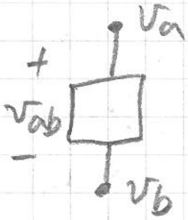
☆ Circuit Analysis using the Node Method

Motivation:

Often, the number of nodes in a circuit is much smaller than that of branches. Node method thus involves fewer number of variables, which means it is often easier to solve.

– definition of node voltage:



$v_{ab} = \underbrace{v_a}_{} - \underbrace{v_b}_{}$ node voltage

Procedure of the node method:

1° select a reference node (接地, $v=0$) $\perp$

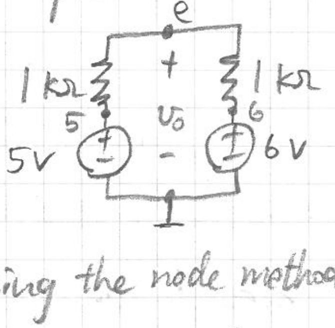
2° assign node variables

3° apply KCL

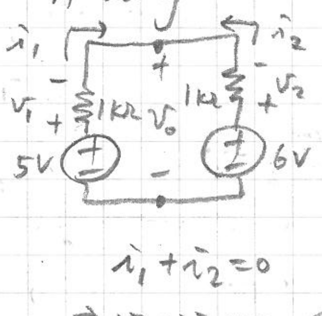
4° solve equations

5° back-solve the needed branch voltage/current

Example: find $v_0 = ?$



if using the basic method,



$i_1 = \dfrac{v_1}{1\,k\Omega}$

$i_2 = \dfrac{v_2}{1\,k\Omega}$

using the node method,

$$\frac{5-e}{1\,k\Omega} + \frac{6-e}{1\,k\Omega} = 0$$

$e = 5.5V \quad v_0 = e - 0 = \underline{5.5\,V}$ ※

$i_1 + i_2 = 0$

$\Rightarrow v_1 + v_2 = 0$

using KVL,

$5 - v_1 + v_2 - 6 = 0$

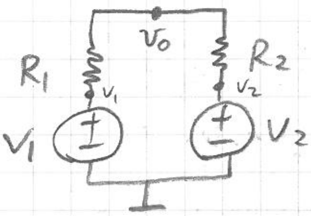
$\Rightarrow v_1 - v_2 = -1$

$\begin{cases} v_1 = -0.5 \\ v_2 = 0.5 \end{cases}$

$v_0 = 6 - v_2$ (KVL)

$= \underline{5.5\,V}$ ※

P22 Symbolic Computation can give us some insights:



KCL: $\frac{V_1 - V_0}{R_1} + \frac{V_2 - V_0}{R_2} = 0$

$$\Rightarrow R_2(V_1 - V_0) + R_1(V_2 - V_0) = 0$$

$$\Rightarrow V_0 = \frac{1}{R_1 + R_2}(R_2 V_1 + R_1 V_2)$$

$$= \frac{R_2}{R_1 + R_2} V_1 + \frac{R_1}{R_1 + R_2} V_2$$

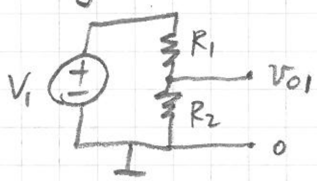
Insights ① V_0 is a linear combination of V_1 and V_2.

The circuit acts as an adder that gives a weighted sum of V_1 and V_2.

② if set $V_2 = 0$, then $V_{01} = \frac{R_2}{R_1 + R_2} V_1$,

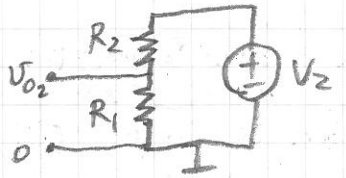
which is equivalent to the result of a voltage divider:

i.e., having the same i-v characteristic.



