Homework 2 solution CSC0001 2020 Fall

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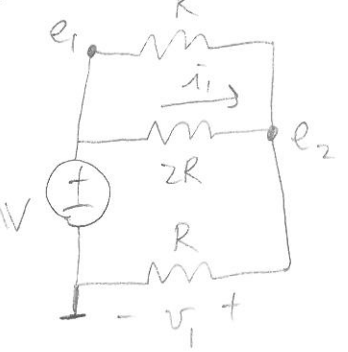

$$\begin{cases} e_1 - 0 = 1V \\ \dfrac{e_1-e_2}{R} + \dfrac{e_1-e_2}{2R} + \dfrac{0-e_2}{R} = 0 \\ i_1 = \dfrac{e_1-e_2}{2R} \\ v_1 = e_2 - 0 \end{cases} \Rightarrow \begin{cases} e_1 = 1V \\ e_2 = \frac{3}{5}1V \\ i_1 = \dfrac{1V}{5R} \\ v_1 = \frac{3}{5}1V \end{cases}$$

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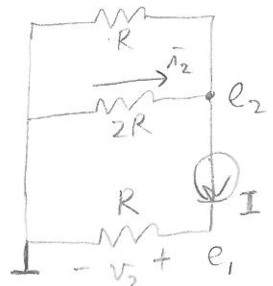

$$\begin{cases} \dfrac{0-e_2}{R} + \dfrac{0-e_2}{2R} = I \\ \dfrac{e_1-0}{R} = I \\ i_2 = \dfrac{0-e_2}{2R} \\ v_2 = e_1 - 0 \end{cases} \Rightarrow \begin{cases} e_1 = IR \\ e_2 = -\frac{2}{3}IR \\ i_2 = \frac{1}{3}I \\ v_2 = IR \end{cases}$$

$i_2 = \frac{1}{3}I$ ← can be verified by current divider.

$v_2 = IR$ ← can be verified by Ohm's law.

Note that we may also see that the branch voltage across this ideal current source is $e_2 - e_1 = -\frac{5}{3}IR$ in this case. Recall the i-v characteristic of an ideal current source is

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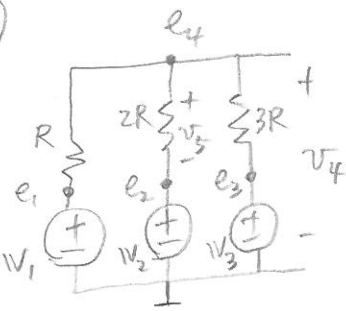
(2.1)



$$\begin{cases} e_1 - 0 = 1V_1 \\ e_2 - 0 = 1V_2 \\ e_3 - 0 = 1V_3 \\ \dfrac{e_1-e_4}{R} + \dfrac{e_2-e_4}{2R} + \dfrac{e_3-e_4}{3R} = 0 \\ v_4 = e_4 - 0 \\ v_5 = e_4 - e_2 \end{cases} \Rightarrow \begin{cases} e_1 = 1V_1 \\ e_2 = 1V_2 \\ e_3 = 1V_3 \\ e_4 = \frac{1}{11}(61V_1 + 31V_2 + 21V_3) \\ v_4 = e_4 = \nearrow \\ v_5 = \frac{1}{11}(61V_1 - 191V_2 + 21V_3) \end{cases}$$

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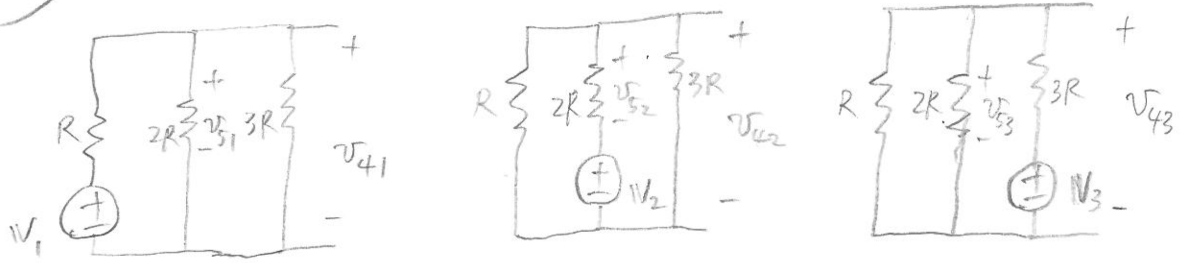
(2.2)



