CSC0056: Data Communication

Lecture 11:
The Slotted Aloha Protocol

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Course information

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- Course meetings: Mondays 9:10-12:10 in C007, Gongguan Campus

Acknowledgement: Some slides' materials in this course are borrowed with permission from the 2014 edition of the course taught by Prof. Yao-Hua Ho 賀耀華

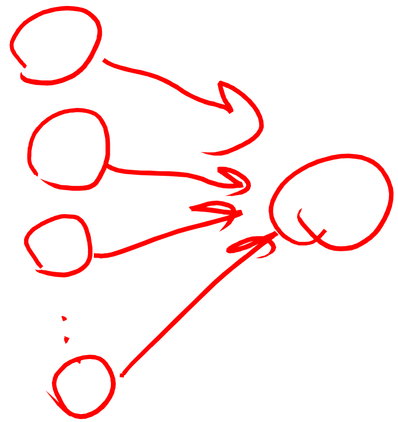
Figures are obtained from the textbook available at http://web.mit.edu/dimitrib/www/datanets.html

Outline of lecture11

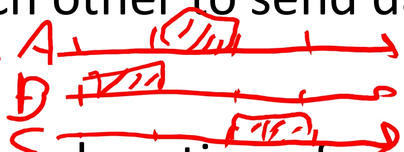
- Multiaccess communication strategies – a big picture
- Slotted Aloha protocol

- Textbook reading assignment for this lecture:
  - Sections 4.2
- Homework 3 will be released tomorrow. Due on 11/25 before class.

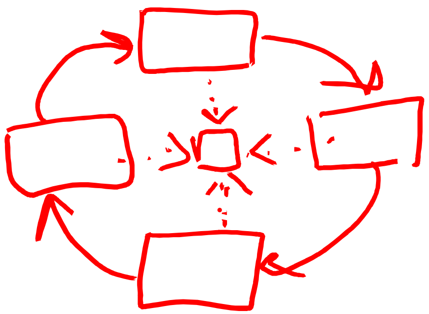
Review of types of multiaccess communication



- Case 1: Free-for-all
  - Senders compete with each other to send data
- Case 2: Scheduled
  - Each sender has a guaranteed portion of resources to communicate