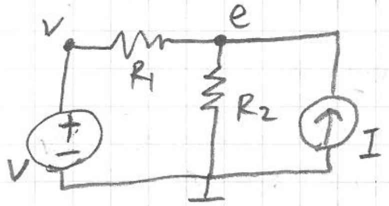
③ similarly, if set $V_1 = 0$, then

$V_{02} = \frac{R_1}{R_1 + R_2} V_2$, equivalent to the result of a voltage divider:



$\Rightarrow$ from ② and ③, the original circuit can be thought of as a superposition of two voltage dividers, with $V_0 = V_{01} + V_{02}$.

Another example:



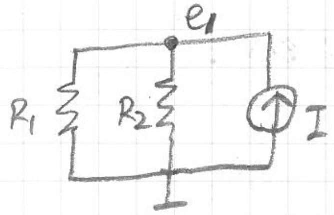
KCL: $\frac{V-e}{R_1} + \frac{0-e}{R_2} + I = 0$

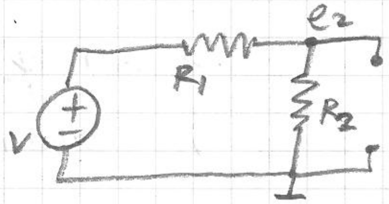
$$\Rightarrow R_2(V-e) + R_1(-e) + IR_1R_2 = 0$$

$$\Rightarrow e = \frac{1}{R_1+R_2}(R_2 V + I R_1 R_2)$$

$$= \frac{R_2}{R_1+R_2} V + \frac{R_1R_2}{R_1+R_2} I$$

Study the case of $V=0$ and $I=0$, respectively, we see that the original circuit can be thought of as a superposistion of one current divider and one voltage divider, where $e = e_1 + e_2$:





$\Downarrow$

$e_1 = I \cdot \left(\frac{R_1R_2}{R_1+R_2}\right)$

$e_2 = \frac{R_2}{R_1+R_2} V.$

(Note: set $V=0$ 相當於將 短路 (short circuit)
set $I=0$ 相當於將 斷路 (open circuit))

In general, for a linear circuit, we can use the concept of superposition to simply our analysis, by first considering one independent source at a time and then adding up the result.

Why does the concept of "superposition" make sense in circuit analysis?

→ Because ① each independent source contributes to the response of circuit "individually" and the contribution is independent from the contribution of any other independent source, and

② independent sources are assumed to have no resistance (see P_{13} of this note).

Why does the concept of "equivalence" make sense in circuit analysis?

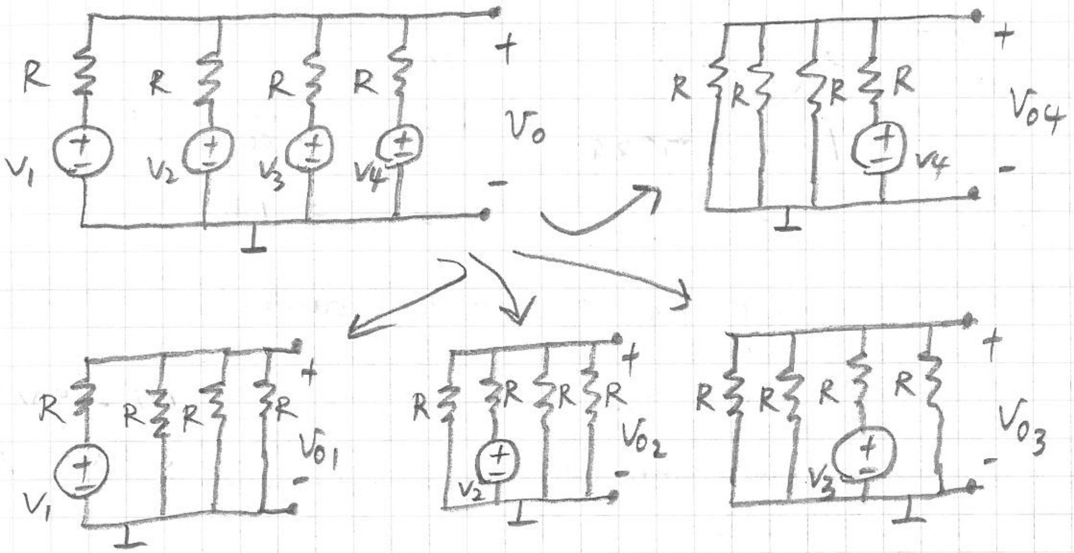
→ Because as long as the i-v characteristic are identical, from input/output viewpoint of a system, what's inside doesn't matter.

