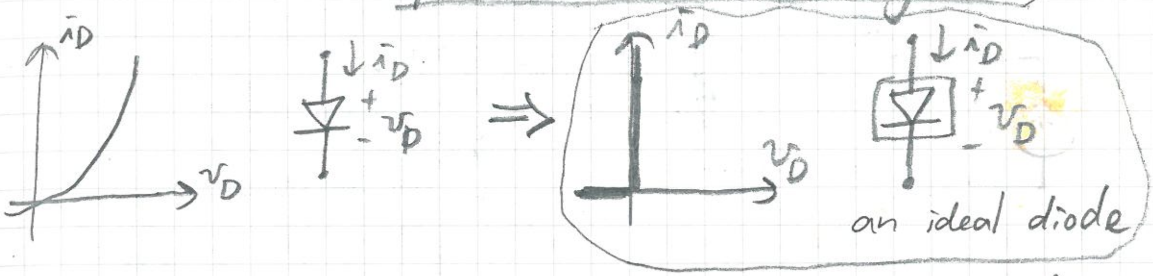
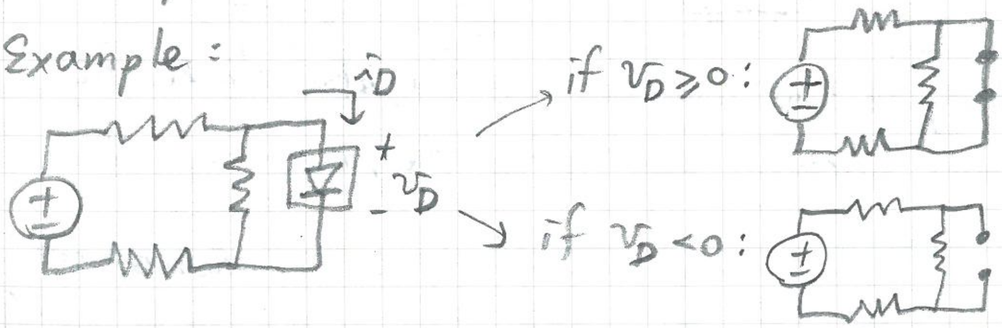
Besides the graphical analysis, in some occasion we may simplify our analysis of a nonlinear circuit by considering an approximated version of the i-v characteristic of a given nonlinear element.

This is called the piecewise linear analysis.



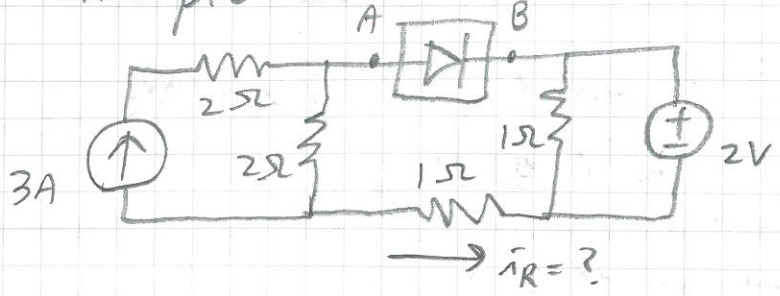
For an ideal diode, depending on the actual voltage (or the actual current direction), we may replace the diode by either a short circuit or an open circuit.

Example :

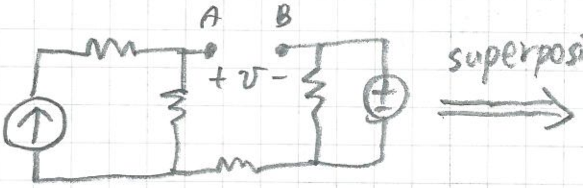


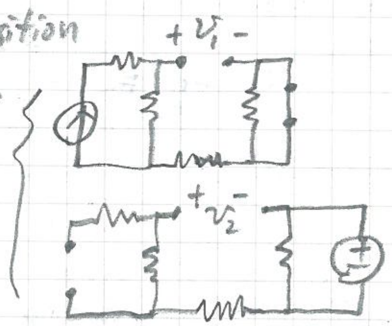
(Note : study textbook P206~209 for an alternative exposition of this subject; study Example 4.11 on P209 for an advanced example.)

P42 Example :

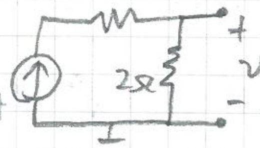


We first find the voltage across A and B.



