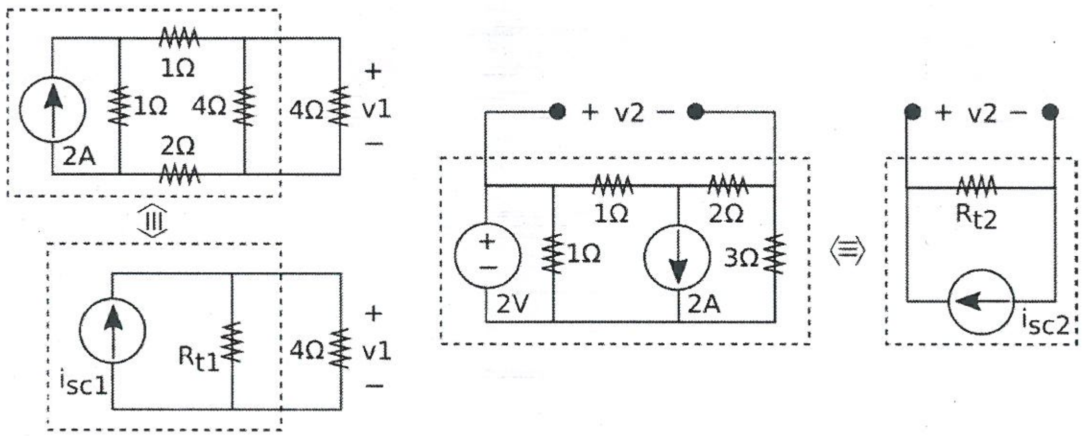
Solution attached

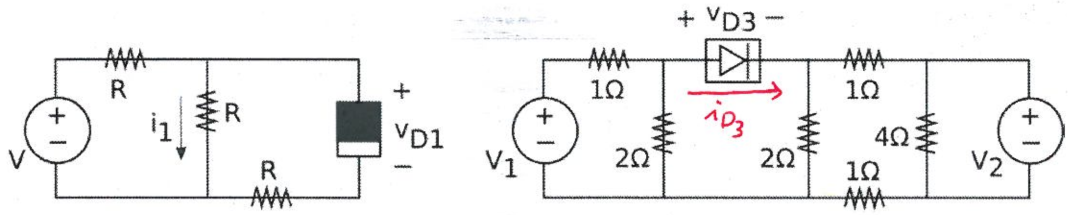
CSU0007 Basic Electronics, Homework 3

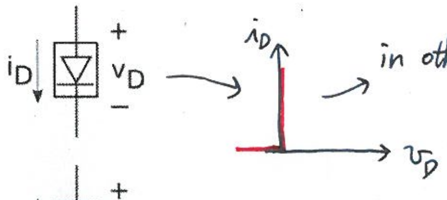
- **IMPORTANT!!** Submit your work via Moodle **before 9AM, Nov 10th, 2020.** We will have a review session on Nov 10th in class. The midterm exam will be on Nov 13th in class.
- Four questions in total. Please clearly label your answer for each question, and clearly state your calculation steps.

1. (42 points) Use Norton's Theorem to find $\{i_{sc1}, R_{t1}, v_1\}$ and $\{i_{sc2}, R_{t2}, v_2\}$. 7 points for each variable.



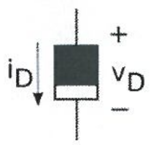
2. (40 points) For the following nonlinear circuits,
   1. (20 points) use the analytical method to determine v_D, and i_1 (the method is covered in class and is described in Section 4.2 in the textbook);
   2. (20 points) if $0 < 2V_2 < V_1$, what would be the value of i_{D3}? Use the piecewise analysis method.





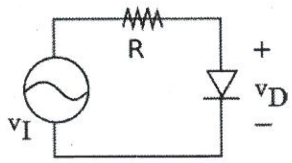
in other word, $v_D \geq 0 \Rightarrow$ short circuit

$v_D < 0 \Rightarrow$ open circuit



$$i_D = \begin{cases} K \cdot v_D^2 & \text{for } v_D > 0 \\ 0 & \text{otherwise} \end{cases}$$

3. (10 points) Use the graphical analysis method to explain the following property: Let $v_{I1} > 0$ and $v_{I2} > 0$, and such that v_{I1} caused v_{D1} and v_{I2} caused v_{D2}. Then we have $\Delta v_I > \Delta v_D$ where $\Delta v_I = |v_{I1} - v_{I2}|$ and $\Delta v_D = |v_{D1} - v_{D2}|$. Illustrate and use your own word to explain why $\Delta v_I > \Delta v_D$.



