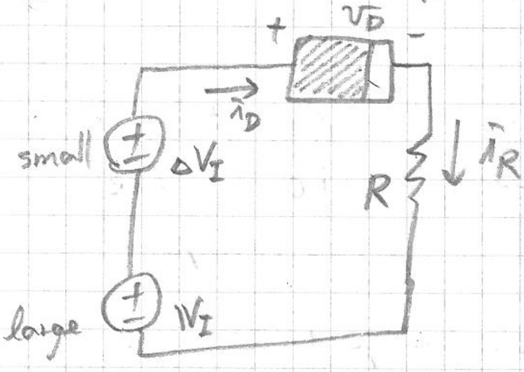
Exercises for small-signal analysis:

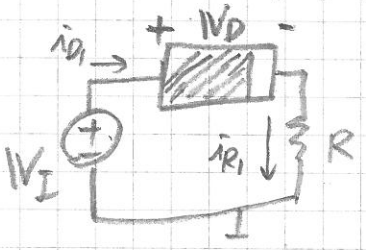
(The diode is defined on page 37)

Exercise ①: find $i_R = ?$



Ans: $i_R = \mathbb{K}\left(\dfrac{-1+\sqrt{1+4\mathbb{K}\mathbb{V}_I R}}{2\mathbb{K}R}\right)^2 + \dfrac{\Delta V_I}{R+\left(\dfrac{R}{-1+\sqrt{1+4\mathbb{K}\mathbb{V}_I R}}\right)}$ #

Solution: First of all, it is clear that $v_D > 0$ in this case.

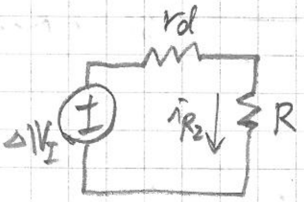


From analytical analysis, we have

$$\frac{\mathbb{V}_I - \mathbb{V}_D}{R} = \mathbb{K}\,\mathbb{V}_D^2$$

$$\Rightarrow \mathbb{V}_D = \frac{-1+\sqrt{1+4\mathbb{K}\mathbb{V}_I R}}{2\mathbb{K}R}$$

$$\Rightarrow i_{R_1} = i_{D_1} = \mathbb{K}\,\mathbb{V}_D^2$$



$$r_d = \frac{1}{f'(v_D)\big|_{v_D=\mathbb{V}_D}} = \frac{1}{2\mathbb{K}\cdot v_D\big|_{v_D=\mathbb{V}_D}}$$

$$= \frac{R}{-1+\sqrt{1+4\mathbb{K}\mathbb{V}_I R}}$$

$$i_{R_2} = \frac{\Delta\mathbb{V}_I}{R+r_d} = \frac{\Delta\mathbb{V}_I}{R+\left(\dfrac{R}{-1+\sqrt{1+4\mathbb{K}\mathbb{V}_I R}}\right)}$$

finally, $i_R = i_{R_1} + i_{R_2}$ ※

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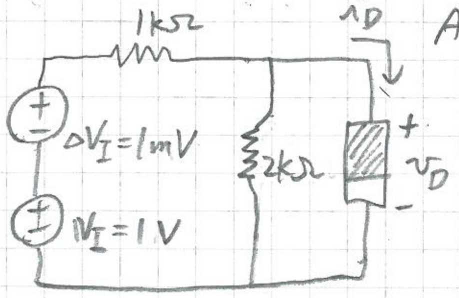
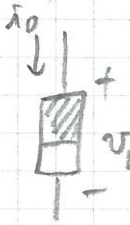
Exercise ②: find $i_D = ?$



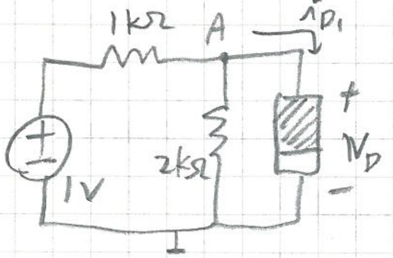
Ans: $\underline{0.2504\ mA}$ ※



$$i_D = \begin{cases} IK\, v_D^2 & \text{for } v_D > 0 \\ 0 & \text{else} \end{cases}$$

$IK = 1 mA/V^2$

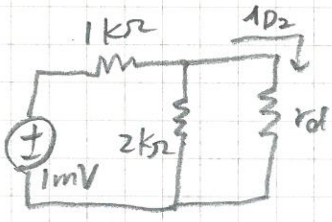
Solution: First of all, from voltage divider we know that $v_D > 0$



