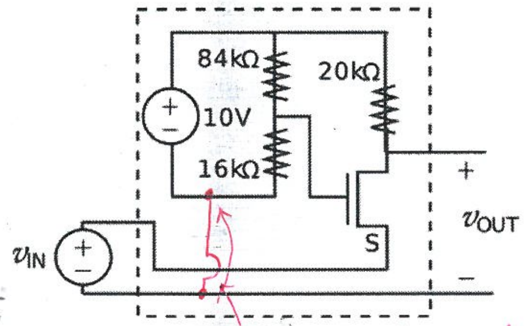


$r_i = \frac{v_{test}}{i_{test}} = \infty$

r_o



$r_o = \frac{v_{test}}{i_{test}} = \frac{1}{g_m}$



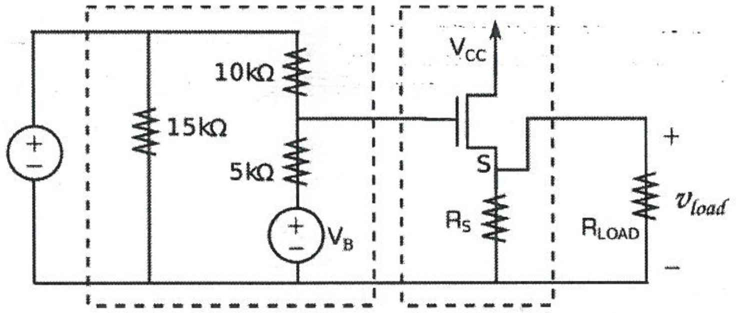
Answer on Page 4

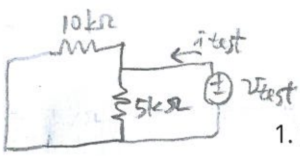
I forgot to ground this. Will give 15 points to all. ~~who have submitted this homework assignment~~.

4. (25 points) In this exercise, we will combine what we've learned so far to analyze a two-stage circuit shown below. Stage 1 is a linear circuit with a DC voltage source, V_B. Stage 2 is a source-follower circuit serving as a voltage buffer. We are interested in the small-signal voltage response on our load R_{LOAD} caused by the small-signal voltage input, v_{in}. We will go through a four-step analysis and will see how a voltage buffer may be useful. Each step is described below.

(4.1) $R_{TH} = \frac{5 \times 10}{5+10} = 3.33\ k\Omega$

$r_o = \frac{v_{test}}{i_{test}} = 10k\Omega // 5k\Omega = 3.33\ k\Omega$

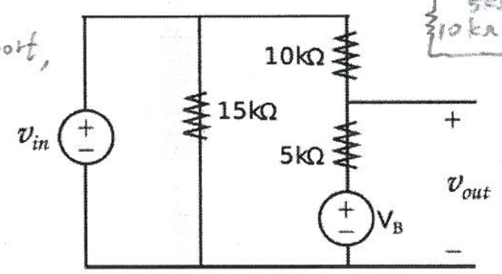




1. (10 points) Let's first consider a circuit *without* a voltage buffer, as shown below. Suppose that $v_{in} = 4$ mV and $V_B = 8$ V. Determine the Thevenin equivalent circuit seen from the output port, and explain why in this case the *small-signal output resistance* is the same as *Thevenin resistance*. Also, compute v_{out}, the small-signal open-circuit voltage.

$r_o = R_{TH}$ because when we determine r_o we are essentially computing the equivalent resistance across the output port, which is the definition of R_{TH}.

