Solution

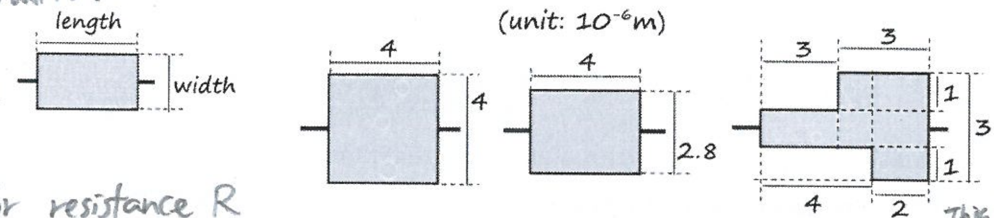
National Taiwan Normal University
Department of Computer Science and Information Engineering

CSU0007 - Basic Electronics

Homework 1

100 points total. Submit your work via Moodle. To receive full score, clearly state each step of your derivation.

1. (10 points) Let r be the resistance of a linear planar resistor with width=length=5×10^{-6}m and thickness of 10^{-6}m. What are the resistance of the following resistors of the same material (and of the same thickness)? ↳ call it r'
   - 1a. (3 points)
   - 1b. (3 points)
   - 1c. (4 points)



1a 1b 1c

Using the fact that

$$R = \rho \cdot \frac{l}{wh} \quad \text{for resistance } R$$

ρ: same for the same material; h: we assume same thickness

(1a) $$\frac{r'}{r} = \frac{\rho \frac{4}{4 \cdot 1}}{\rho \frac{5}{5 \cdot 1}} \Rightarrow r' = r$$ ※

(1b) $$\frac{r'}{r} = \frac{\frac{4}{2.8}}{\frac{5}{5}} \Rightarrow r' \approx 1.428\, r$$ ※

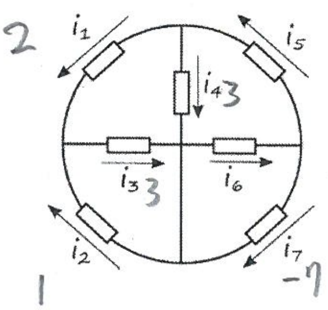
(1c) We may consider that equivalently it is as if we connect three resistors in series

$$\frac{r'}{r} = \frac{\frac{3}{1} + \frac{1}{2} + \frac{2}{3}}{\frac{5}{5}}$$

$r' \approx 4.16\, r$ ※

This is a generalization of equation 1.5 on section 1.5 in the textbook. Also, see Appendix A.3 in the textbook for detail.

2. (10 points) In the following figure, use KCL to find the branch current with respect to each lumped element. In particular, suppose i_1=2mA, i_2=1mA, i_4=3mA, i_7=-7mA. Determine i_3, i_5, and i_6.



⇕

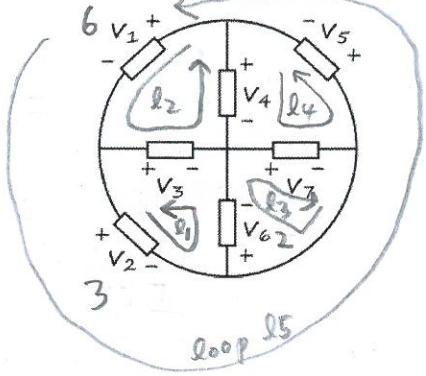
$i_3 = i_1 + i_2 = 3$ mA

$i_6 = i_3 + i_4 - i_2 + i_7 = 3 + 3 - 1 + (-7) = -2$ mA

$i_5 = i_1 + i_4 = 2 + 3 = 5$ mA

Double-check: $i_5 = i_6 - i_7 = -2 - (-7) = 5$ mA

3. (10 points) In the following figure, use KVL to find the branch voltage with respect to each lumped element. In particular, suppose v_1=6V, v_2=3V, and v_6=2V. Determine v_3, v_4, v_5, and v_7.



loop l_1: $3+2-v_3=0 \Rightarrow \underline{v_3 = 5V}$ #

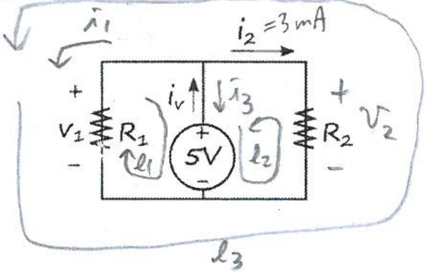
loop l_2: $6+5-v_4=0 \Rightarrow \underline{v_4 = 11\ V}$ #

loop l_3: $2+v_7=0 \Rightarrow \underline{v_7 = -2\ V}$ #

loop l_4: $11+(-2)+v_5=0 \Rightarrow \underline{v_5=-9\ V}$ #

Double-check: loop l_5: $6+3+(-9)=0$

4. (10 points) In Figure 4c, suppose we know i_2=3mA and R_1=10kΩ. Determine v_1, i_v, and R_2.