$$v_4 = v_{41} + v_{42} + v_{43}$$

$$= V_1 \times \frac{\frac{2R\times 3R}{2R+3R}}{\frac{2R\times 3R}{2R+3R}+R} + V_2 \times \frac{\frac{R\times 3R}{R+3R}}{\frac{R\times 3R}{R+3R}+2R} + V_3 \times \frac{\frac{R\times 2R}{R+2R}}{\frac{R\times 2R}{R+2R}+3R}$$

$$= V_1 \times \frac{\frac{6}{5}}{\frac{6}{5}+1} + V_2 \times \frac{\frac{3}{4}}{\frac{3}{4}+2} + V_3 \times \frac{\frac{2}{3}}{\frac{2}{3}+3}$$

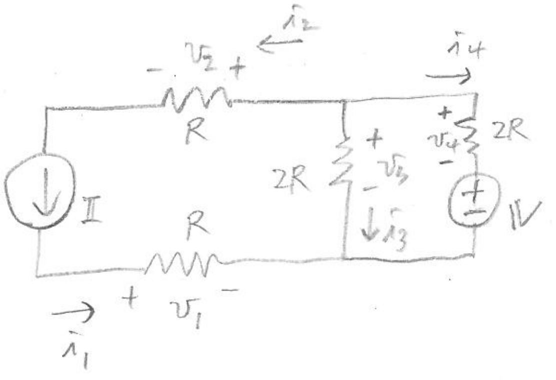
$$= \frac{6}{11} V_1 + \frac{3}{11} V_2 + \frac{2}{11} V_3 \quad ※$$

$$v_5 = v_{51} + v_{52} + v_{53}$$

$$= v_{41} + (v_{42} - V_2) + v_{43}$$

$$= \frac{6}{11} V_1 - \frac{19}{11} V_2 + \frac{2}{11} V_3 \quad ※$$

(3.1)



$$\begin{cases} v_1 = i_1 R \\ v_2 = i_2 R \\ v_3 = i_3 (2R) \\ v_4 = i_4 (2R) \\ i_1 = I = i_2 \\ i_2 + i_3 + i_4 = 0 \\ v_3 = v_4 + V \end{cases}$$

with these it is sufficient to conclude that $v_1 = IR$

$$\Rightarrow \begin{cases} v_1 = IR, & i_1 = I \\ v_2 = IR, & i_2 = I \\ v_3 = -IR + \frac{1}{2}V, & i_3 = -\frac{1}{2}I + \frac{V}{4R} \\ v_4 = -IR - \frac{1}{2}V, & i_4 = -\frac{1}{2}I - \frac{V}{4R} \end{cases}$$

Note that $v_3 \neq v_2 + v_1$ because we will need to consider the branch voltage across the current source. We will verify this in (3.2)

(You can check that the branch voltage is $-3IR + \frac{1}{2}V$ essentially, $V_3 - V_2 - V_1$, following KVL.)

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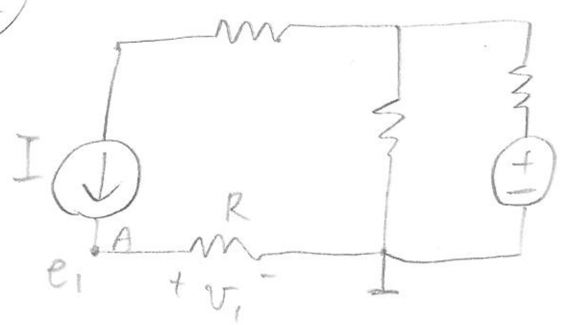
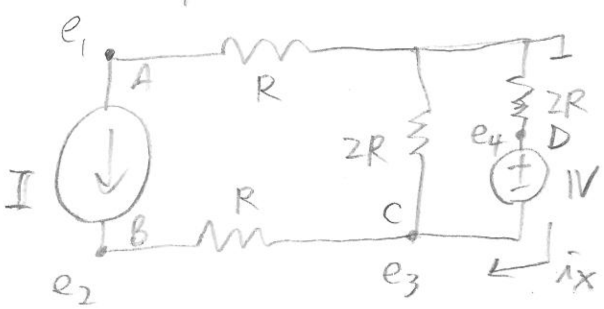
(3.2)



