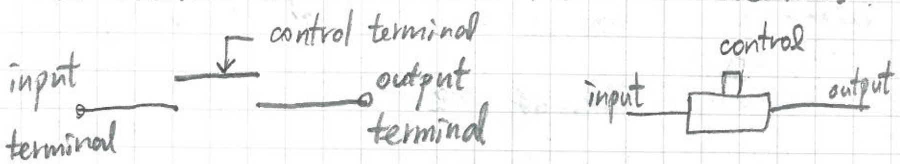
★ The MOSFET switch

(Metal Oxide Semiconductor Field-Effect Transistor)

A switch is a three-terminal element in a circuit.



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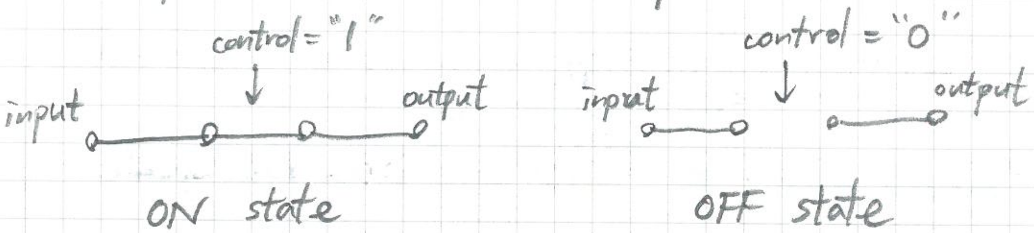
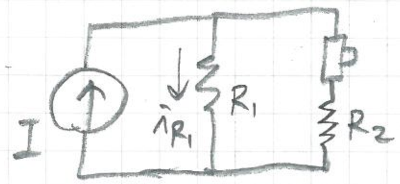
Ideally, input to the control terminal determines whether or not there is a short circuit between the input terminal and the output terminal:



(compare this with an ideal diode 二極, P39)

Such a simple idea of a switch enables many ways to control the response of certain elements in a circuit. First, let's look at two examples showing how a switch may impact the response of another element in a circuit:

Example 1.

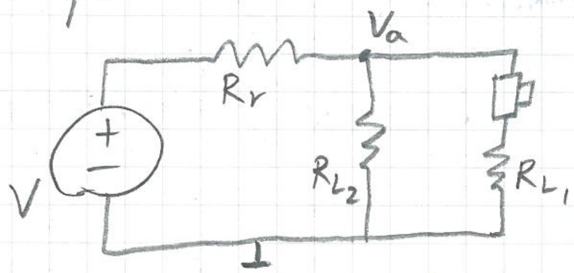


A switch connected in series with resistor R_2 will impact the current flowing through resistor R_1.

ON state : $i_{R_1} = I \times \frac{R_2}{R_1+R_2}$

OFF state : $i_{R_1} = I$ ※

Example 2.



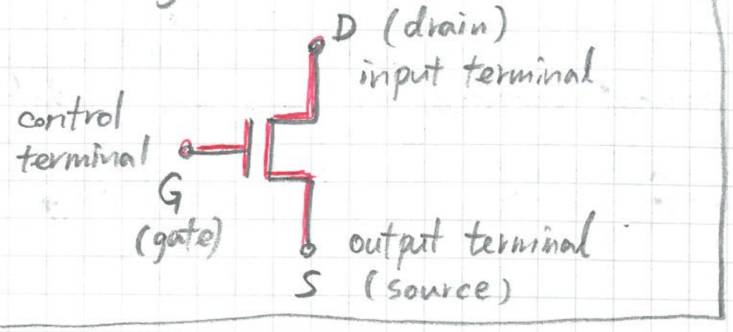
ON state : $V_a = V \times \frac{R_{L_2} /\!/ R_{L_1}}{R_r + R_{L_2} /\!/ R_{L_1}}$

OFF state : $V_a = V \times \frac{R_{L_2}}{R_r + R_{L_2}}$

The voltage across the resistor R_{L_2} will drop if the switch is in the ON state, because of the existance of R_r. ※

Conceptually, by connecting two switches in series we may implement the AND logic; by connecting two switches in parallel, we may implement the OR logic. In both cases, the control terminals take the input logical values.

