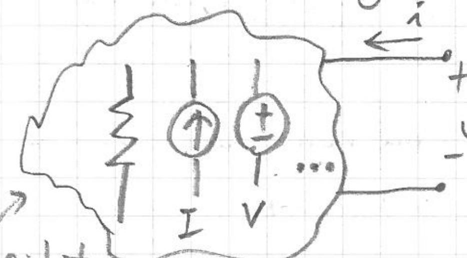
☆ Thévenin's Theorem

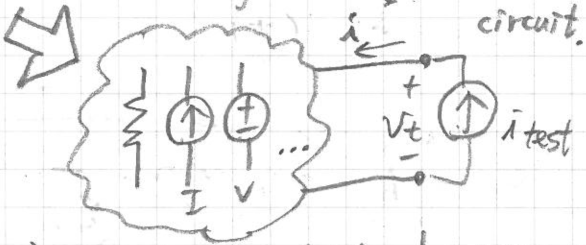
goal: Given an arbitrary linear circuit, we would like to know how it would respond to external excitation; in other word, we'd like to know its i-v characteristic.

Approach: leverage the concept of superposition!



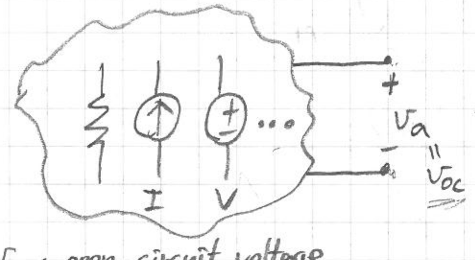
an arbitrary circuit

We append a testing current source, with the original circuit together they form a new circuit.

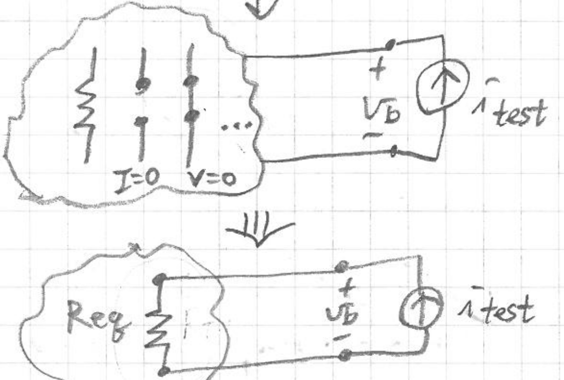


set $i_{test} = 0$

set internal sources $= 0$



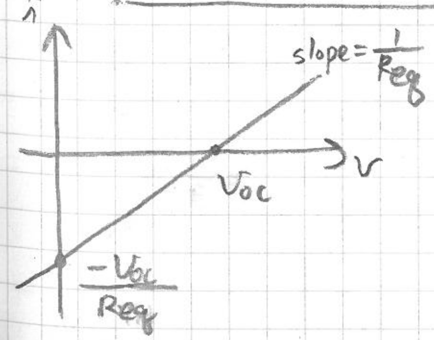
v_{oc} : open-circuit voltage

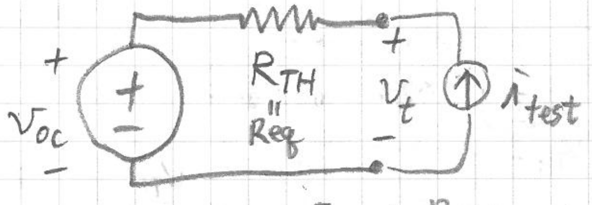


By superposition,

$$v_t = v_a + v_b$$

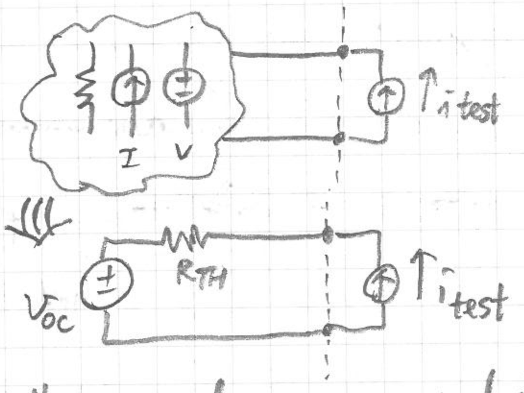
$$\Rightarrow \boxed{v_t = v_{oc} + i_{test} R_{eq}} \Rightarrow$$ equivalently, this relation describes the following circuit:



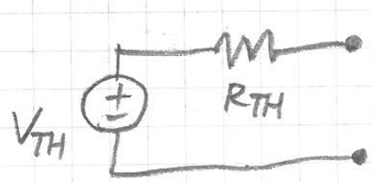


We rename R_{eq} to R_{TH} to honor Thévenin

P28 Therefore, we have the following equivalence:

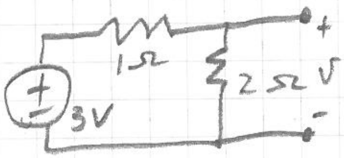


In other word, we may reduce an arbitrary linear circuit to an equivalent circuit of the form:

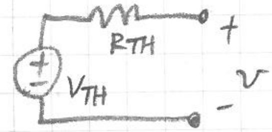


we name $V_{TH} = V_{OC}$ in honor of Thévenin.