Note that $V_D \neq 1 \times \frac{2}{1+2}$

$$\begin{cases} \frac{1 - V_D}{1} = \frac{V_D - 0}{2} + i_D & \text{(KCL at A)} \\ i_D = IK\, V_D^2 & \text{(element's law)} \end{cases}$$

$\Rightarrow V_D = \frac{1}{2} V \Rightarrow i_D = 0.25\ mA$



$$r_d = \frac{1}{f'(v_D)\big|_{v_D = V_D}} = \frac{1}{2IK\, V_D} = \frac{1}{IK} = 1\ k\Omega$$

$$i_{D_2} = \underbrace{\left(\frac{2}{1+2}\right)}_{\text{current divider}} \times \frac{1mV}{1 + \frac{1\times 2}{1+2}} = 0.4\ \mu A$$

$$= 0.4 \times 10^{-3}\ mA$$

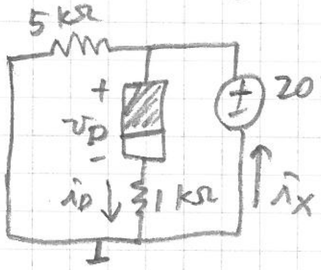
finally, $i_D = i_{D_1} + i_{D_2}$

$= 0.25\ mA + 0.0004\ mA$

$= 0.2504\ mA$

※

Exercise ③ : find $i_x = ?$ Ans : $\underline{20\ mA}$ ※

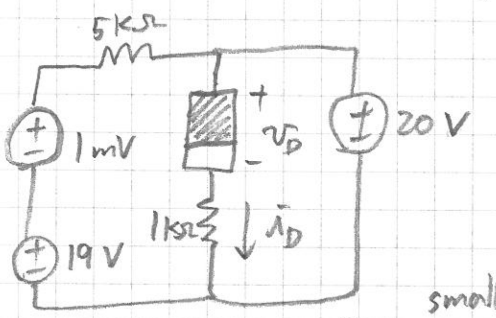


Solution: It's clear that $v_D > 0$

Apply the node analysis method, we have

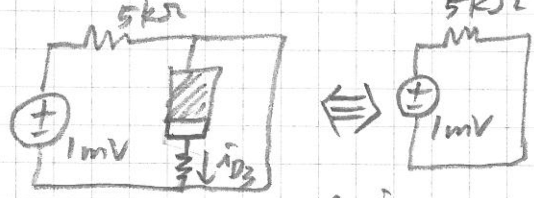
$$\begin{cases} \dfrac{20}{5k\Omega} + i_D = i_x \\ i_D = \dfrac{20 - v_D}{1k\Omega} \\ i_D = 1K \cdot v_D^{\,2} \end{cases} \Rightarrow \begin{cases} v_D = 4V \\ i_D = 16\ mA \\ i_x = 20\ mA \end{cases}$$

Exercise ④ find $i_D = ?$ Ans : $\underline{16\ mA}$ ※



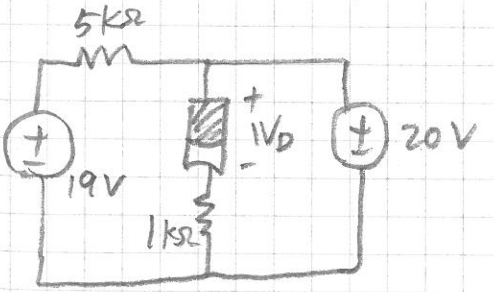
Solution: see the following decomposition

small signal



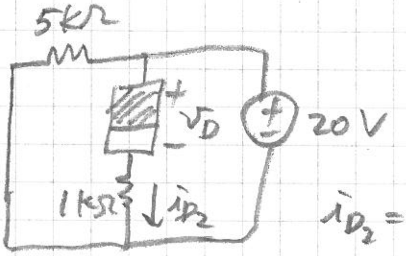
$\therefore i_{D_3} = 0$

large signal



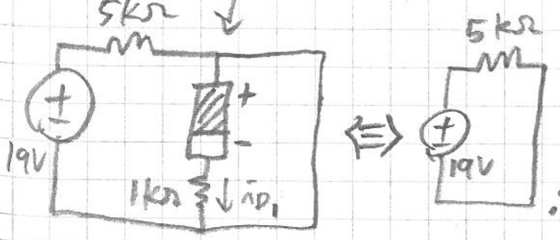
$$\underline{i_D = i_{D_1} + i_{D_2} + i_{D_3} = 16\ mA}$$ ※

using superposition



$i_{D_2} = 16\ mA$

which we solved in exercise ③ ※



$\therefore i_{D_1} = 0$