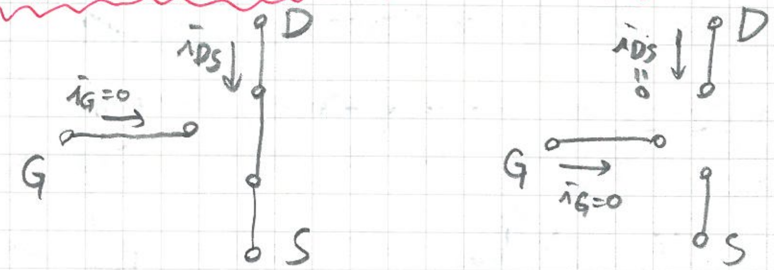
The symbol of a MOSFET



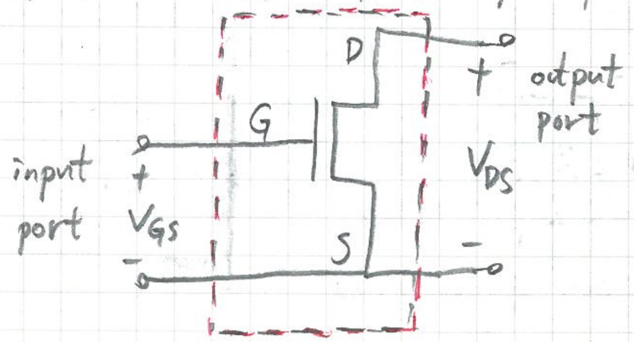
Let i_{DS} be the current flowing from the drain to the source, V_{GS} be the voltage across the gate and the source, and V_T be a threshold voltage.

In its simplest model (S model; S = switch), the MOSFET behaves as follows:

$V_{GS} \geq V_T \Rightarrow$ ON state $\qquad$ $V_{GS} < V_T \Rightarrow$ OFF state

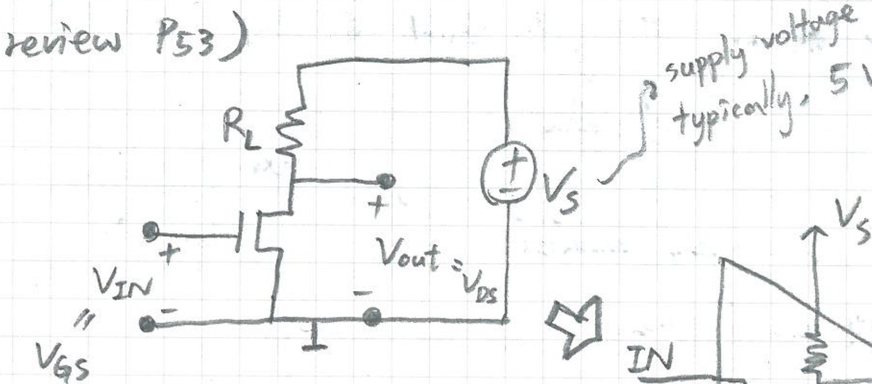


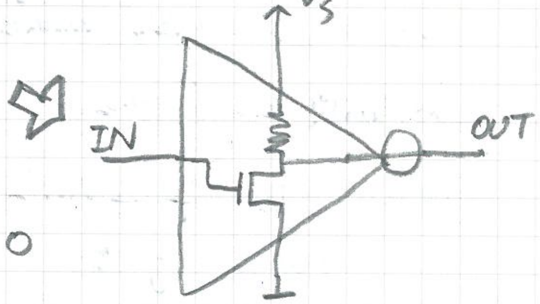
And in terms of input/output ports:



Later, we will study some more realistic models of MOSFET (for example, explicitly consider its internal resistance). For now, let's focus on the S model and its switching behavior.

