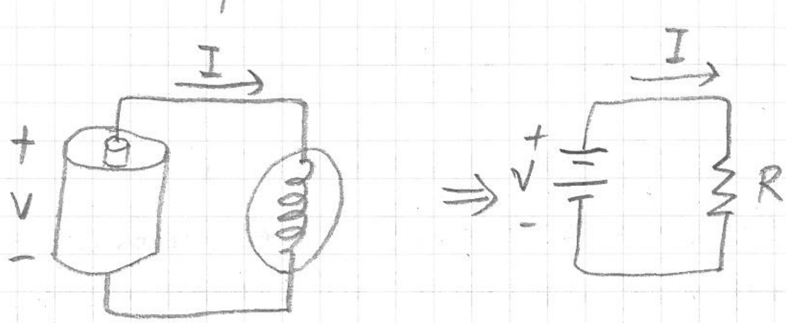
- The lumped circuit abstraction



for now, let's just consider elements having two terminals, e.g.,
battery,
lightbulb

We may ignore the internal structure of an element and consider it as a "lump", where we may completely describe relevant properties (such as <u>voltage</u> and <u>current</u>) by only observing its terminals.

For example, if a resistor with resistance R obeys Ohm's law, then we may compute the current flowing through the resistor by the voltage across its terminals : $I = V/R$.

The lumped circuit abstraction works only under certain constraints, which we call the lumped matter discipline (LMD).

From Physics to Electronic Circuit:

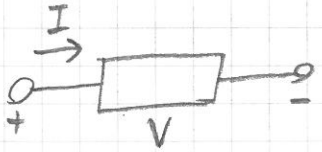
☆ The lumped matter discipline (LMD)

discipline → a set of rules that control an activity or situation.

→ simplify the analysis of electronic circuit

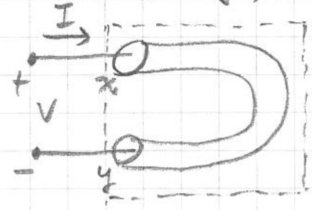
→ modularize a complex circuit into analyzable elements

Derivation of LMD:



$$\boxed{\text{goal\#1: able to ascribe a unique voltage across the terminals } x \text{ and } y}$$

example element
燈絲, U型, 長度為 l



definition of voltage:

$$V_{yx} = -\int_x^y \mathbb{E} \cdot dl$$

where $\mathbb{E}$ is the electrical field (a vector)

dl is a tiny portion of l

and Faraday's law of induction:

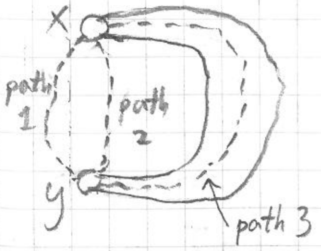
$$\oint \mathbb{E} \cdot dl = -\frac{\partial \phi_B}{\partial t}$$

where ϕ_B is the magnetic flux.

$\oint$ represents a closed path integral

Then we see that if there's no time-varying magnetic flux,

$$\oint \mathbb{E} \cdot dl = 0 \Rightarrow \underbrace{\int_x^y \mathbb{E} \cdot dl}_{\text{along path 1}} + \underbrace{\int_y^x \mathbb{E} \cdot dl}_{\text{path 2}} = 0$$



$$\Rightarrow \int_{x \atop \text{path1}}^y \mathbb{E} \cdot dl = \int_{x \atop \text{path 2}}^y \mathbb{E} \cdot dl$$

也就是說 $\int_x^y \mathbb{E} \cdot dl$ 的值和路徑無關! $\left(= \int_{x \atop \text{path3}}^y \mathbb{E} \cdot dl\right)$

Therefore, the constraint for goal #1 is

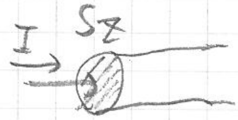
$$\frac{\partial \phi_B}{\partial t} = 0 ,$$

and we assumed that holds for all time.
(To make sure this constraint holds, we may need to revise the model and introduce an element called "inductor".)

$$\boxed{\begin{array}{l}\text{goal \#2 :}\\ \text{able to define a unique current}\\ \text{through the terminals } x \text{ and } y\end{array}}$$

First of all, the definition of current :

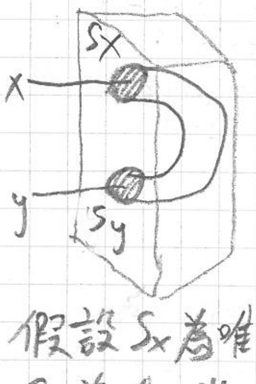
$$I = \int_{S_z} \mathbf{J} \cdot d\mathbf{s}$$



where $\mathbf{J}$ is the current density at a given point within the filament and S_z is the cross-sectional surface of the filament at point z.