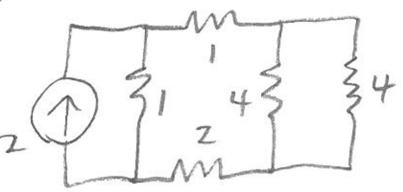
4. (8 points) In class we've talked about an approach to determine R_{TH} in Thevenin's Theorem by using a potentiometer (i.e., a variable resistor 可變電阻). Now, with the additional help of Norton's Theorem, we may determine R_{TH} without using any potentiometer. Think about it and describe an approach to determine R_{TH} by only using a multimeter (i.e., a volt-ohm-milliammeter 三用電表).

Name: CSU0007 **Student ID:** solution for Homework 3

Page 1

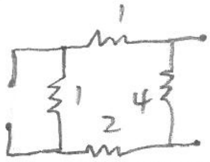
①

PART I



⇓

from

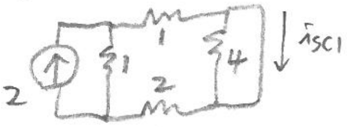


$$R_{t_1} = \frac{4\times(1+1+2)}{4+(1+1+2)} = 2\,\Omega \quad ※$$

⇓

$$i_{sc1} = 2\times\frac{1}{1+(1+2)} = \frac{1}{2}A \quad ※$$

from

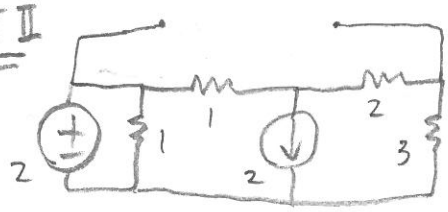


⇕

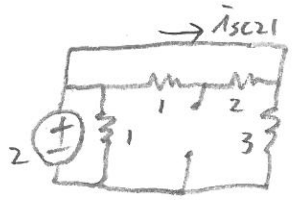
⇒

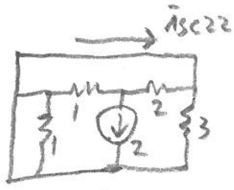
$$v_1 = \left(\frac{1}{2}\times\frac{2}{4+2}\right)\times 4 = \frac{2}{3}V \quad ※$$

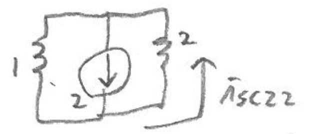
PART II



$i_{sc2} = i_{sc21} + i_{sc22}$ by superposition

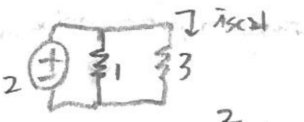




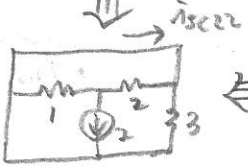


$$i_{sc22} = 2\times\frac{1}{1+2} = \frac{2}{3}A$$

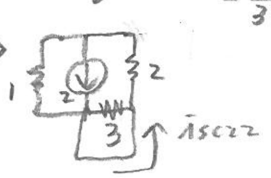
⇕ ⇕ ⇕



$$i_{sc21} = \frac{2}{3}A$$

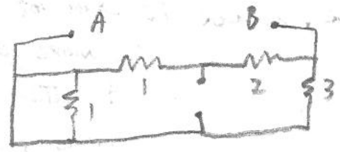


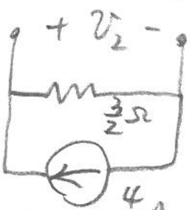
⇔



⇓

from



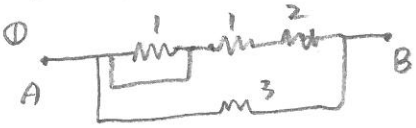


$$v_2 = \frac{4}{3}\times\frac{3}{2} = 2\,V \quad ※$$

$$i_{sc2} = \frac{4}{3}A$$

There are at least two ways to analyze this:

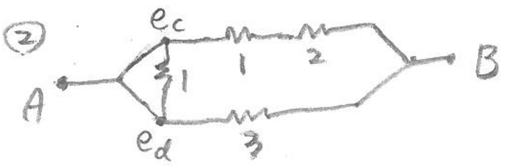
①



short circuit has resistance = 0

$$\Rightarrow R_{t_2} = \frac{3\times\left(\frac{1\times 0}{1+0}+1+2\right)}{3+\left(\frac{1\times 0}{1+0}+1+2\right)} = \frac{3}{2}\,\Omega \quad ※$$

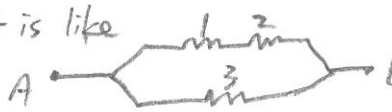
②



since node voltage $e_c = e_d$

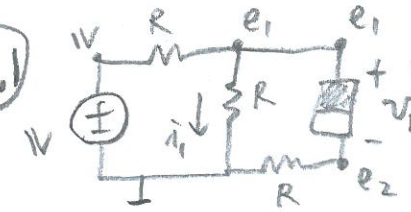
there will be no current flowing through 1 if we attached A, B with a voltage source.