Using the basic analysis method, we have

element law: $\begin{cases} i_1 = \dfrac{v_1}{R_1} \\ i_2 = \dfrac{v_2}{R_2} = 3mA \text{ according to our assumption} \end{cases}$

KVL: $\begin{cases} \text{loop } l_1: 5 - v_1 = 0 \\ \text{loop } l_2: 5 - v_2 = 0 \end{cases}$

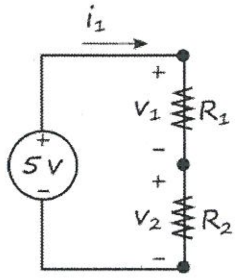
alternatively, loop l_3: $v_1 - v_2 = 0$

KCL: $i_1 + i_2 + i_3 = 0$

$$\Rightarrow \begin{cases} v_1 = v_2 = \underline{5V} \text{ and } R_2 = 5/3mA = \underline{\tfrac{5}{3} k\Omega} \\ i_1 = 5/10k\Omega = 0.5\ mA \\ i_V = -i_3 = i_1 + i_2 = \underline{3.5\ mA} \end{cases}$$ #

5. (10 points) Consider the following voltage dividers.

5a. (5 points) Suppose R_1=9kΩ and R_2=6kΩ. Determine i_1 and v_2.



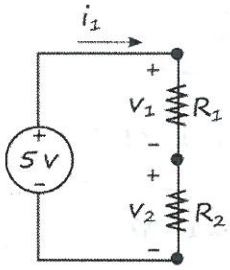
To determine i_1, we may conveniently consider the equivalent resistance $R' = (R_1 + R_2) = 15 k\Omega$, and by Ohm's law we have

$$\underline{i_1} = \frac{5V}{R'} = \underline{\frac{1}{3} mA}$$ #

For v_2, according to voltage-divider formula we have

$$\underline{v_2} = \frac{R_2}{R_1+R_2} \cdot 5V = \frac{6}{15} \cdot 5V = \underline{2V}$$ #

5b. (5 points) In the following circuit, suppose R_1=2.7kΩ. If we want to create a branch voltage of 3.3V across resistor R_2 (i.e., to make v_2=3.3V), what should be the resistance of R_2?



According to the formula of a voltage-divider circuit, we have

$$V_2 = \frac{R_2}{R_1+R_2}\times(5V)$$

$$\Rightarrow 3.3V = \frac{R_2}{2.7\,k\Omega + R_2}\cdot 5\,V$$

$$\Rightarrow 3.3(2.7k + R_2) = 5R_2$$

$$R_2 \approx 5.24\,k\Omega$$ ※

Double-check: $\frac{5.24}{2.7+5.24}\times 5 = \frac{26.2}{7.94} \approx 3.3$

6. (10 points) In class, we have shown that the equivalent resistance of two resistances R_a and R_b in parallel is $R_p=R_aR_b/(R_a+R_b)$.

6a. (5 points) Use mathematical induction (i.e., 數學歸納法) to prove that for *n* resistors connected in parallel, the following equation holds for the equivalent resistance R_p: $1/R_p = \sum_{k=1}^{n}(1/R_k)$

equivalently

$$\frac{1}{R_p} = \frac{1}{R_a} + \frac{1}{R_b}$$

1° For $n=1$, $\frac{1}{R_p} = \sum_{K=1}^{1}\frac{1}{R_K} = \frac{1}{R_p}$.

2° Suppose the equation holds for $n = \mathbb{K}$.

That is, $\frac{1}{R_p} = \sum_{K=1}^{\mathbb{K}}\frac{1}{R_K}$

Then for $n = \mathbb{K}+1$, we have

$$\frac{1}{R_p'} = \frac{1}{R_p} + \frac{1}{R_{\mathbb{K}+1}} = \sum_{K=1}^{\mathbb{K}}\frac{1}{R_K} + \frac{1}{R_{\mathbb{K}+1}} = \sum_{K=1}^{\mathbb{K}+1}\frac{1}{R_K}$$

Q.E.D. by induction ※

6b. (5 points) Prove that when N resistors, each with resistance R, are connected in parallel, the equivalent resistance is R_p=R/N.

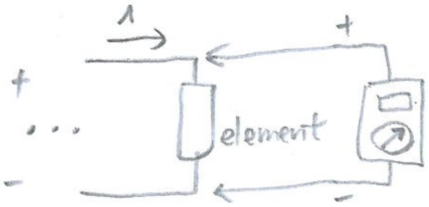
Using the result above, we have

$$\frac{1}{R_p} = \sum_{K=1}^{N}\frac{1}{R_K} = N\cdot\frac{1}{R}$$

$$\Rightarrow R_p = R/N$$ ※

7. (10 points) Use your own words to explain why that, typically, a voltage meter is designed to have a relatively large internal resistance (say, 1MΩ)?

