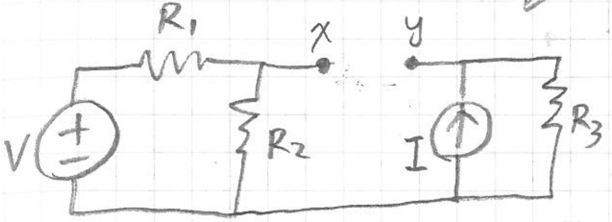
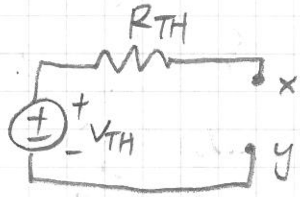
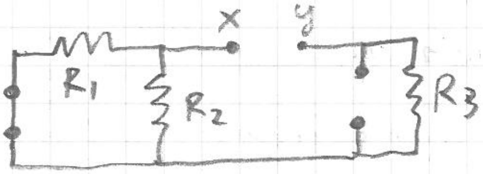
Exercise: create the Thévenin Equivalent Circuit for the following circuit:



$\Rightarrow$



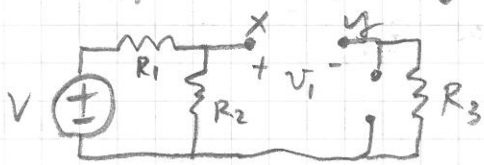
first of all, for R_{TH}:

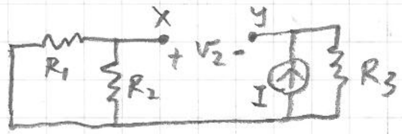


$$\Rightarrow R_{TH} = (R_1 /\!/ R_2) + R_3 = \frac{R_1 R_2}{R_1 + R_2} + R_3$$

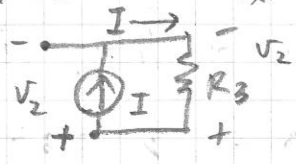
then, for V_{TH}:

approach ①, using superposition

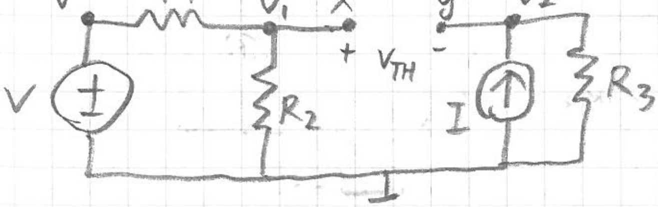




$$V_{TH} = V_1 + V_2 = \frac{R_1}{R_1+R_2} V + (-IR_3) = \frac{R_1}{R_1+R_2} V - IR_3 \quad ※$$



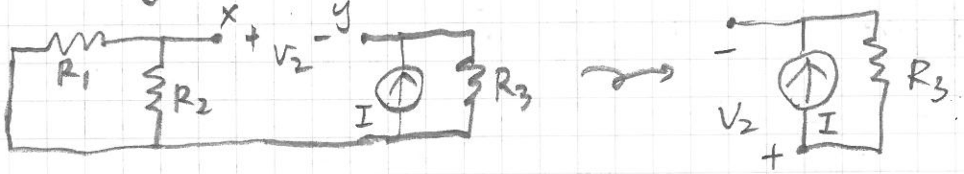
approach ②, using the node method (Page 21)



$$V_{TH} = V_1 - V_2 = \frac{R_2}{R_1+R_2} V - IR_3 \quad ※$$

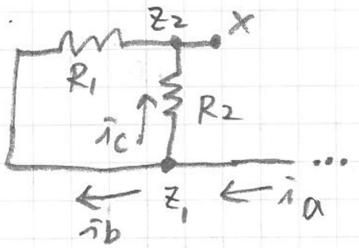
KCL:
$$\begin{cases} \dfrac{V - V_1}{R_1} + \dfrac{0 - V_1}{R_2} = 0 \\ I + \dfrac{0 - V_2}{R_3} = 0 \end{cases} \Rightarrow \begin{cases} V_1 = \dfrac{R_2}{R_1+R_2} V \\ V_2 = IR_3 \end{cases}$$

In the exercise on the previous page, you might wonder why we can do the following transformation:



Could it be possible that there are some current flowing through R_1 and/or R_2?