Due to the conservation of charge, we have

$$\underbrace{\oint \mathbf{J} \cdot d\mathbf{s}}_{\text{流出的电量}} = \underbrace{-\frac{\partial q}{\partial t}}_{\text{減少的电量}} \quad \text{for a closed surface}$$



假設 S_x 為唯一入口
S_y 為唯一出口.

Thus, if there's no time-varying charge within the closed surface, we have

$$\oint \mathbf{J} \cdot d\mathbf{s} = 0 \Rightarrow -\int_{S_x} \mathbf{J} \cdot d\mathbf{s} + \int_{S_y} \mathbf{J} \cdot d\mathbf{s} = 0$$

$$\Rightarrow \int_{S_y} \mathbf{J} \cdot d\mathbf{s} = \int_{S_x} \mathbf{J} \cdot d\mathbf{s}$$

$$I_{in} = I_{out}$$

Therefore, the constraint for goal # 2 is

$$\frac{\partial q}{\partial t} = 0,$$

and we assume that holds for all time.
(To make sure this constraint holds, we may need to revise the model and introduce an element called "capacitor".)

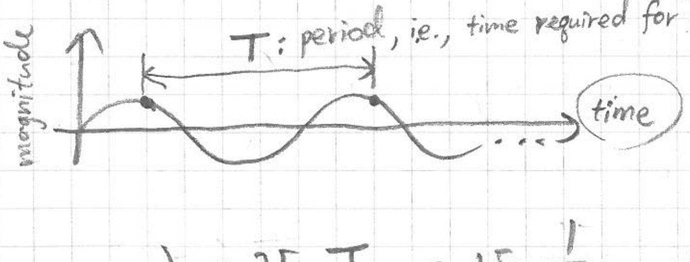
Besides goals #1 and #2, we also need to assume that the signal timescale must be much larger than the propagation delay of electromagnetic waves across the lumped elements. (Otherwise, ... see textbook P11)

rate of change, i.e., frequency

...0110100... for example

⇒ the size of our lumped elements must be much smaller than the wavelength associated with the V and I signals, and such a condition may be challenging to hold as we reduce the element size and increase the operating frequency (e.g., a 2GHz CPU).

definition of wavelength (λ): the distance between two adjacent points in the wave having the same phase.



, i.e., time required for a wave to travel for the distance of one wavelength.

$$\lambda = v \cdot T = v \cdot \frac{1}{f} \Rightarrow f \uparrow \text{ then } \lambda \downarrow$$

wavelength — wave speed — period — frequency for example, electromagnetic waves travel at about 15×10^4 km/s, or 15×10^9 cm/s, within a microprocessor (to be specific, through silicon dioxide). Now, suppose that the microprocessor operates at a clock rate of 2 GHz. This translates to the wavelength equal to

$$\lambda = 15 \times 10^9 / 2 \times 10^9 = 7.5 \text{ cm},$$

which means that LMD may not hold if the microprocessor chip is larger than 7.5 cm on a side.

Think about it : what if ① clock rate ↑ ?

② wave speed ↑ ?

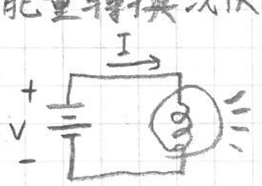
In general, in computer engineering, people are often working to meet various constraints (such as this) so that they may apply a previously established model (such as LMD) and make use of known results / properties that depend on the given model. This is like "Standing on the shoulders of giants."

- Basic lumped element 1: batteries

Two key properties
- energy 能量 (unit: joule or ampere-hours or watt-hours)
- power 功率 (unit: watt)

功率 → 能量轉換或使用的速率

$$\boxed{\begin{array}{c} P = V \cdot I \\ 1 \text{ watt} = 1 \text{ volt} \cdot 1 \text{ ampere} \end{array}}$$



Let ε be the amount of <u>energy</u> supplied to an element over an interval T, then we have $\varepsilon = p \cdot T$.
In general, the amount of energy supplied is the time integral of the power.

1 joule = 1 watt - second

Example: Suppose a Raspberry Pi consumes 2W of power and its energy is supplied by a 3.7V, 2600 mA-h battery. For how long can the battery power the Raspberry Pi?

$$p = V \cdot I = 3.7 \times 2600 \times 10^{-3} \text{ W-h}$$
$$= 9.62 \text{ W-h}$$

$$\frac{9.62 \text{ W-h}}{2 \text{ W}} = 4.81 \text{ hours}$$

- Basic lumped element 2: resistors (linear)

Ohm's law: the voltage measured across the terminals of a resistor is linearly proportional to the current flowing through the resistor.