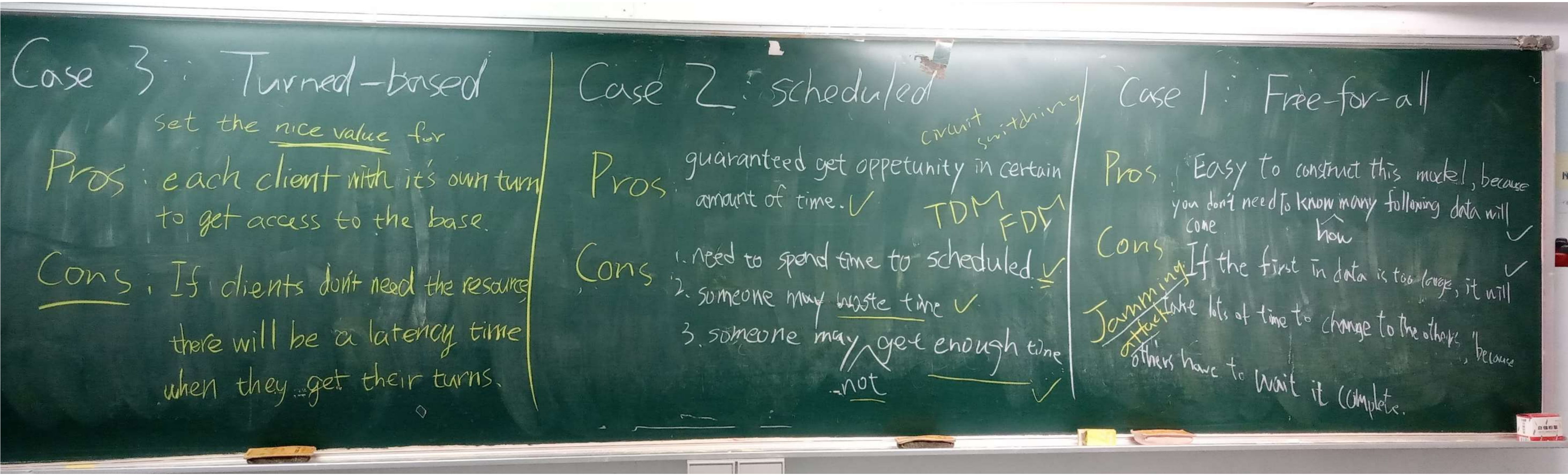


- Case 3: Turn-based
  - Each sender is guaranteed to have a chance to send data



What we've discussed last Monday

- Pros and cons of each type of multiaccess protocols



A summary

- Case 1: Free-for-all
  - Pros: Flexible; simple (relatively)
  - Cons: Higher variety in latency
- Case 2: Scheduled
  - Pros: Performance guaranteed
  - Cons: Pre-processing; longer latency
- Case 3: Turn-based
  - Pros: Cooperative
  - Cons: latency penalty

The slotted Aloha protocol

- A free-for-all multiaccess communication protocol
- Developed in 1970s at University of Hawaii, for radio communication from its campuses to a central computer
- Each un-backlogged node transmits a newly arriving packet at the first slot after arrival
  - If packets collided,
    wait for a random number of slots
    before re-transmission.

Source: Google Maps

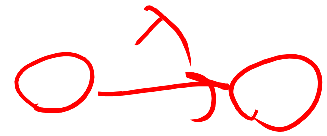
⇨ Performance analysis of the slotted Aloha:
The rate of departures (successful transmissions)?

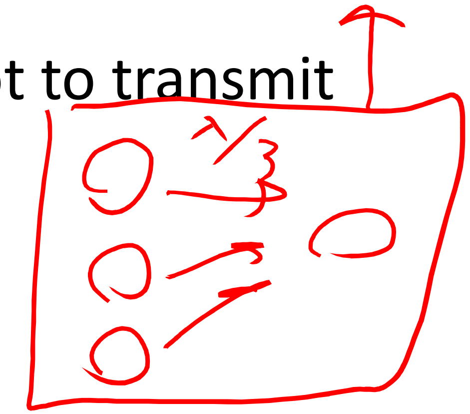
A slotted multiaccess model for analysis

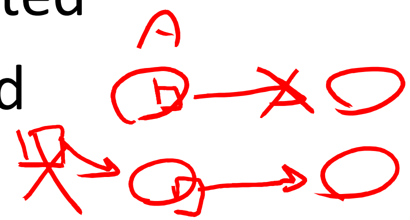
- Consider *m* senders and one receiver

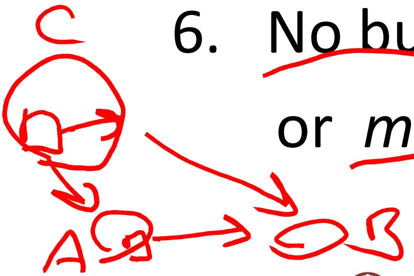
1. Slotted system: each packet needs only one time slot to transmit
2. Poisson arrival: each sender with arrival rate λ/m
3. Collision or perfect reception
4. Immediate feedback at the end of each slot
5. Retransmissions: collided packets must be re-transmitted
6. No buffering: a newly arriving packet may be discarded