We may use a MOSFET to construct a logical NOT gate , so-called an "inverter":
(review P53)



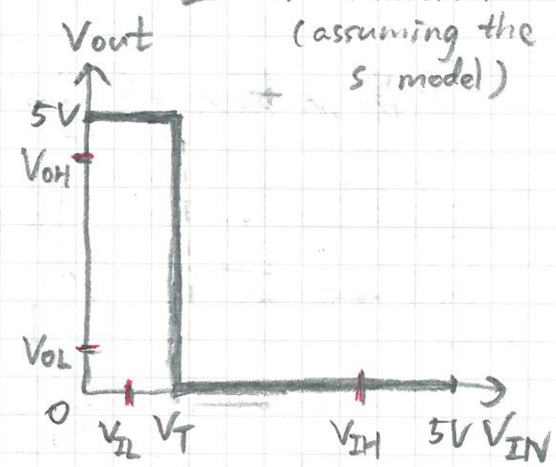


$V_{GS} \geq V_T \Rightarrow V_{out} = 0$
(logic 1) $\qquad$ (logic 0)

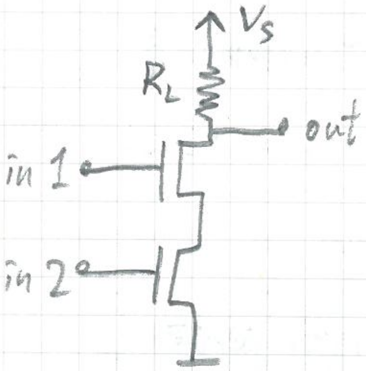
$V_{GS} < V_T \Rightarrow V_{out} = V_S$
(logic 0) $\qquad$ (logic 1)

transfer characteristic of an inverter (assuming the S model)

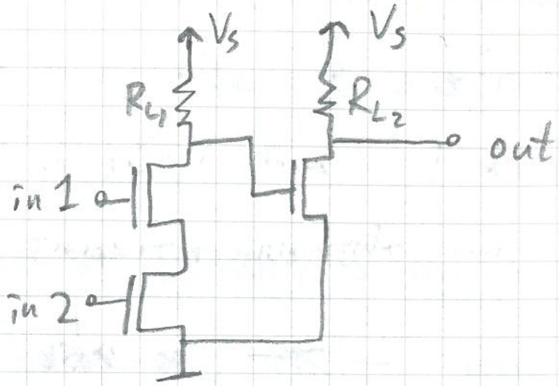
A valid mapping of voltage levels specified in the static discipline →
(review P52)



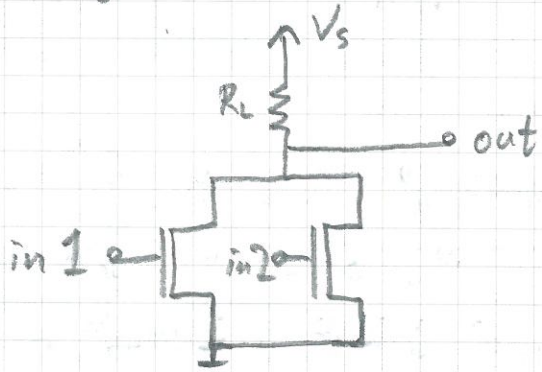
Similarly, we may use MOSFETs to construct other logic gates:



NAND gate



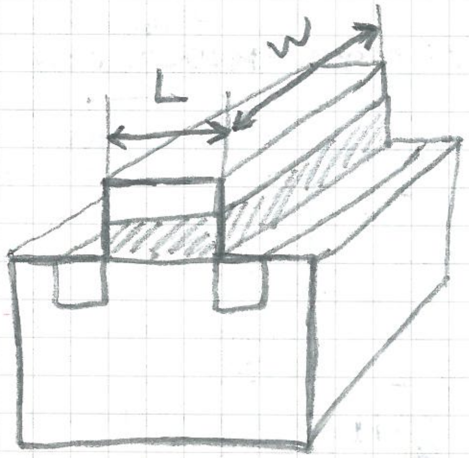
AND gate (NAND + NOT)