Therefore, we may replace some part of a circuit by its equivalence, solely for the purpose of simplifying our analysis. It is an extremely useful trick in engineering!

For example, we may use 訊號產生器 to feed an equivalent input to a system, emulating some physical input circuit.

Example of the use of superposition:

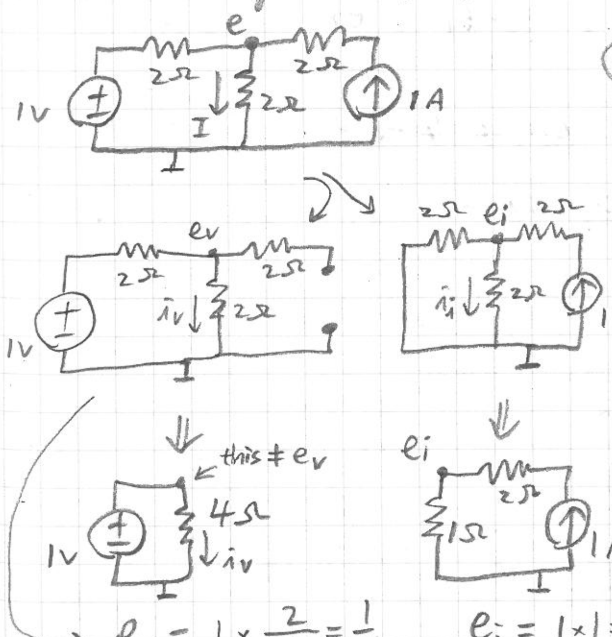
find $V_0 = ?$



$$V_{01} = \frac{\frac{1}{3}}{1+\frac{1}{3}} V_1 = \frac{1}{4} V_1, \quad V_{02} = \frac{\frac{1}{3}}{1+\frac{1}{3}} V_2 = \frac{1}{4} V_2, \quad V_{03} = \frac{\frac{1}{3}}{1+\frac{1}{3}} V_3 = \frac{1}{4} V_3, \quad V_{04} = \frac{\frac{1}{3}}{1+\frac{1}{3}} V_4 = \frac{1}{4} V_4$$

$$\Rightarrow V_0 = V_{01} + V_{02} + V_{03} + V_{04} = \frac{1}{4}(V_1 + V_2 + V_3 + V_4)$$ ✻

Another example: find $I = ?$



$e_v = 1 \times \frac{2}{2+2} = \frac{1}{2}$ $\quad$ $e_i = 1 \times 1 = 1$

$e = e_v + e_i = \frac{3}{2}$

$I = \frac{e - 0}{2\Omega} = 0.75\ A$ ✻

Alternatively, we may compute I directly:

$i_v = \frac{1}{4}$ $\quad$ $i_i = \underbrace{\frac{2}{2+2}}_{\text{current divider}} \times 1 = \frac{1}{2}$

$\Rightarrow I = i_v + i_i$

$= \frac{1}{4} + \frac{1}{2} = 0.75\ A$ ✻

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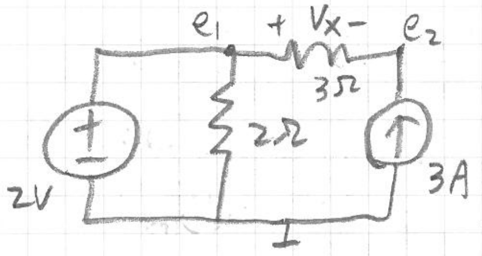
Some further example of the use of node method:



find $V_x = ?$

$e_1 = 2V$

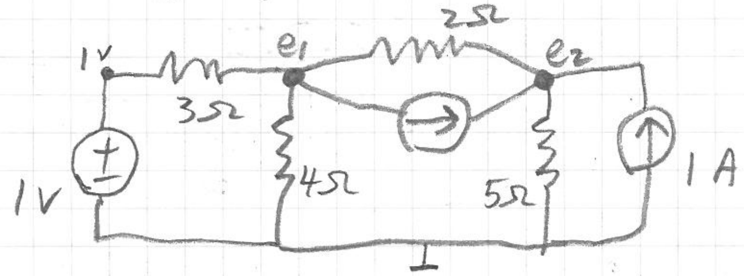
KCL: $\dfrac{e_1 - e_2}{3\Omega} + 3 = 0$