   or $m = \infty$: each new packet arrives at a new sender



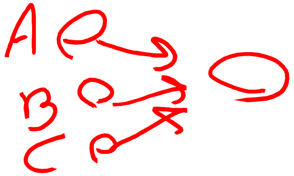




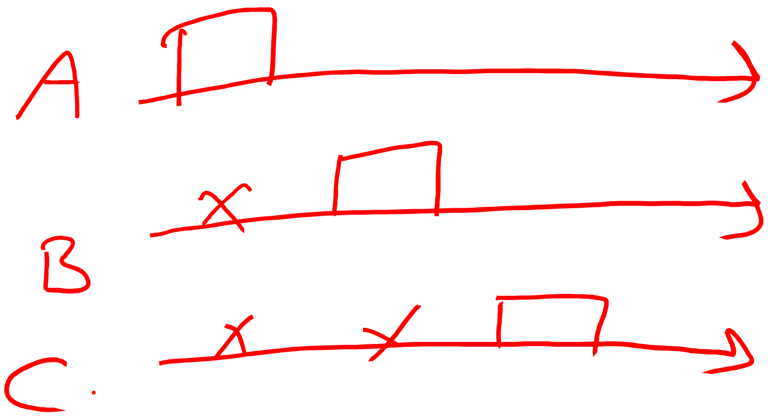


A slotted multiaccess model for analysis

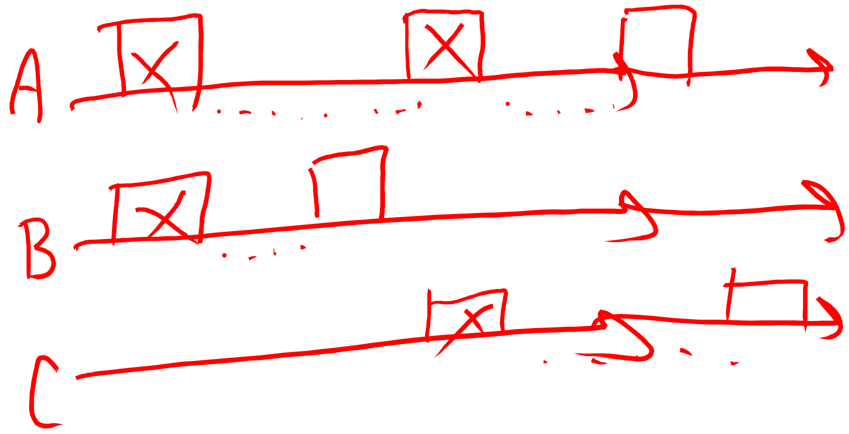
- An illustration:



Successful transmission



Unsuccessful transmission



Departure rate of the slotted Aloha

- A quick review of Poisson process:

$$P\{A(t+1)-A(t)=n\}=\frac{\lambda^n}{n!}e^{-\lambda}$$

$$\Rightarrow \frac{G^n}{n!}e^{-G}$$

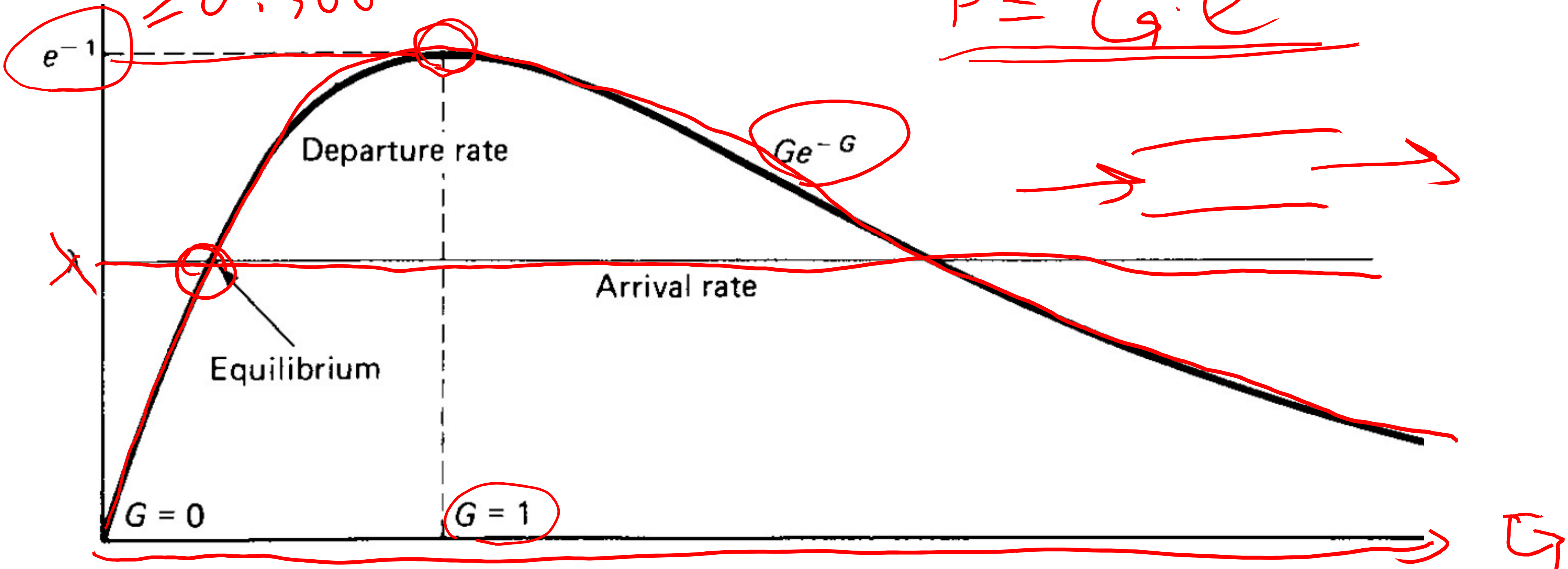
- With re-transmissions, we may approximate the total number of re-transmissions and new transmissions as a Poisson random variable with parameter $G > \lambda$
  - The probability of a successful transmission in a slot is Ge^{-G}