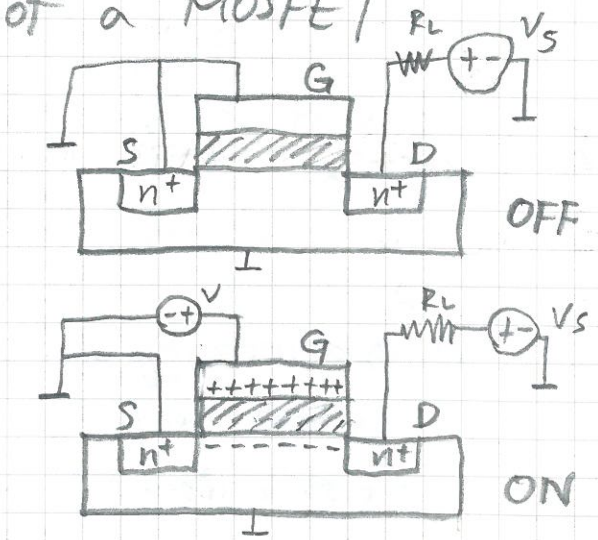
NOR gate

Exercise: OR gate

☆ Physical structure of a MOSFET



3D view



2D view

When a positive voltage is applied at the control terminal G, a conducting channel will be built up between the two n^+ regions, and therefore some nontrivial current may flow from D to S. Read Section 6.7 in the textbook for a more detailed account.

The physical structure also implies that in the ON state there really will exist some resistance between D and S, and the value of the resistance is characterised by the dimensions of that conducting channel. Let R_n be the resistance per square of the channel, and L be the channel length and W the channel width. We have

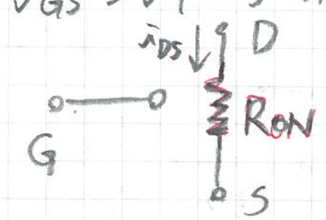
$$R_{ON} = R_N \cdot \frac{L}{W} \qquad \text{(review P6)}$$

↳ the resistance of the channel

The S model neglects R_{ON}. A more realistic model takes R_{ON} into account, and we call it the SR model:

$V_{GS} \geq V_T \Rightarrow$ ON state

(the OFF state is same as that in the S model)

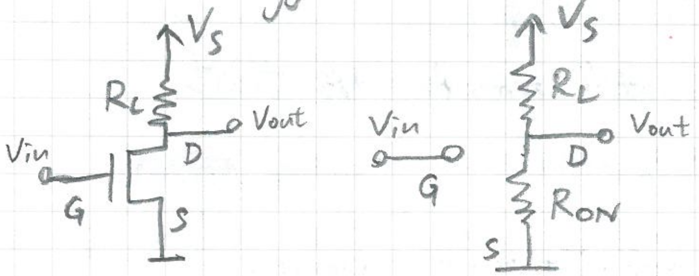


$$i_{DS} = \frac{V_{DS}}{R_{ON}}$$

✱ Analyzing a MOSFET circuit using the SR model

Taking into account the impact from R_{ON}, the analysis may seem complicated, but it is still based on what we've learned so far.

Let's analyze an inverter to illustrate this:

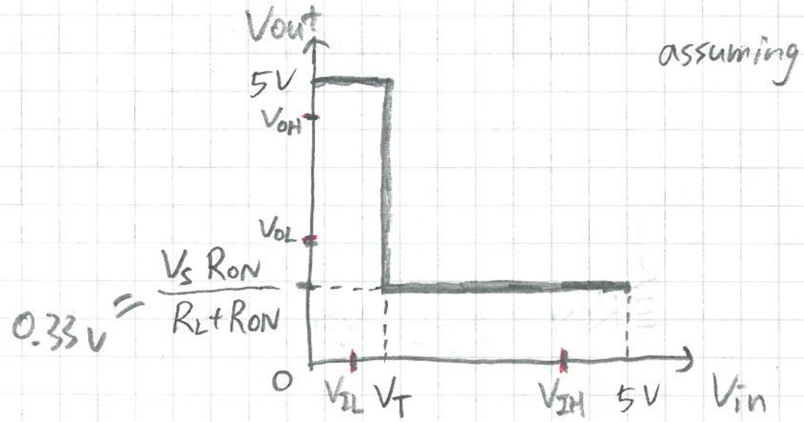


Because now we consider R_{ON}, the output voltage V_{out} will not be zero when $V_{GS} \geq V_T$; instead,

$$\boxed{V_{out} = V_S \times \frac{R_{ON}}{R_L + R_{ON}}}$$

following the voltage-divider relationship.