We can use KCL to figure it out:



for node z_1, we have

$$i_a - i_b - i_c = 0$$

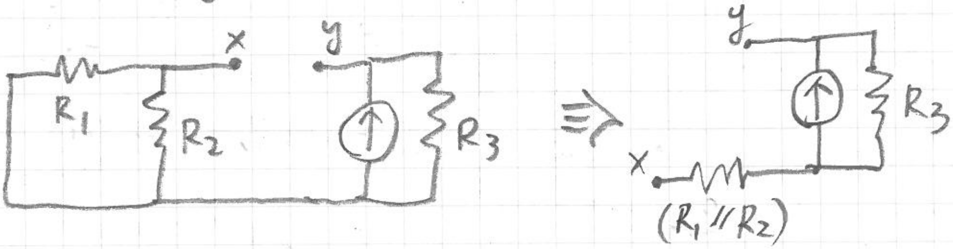
for node z_2, we have

$$i_b + i_c = 0$$

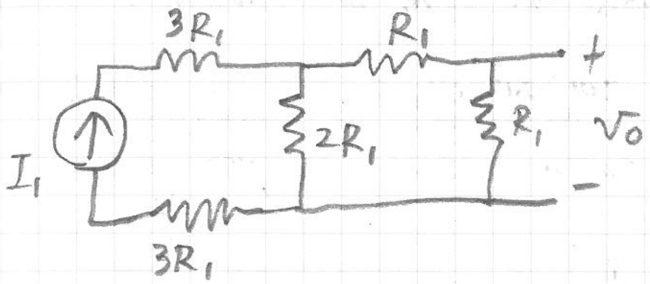
compare

Therefore, we see that $i_a = 0$.

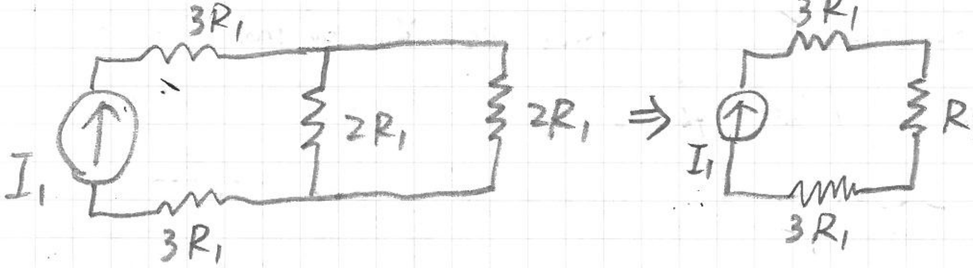
In practice, it is often helpful to think in terms of equivalent resistance, which may make the situation much more obvious:



Exercise : In the following circuit, determine voltage v_o :



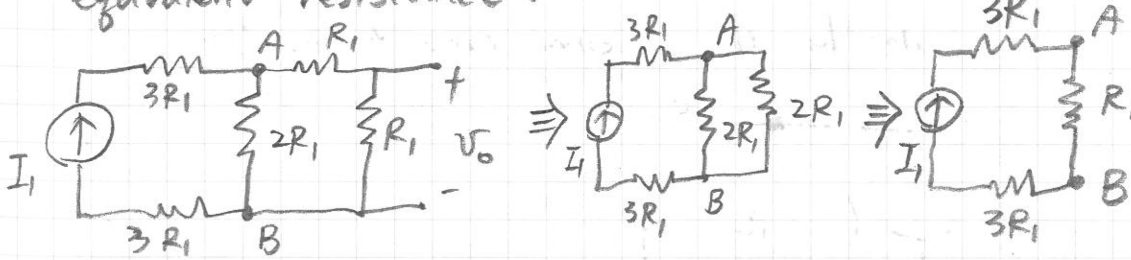
We may do the following transformation :



then we might conclude that $v_o = I_1 \cdot R_1$,

but it is wrong.

