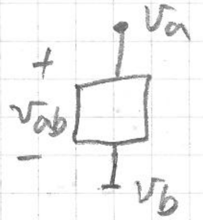
☆ Circuit Analysis using the Node Method

Motivation:

Often, the number of nodes in a circuit is much smaller than that of branches. Node method thus involves fewer number of variables, which means it is often easier to solve.

– definition of node voltage:



$$v_{ab} = \underbrace{v_a}_{\text{node voltage}} - \underbrace{\overset{0}{v_b}}_{}$$

Procedure of the node method:

1° select a reference node (接地, $v=0$) $\perp$

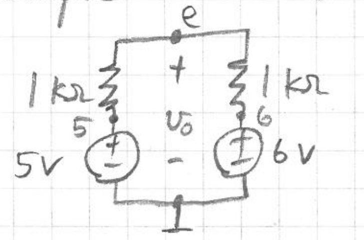
2° assign node variables

3° apply KCL

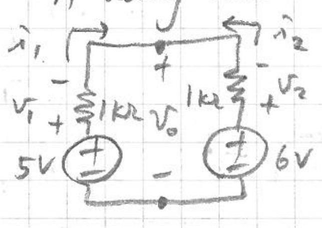
4° solve equations

5° back-solve the needed branch voltage/current

Example: find $v_0 = ?$



if using the basic method,



$$i_1 = \frac{v_1}{1\,k\Omega}$$

$$i_2 = \frac{v_2}{1\,k\Omega}$$

using the node method,

$$\frac{5-e}{1\,k\Omega} + \frac{6-e}{1\,k\Omega} = 0$$

$e = 5.5V$ $\quad v_0 = e - 0 = \underline{5.5\,V}$ ※

$i_1 + i_2 = 0$

$\Rightarrow v_1 + v_2 = 0$

using KVL,

$5 - v_1 + v_2 - 6 = 0$

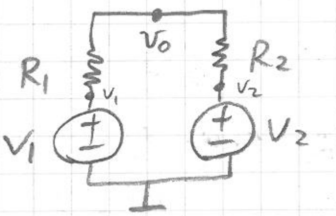
$\Rightarrow v_1 - v_2 = -1$

$$\begin{cases} v_1 = -0.5 \\ v_2 = 0.5 \end{cases}$$

$v_0 = 6 - v_2$ (KVL)

$= \underline{5.5\,V}$ ※

P22 Symbolic Computation can give us some insights:



KCL: $\dfrac{v_1 - v_o}{R_1} + \dfrac{v_2 - v_o}{R_2} = 0$

$\Rightarrow R_2(v_1 - v_o) + R_1(v_2 - v_o) = 0$

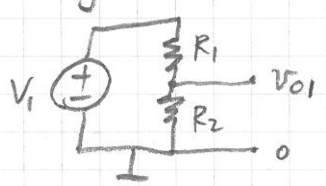
$\Rightarrow v_o = \dfrac{1}{R_1 + R_2}(R_2 v_1 + R_1 v_2)$

$= \dfrac{R_2}{R_1 + R_2} v_1 + \dfrac{R_1}{R_1 + R_2} v_2$

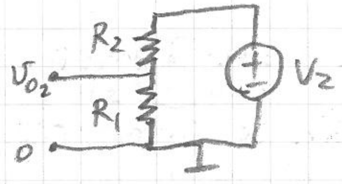
Insights ① v_o is a linear combination of v_1 and v_2.
The circuit acts as an adder that gives a weighted sum of v_1 and v_2.

② if set $v_2 = 0$, then $v_{o1} = \dfrac{R_2}{R_1 + R_2} v_1$,
which is equivalent to the result of a voltage divider:

i.e., having the same i-v characteristic.



③ similarly, if set $v_1 = 0$, then
$v_{o2} = \dfrac{R_1}{R_1 + R_2} v_2$, equivalent to the result of a voltage divider:



$\Rightarrow$ from ② and ③, the original circuit can be thought of as a superposition of two voltage dividers, with $v_o = v_{o1} + v_{o2}$.

we will talk more about it on October 6!