And thus the transfer characteristic will become as follows (compare to one on P57):



assuming $V_S = 5\text{V}$

$V_T = 1\text{V}$

$R_{ON} = 1\,\text{k}\Omega$

$R_L = 14\,\text{k}\Omega$

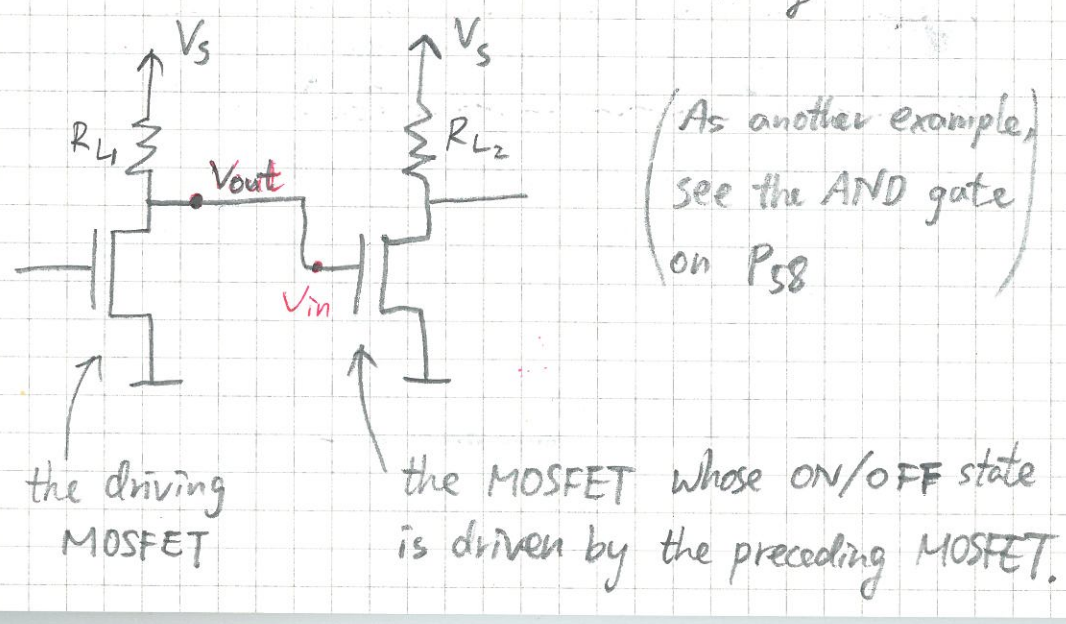
This change of V_{out} value has at least two impacts:

① To meet the static discipline, V_{out} (in ON state) needs to be lower than V_{OL}; otherwise, we may observe that even though the magnitude of noise is within the specified margin, the value of V_{in} still exceeds V_{IL}.

⟶ solution A: redesign to reduce V_{out}.
(from the manufacturer's viewpoint)

⟶ solution B: replace by a compatible device.
(from the consumer's viewpoint)

② To drive another MOSFET, V_{out} in ON state needs to be lower than V_T, since V_{out} in OFF state equals $V_S \geq V_T$ (usually).



(As another example, see the AND gate on P58) study Examples 6.5 and 6.6 in the textbook.

In general, since $\underset{\text{ON state}}{V_{out}} = V_S \times \dfrac{R_{ON}}{R_L + R_{ON}}$,

to change V_{out}, we may
① change V_S
② change R_L
③ change R_{ON}

① ⇒ as a side-effect, it will also change V_{out} in OFF state!

