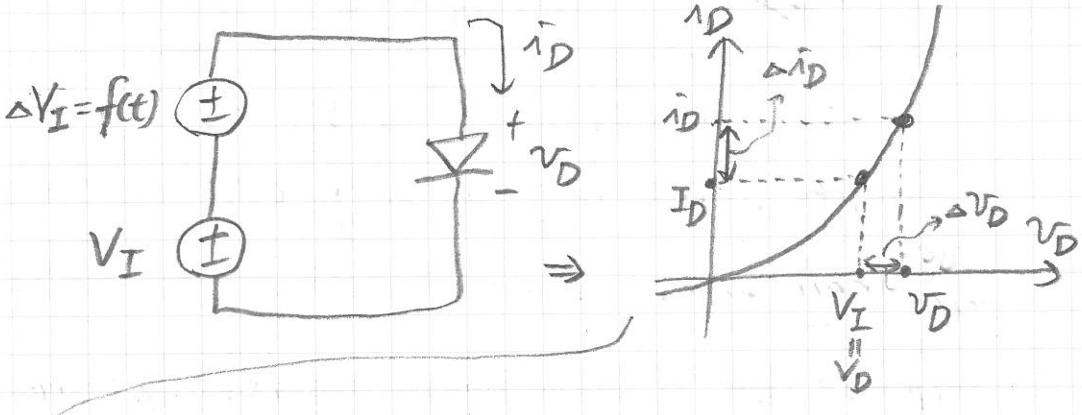
☆ Small-Signal Analysis for Nonlinear Devices

- In many sensor applications and most audio amplifiers, the input voltage/current to a circuit often consists of two parts:
  - ① a time-invariant source (large signal)
  - ② a time-varying source (small signal)

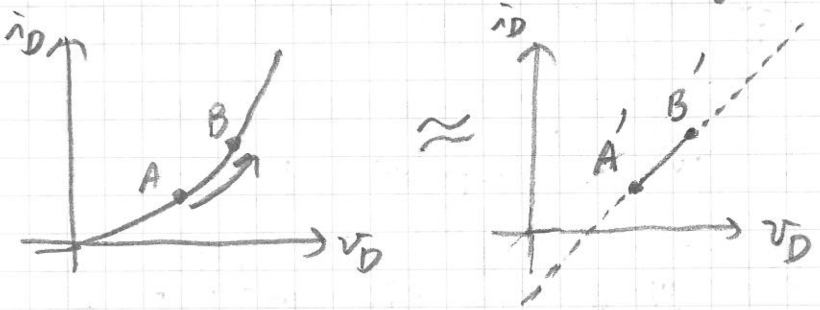


The large signal is used to determine the region of operation (i.e., which part of the i_D-v_D curve), and the small signal is the "real" input (e.g., those induced by human voice, as in the case of a microphone).

$$v_D = \underbrace{V_I}_{\text{large}} + \underbrace{\Delta v_D}_{\text{small signal}}$$

$$i_D = I_D + \Delta i_D$$

P44 as we will see, moving along a small distance on "the $i_D - v_D$ curve" can be approximated as moving along a small distance on "a straight line"

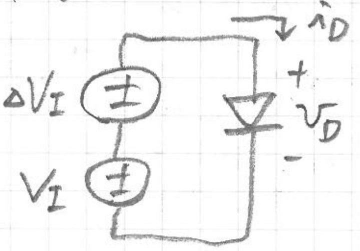


Therefore, we may simplify our analysis of small signal by considering the signal's response on a nonlinear device as if it is the response on a linear device (resistor).

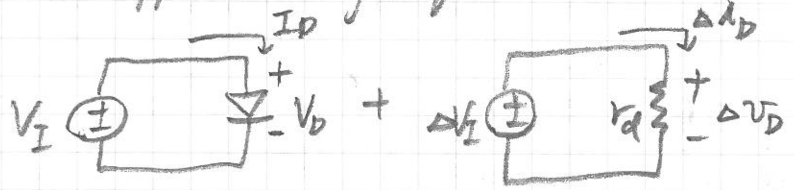
Review P13, where we've shown that the i-v characteristic of a resistor on the i-v plot is a straight line; further, the slope of the line is equal to the reciprocal of the resistance (R) of the resistor.

Now, a question is: how do we determine the resistance of that linear device?

Let r_d be the resistance of the linear device. Using small-signal analysis, we essentially transform the original circuit



into an approximately equivalence:

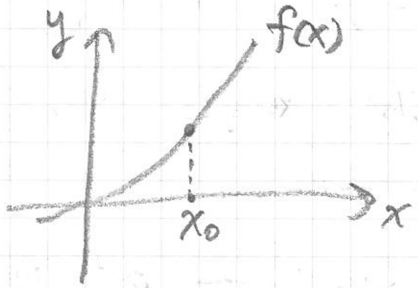


where $i_D = I_D + \Delta i_D$

$$'' \ \frac{V_{THE}}{I_D}$$

Now, let's see how to determine r_d!