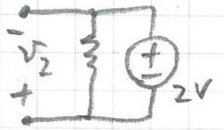


for v_1 :



$$v_1 = 3A \times 2\Omega = 6\ V$$

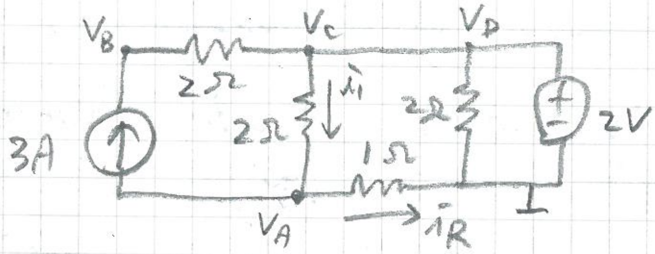
for v_2 :



$v_2 = -2V$

$$v = v_1 + v_2 = 6 - 2 = 4\ v$$

therefore we may replace the ideal diode by a short circuit, leading to the following equivalence :



(exercise : try to analyze this using superposition !)

$V_C = V_D = 2$

$i_1 = 3 + i_R$

$$\Rightarrow \frac{2 - V_A}{2} = 3 + \frac{V_A - 0}{1}$$

$$2 - V_A = 6 + 2V_A$$

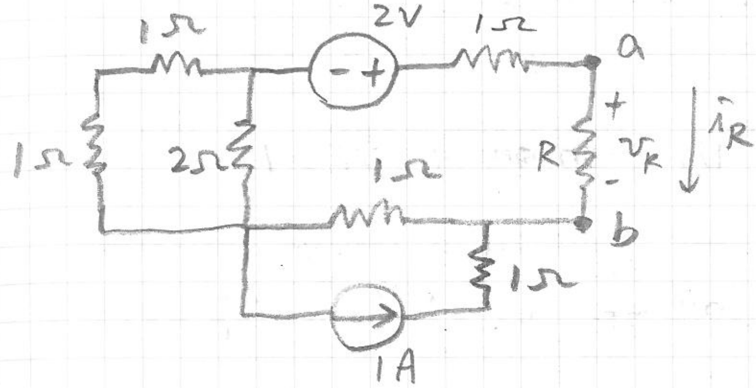
$$\Rightarrow V_A = \frac{-4}{3}$$

$$\Rightarrow i_R = \frac{V_A - 0}{1} = \frac{-4}{3}\ A$$

※

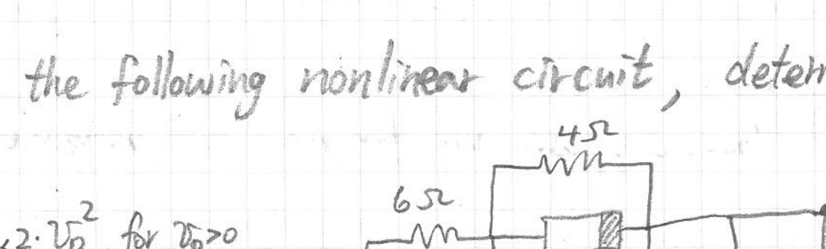
Some exercise problems since P_{32}:

A. Use Norton's Theorem to find i_R and v_R for ① $R=2$ and ② $R=4$

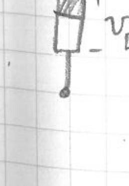


B. Following A, rather than using Norton's Theorem, directly apply superposition to find i_R for $R=2$.

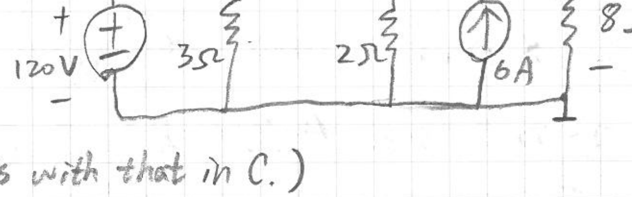
C. Determine the Norton equivalent at terminals a, b and at terminals c, d, for the following circuit:



D. For the following nonlinear circuit, determine v_{R_1}.



$$i_D = \begin{cases} 2 \cdot v_D^2 & \text{for } v_D > 0 \\ 0 & \text{otherwise} \end{cases}$$

