Solution attached

CSC0056 Data Communication, Homework 3

- **IMPORTANT**:
  - Submit your answer of "Part 1" before 9PM, Nov 6th (Friday). Clearly label your answer.
  - Submit your answer of "Part 2" before 9PM, Nov 12th (Wednesday).
  - We will have the midterm exam on Nov 9th, in class.

Part 1: Writing Tasks (60%)

1. (20%) Basics of the discrete-time Markov chain (DTMC):

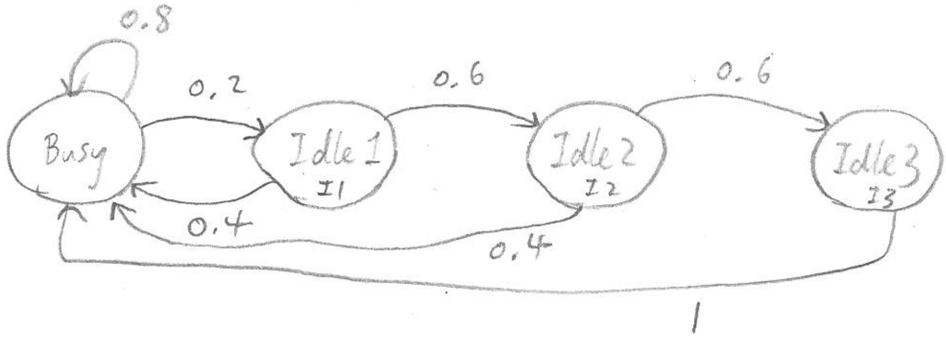
   A networking server is either busy or idle. If it is currently busy, with 80% probability it will be busy at the next time step; if it is currently idle, with 60% probability it will be idle at the next time step; but after the server remains idle for three consecutive time steps, at the next time step it will perform some garbage collection and thus will be busy.

   1. (5%) Draw the DTMC diagram. There are four states in total.
   2. (5%) State the transition probability matrix (please use the format introduced in this course).
   3. (10%) Compute the limiting probability for each state.
2. (15%) The exponential distribution and its memoryless property:
   1. (10%) Suppose that a system has three servers, and that packet P arrived at the system when all the servers were processing some other packets. Assume that (1A) all packets have independent, identical, exponential distribution of service time, (1B) packet P is the only one waiting in the system (the other three packets are being processed), and (1C) packet P will start being processed right after any of the servers is available. Among this four packets, what is the probability that packet P will be the last to complete the process?
   2. (5%) Following the above question, what is the average time packet P will spend in the system, assuming that the average service time is 30 milliseconds?
3. (25%) M/M/1 system, with arrival rate λ and service rate μ:
   1. (5%) Explain in your own word, conceptually, why we have $p_n \cdot \lambda = p_{n+1} \cdot \mu$ for the corresponding continuous-time Markov chain.
   2. (10%) Suppose $\lambda = 44$ packets/second and $\mu = 50$ packets/second. What would be the steady-state probability that there are 10 packets in the system?
   3. (10%) Now let's consider the end-to-end delay in delivering a packet, where this M/M/1 queue sit on the end-to-end delivery path. Suppose a networking application's service-level agreement (SLA) demands an end-to-end delay of no more than 100 milliseconds. If we know that each packet will take no more than 35 milliseconds to reach this M/M/1 system from the sender, and it will take no more than 40 milliseconds for the network to forward a packet from the output of the system to the destination. Suppose service rate $\mu = 0.25$ packets/millisecond for this system. Determine the range of admissible arrival rate to this system so that such a configuration can meet the SLA of 100 milliseconds.

Name: Solution for Homework 3 Student ID: ______ P1

1.

1.1



1.2

$$\mathbb{P} = \begin{array}{c} \\ B \\ I1 \\ I2 \\ I3 \end{array} \begin{array}{cccc} B & I1 & I2 & I3 \\ \end{array}$$

$$\mathbb{P} = \begin{array}{c|cccc} & B & I1 & I2 & I3 \\ \hline B & 0.8 & 0.2 & 0 & 0 \\ I1 & 0.4 & 0 & 0.6 & 0 \\ I2 & 0.4 & 0 & 0 & 0.6 \\ I3 & 1 & 0 & 0 & 0 \end{array}$$