To see this, it could be helpful to <u>clearly label</u> the nodes between which you calculate the equivalent resistance :



thus we see that v_o is not the voltage across A, B,

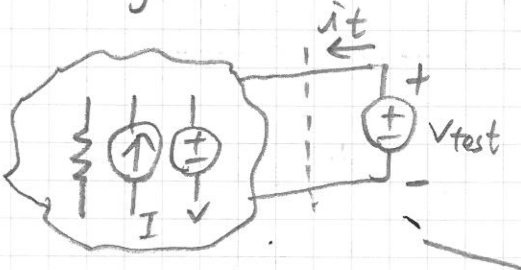
and $v_o = (\text{voltage across } A, B) \times \dfrac{R_1}{R_1 + R_1}$

$$= (I_1 \cdot R_1) \times \frac{R_1}{R_1 + R_1} = \frac{1}{2} I_1 R_1$$ is the correct answer.

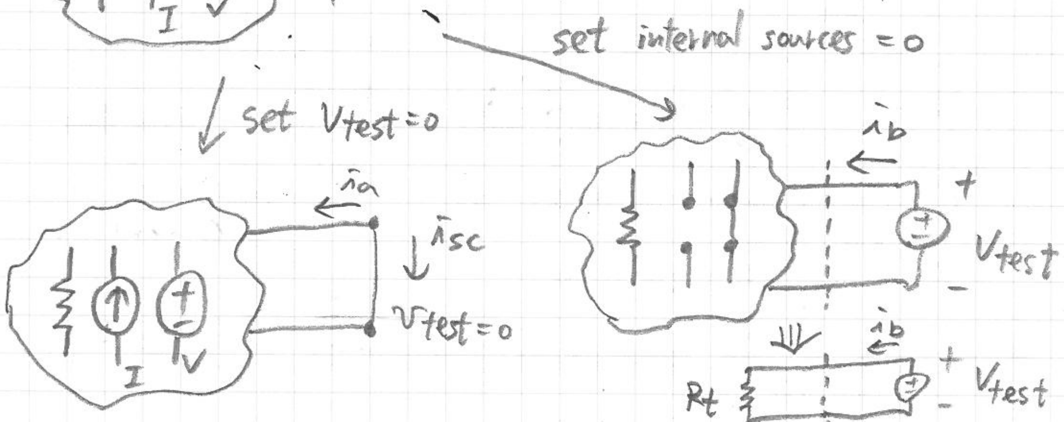
\#

P32 ☆ Norton's Theorem

For the same goal as that of Thévenin's Theorem, here we choose to attach a testing independent voltage source to a possibly complex circuit:



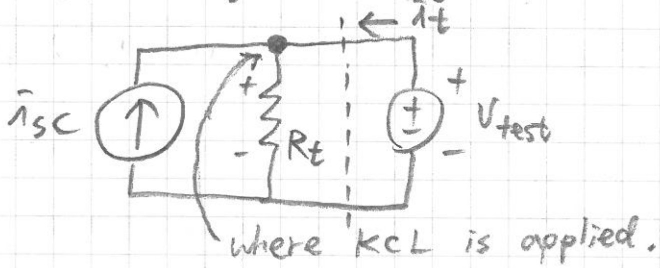
and leverage superposition



then $i_t = i_a + i_b = -i_{sc} + \dfrac{V_{test}}{R_t}$

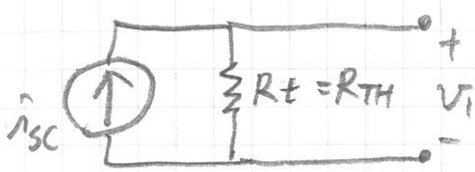
$$\Rightarrow i_t + i_{sc} - \frac{V_{test}}{R_t} = 0$$

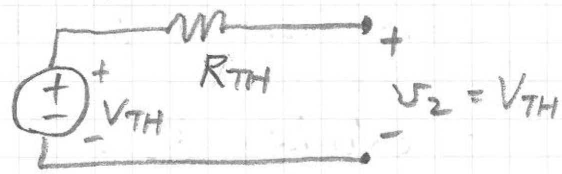
Think of above in terms of KCL, then, equivalently the original circuit is like



And $R_t = R_{TH}$, since we applied same procedure of decomposition as we did in Thévenin's Theorem.

