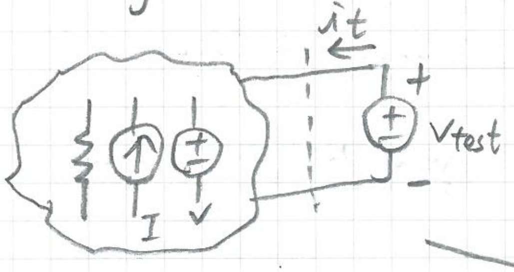
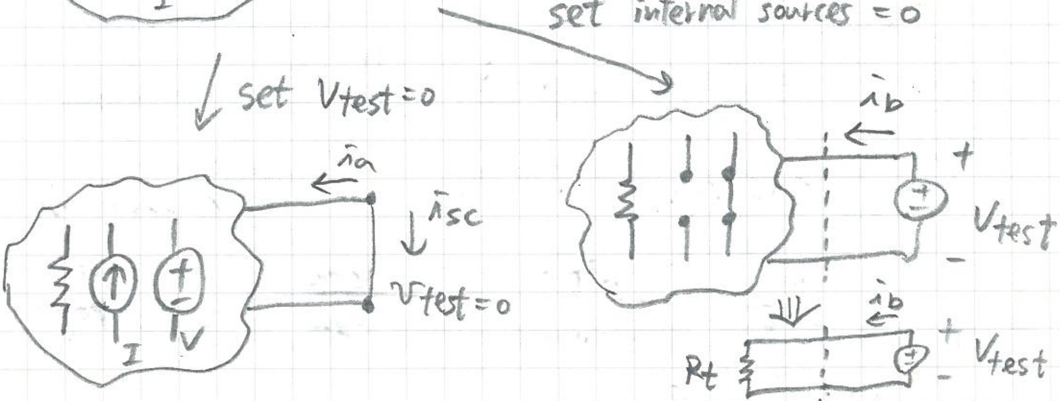
P32 ☆ Norton's Theorem

For the same goal as that of Thévenin's Theorem, here we choose to attach a testing independent voltage source to a possibly complex circuit:



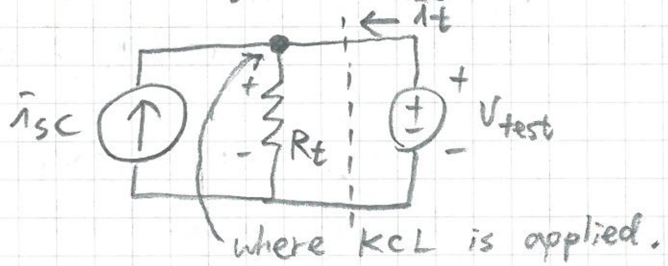
and leverage superposition



then $i_t = i_a + i_b = -i_{sc} + \dfrac{V_{test}}{R_t}$

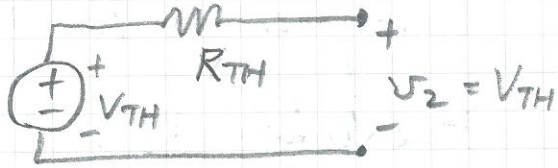
$$\Rightarrow i_t + i_{sc} - \frac{V_{test}}{R_t} = 0$$

Think of above in terms of KCL, then, equivalently the original circuit is like



And $R_t = R_{TH}$, since we applied same procedure of decomposition as we did in Thévenin's Theorem.

Relation between Norton's Equivalent Circuit and Thévenin's Equivalent Circuit:



since $v_1 = v_2$, we have

$$i_{sc} \cdot R_t = v_2 = V_{TH}$$

$$\Rightarrow R_t = \frac{V_{TH}}{i_{sc}} = \frac{v_{oc}}{i_{sc}}$$

(recall that $V_{TH} = v_{oc}$ page 28)