$v_{out} = 4 \times \frac{5}{10+5} = 1.33\ mV$ #

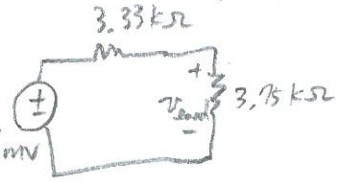


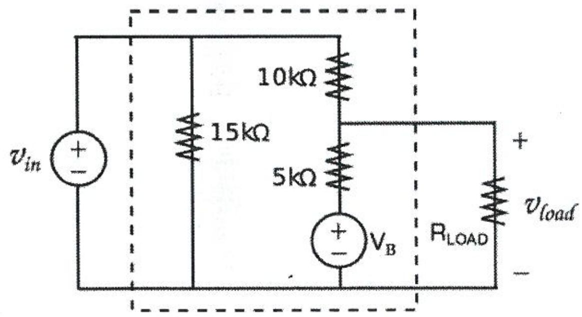
$V' = 8 \times \frac{10}{10+5} = 5.33\ V$

$V_{TH} = v_{out} + V'$ (Superposition) $= 5.33V + 1.33\ mV$ #

2. (5 points) Following the question above, now consider the circuit with $R_{LOAD} = 3.75$ $k\Omega$ attached, as shown below. Compute v_{load}, the small-signal voltage response on R_{LOAD} caused by v_{in}. You should see that v_{load} is smaller than v_{out} computed above (due to the non-zero output resistance).

(4-2)





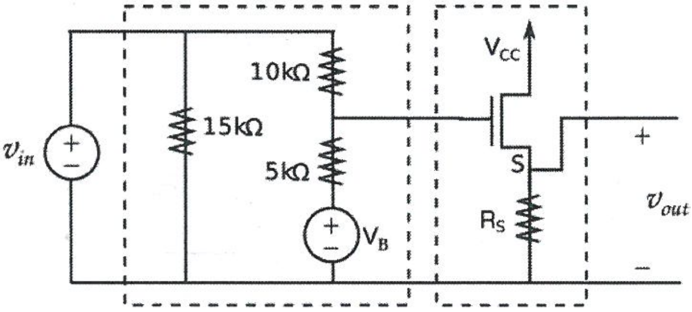
$v_{load} = 1.33 \times \frac{3.75}{3.33+3.75} \approx 0.7\ mV$ #

P3

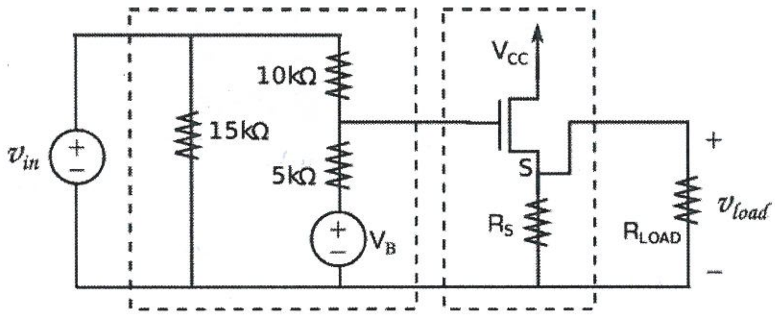
$V_T = 1V$

3. (5 points) Now, we insert a source-follower circuit, as shown below. Suppose that $R_S = 10\ k\Omega$ and $V_{CC} = 10$ V and $\cancel{K} = 1$ mA/V^2. Find the small-signal output resistance. You should find it smaller than that in Question 4.1.

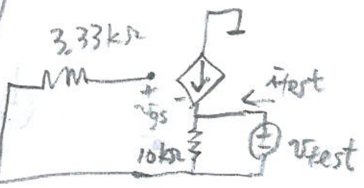
We will give 10 points to all students, because we didn't give the right parameter.

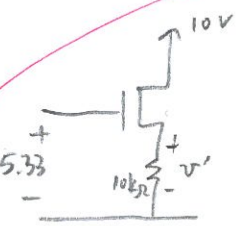


4. (5 points) Now we attach R_{LOAD} again, as the circuit shown below. Determine v_{load}, the small-signal voltage response on R_{load} caused by v_{in}. You should find this voltage *larger* than that in Question 4.2. This demonstrates a benefit of using a voltage buffer: because the voltage buffer has a smaller output resistance, the circuit may waste less energy internally, and more energy may be applied to the actual load (which is R_{LOAD} here).