Relation between Norton's Equivalent Circuit and Thévenin's Equivalent Circuit:





since $v_1 = v_2$, we have

$$\hat{\imath}_{SC} \cdot R_t = v_2 = V_{TH}$$

$$\Rightarrow R_t = \frac{V_{TH}}{\hat{\imath}_{SC}} = \frac{v_{OC}}{\hat{\imath}_{SC}}$$

(recall that $V_{TH} = v_{OC}$ page 28)

in other word,

$$\text{等效電阻} = \frac{\text{開路電壓}}{\text{短路電流}}$$

Example: find $v = ?$

approach ①: voltage divider

$$v = 3 \times \frac{2}{1+2} = \underline{2\text{ V}} \quad ※$$

approach ②: Thévenin's Theorem

$V_{TH} = 2\text{V}$

$v = V_{TH} = \underline{2\text{ V}}$ ※

approach ③: Norton's Theorem

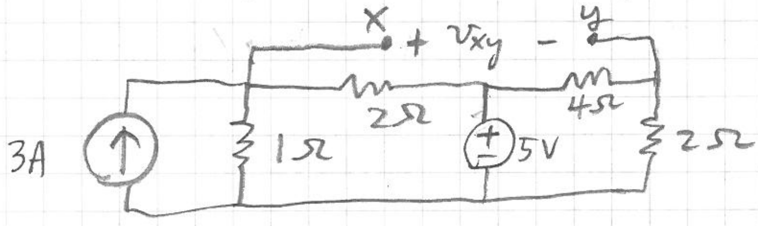
$\hat{\imath}_{SC} = 3\text{A}$

$R_t = \frac{1 \times 2}{1+2} = \frac{2}{3}\,\Omega$

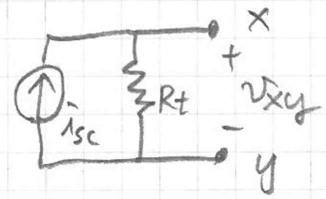
$v = 3 \times \frac{2}{3} = \underline{2\text{ V}}$ ※

$\Rightarrow$ these three agree in one.

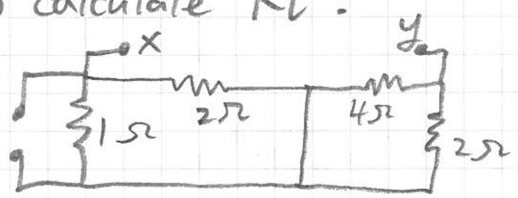
P34 Example : find $v_{xy} = ?$



Using Norton's Theorem, we have

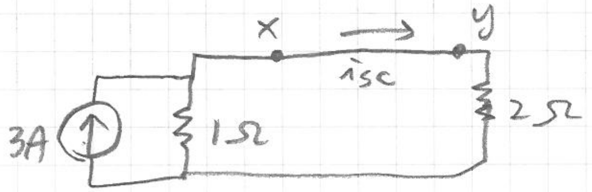


to calculate R_t :



$$R_t = (1\Omega // 2\Omega) + (4\Omega // 2\Omega) = 2\,\Omega$$

to calculate i_{sc} :

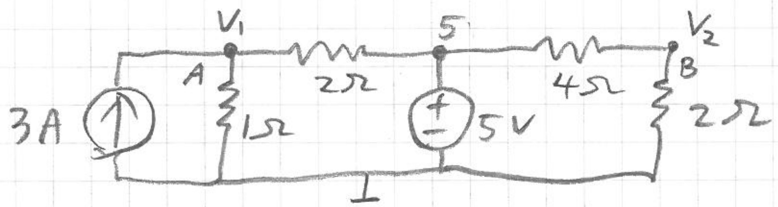


$$i_{sc} = 3 \times \frac{1}{1+2} = 1\ A$$

(current divider)