Departure rate vs. transmission rate

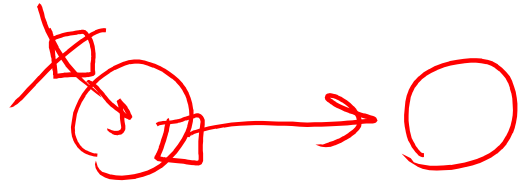
$G(n)$

$P = G \cdot e^{-G}$

$= 0.368$

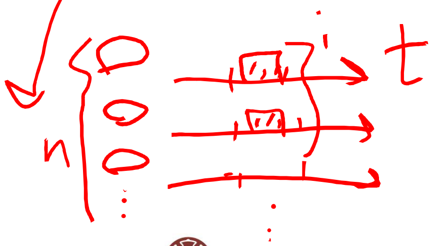


- Equilibrium happens when the departure rate equals the transmission rate



A more precise analysis

- Each backlogged sender re-transmits with probability q_r in each successive slot until success.
  - Will succeed in the *i*-th slot with probability $q_r(1-q_r)^{i-1}$
- With the no-buffering assumption, the behavior of slotted Aloha can be described as a discrete-time Markov chain (DTMC).
  - Let *n* be the number of backlogged senders at the beginning of a slot
  - Let $Q_a(i,n)$ be the probability that *i* un-backlogged senders sends packets in a slot
  - Let $Q_r(i,n)$ be the probability that *i* backlogged senders sends packets in a slot



A more precise analysis (cont.)