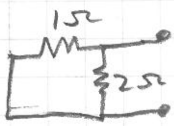
Example:



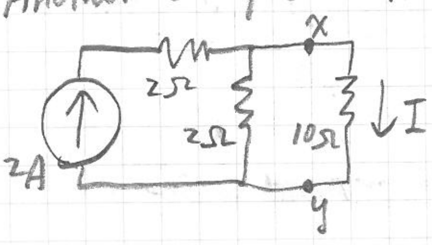
$\Rrightarrow$



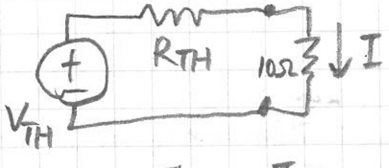
$V_{TH} = v = 3 \times \frac{2}{1+2} = 2V$, $R_{TH} = \frac{1 \cdot 2}{1+2} = \frac{2}{3}\Omega$ from



Another example: find $I = ?$



We may replace the left side of x-y by an equivalent circuit:



Then $I = \frac{V_{TH}}{R_{TH} + 10\Omega}$

calculating V_{TH}:

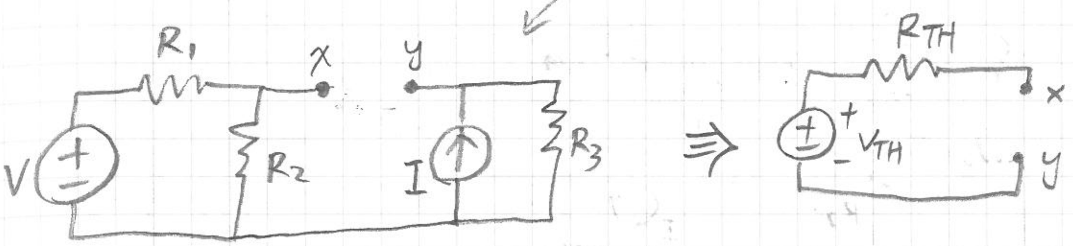
$V_{TH} = 2A \times 2\Omega = 4V$

calculating R_{TH}:

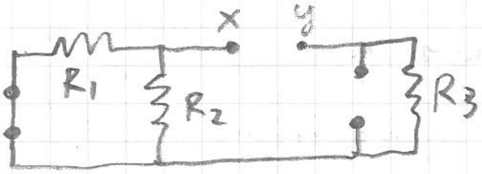
$\leftarrow R_{TH} = 2\Omega$

$\Rightarrow I = \frac{4}{2+10} = \frac{1}{3}A$ ※

Exercise: create the Thévenin Equivalent Circuit for the following circuit:



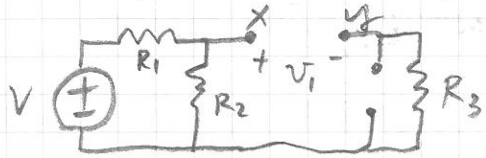
first of all, for R_{TH}:



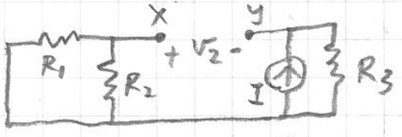
$$\Rightarrow R_{TH} = (R_1 /\!/ R_2) + R_3 = \frac{R_1 R_2}{R_1 + R_2} + R_3$$

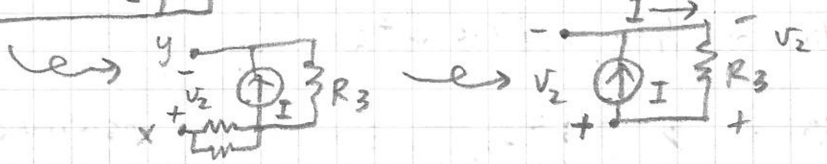
then, for V_{TH}:

approach ①, using superposition

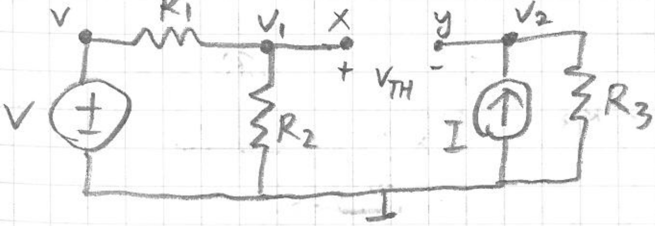


$$V_{TH} = V_1 + V_2 = \frac{R_2}{R_1 + R_2} V + (-IR_3) = \frac{R_2}{R_1 + R_2} V - IR_3 \quad ※$$





approach ②, using the node method (Page 21)