→ so, equivalently, it is like



$$R_{t_2} = \frac{3}{2}\,\Omega$$

(2.1)



$$\begin{cases} \frac{1V - e_1}{R} = \frac{e_1}{R} + i_{D_1} \\ e_1 - e_2 = v_{D_1} \\ i_{D_1} = \frac{e_2}{R} \\ i_{D_1} = 1K\, v_{D_1}^2 \end{cases} \Rightarrow \begin{cases} 1V - 2e_1 = R\cdot 1K \cdot v_{D_1}^2 \\ e_1 = v_{D_1} + e_2 \\ e_2 = R\,1K \cdot v_{D_1}^2 \end{cases}$$

$$\Rightarrow 3R\,1K\, v_{D_1}^2 + 2v_{D_1} - 1V = 0$$

$$\Rightarrow v_{D_1} = \frac{-2 + \sqrt{4 + 12\,1K\,1V R}}{6R\,1K} \quad \#$$

equation 1

first of all, observe that v_{D_1} must be larger than zero.

$$i_1 = \frac{e_1}{R} = \frac{v_{D_1} + e_2}{R} = \frac{v_{D_1} + R\,1K\, v_{D_1}^2}{R}$$

$$= \frac{-2 + \sqrt{4 + 12\,1K\,1VR}}{6\,1K R^2} + 1K\left(\frac{-2 + \sqrt{4 + 12\,1K\,1VR}}{6R\,1K}\right)^2$$

$$= \frac{-2 + \sqrt{4 + 12\,1K\,1VR}}{6\,1K R^2} + \frac{4 + (4 + 12\,1K\,1VR) - 4\sqrt{4 + 12\,1K\,1VR}}{36\,1K R^2}$$

$$= -\frac{1}{3}\frac{1}{1KR^2} + \frac{2}{9}\frac{1}{1KR^2} + \frac{1V}{3R} + \frac{1}{18}\left(\sqrt{4 + 12\,1K\,1VR}\right)/(1KR^2)$$

$$\Rightarrow i_1 = -\frac{1}{9}\frac{1}{1KR^2} + \frac{1V}{3R} + \frac{1}{18}\left(\sqrt{4 + 12\,1K\,1VR}\right)/(1KR^2) \quad \#$$

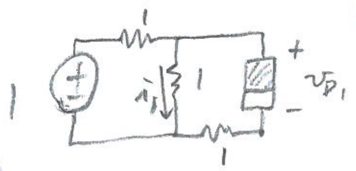
equation 2

A way to verify our derivation is to plug in some good numbers and analyze the circuit with those numbers.

For example, let $\underset{mA/V^2}{1K = 1}$, $\underset{V}{1V = 1}$, $\underset{k\Omega}{R = 1}$ we have $v_{D_1} = \frac{-2 + \sqrt{4 + 12}}{6} = \frac{1}{3}$ V

$\Rightarrow i_{D_1} = \frac{1}{9}$ mA

↳ This is actually a poor choice in this case, because it could not catch the above error. A better choice we should pick $1K \neq 1V \neq R$.



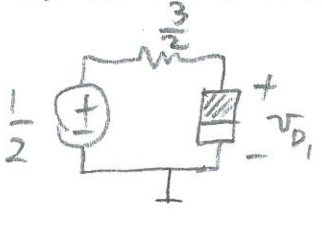
$$i_1 = \left(\frac{1}{3} + \left(\frac{1}{9} \times 1k\Omega\right)\right)/1k\Omega = \frac{4}{9} \text{ mA}$$

and $i_1 = -\frac{1}{9}\left(\frac{1}{1\times 1^2}\right) + \frac{1}{3\cdot 1} + \frac{1}{18}\sqrt{4 + 12\cdot 1\cdot 1\cdot 1}$

$$= -\frac{1}{9} + \frac{1}{3} + \frac{2}{9} = \frac{4}{9} \text{ mA}$$

which gives us some assurance that equation 2 is the correct result given that equation 1 is correct.

To see that equation 1 is correct (and for the sake of practice), we can start from the Thévenin's equivalence:



and use the node analysis

$$\begin{cases} \frac{3}{2} i_D + v_{D_1} = \frac{1}{2} \\ i_D = v_{D_1}^2 \end{cases} \Rightarrow v_{D_1} = \frac{1}{3} V,$$ which is the same as we plugged those numbers into equation 1.