- We have

$$Q_a(i,n) = \binom{m-n}{i}(1-q_a)^{m-n-i} q_a^i$$

$$Q_r(i,n) = \binom{n}{i}(1-q_r)^{n-i} q_r^i$$

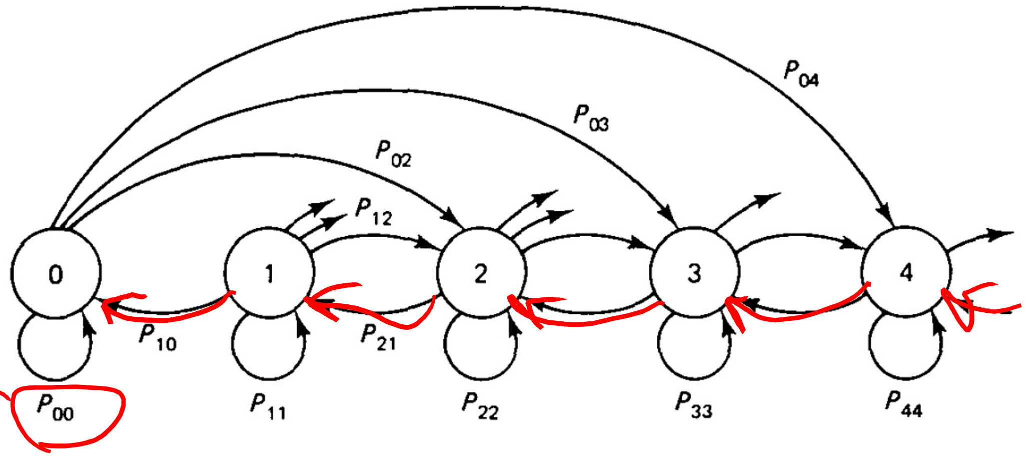
where $\lambda' = \frac{\lambda}{m}$

where $q_a = 1 - e^{-\lambda/m}$

$P\{A(t+1)-A(t)=n\} = \frac{\lambda'^n}{n!} e^{-\lambda'}$

The Markov chain and transition probabilities

n = 0
i = 0



$P_{n,n+i}$

$$P_{n,n+i} = \begin{cases} Q_a(i,n), & 2 \le i \le (m-n) \\ Q_a(1,n)[1 - Q_r(0,n)], & i = 1 \\ Q_a(1,n)Q_r(0,n) + Q_a(0,n)[1 - Q_r(1,n)], & i = 0 \\ Q_a(0,n)Q_r(1,n), & i = -1 \end{cases}$$

0 new transmission

1 retransmission

Stability of the slotted Aloha system

- One may compute p_n and p_0 for the Markov chain in the previous page, and then calculate the expected number of backlogged senders, and finally calculate the average delay using Little’s theorem.

- In practice, however, a run of bad luck may cause the slotted Aloha system to remain heavily backlogged for a long time

Stability of the slotted Aloha system (cont.)

- Define the *drift* in state *n*, (D_n), as the expected change in backlog over one slot time
  - D_n = expected number of new arrivals minus the expected number of successful transmissions in a slot

$$D_n = \boxed{(m-n)q_a} - \boxed{P_{succ}}$$

where $P_{succ} = Q_a(1,n)Q_r(0,n) + Q_a(0,n)Q_r(1,n)$

- Define the attempt rate, $G(n)$, as the expected number of transmissions in a slot when the system is in state n

$$G(n) = (m-n)q_a + nq_r$$

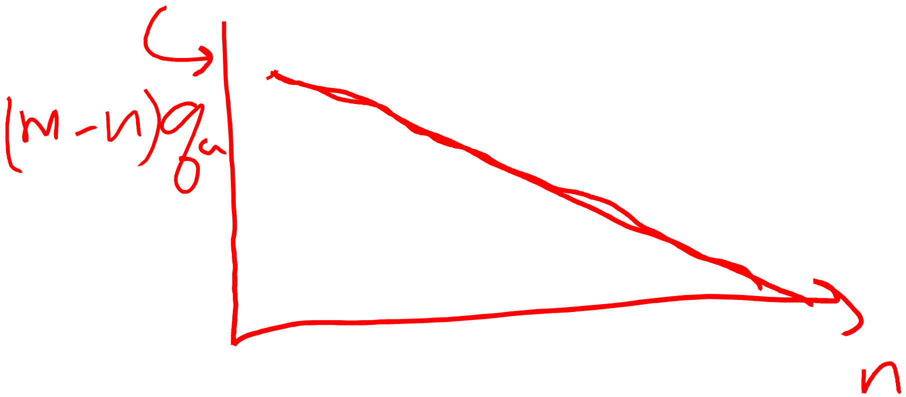
and $P_{succ} \approx G(n)e^{-G(n)}$ for small q_a and q_r

Stability of the slotted Aloha system (cont.)