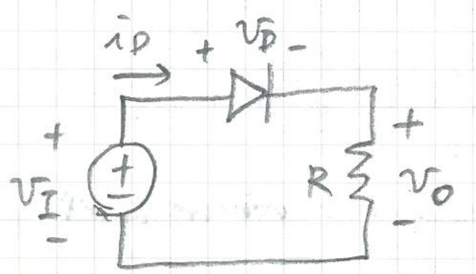


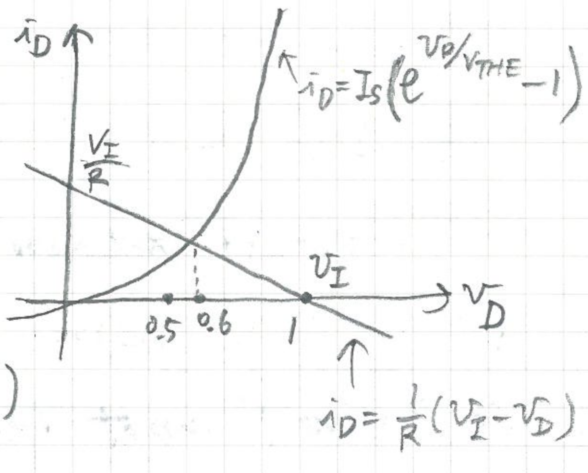
(hint: compare this with that in C.)

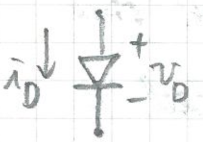
P44 E. For the following nonlinear circuit, with some initial analysis of its $i_D - v_D$ relation at hand, try to answer three questions:

① if for some need we changed the input voltage v_I, so that $v_I > 1$, would that lead to a change to the output voltage $v_O > 0.4$ or $v_O < 0.4$?

② Now, suppose we operate the circuit at region $v_I \gg 1$. What can we say about v_O?







$i_D = I_s(e^{v_D/V_{THE}} - 1)$

③ Now, suppose we fix v_I but replace the linear resistor by a very heavy load, such that $R \gg 1$. What would i_D become?

Answers to Problems A, B, C, D:

A. ① $i_R = \frac{1}{5}A$, $v_R = \frac{2}{5}V$ ② $i_R = \frac{1}{7}A$, $v_R = \frac{4}{7}V$ B. same as A.

C. $R_{tab} = 2\Omega$, $i_{scab} = 7\,A$; $R_{tcd} = \frac{3}{2}\Omega$, $i_{scd} = \frac{38}{3}A$

D. 16 V #

Interlude: A note on an extremely useful tool, "the Taylor's Theorem" (A.K.A. Taylor Expansion)

Motivation: To approximate a complex function by a simplier one, given some input.

Idea: for function $f(x)$, we may say that function $g(x)$ is approximately the same as $f(x)$ at $x = x_0$ if $f(x)$ and $g(x)$ have similar trend around $x = x_0$.

Approach: Construct $g(x)$ such that

$$g(x_0) = f(x_0)$$

$g'(x_0) = f'(x_0)$ ← first derivative, i.e., the change rate of a function.

$g''(x_0) = f''(x_0)$ ← second derivative, i.e., the change rate of the change rate of the function.

$\vdots$

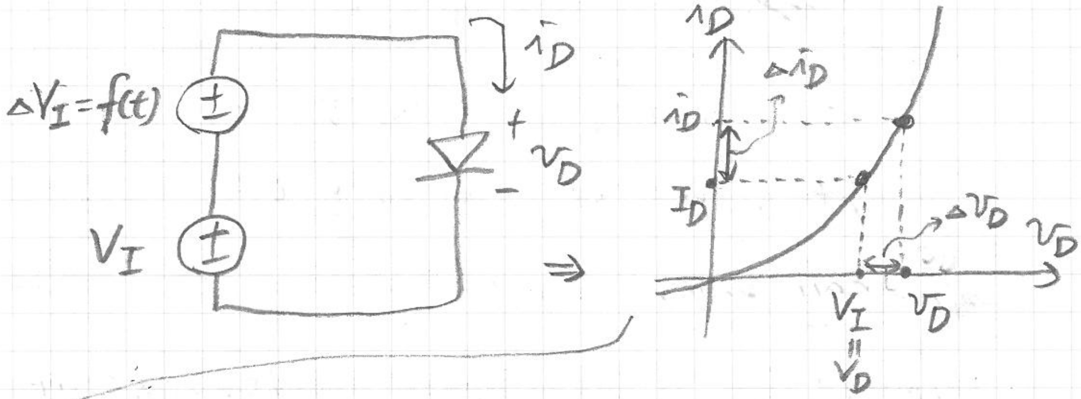
So we write
$$\underset{x \text{ near } x_0}{g(x)} = f(x)\Big|_{x=x_0} + f'(x)\Big|_{x=x_0}(x-x_0) + \frac{1}{2!}f''(x)\Big|_{x=x_0}(x-x_0)^2 + \frac{1}{3!}f'''(x)\Big|_{x=x_0}(x-x_0)^3 + \cdots$$

① Try to compute and compare $\{g(x), f(x)\}$, $\{g'(x), f'(x)\}$ and $\{g''(x), f''(x)\}$ to see this really makes sense.