We use Taylor's Theorem, which provides a way to approximate a curve near a certain point $x = x_0$:



$$\underset{x \text{ near } x_0}{y} = \underset{x \text{ near } x_0}{f(x)} = f(x)\Big|_{x=x_0} + f'(x)\Big|_{x=x_0}(x-x_0)$$

$$+ \frac{1}{2!}f''(x)\Big|_{x=x_0}(x-x_0)^2 + \frac{1}{3!}f'''(x)\Big|_{x=x_0}(x-x_0)^3 + \cdots$$

in our case of a nonlinear diode,

recall that

$$i_D = I_S\left(e^{v_D/v_{THE}} - 1\right) \underset{\substack{\uparrow \\ \text{we define it}}}{=} f(v_D)$$

$$\Rightarrow \underset{v_D \text{ near } V_D}{i_D} = f(v_D)\Big|_{v_D=V_D} + f'(v_D)\Big|_{v_D=V_D}(v_D - V_D)$$

$$+ \frac{1}{2!}f''(v_D)\Big|_{v_D=V_D}(v_D - V_D)^2$$

$$+ \frac{1}{3!}f'''(v_D)\Big|_{v_D=V_D}(v_D - V_D)^3 + \cdots$$

$$= f(v_D)\Big|_{v_D=V_D} + (v_D - V_D)\Big(f'(v_D)\Big|_{v_D=V_D}$$

we choose to ignore these terms $\leftarrow$

$$\left.\begin{aligned} &+ \frac{1}{2!}f''(v_D)\Big|_{v_D=V_D}(v_D - V_D) \\ &+ \frac{1}{3!}f'''(v_D)\Big|_{v_D=V_D}(v_D - V_D)^2 \\ &+ \cdots \Big) \end{aligned}\right.$$

using the chain rule, we get $f'(v_D) = \frac{I_S}{v_{THE}} e^{v_D/v_{THE}}$

$$\Rightarrow \underset{v_D \text{ near } V_D}{i_D} \approx f(v_D)\Big|_{v_D=V_D} + (v_D - V_D)\cdot \frac{I_S}{v_{THE}} \cdot e^{v_D/v_{THE}}\Big|_{v_D=V_D}$$

$$= I_S\left(e^{V_D/v_{THE}} - 1\right) + \Delta v_D \cdot \frac{I_S}{v_{THE}} \cdot e^{V_D/v_{THE}}$$

since we know that

$$e^{V_D/v_{THE}} \gg 1$$

so we can think of $e^{V_D/v_{THE}} \approx e^{V_D/v_{THE}} - 1$

With that, we may rewrite the equation as

$$\underset{v_D \text{ near } V_D}{i_D} = I_s\left(e^{V_D/v_{THE}} - 1\right) + \Delta v_D \frac{I_s}{v_{THE}}\left(e^{V_D/v_{THE}} - 1\right)$$

Now, by observation we see $I_s\left(e^{V_D/v_{THE}} - 1\right) = I_D$

Thus, $\underset{v_D \text{ near } V_D}{i_D} = I_D + \frac{\Delta v_D}{v_{THE}} I_D$

Compare to $i_D = I_D + \Delta i_D$, we have $\Delta i_D = \frac{\Delta v_D}{v_{THE}} I_D$

Think in terms of relation of Δi_D and Δv_D

and we may choose to define

$$\boxed{r_d = \frac{v_{THE}}{I_D}}$$ ※

In the hindsight, we may generalize our result by saying that for an arbitrary nonlinear element, we have

$$\boxed{r_d = \frac{1}{f'(v_D)\big|_{v_D = V_D}} = \frac{1}{\left.\frac{df(v_D)}{dv_D}\right|_{v_D = V_D}}}$$

※ (finally!)

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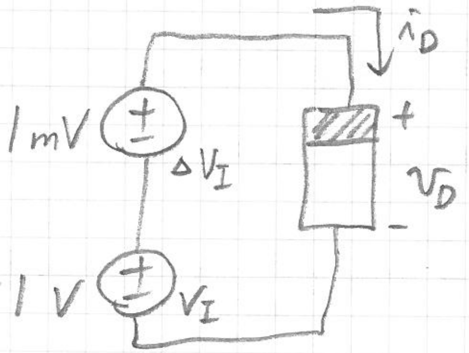
Example:

find $i_D = ?$



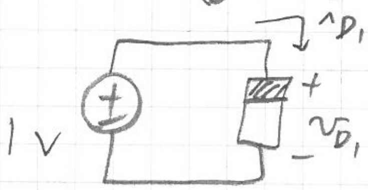
recall that for (P35)

$i_D = K \cdot v_D^2$ for $v_D > 0$

and here we suppose

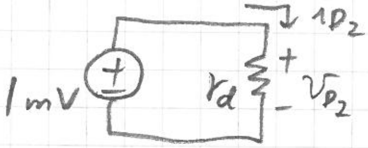
$K = 1\,mA/V^2$

Ans:



$v_{D_1} = 1$ (KVL)

$\Rightarrow i_{D_1} = K \cdot v_{D_1}^2 = 1\,mA$



$v_{D_2} = 1\,mV$ (KVL)

$$r_d = \frac{1}{f'(v_D)\big|_{v_D = V_D}} = \frac{1}{2 \cdot K \cdot v_D\big|_{v_D = 1V}}$$

$$= 500\,\Omega$$

$$\Rightarrow i_{D_2} = \frac{v_{D_2}}{r_d} = 2\,\mu A$$

$$\Rightarrow i_D = i_{D_1} + i_{D_2} = 1.002\,mA$$

※