(4.3)





$\frac{1}{2}1K(5.33 - v' - 1)^2 = \frac{v'}{10}$

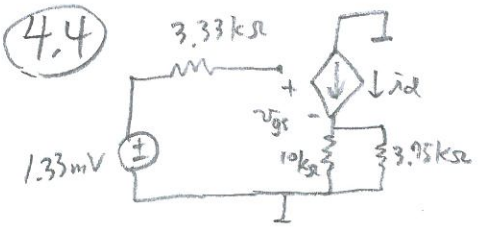
$v' \simeq 3.5$

$v_{test} = (i_{test} + g_m(-v_{test}))\cdot 10$

$\Rightarrow v_{test}(1 + 10g_m) = i_{test} \times 10$

$\Rightarrow r_o = \frac{v_{test}}{i_{test}} = \frac{10}{1+10g_m} = \frac{10}{1+10\times 1K\,(5.33-1)\;(-3.5)} = \cancel{0.225\ k\Omega}$ 1.075 kΩ #

(4.4)



$v_{load} = i_d \cdot (10k\Omega \,//\, 3.75k\Omega)$

$= g_m v_{gs} \cdot \frac{10\times 3.75}{10+3.75}$

$= 4.33 \times v_{gs} \times 2.727$

$= 4.33(1.33 - v_{load}) \times 2.727$

$= 11.8(1.33 - v_{load})$

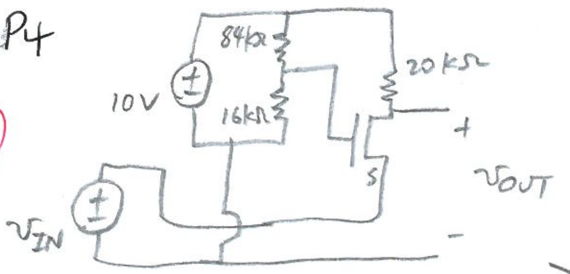
$\Rightarrow v_{load} = \frac{11.8\times 1.33}{12.8} = 1.226$ mV #

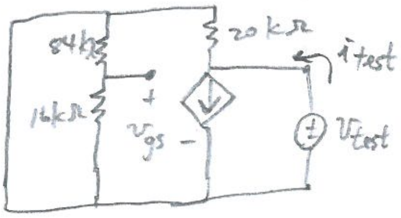
(recall in (4.2) we have $v_{load} = 0.7$ mV)

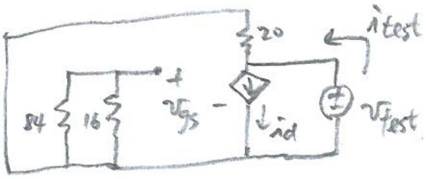
4

P4

3

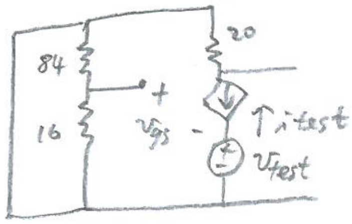


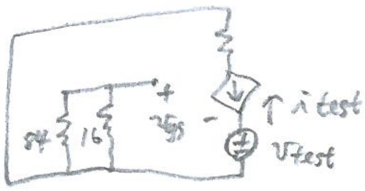




$\because v_{gs} = 0 \Rightarrow i_d = 0$

$\Rightarrow r_o = \dfrac{v_{test}}{i_{test}} = 20k\Omega$ #





$v_{gs} = -v_{test}$

$$r_i = \frac{v_{test}}{i_{test}} = \frac{v_{test}}{-g_m \cdot v_{gs}}$$

$$= \frac{v_{test}}{-g_m(-v_{test})}$$

$$= \frac{1}{g_m}$$ #