- Given $D_n = (m-n)q_a - P_{succ}$ and $\boxed{P_{succ} \approx G(n)e^{-G(n)}}$

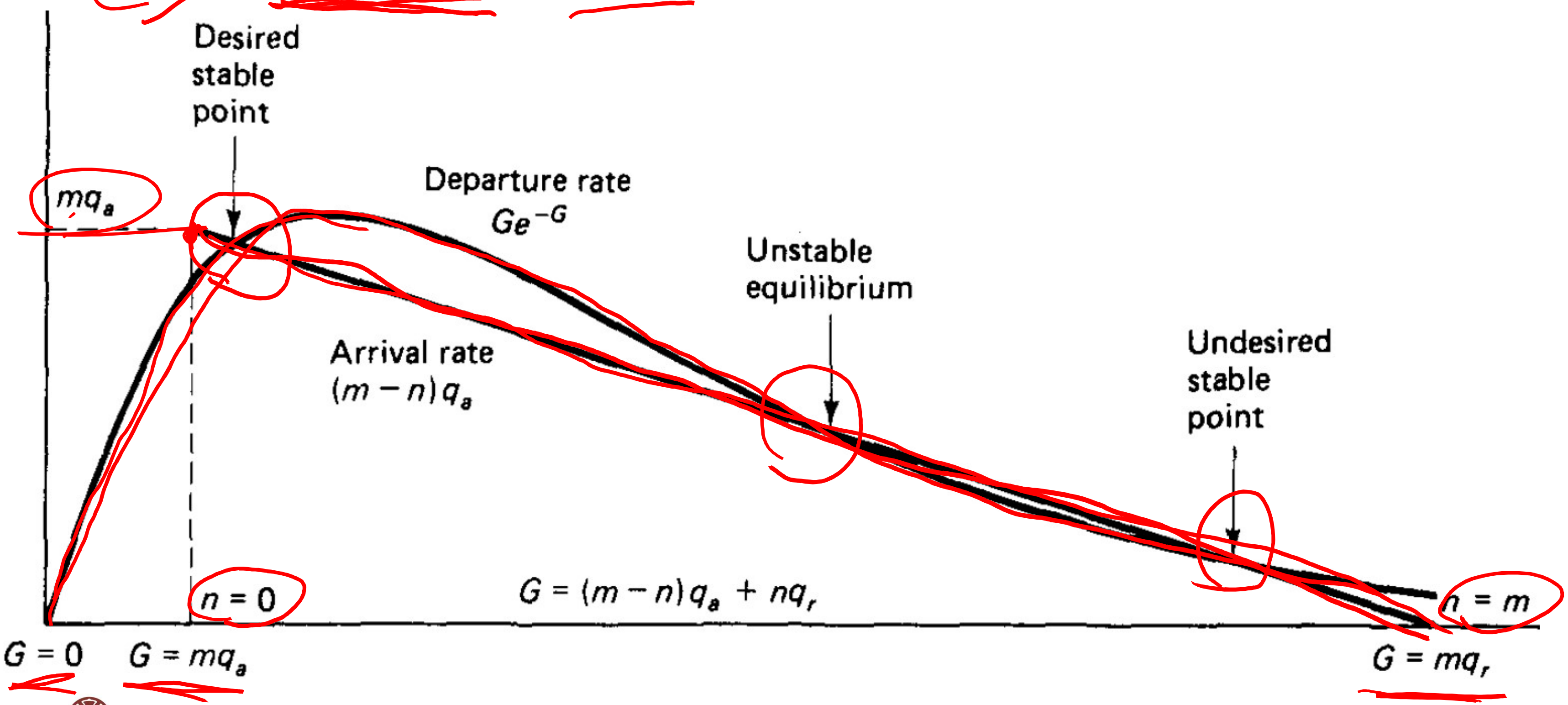
data arrival

data departure



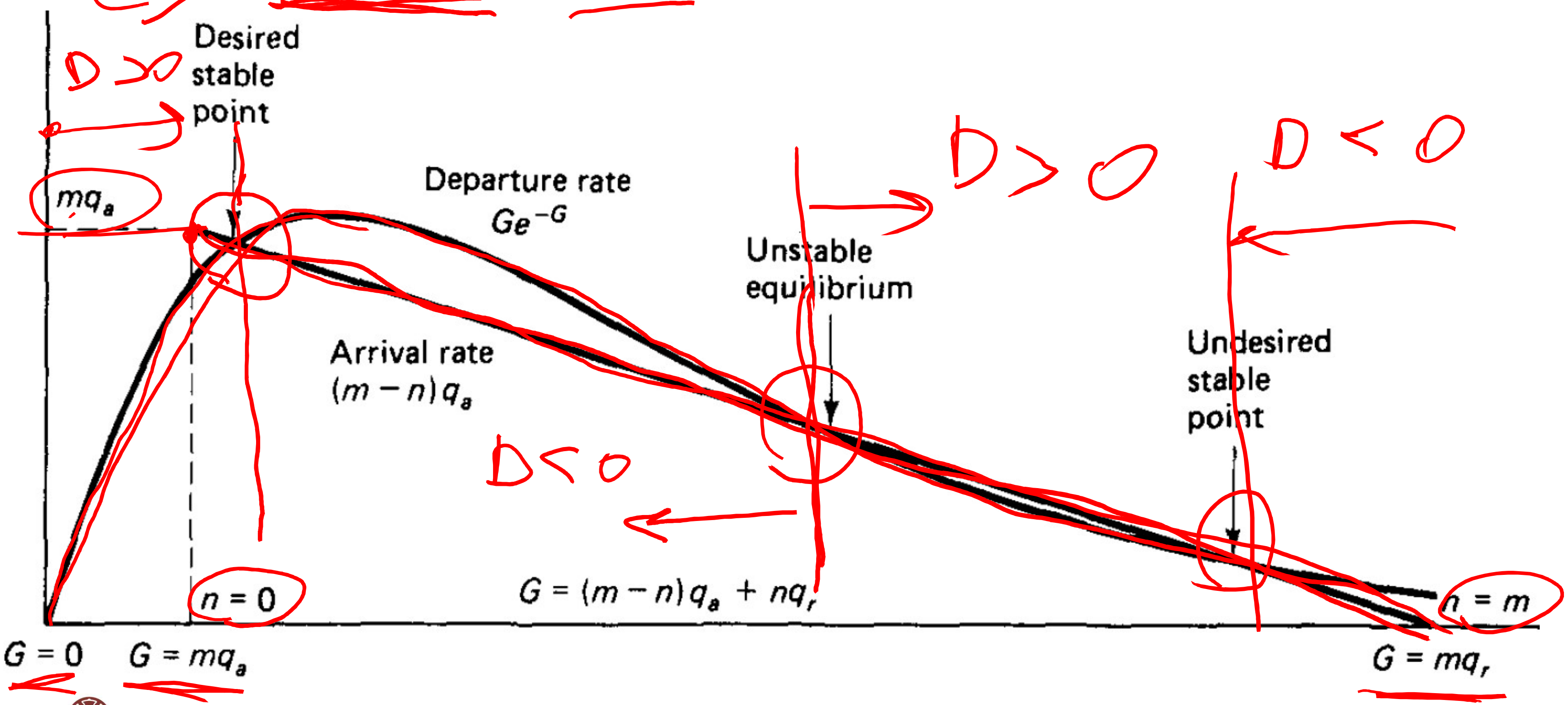
Stability of the slotted Aloha system (cont.)