$\Rightarrow e_2 = 11\,V$

$\Rightarrow V_x = e_1 - e_2 = -9\,V$ ※

Compare this with the use of basic method as we did on P20 of this note! ($V_x = -9V$ there)

Another example: find e_1 and e_2



KCL on e_1: $\dfrac{e_1 - 1}{3} + \dfrac{e_1}{4} + \dfrac{e_1 - e_2}{2} + 2 = 0$

KCL on e_2: $-2 + \dfrac{e_2 - e_1}{2} + \dfrac{e_2}{5} - 1 = 0$

$$\Rightarrow \begin{cases} 4e_1 - 4 + 3e_1 + 6e_1 - 6e_2 + 24 = 0 \\ -20 + 5e_2 - 5e_1 + 2e_2 - 10 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} 13e_1 - 6e_2 = -20 \\ -5e_1 + 7e_2 = 30 \end{cases}$$

$\Rightarrow 13e_1 - 6\left(\frac{1}{7}(30 + 5e_1)\right) = -20$

$\Rightarrow 91e_1 - 180 - 30e_1 = -140$

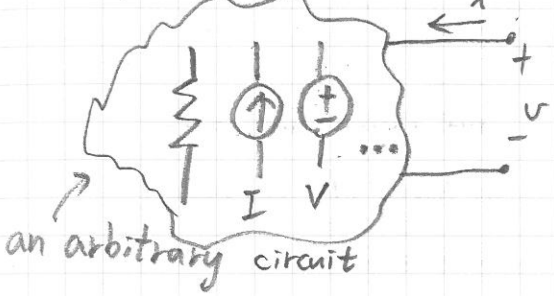
$\Rightarrow 61e_1 = 40 \Rightarrow e_1 \approx 0.655$, $e_2 = \frac{1}{6}(13e_1 + 20)$

$= \frac{1}{6}(13 \times 0.655 + 20) = 4.75$ ※

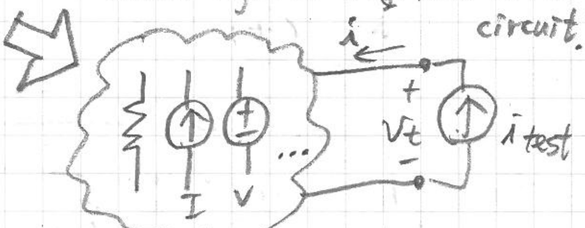
☆ Thévenin's Theorem

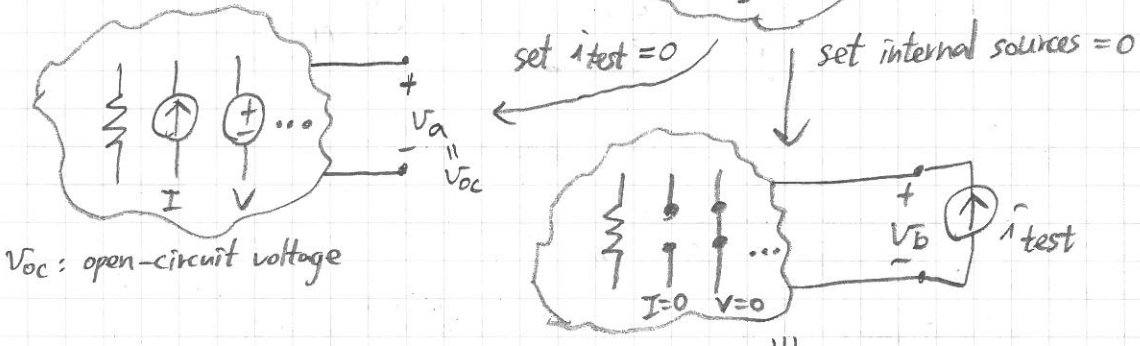
goal: Given an arbitrary linear circuit, we would like to know how it would respond to external excitation; in other word, we'd like to know its i-v characteristic.