in other word,

$$等效電阻 = \frac{開路電壓}{短路電流}$$

Example: find $v = ?$

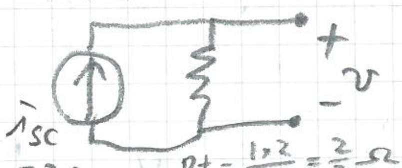
approach ①: voltage divider

$$v = 3 \times \frac{2}{1+2} = \underline{2\ V}$$ ※

approach ②: Thévenin's Theorem

$v = V_{TH} = \underline{2\ V}$ ※

approach ③: Norton's Theorem

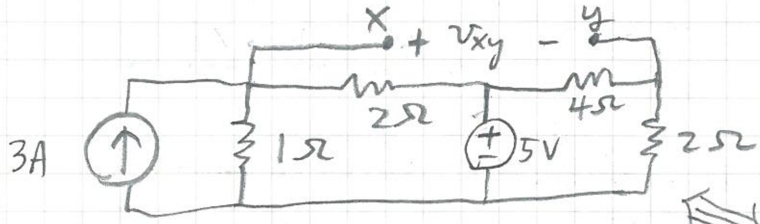


$R_t = \frac{1 \times 2}{1+2} = \frac{2}{3}\ \Omega$

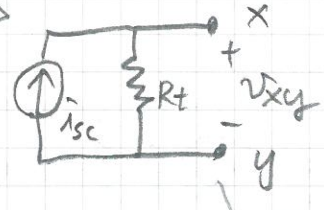
$v = 3 \times \frac{2}{3} = \underline{2\ V}$ ※

$\Rightarrow$ these three agree in one.

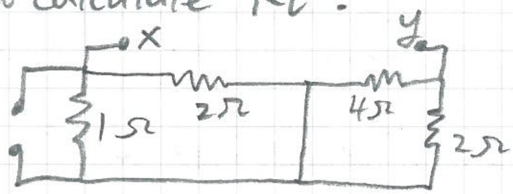
P34 Example: find $v_{xy} = ?$



Using Norton's Theorem, we have:



Now, to calculate R_t:

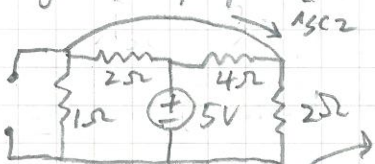


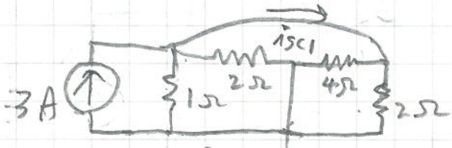
$$R_t = (1\Omega // 2\Omega) + (4\Omega // 2\Omega)$$

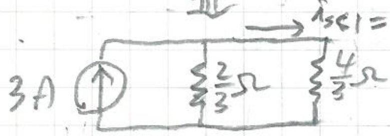
$$= 2\,\Omega$$

To calculate i_{sc}, we may use superposition:

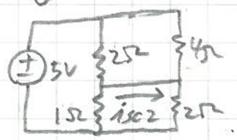
$i_{sc} = i_{sc1} + i_{sc2}$







$$i_{sc1} = 3 \times \frac{\frac{2}{3}}{\frac{2}{3} + \frac{4}{3}} = 1\,A$$



from KCL, we may see that $i_{sc2} = 0$

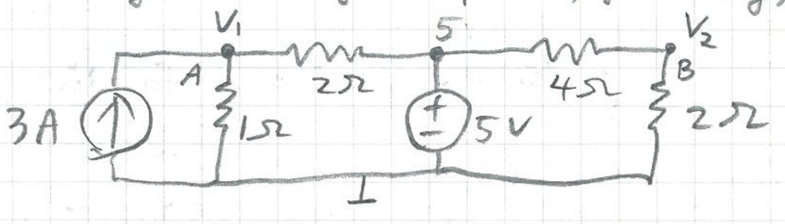
therefore,

$i_{sc} = 1\,A$

$v_{xy} = i_{sc} \times R_t$

$= 2V$ ※

Alternatively, we may compute V_{xy} directly, using the node analysis method:



KCL at node A: $3 + \frac{0 - V_1}{1} + \frac{5 - V_1}{2} = 0 \Rightarrow V_1 = \frac{11}{3}$

KCL at node B: $\frac{5 - V_2}{4} + \frac{0 - V_2}{2} = 0 \Rightarrow V_2 = \frac{5}{3}$

$$V_{xy} = V_1 - V_2 = \frac{11}{3} - \frac{5}{3} = 2\,V$$

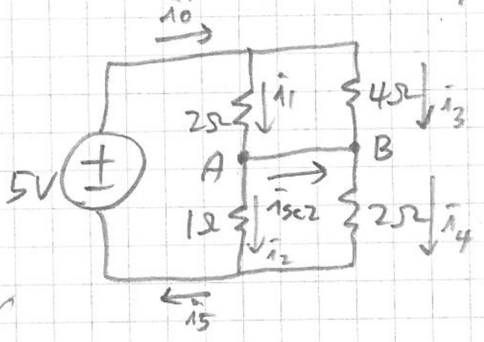
#

P35

From the above example, we see that in order to use Norton's Theorem correctly, we need to be very careful when determining the short-circuit current i_{sc}.

