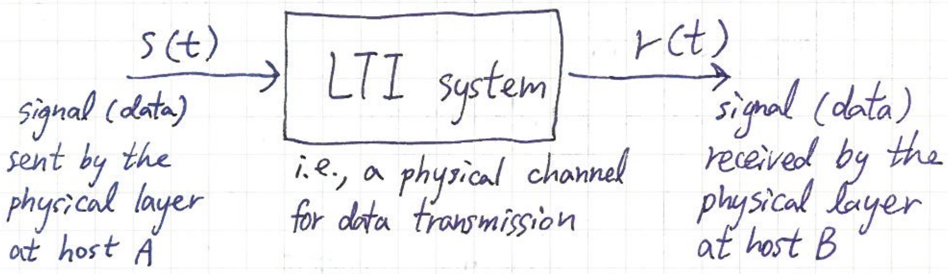
Note for lecture 02:



Goal: to have $r(t) = s(t)$,
i.e., to have host B correctly receive the data sent by host A.

Challenges: ① intersymbol interference (Sec. 2.2.1)
② a channel may distort signal (its frequency response; Sec. 2.2.2)

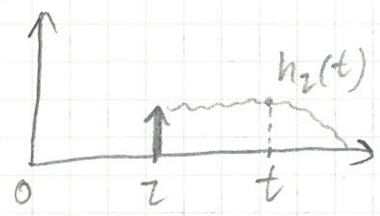
Analysis: 1°

$$s(t) = \int_{-\infty}^{\infty} s(\tau)\cdot\delta(t-\tau)\,d\tau$$

where $\delta(t-\tau) = \begin{cases} 1 & \text{if } t=0 \\ 0 & \text{otherwise} \end{cases}$

$$r(t) = \int_{-\infty}^{\infty} s(\tau)\cdot h_\tau(t)\,d\tau$$

where $h_\tau(t)$ is the response of the unit impulse at time τ



in fact, $h_\tau(t) = h_0(t-\tau)$
$\because$ time-invarient property

Let $h(t) = h_0(t)$, we can rewrite $r(t)$ to be

$$\boxed{r(t) = \int_{-\infty}^{\infty} s(\tau)\cdot h(t-\tau)\,d\tau} \qquad \text{Eq.}(2.1)$$

(interpretation: $r(t)$ is the superposition of responses of all previous inputs!)

2° Now consider a common type of signal

$$s(t) = e^{j2\pi f\tau}$$

using Eq. (2.1), we have

$$r(t) = H(f)\cdot e^{j2\pi ft}$$

where

$$\boxed{H(f) = \int_{-\infty}^{\infty} h(\tau)\cdot e^{-j2\pi f\tau}\,d\tau}$$

Follow the derivation outlined in Sec 2.2.2, we have

$$R(f) = H(f) \cdot S(f)$$

So, if we can get rid of $H(f)$ we can achieve our goal, since $R(f) = S(f)$ implies $r(t) = s(t)$.

$\longrightarrow$ We may add an additional filter of frequency response $H^{-1}(f)$ to the channel and get $H'(f) = H^{-1}(f) \cdot H(f) = 1$, making $R(f) = S(f)$.

The above analysis also gives us insight regarding what frequency range should be used by signal $s(t)$. For example, consider $h(t) = \alpha e^{-\alpha t}$ for a channel. Then $H(f) = \int_{-\infty}^{\infty} \alpha e^{-\alpha \tau} e^{-j2\pi f \tau} d\tau$

$$= \frac{\alpha}{\alpha + j2\pi f} \Rightarrow \begin{cases} f \ll 1 \text{ gives } H(f) \approx 1 \\ f \gg 1 \text{ gives } H(f) \approx \frac{1}{f} \end{cases}$$

This corresponds to our observation that with a lower frequency (i.e., longer interval in between), we will have less intersymbol interference, and vice versa.

The sampling theorem (Sec. 2.2.3) offers another look at the signal rate.

Motivation to modulation:

Many real-world channels are bandpass channels, i.e., $|H(f)|$ is significantly nonzero only within some frequency band $f_1 \leq f \leq f_2$. To reduce signal distortion, we may multiply the signal by a sinusoidal carrier, say $\cos(2\pi f_0 t)$, to make the modulated signal fall within the desirable frequency band.

Then we send the modulated signal $s(t) \cdot \cos(2\pi f_0 t)$ over the channel. The receiving side of the channel may obtain $r(t) \approx s(t)$ by the following demodulation:

1° multiply $s(t)\cos(2\pi f_0 t)$ by another $\cos(2\pi f_0 t)$, giving $r(t) = s(t) \cdot \cos^2(2\pi f_0 t)$

$$= \frac{1}{2} s(t) + \frac{1}{2}(s(t) \cdot \cos(4\pi f_0 t))$$

by applying the Double-angle formulae

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

2° filter out the high-frequency component $\frac{1}{2}(s(t) \cdot \cos(4\pi f_0 t))$ using, e.g., a low-pass filter.

Summary (Take-home messages):

(1) It is nontrivial to send data over a physical channel and have it correctly received at the other end of the channel.

(2) In the layered network architecture, physical channel "can be regarded simply as unreliable bit pipes by the higher layers." (Sec. 2.2.9)

(3) Therefore, we must do error detection (and possibly error correction) at the higher layers. We will cover this topic next week in lecture 03 !