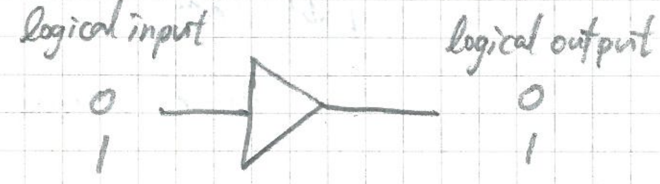
② ⇒
- larger resistance is hard to achieve in VLSI;
- larger resistance would cause nontrivial voltage drop in the presence of leakage current;

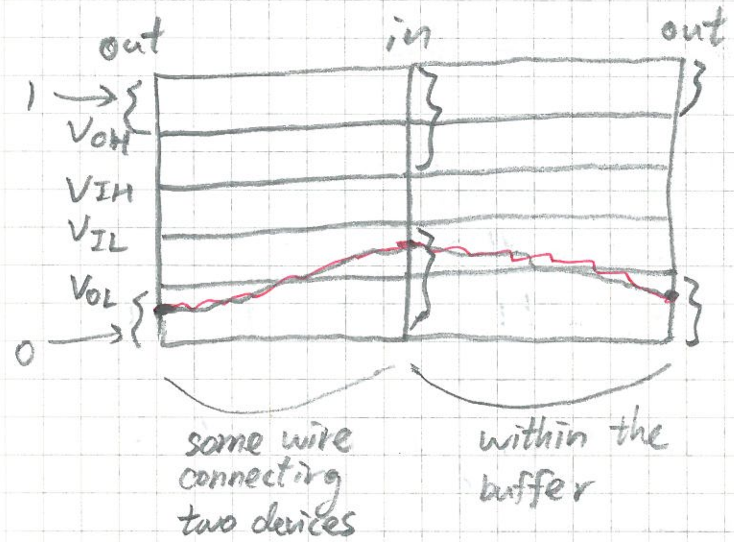
③ ⇒ may be achieved by changing the W/L ratio of the MOSFET.

⇨ An engineer's job often involves in finding the best solution among "multiple" options!!

☆ Signal restoration, gain, and nonlinearity (Section 6.9 in the textbook)

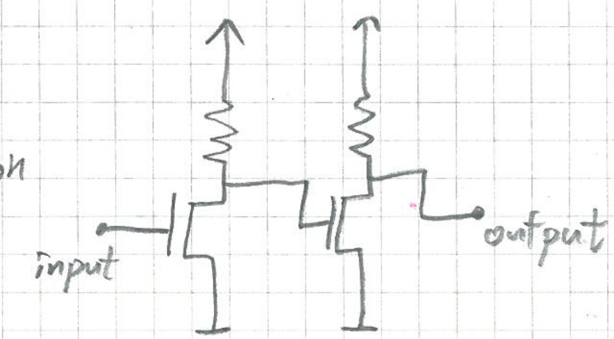
We may use a device called "buffer" to restore a distorted signal (distorted by noise, for example):



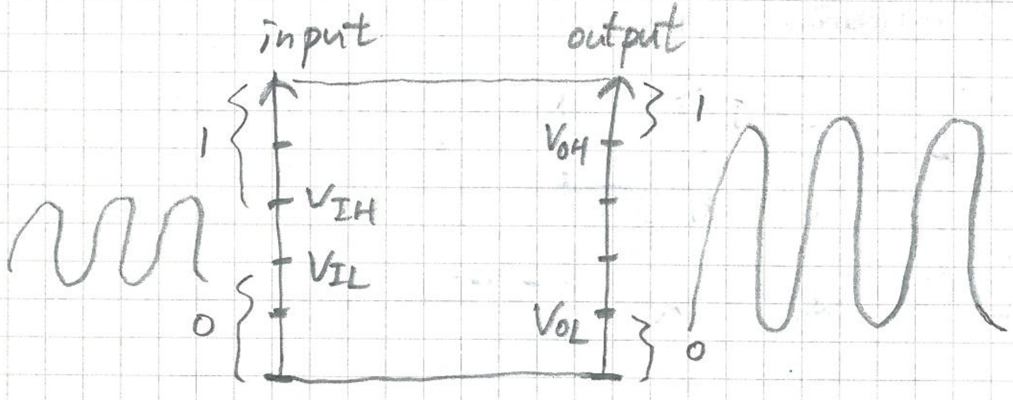


As long as the system follows the static descipline, the output of a buffer will be closer to the output of the device that precedes it.