For v_1, it is sufficient to consider $\begin{cases} \dfrac{e_1 - 0}{R} = I & \text{(KCL at node A)} \\ v_1 = e_1 - 0 \end{cases}$

$\Rightarrow v_1 = e_1 = IR$

For your interest, we may use node analysis to completely derive all node voltages of the circuit in one shot, and we may assign the ground node at any node. Here is an example: (which would in turn makes the derivation of all branch variables trivial)



$$\begin{cases} \text{(node A)}: & \dfrac{0 - e_1}{R} = I \\ \text{(node B)}: & I = \dfrac{e_2 - e_3}{R} \\ \text{(node C)}: & \dfrac{e_2 - e_3}{R} + \dfrac{0 - e_3}{2R} + i_x = 0 \\ \text{(node D)}: & \dfrac{0 - e_4}{2R} = i_x \\ & e_4 - e_3 = 1V \end{cases}$$

Now, rearrange the above equations and we may use Gaussian elimination method to solve all node voltages (and i_x):

$$\begin{array}{ccccc|c} e_1 & e_2 & e_3 & e_4 & i_x & \text{constant} \\ 1 & 0 & 0 & 0 & 0 & IR \\ 0 & 1 & -1 & 0 & 0 & -IR \\ 0 & 2 & -3 & 0 & 2R & 0 \\ 0 & 0 & 0 & 1 & 2R & 0 \\ 0 & 0 & 1 & -1 & 0 & 1V \end{array}$$

$$\Rightarrow \begin{array}{cccc|c} e_2 & e_3 & e_4 & i_x & \text{constant} \\ 1 & -1 & 0 & 0 & -IR \\ 0 & 1 & 0 & -2R & -2IR \\ 0 & 1 & -1 & 0 & 1V \\ 0 & 0 & 1 & 2R & 0 \end{array} \quad \begin{bmatrix} e_2 \\ e_3 \\ e_4 \\ i_x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2R & -3IR \\ 0 & 1 & 0 & -2R & -2IR \\ 0 & 0 & 1 & -2R & -2IR - V \\ 0 & 0 & 1 & 2R & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2R & -3IR \\ 0 & 1 & 0 & -2R & -2IR \\ 0 & 0 & 1 & -2R & -2IR - V \\ 0 & 0 & 0 & 1 & \frac{1}{2}I + \frac{1V}{4R} \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -2IR + \frac{1}{2}1V \\ 0 & 1 & 0 & 0 & -IR + \frac{1}{2}1V \\ 0 & 0 & 1 & 0 & -IR - \frac{1}{2}1V \\ 0 & 0 & 0 & 1 & \frac{1}{2}I + \frac{1V}{4R} \end{array}\right]$$

$$\Rightarrow \begin{cases} e_1 = -IR \\ e_2 = 2IR - \frac{1}{2}1V \\ e_3 = IR - \frac{1}{2}1V \\ e_4 = IR + \frac{1}{2}1V \\ i_x = -\frac{1}{2}IR - \frac{1V}{4R} \end{cases}$$

$\Rightarrow v_1 = e_2 - e_3 = IR$, and the branch voltage across (↓) is $e_1 - e_2 = -3IR + \frac{1}{2}1V$; the branch current across (±) is $i_x = -\frac{1}{2}IR - \frac{1V}{4R}$