KCL is a great tool here to help us.

To be specific, when we were determining i_{sc2} in the previous example:



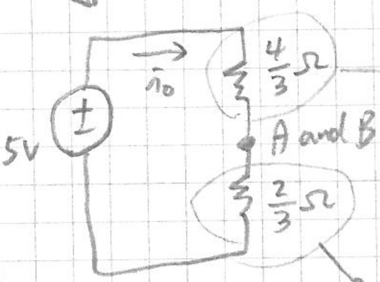
Applying KCL at nodes A and B, we have

$i_{sc2} = i_2 - i_1$

$i_{sc2} = i_4 - i_3$

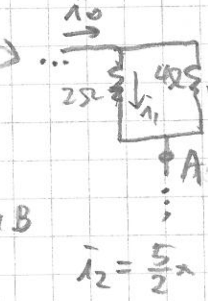
The above two equalities must hold, and we can use one of them to calculate i_{sc2} and then use the other one to verify our answer.

equivalently



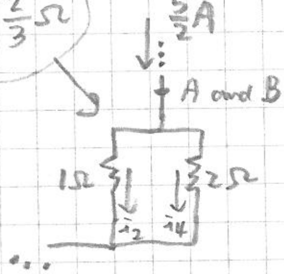
Hence, $i_0 = \dfrac{5}{\frac{4}{3}+\frac{2}{3}} = \dfrac{5}{2}A$

according to Ohm's law.



$i_1 = i_0 \times \dfrac{4}{2+4} = \dfrac{5}{3}A$

$i_3 = i_0 \times \dfrac{2}{2+4} = \dfrac{5}{6}A$

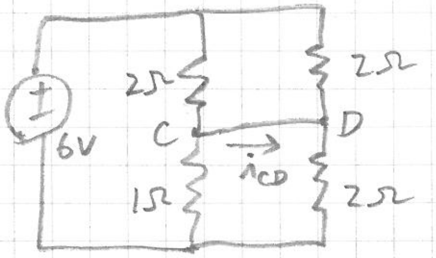


$i_2 = \dfrac{5}{2} \times \dfrac{2}{1+2} = \dfrac{5}{3}A$

$i_4 = \dfrac{5}{2} \times \dfrac{1}{1+2} = \dfrac{5}{6}A$

※

Interestingly, if we have the following circuit instead, the current flowing from node C to node D would be non-zero:



You can verify, as an exercise, that $i_{CD} = -\frac{3}{5}$ A

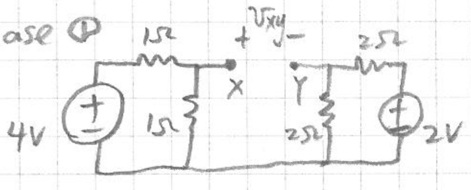
In essence, the condition to have no current flowing from node X to node Y is to have the node voltage at node X equal to that at node Y. Starting from here and using the symbolic computation, you might rediscover the "Wheatstone Bridge" circuit :)

One final note on circuit transformation:

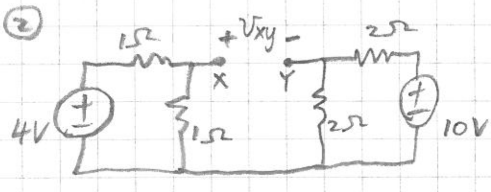
When we are using either Thévenin's Theorem or Norton's Theorem to transform a circuit into its equivalence, it is possible that we might define the Voc or isc variable in the direction opposite to what's really happening in the circuit, since for a complex circuit we might not readily perceive the real direction of the voltage or current. But that's fine, because our computed result will have a negative sign if the direction was wrong.

Example: determine v_{xy} in these two cases:

case ①



case ②



(ans: $v_{xy} = 1$ V)

(ans: $v_{xy} = -3$ V)