- Given $D_n = (m-n)q_a - P_{succ}$ and $P_{succ} \approx G(n)e^{-G(n)}$



Stability of the slotted Aloha system (cont.)

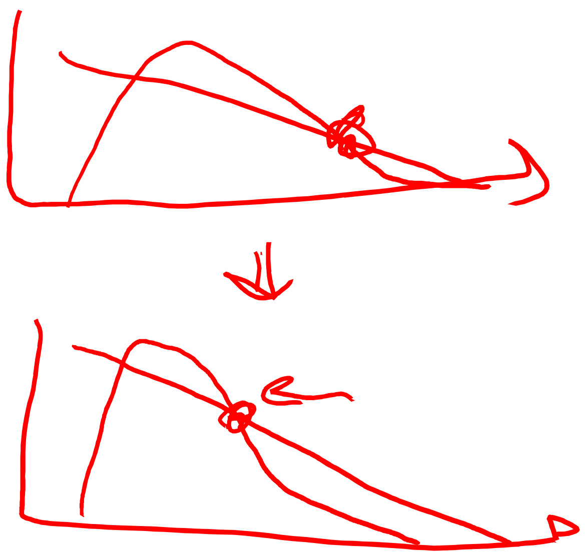
- Given $D_n = (m - n)q_a - P_{succ}$ and $P_{succ} \approx G(n)e^{-G(n)}$