P4 (3.3) $v_1 = IR + 0 = \underline{IR}$ #

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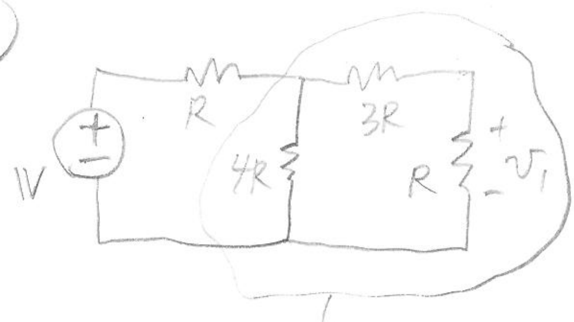
(4.1)

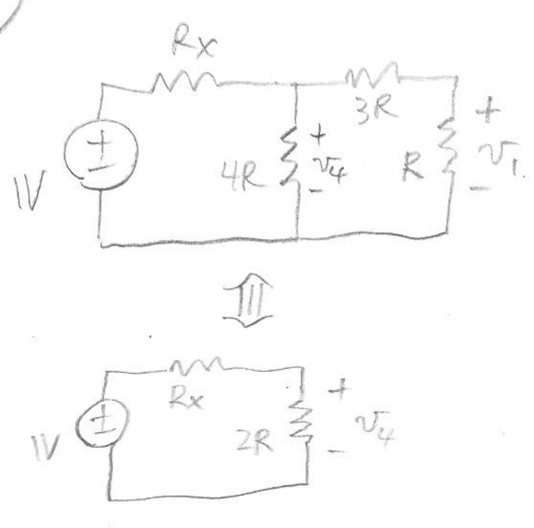


component 1

$$v_1 = \frac{1}{3+1} \times \left(\frac{\frac{4\times4}{4+4}}{\frac{4\times4}{4+4}+1} \times 1V \right)$$
$$= \frac{1}{6} 1V$$ #

(4.2) In order to increase v_1, we should increase the voltage across component 1. This means we should decrease R_x, since $\underline{R_x}$ and $\underline{\text{the resistance of component 1}}$ divide 1V. You may also use symbolic computation to get the same conclusion.

(4.3)



Here we can use the concept of voltage divider in the reverse way:

$v_1 = \frac{1}{5} 1V$ implies that

$$v_4 = \frac{4}{1} v_1 = \frac{4}{5} 1V$$

Now, $v_4 = \frac{2R}{R_x + 2R} 1V = \frac{4}{5} 1V$

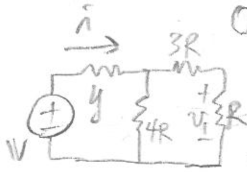
$$\Rightarrow 4R_x + 8R = 10R$$
$$\Rightarrow R_x = \frac{1}{2} R$$ #

(4.4) Symbolic computation is a great help in answering such a question:

$$\frac{1}{3} 1V = 1V \times \frac{2R}{y+2R} \times \frac{R}{3R+R} \Rightarrow \frac{1}{3} = \frac{2R}{y+2R} \times \frac{1}{4} \Rightarrow y = -\frac{1}{2}R$$

$\Rightarrow R > 0$ by definition and thus y cannot be larger than zero.

Alternatively, we may start from the condition, $y > 0$:



Ohm's law $\Rightarrow i = \frac{1V}{y+2R} < \frac{1V}{2R}$, since $y>0$ → which means v_1 must be less than $\frac{1}{4}$ 1V

Current divider $\Rightarrow v_1 = (\frac{1}{2} i) \cdot R < (\frac{1}{2} \cdot \frac{1V}{2R}) \cdot R = \frac{1}{4}$ 1V. $\Rightarrow v_1$ cannot equal $\frac{1}{3}$ #