② Take a look at P218 in the textbook and Equation 4.66 to see how we may control the error.

☆ Small-Signal Analysis for Nonlinear Devices

- In many sensor applications and most audio amplifiers, the input voltage/current to a circuit often consists of two parts:
  - ① a time-invariant source (large signal)
  - ② a time-varying source (small signal)

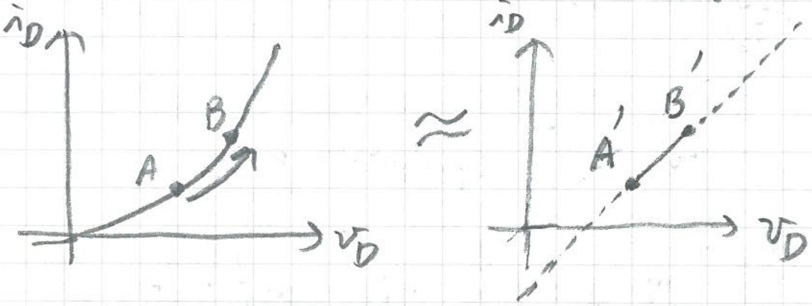


The large signal is used to determine the region of operation (i.e., which part of the i_D–v_D curve), and the small signal is the "real" input (e.g., those induced by human voice, as in the case of a microphone).

$$v_D = \underset{\text{large}}{V_I} + \underset{\text{small signal}}{\Delta v_D}$$

$$i_D = I_D + \Delta i_D$$

as we will see, moving along a small distance on "the $i_D - v_D$ curve" can be approximated as moving along a small distance on "a straight line"

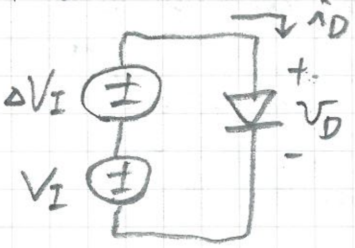


Therefore, we may simplify our analysis of small signal by considering the signal's response on a nonlinear device as if it is the response on a linear device (resistor).

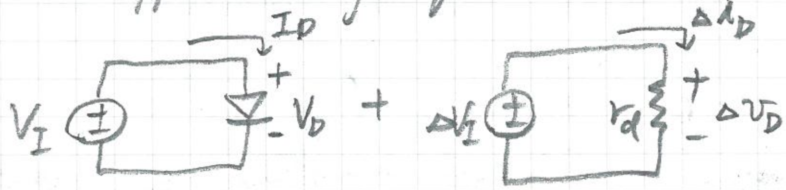
Review P13, where we've shown that the i-v characteristic of a resistor on the i-v plot is a straight line; further, the slope of the line is equal to the reciprocal of the resistance (R) of the resistor.

Now, a question is: how do we determine the resistance of that linear device?

Let r_d be the resistance of the linear device. Using small-signal analysis, we essentially transform the original circuit



into an approximately equivalence:

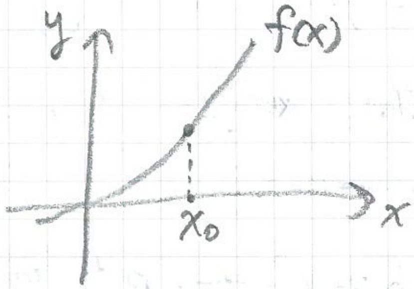


where $i_D = I_D + \Delta i_D$

" $\frac{V_{THE}}{I_D}$

Now, let's see how to determine r_d!

We use Taylor's Theorem, which provides a way to approximate a curve near a certain point $x = x_0$:



$$\underset{x \text{ near } x_0}{y} = \underset{x \text{ near } x_0}{f(x)} = f(x)\Big|_{x=x_0} + f'(x)\Big|_{x=x_0}(x-x_0)$$

$$+ \frac{1}{2!}f''(x)\Big|_{x=x_0}(x-x_0)^2 + \frac{1}{3!}f'''(x)\Big|_{x=x_0}(x-x_0)^3 + \cdots$$