1.3 Let $\vec{\pi} = (\pi_B, \pi_{I1}, \pi_{I2}, \pi_{I3})$ be the vector of limiting probability.

from $\vec{\pi} \cdot \mathbb{P} = \vec{\pi}$ and $\sum_{i=0}^{3} \pi_i = 1$, we have

$$\begin{cases} \pi_B = \pi_B \times 0.8 + \pi_{I1} \times 0.4 + \pi_{I2} \times 0.4 + \pi_{I3} \\ \pi_{I1} = \pi_B \times 0.2 \\ \pi_{I2} = \pi_{I1} \times 0.6 = \pi_B \times 0.12 \\ \pi_{I3} = \pi_{I2} \times 0.6 = \pi_{I1} \times 0.36 = \pi_B \times 0.072 \\ \pi_B + \pi_{I1} + \pi_{I2} + \pi_{I3} = 1 \end{cases}$$

$$\Rightarrow \begin{cases} \pi_B = \frac{1}{1.392} \approx 0.718 \\ \pi_{I1} \approx 0.718 \times 2 \approx 0.143 \\ \pi_{I2} \approx 0.718 \times 0.12 \approx 0.086 \\ \pi_{I3} \approx 0.718 \times 0.072 \approx 0.051 \end{cases}$$

Name: Solution for homework 3 Student ID: ______

(2) Recall that the memoryless property is

(2.1)

$$\mathbb{P}\{X > s+t \mid X > s\} = \mathbb{P}\{X > t\}$$

Let random variable X denote the total service time of a packet. $\mathbb{P}\{X > s+t \mid X > s\}$ means the probability that it will take t more time units to complete, given that it has been in service for s time units. Now, the memoryless property implies that the value of s will not change $\mathbb{P}\{X > t\}$. Therefore, at the time packet P starts its service, it will have the same probability to be the last to complete as the probability for each of the other two. These three probabilities sums up to 1. Therefore, each probability is $\frac{1}{3}$ #

(2.2) $E[P_{total}] = E[P_{waiting}] + \underbrace{E[P_{inservice}]}_{30\ ms}$

To compute $E[P_{waiting}]$, we may consider first the probability that packet P needs to wait for at least t time units:

$\mathbb{P}\{P \text{ waits for at least } t \text{ time units}\} = 1 - \mathbb{P}\{\text{all packets in the servers haven't completed at } t\}$

$$= 1 - (\mathbb{P}\{X > t\})^3 = 1 - (e^{-\frac{1}{30}t})^3 = 1 - e^{-\frac{1}{10}t}$$

Let Y be a random variable denoting P's waiting time, and then we have $\mathbb{P}\{Y \le t\} = 1 - e^{-\frac{1}{10}t}$. This suggests a p.d.f. of $f(t) = \frac{1}{10}e^{-\frac{1}{10}t}$, which is an exponential distribution of rate $\frac{1}{10}$: So the mean is $10\ ms = E[P_{waiting}]$.

$\Rightarrow E[P_{total}] = 10 + 30 = 40\ ms$ #

Name: Solution for homework 3 Student ID: ________ P_3

③

(3.1) You may use the concept of "traversing between states for many times", as we did in class, to explain the frequency of going from state n to state $n+1$ is equal to that of going from state $n+1$ to state n. Then

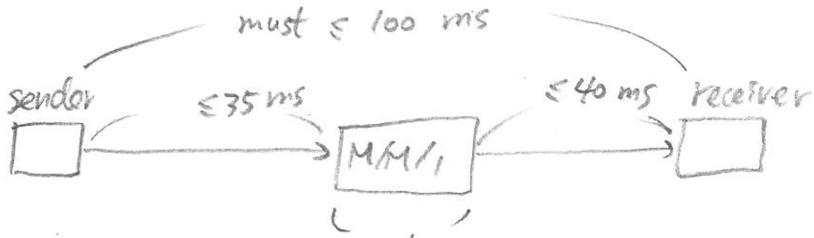
$$\frac{\#\text{ of transitions from } n \text{ to } n+1}{\text{total } \#\text{ of transitions}} = \frac{\#\text{ of transitions from } n+1 \text{ to } n}{\text{total } \#\text{ of transitions}}$$

$\Rightarrow \quad P_n \lambda = P_{n+1} \mu$ ※

(3.2) $P_{10} = \rho^{10}(1-\rho) \qquad \rho = \frac{\lambda}{\mu} = 0.88$

$\approx 3.3\%$ ※

(3.3)



must $\leq 100-35-40$
$= 25$ ms

$$T = \frac{1}{\mu - \lambda} \leq 25 \text{ ms}$$

$$\Rightarrow \frac{1}{0.25 - \lambda} \leq 25 \text{ ms}$$

$\Rightarrow \lambda \leq 0.21$ packets/ms

\#