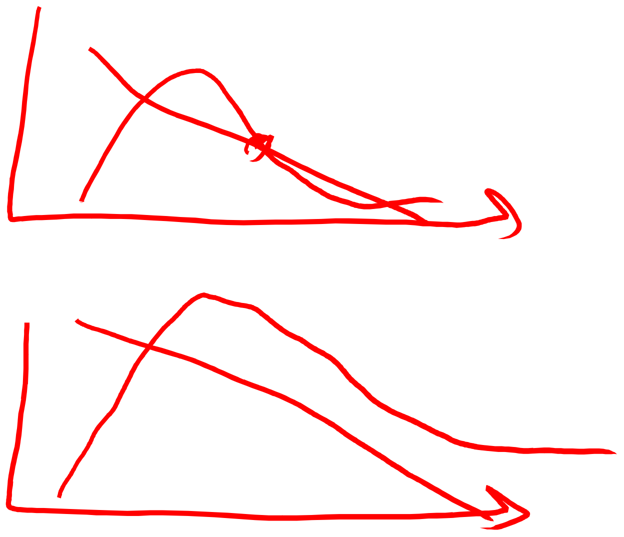
Stability of the slotted Aloha system (cont.)

- Now consider an increase/decrease of q_r:

case 1 : increase q_r

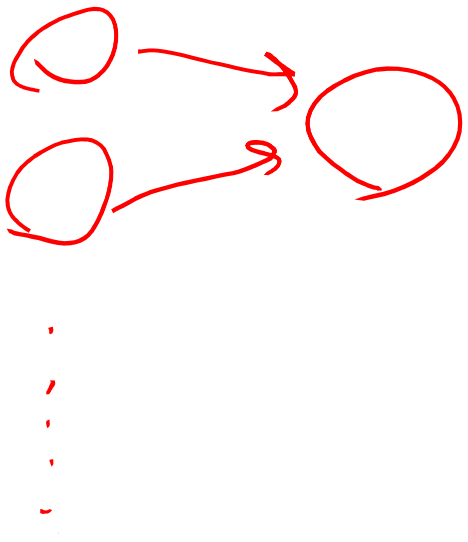


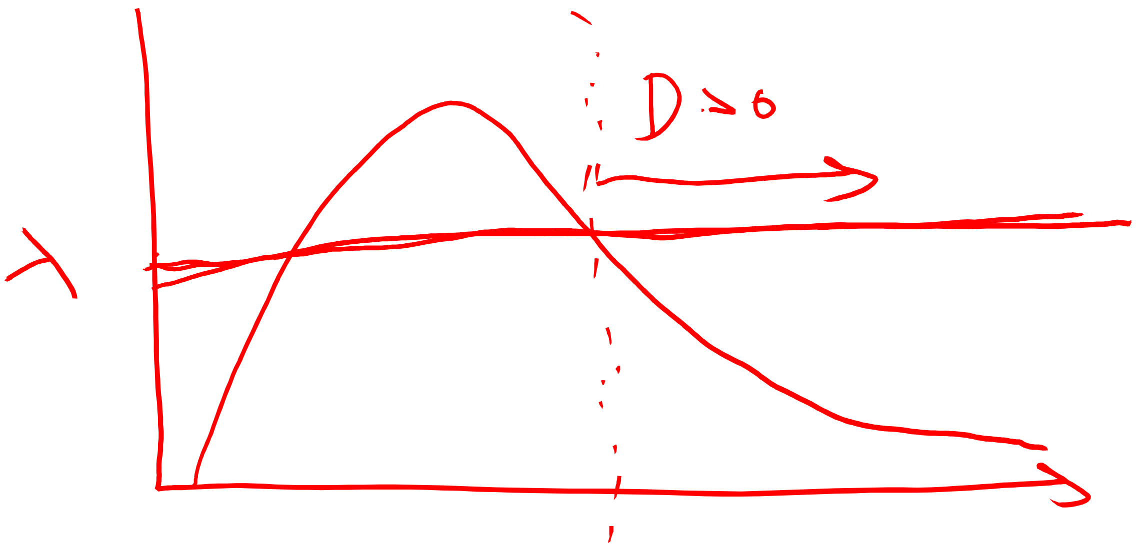
case 2 : decrease q_r



Stability of the slotted Aloha system (cont.)

- Finally, consider the infinite-sender assumption ($m = \infty$) instead





Versions of Aloha

- Pure Aloha (the original version)
- Stabilized Aloha
- Aloha with binary exponential backoff
- CSMA slotted Aloha
- …