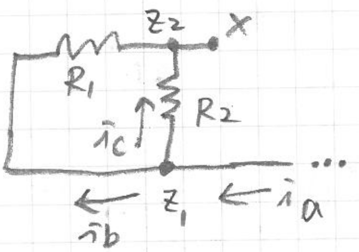
$$V_{TH} = V_1 - V_2 = \frac{R_2}{R_1 + R_2} V - IR_3 \quad ※$$

$$\text{KCL: } \begin{cases} \dfrac{V - V_1}{R_1} + \dfrac{0 - V_1}{R_2} = 0 \\ I + \dfrac{0 - V_2}{R_3} = 0 \end{cases} \Rightarrow \begin{cases} V_1 = \dfrac{R_2}{R_1 + R_2} V \\ V_2 = IR_3 \end{cases}$$

In the exercise on the previous page, you might wonder why we can do the following transformation:

Could it be possible that there are some current flowing through R_1 and/or R_2?

We can use KCL to figure it out:



for node z_1, we have

$$i_a - i_b - i_c = 0$$

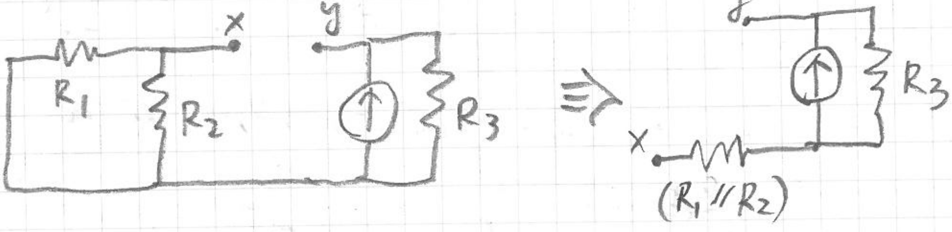
for node z_2, we have

$$i_b + i_c = 0$$

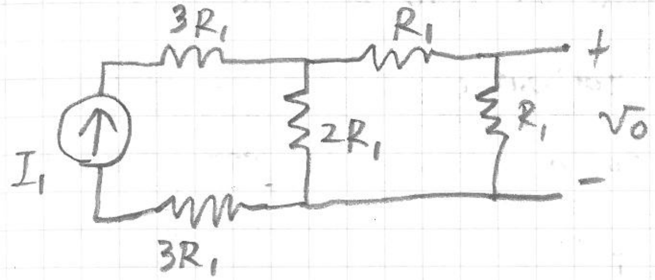
compare

Therefore, we see that $i_a = 0$.

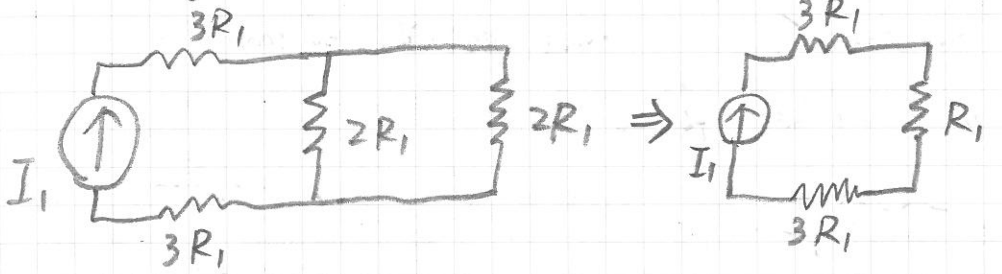
In practice, it is often helpful to think in terms of equivalent resistance, which may make the situation much more obvious:



Exercise : In the following circuit, determine voltage v_o :



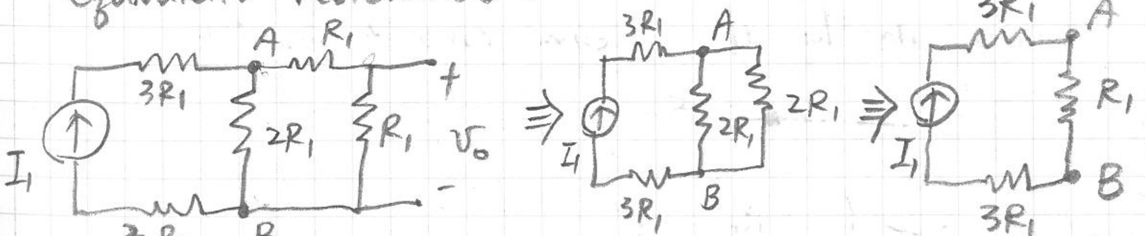
We may do the following transformation :



then we might conclude that $v_o = I_1 \cdot R_1$,

but it is wrong.

To see this, it could be helpful to <u>clearly label</u> the nodes between which you calculate the equivalent resistance :



thus we see that v_o is not the voltage across A, B,

and $v_o = (\text{voltage across } A, B) \times \dfrac{R_1}{R_1 + R_1}$

$$= (I_1 \cdot R_1) \times \frac{R_1}{R_1 + R_1} = \frac{1}{2} I_1 R_1$$ is the correct answer.

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