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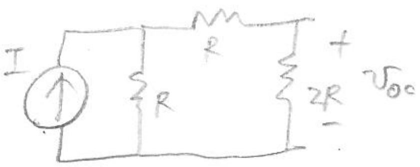
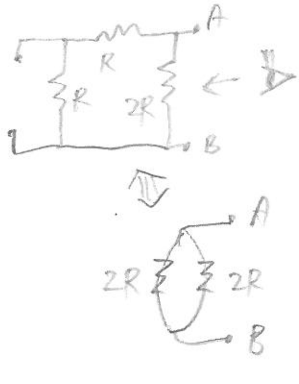
(5)

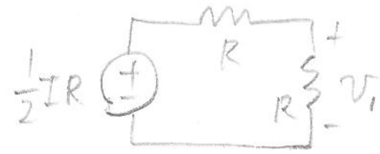
$$V' = V_{OC} = \left(I \times \frac{R}{R+(R+2R)}\right) \times 2R = \frac{1}{2} IR \Leftarrow$$



$$R' = R_{TH} = \frac{(R+R)\cdot 2R}{(R+R)+2R} = R \Longleftarrow$$



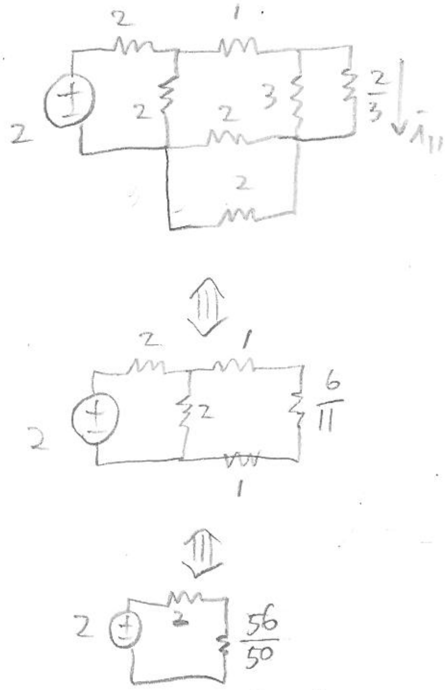
$$V_1 = \frac{1}{2} IR \times \frac{R}{R+R} = \frac{1}{4} IR$$

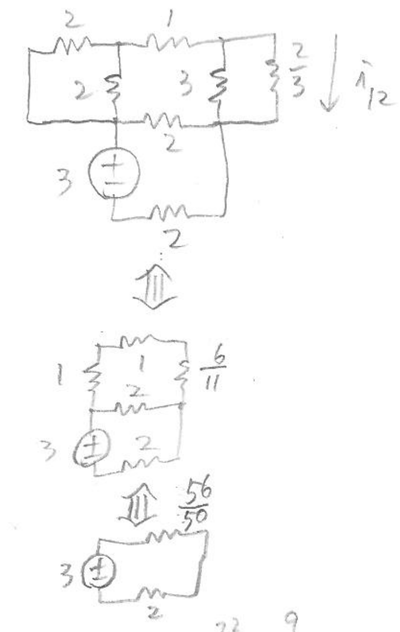


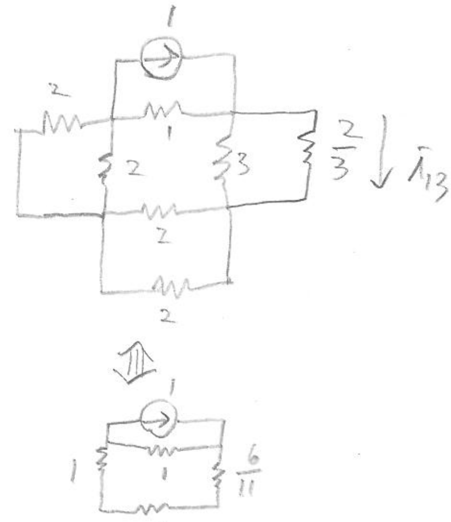
(6) Early on we've made a correction saying that the current source in this case should have been 1 mA, not 1 A. You will still receive score if you used 1 A, though the computation in that case is more complicated.