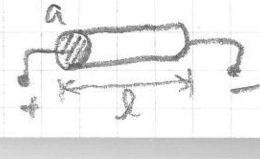
That is, $\boxed{v = i \cdot \underbrace{R}}$

we call it the <u>resistance</u> of a resistor.

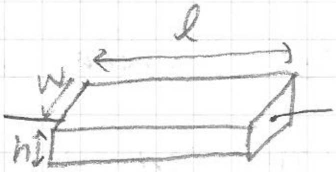
Further, $\boxed{R = \underbrace{\rho} \frac{l}{a}}$

resistivity

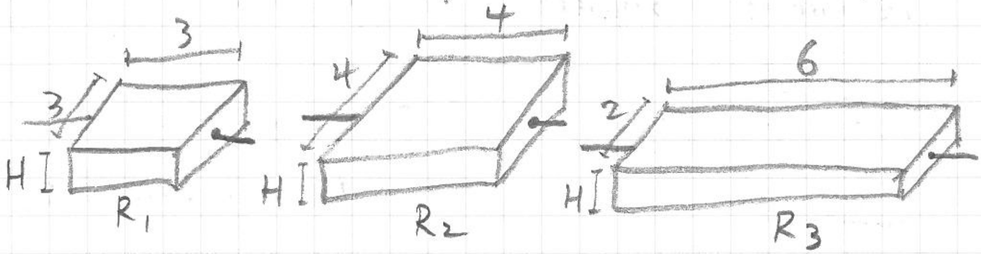
(see Appendix A.3 in the textbook)



also, $\boxed{R = \rho \frac{l}{wh}}$



Example : Consider three planar resistors as follows



Let $R_0 = \rho_0 \frac{1}{1 \cdot H} = 2\,k\Omega$ and assume $\rho_0 = \rho_1 = \rho_2 = \rho_3$

Then $R_1 = \rho_1 \frac{3}{3 \cdot H} = R_0 = 2\,k\Omega$

$$\frac{R_1}{R_2} = \frac{\rho_1 \frac{3}{3H}}{\rho_2 \frac{4}{4H}} = 1 \quad \text{and } R_2 = R_1 = 2\,k\Omega$$

$$\frac{R_2}{R_3} = \frac{\rho_2 \frac{4}{4H}}{\rho_3 \frac{6}{2H}} = \frac{1}{3} \Rightarrow R_3 = 6\,k\Omega$$

exercise : you can verify that $\frac{R_1}{R_3} = \frac{R_2}{R_3}$.

⇒ 等比例縮小長及寬，則相對電阻值 ____ ?

A. ↓ B. ↑ C. 不變

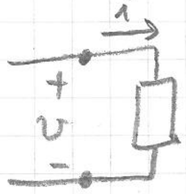
⇒ 縮小晶片的大小不會改變相對電阻值

⇒ Often, signal values are derived as a function of resistance ratios. Therefore, by such a process shrink the chip may continue to function as before!

example : a voltage divider, which we will study later this semester.

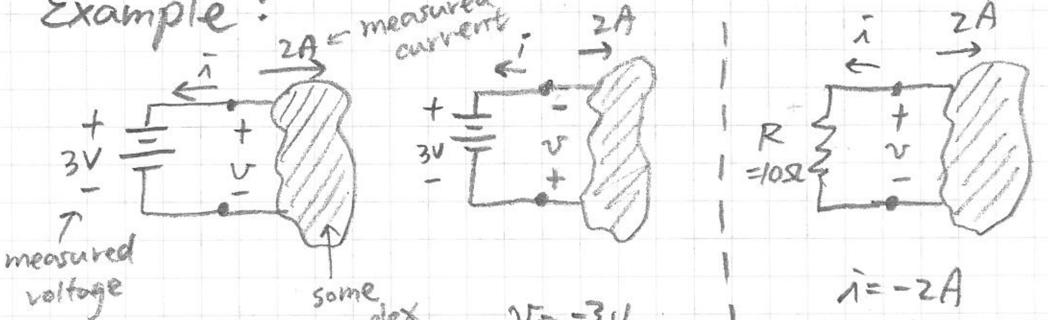
– Associated Variable Convention : (約定俗成)

For a two-terminal lumped element, define current to flow in at the element terminal assigned to be positive in voltage.



v and i are called "terminal variables."

Example :



$v = 3A$
$i = -2A$
$P = v \cdot i = -6W$

$v = -3V$
$i = 2A$
$P = v \cdot i = -6W$

⇒ power supplied "from" battery
by

$i = -2A$
$v = iR = -20V$
$P = v \cdot i = 40W$

⇒ power supplied "to" resistor (and then is dissipated in the form of heat)

Note that the above is from the viewpoint of the battery; from the viewpoint of that complex circuit, you may try and see that now power is supplied "to" it.