Therefore $V_{xy} = i_{sc} \times R_t = 2\ V$ #

Alternatively, we may use the node method:



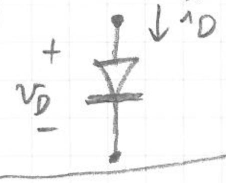
KCL at node A : $3 + \frac{0-V_1}{1} + \frac{5-V_1}{2} = 0 \Rightarrow V_1 = \frac{11}{3}$

KCL at node B : $\frac{5-V_2}{4} + \frac{0-V_2}{2} = 0 \Rightarrow V_2 = \frac{5}{3}$

$V_{xy} = V_1 - V_2 = \frac{11}{3} - \frac{5}{3} = 2\ V$ #

☆ Nonlinear Devices

Silicon Diode



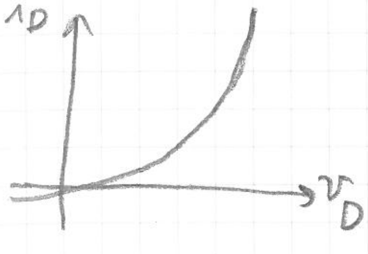
$$i_D = I_S\left(e^{v_D/v_{THE}} - 1\right)$$

I_S : saturation current $\approx 10^{-12}$ A

v_{THE} : thermal voltage

$v_{THE} = \frac{kT}{q}$ T: temperature in kelvins

$°K = °C + 273.15$

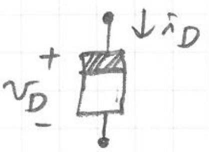


k : Boltmann's constant $= 1.38 \times 10^{-23}$ J/°K

q : charge of an electron $= 1.602 \times 10^{-19}$ C

Example : CPU 溫度計, see example 16.1 in the textbook.

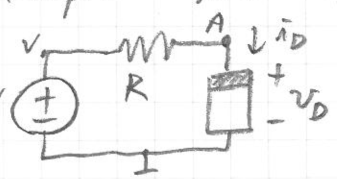
Hypothetical Nonlinear Device



$$i_D = \begin{cases} \mathbb{K} \cdot v_D^2 & \text{for } v_D > 0 \\ 0 & \text{otherwise} \end{cases}$$

where $\mathbb{K}$ is a positive constant

Example : find v_D and i_D :



KCL at A : $\frac{V - v_D}{R} + (-i_D) = 0$

Plug in the i-v relation, we have

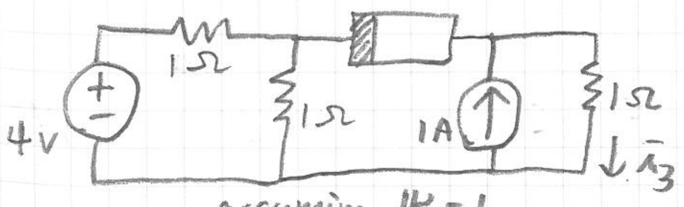
$$\begin{cases} \frac{V - v_D}{R} - \mathbb{K} v_D^2 = 0 & \text{for } v_D > 0 \\ \frac{V - v_D}{R} = 0 & \text{otherwise} \end{cases}$$

$\Rightarrow v_D = \frac{-1 + \sqrt{1 + 4R\mathbb{K}V}}{2R\mathbb{K}}$ and $i_D = \mathbb{K}\left(\frac{-1+\sqrt{1+4R\mathbb{K}V}}{2R\mathbb{K}}\right)^2$ for $v_D > 0$

※

Exercise : find i_3 = ?

(hint : Page 29 and the above example)



assuming $\mathbb{K} = 1$

You can do it !

Ans: $i_3 = \left(\frac{\sqrt{7} - 1}{3}\right)^2 + 1$

※