(6.1) (using 1mA) (using superposition)

$$i_1 = i_{11} + i_{12} + i_{13}$$







$$i_{13} = 1 \times \frac{1}{\frac{28}{11}+1} \times \frac{9}{11} = \frac{9}{39}$$

$$i_{11} = 2 \times \frac{50}{156} \times \frac{\frac{22}{11}}{\frac{50}{11}} \times \frac{\frac{9}{3}}{\frac{11}{3}} = 2 \times \frac{9}{78}$$

$$i_{12} = 3 \times \frac{50}{156} \times \frac{\frac{22}{11}}{\frac{50}{11}} \times \frac{\frac{9}{3}}{\frac{11}{3}} = 3 \times \frac{9}{78}$$

$$i_1 = i_{11} + i_{12} + i_{13} = 2 \times \frac{9}{78} + 3 \times \frac{9}{78} + \frac{2 \times 9}{78} = \frac{21}{26} \text{ mA}$$

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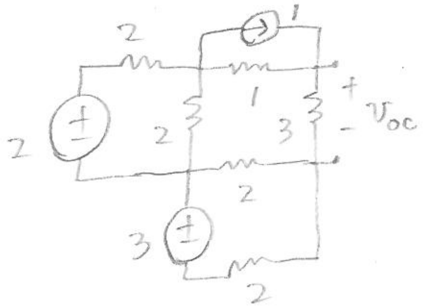
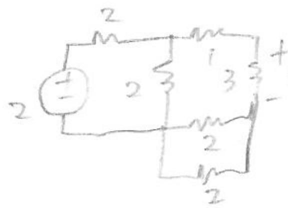
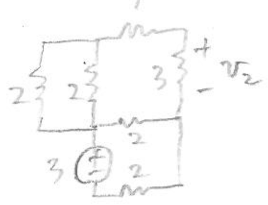
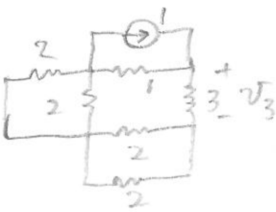
6.2 (using 1 mA)

$$V' = V_{oc} = v_1 + v_2 + v_3 = \left(2 \times \frac{\frac{5\times2}{5+2}}{\frac{5\times2}{5+2}+2} \times \frac{3}{5}\right) + \left(3 \times \frac{\frac{5\times2}{5+2}}{\frac{5\times2}{5+2}+2} \times \frac{3}{5}\right) + \frac{1}{5+1} \times 3 = \underline{\frac{7}{4}\,V}$$

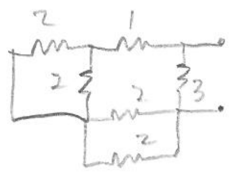




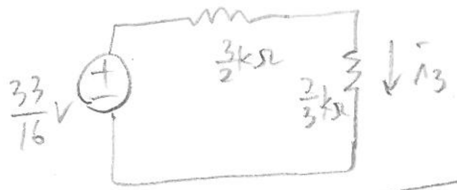




$$\underline{R' = R_{TH} = \frac{3}{2}\,k\Omega}$$



$$i_3 = \frac{7}{4} \times \frac{\frac{2}{3}}{\frac{3}{2}+\frac{2}{3}} \times \frac{1}{\frac{2}{3}} = \frac{7}{4} \times \frac{6}{13} = \underline{\frac{21}{26}\ mA}$$



$$i_1 = i_{11} + i_{12} + i_{13} = \frac{45}{78} + \frac{9}{39} \times 1000 = \frac{18045}{78}\ mA \approx \underline{231\ mA}$$ #

6.1 (using 1 A)

6.2 (using 1 A) $V' = V_{oc} = v_1 + v_2 + v_3 = \frac{15}{12} + \frac{1}{2} \times 1000 \approx 500\ V$

$R' = R_{TH} = \frac{3}{2}\,k\Omega$

$$i_3 \approx \frac{500}{\frac{3}{2}+\frac{2}{3}} = \frac{500}{\frac{13}{6}} \approx \underline{230\ mA}$$ #

Basically, thanks to superposition, changing from 1 mA to 1 A we may reuse most of our results for the case 1 mA.

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