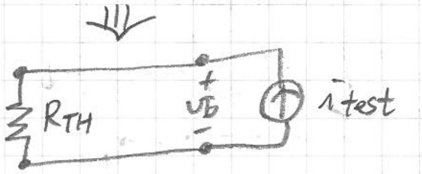
Approach: leverage the concept of superposition!



We append a testing current source, with the original circuit together they form a new circuit.





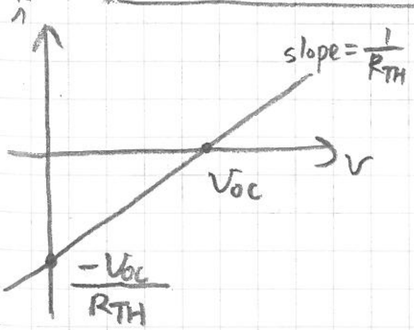


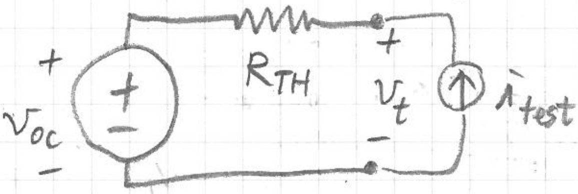
By superposition,

$$v_t = v_a + v_b$$

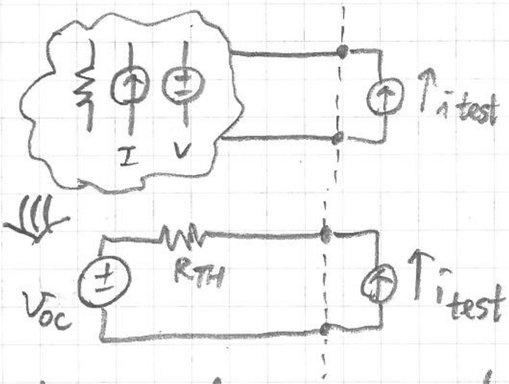
$$\Rightarrow \boxed{v_t = v_{oc} + i_{test} R_{TH}}$$

$\Rightarrow$ equivalently, this relation describes the following circuit:





P28 Therefore, we have the following equivalence:



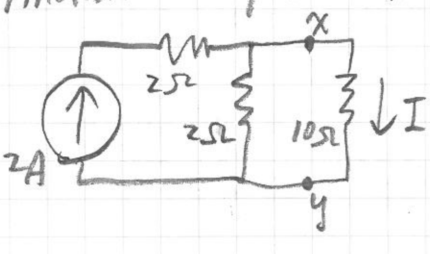
In other word, we may reduce an ~~arbitrary~~ linear circuit to an equivalent circuit of the form:

we name $V_{TH} = V_{OC}$ in honor of Thévenin.

Example:

$V_{TH} = v = 3 \times \frac{2}{1+2} = 2V$, $R_{TH} = \frac{1 \cdot 2}{1+2} = \frac{2}{3}\Omega$ from

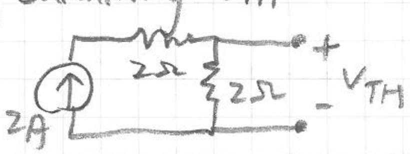
Another example: find $I = ?$



We may replace the left side of x-y by an equivalent circuit:

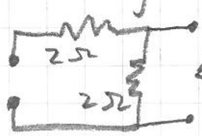
Then $I = \frac{V_{TH}}{R_{TH} + 10\Omega}$

calculating V_{TH}:



$V_{TH} = 2A \times 2\Omega = 4V$

calculating R_{TH}:



$\leftarrow R_{TH} = 2\Omega$

$\Rightarrow I = \frac{4}{2+10} = \frac{1}{3} A$ ※