P49 in our case of a nonlinear diode,

recall that

$$i_D = I_s\left(e^{v_D/v_{THE}} - 1\right) \underset{\substack{\uparrow \\ \text{we define it}}}{=} f(v_D)$$

$$\Rightarrow \underset{v_D \text{ near } V_D}{i_D} = f(v_D)\Big|_{v_D=V_D} + f'(v_D)\Big|_{v_D=V_D}(v_D - V_D)$$

$$+ \frac{1}{2!} f''(v_D)\Big|_{v_D=V_D} (v_D - V_D)^2$$

$$+ \frac{1}{3!} f'''(v_D)\Big|_{v_D=V_D} (v_D - V_D)^3 + \cdots$$

$$= f(v_D)\Big|_{v_D=V_D} + (v_D - V_D)\Big(f'(v_D)\Big|_{v_D=V_D}$$

we choose to ignore these terms $\leftarrow$ $\left\{ \begin{array}{l} + \frac{1}{2!} f''(v_D)\Big|_{v_D=V_D}(v_D - V_D) \\ + \frac{1}{3!} f'''(v_D)\Big|_{v_D=V_D}(v_D - V_D)^2 \\ + \cdots \Big) \end{array} \right.$

using the chain rule, we get $f'(v_D) = \frac{I_s}{v_{THE}} e^{v_D/v_{THE}}$

$$\Rightarrow \underset{v_D \text{ near } V_D}{i_D} \approx f(v_D)\Big|_{v_D=V_D} + (v_D - V_D) \cdot \frac{I_s}{v_{THE}} \cdot e^{v_D/v_{THE}}\Big|_{v_D=V_D}$$

$$= I_s\left(e^{V_D/v_{THE}} - 1\right) + \Delta v_D \cdot \frac{I_s}{v_{THE}} \cdot e^{V_D/v_{THE}}$$

since we know that

$$e^{V_D/v_{THE}} \gg 1$$

so we can think of $e^{V_D/v_{THE}} \approx e^{V_D/v_{THE}} - 1$

With that, we may rewrite the equation as

$$\underset{v_D \text{ near } V_D}{i_D} = I_s\left(e^{V_D/v_{THE}} - 1\right) + \Delta v_D \frac{I_s}{v_{THE}}\left(e^{V_D/v_{THE}} - 1\right)$$

Now, by observation we see $I_s(e^{V_D/v_{THE}} - 1) = I_D$

Thus, $\underset{v_D \text{ near } V_D}{i_D} = I_D + \frac{\Delta v_D}{v_{THE}} I_D$

Compare to $i_D = I_D + \Delta i_D$, we have $\Delta i_D = \frac{\Delta v_D}{v_{THE}} I_D$

Think in terms of relation of Δi_D and Δv_D

$$\underbrace{\frac{\Delta v_D}{\Delta i_D}}_{r_d} = \frac{v_{THE}}{I_D}$$

and we may choose to define

$$\boxed{r_d = \frac{v_{THE}}{I_D}}$$ ※

In the hindsight, we may generalize our result by saying that for an arbitrary nonlinear element, we have

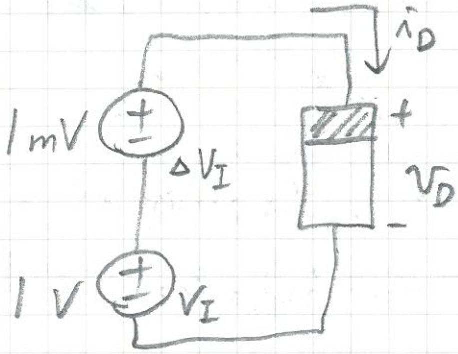
$$\boxed{r_d = \frac{1}{f'(v_D)\big|_{v_D = V_D}} = \frac{1}{\frac{df(v_D)}{dv_D}\big|_{v_D = V_D}}}$$

※ (finally!)

P51

Example :

find $i_D = ?$



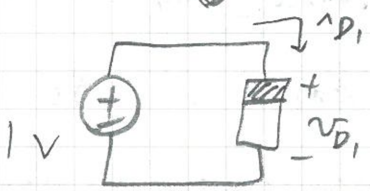
recall that for  (P35)

$i_D = K \cdot v_D^2$ for $v_D > 0$

and here we suppose

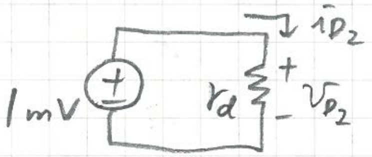
$K = 1\,mA/V^2$

Ans :



$v_{D_1} = 1$ (KVL)

$\Rightarrow i_{D_1} = K \cdot v_{D_1}^2 = 1\,mA$



$v_{D_2} = 1\,mV$ (KVL)

$$r_d = \frac{1}{f'(v_D)\big|_{v_D = V_D}} = \frac{1}{2 \cdot K \cdot v_D\big|_{v_D = 1V}}$$

$$= 500\,\Omega$$

$$\Rightarrow i_{D_2} = \frac{v_{D_2}}{r_d} = 2\,\mu A$$

$$\Rightarrow i_D = i_{D_1} + i_{D_2} = 1.002\,mA$$

\#