Name: CSU0007 Student ID: Solution for homework 3

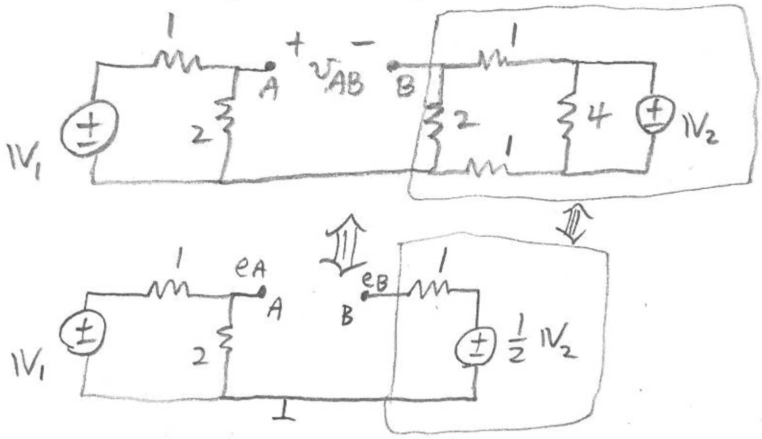
page 3

(2.2) First of all, we need to determine

whether we can replace the diode by a short circuit

or we can replace the diode by an open circuit

We analyze this by first taking away the diode:



→ we may use Thévenin Theorem to simplify this part.

Then we may use either the node analysis method or superposition to obtain that $v_{AB} = \frac{2}{3}V_1 - \frac{1}{2}V_2$.

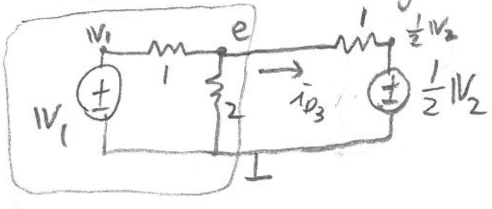
Given condition $0 < 2V_2 < V_1$, we have $v_{AB} = \frac{2}{3}V_1 - \frac{1}{2}V_2$

$\Rightarrow V_1 > 2V_2$ $> \frac{4}{3}V_2 - \frac{1}{2}V_2 > 0$

Thus, we now see we should consider the diode as a short circuit

because the i-v characteristic of the diode told us so.

Now we are ready to compute i_{D_3}!



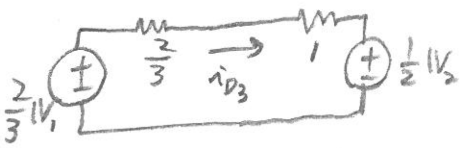
$$\begin{cases} \dfrac{V_1 - e}{1} = \dfrac{e-0}{2} + \dfrac{e - \frac{1}{2}V_2}{1} \\ i_{D_3} = \dfrac{e - \frac{1}{2}V_2}{1} \end{cases}$$

$$\Rightarrow \begin{cases} 2V_1 - 2e = e + 2e - V_2 \\ i_{D_3} = e - \frac{1}{2}V_2 \end{cases}$$

$$\Rightarrow \begin{cases} e = \frac{1}{5}(2V_1 + V_2) \\ i_{D_3} = \frac{2}{5}V_1 - \frac{3}{10}V_2 \end{cases}$$

\#

alternatively, you may transform the circuit further:

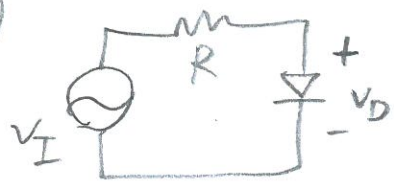


$$i_{D_3} = \frac{\frac{2}{3}V_1 - \frac{1}{2}V_2}{\frac{2}{3}+1} = \frac{2}{5}V_1 - \frac{3}{10}V_2$$

Name: Solution **Student ID:** CSU0007 homework 3

page 4

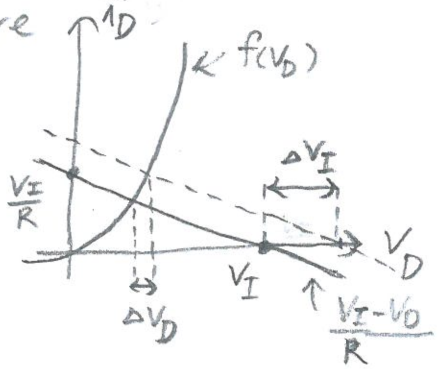
3



let $f(V_D)$ be the branch current flowing through ⏚ (i.e., i_D)

By KCL we have $\frac{V_I - V_D}{R} = f(V_D)$

Therefore we have



$\Delta V_I > \Delta V_D$

because the tangent slope of $f(V_D)$ for all $V_D > 0$ is larger than zero.

4 Refer to page 33 of the lecture note.

Because $R_{TH} = \frac{V_{open\text{-}circuit}}{I_{short\text{-}circuit}}$, it is sufficient to just measure $V_{open\text{-}circuit}$ and $I_{short\text{-}circuit}$:)