We connect our voltage meter in parallel to the element of which the branch voltage is of interest.



From the result of equivalent resistance, we see that $\frac{1}{R_p} = \frac{1}{R_{element}} + \frac{1}{R_{in\ voltage\ meter}}$

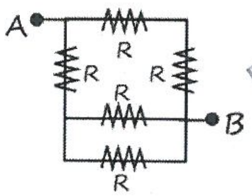
With a large internal resistance, we will have $R_p \approx R_{element}$, thus reducing interference caused by the voltage meter.

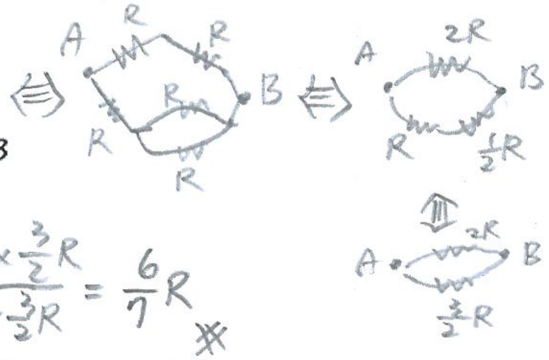
Otherwise, suppose that the internal resistance is very small (e.g., $\to 0$).

Then since $R_p = \frac{R_A R_B}{R_A + R_B} = \frac{R_A}{\frac{R_A}{R_B} + 1}$

if $R_B \to 0$ then $R_p \to 0$ which is far from the original $R_{element}$.

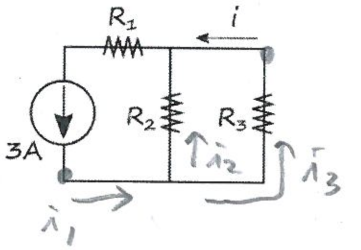
8. (5 points) For the following circuit, find the equivalent resistance from the viewpoint of A-B.





$$R_{AB} = \frac{2R \times \frac{3}{2}R}{2R + \frac{3}{2}R} = \frac{6}{7}R$$

9. (10 points) For the following circuit, determine current i.



According to KCL, we see

$i_1 = 3A \quad i_3 = i$

and from the result of current divider we see

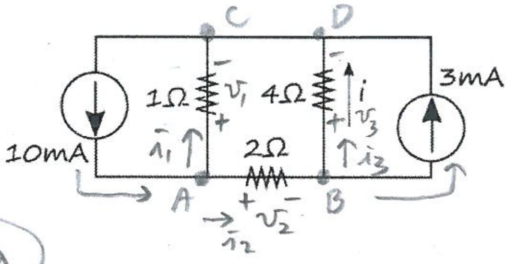
$i_2 = \frac{R_3}{R_2 + R_3} i_1$ and $i_3 = \frac{R_2}{R_2 + R_3} i_1$

$$\Rightarrow i = i_3 = \frac{3R_2}{R_2 + R_3}$$

10. (15 points) For the following circuit,

9a. (5 points) use the basic analysis method to find current i.

9b. (10 points) use the node analysis method to find current i.



six branch variables in total

9a

element law:

$$i_1 = \frac{v_1}{1} \quad \text{—①}$$

$$i_2 = \frac{v_2}{2} \quad \text{—②}$$

$$i_3 = \frac{v_3}{4} \quad \text{—③}$$

KCL:

$i_1 + i_2 = 10$ ← node A —④ (alternatively, $i_1 + i_3 + 3 = 10$ at node C)

$i_2 - i_3 = 3$ ← node B —⑤

KVL: loop A→C→D→B→A : $v_1 - v_3 - v_2 = 0$ —⑥

$$\Rightarrow i_1 - 4i_3 - 2i_2 = 0 \quad \text{—⑥}'$$

$$\Rightarrow (7 - i_3) - 4i_3 - 2(3 + i_3) = 0$$

$$\Rightarrow i_3 = \frac{1}{7}\text{ mA}$$

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9b

Beside the ground node, two meaningful nodes are nodes E and F. Let their node voltage be e_1 and e_2, respectively.

From KCL:

(ground node) $i + 3 = \frac{e_1 - 0}{2}$ —①

(node E) $10 = \frac{e_1 - e_2}{1} + \frac{e_1 - 0}{2}$ —②

(node F) $10 = \frac{e_1 - e_2}{1} + i + 3$

There are actually only two independent equations, and we cannot use them to solve three unknowns. $i = \frac{0 - e_2}{4}$ is the missing equation, and with that we can get

$$\frac{-e_2}{4} + 3 = \frac{e_1}{2} \quad \text{from ① and ②}$$

$$3e_1 - 2e_2 = 20$$

$$\Rightarrow e_2 = -\frac{4}{7} \Rightarrow i = \frac{-e_2}{4} = \frac{1}{7}\text{ mA}$$

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