A rough implementation of a buffer:



In order for an electronic device to satisfies the system's specification of the static discipline, it turns out that such a device must be capable of <u>amplifying</u> an input signal. Why? Because a signal may fluctuate between logic 0 and logic 1:



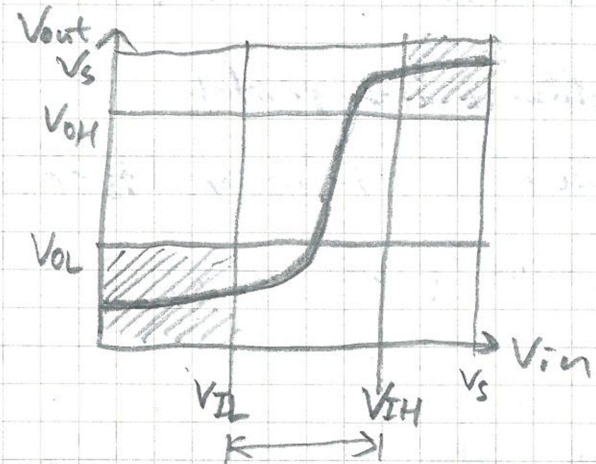
Definition:

$$\text{Gain} = \frac{V_{OH} - V_{OL}}{V_{IH} - V_{IL}} \quad \text{for } V_{IL} \to V_{IH} \text{ transition}$$

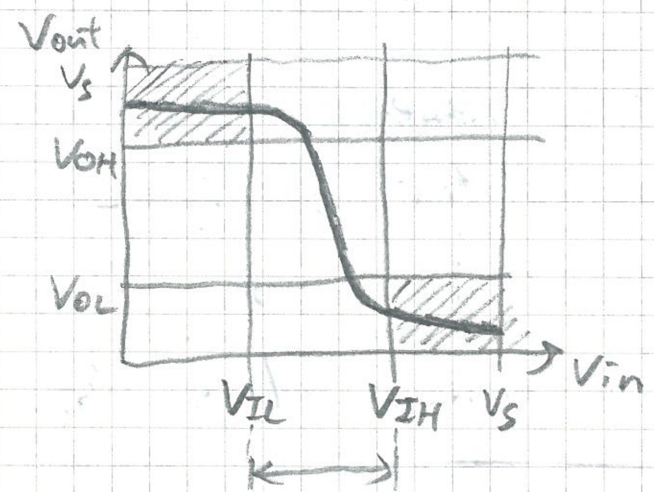
for example, if $V_{OH}=4$, $V_{IH}=3$, $V_{IL}=2$, $V_{OL}=1$

then the gain is 3.

The transfer characteristic of a buffer:



The transfer characteristic of an inverter:

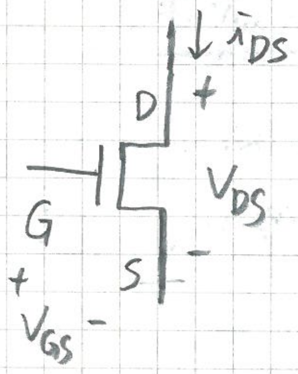


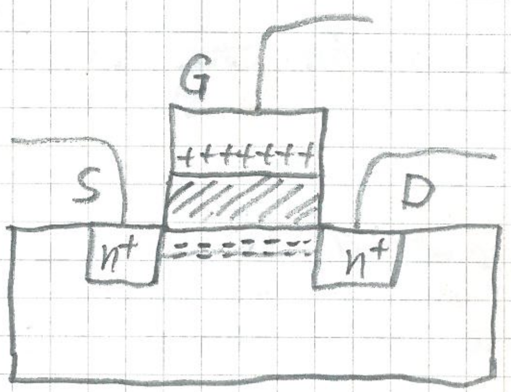
In both cases, the shaded region represents the valid region for the transfer curve.

Note that since $V_{OL} < V_{IL}$ and $V_s - V_{OH} < V_s - V_{IH}$, the magnitude of the slope of the curve in the valid region is smaller than 1.

So far, our discussion of MOSFET include its behavior as a switch (S model) as well as its linear $v_{DS}-i_{DS}$ relation in its ON state (SR model, with resistor R_{ON}).

In science and engineering, we are often curious about how a system would behave should we increase/decrease the value of a certain parameter. We have seen that a MOSFET will go from OFF state to ON state as we increase voltage V_{GS}. Now, let's consider what will happen if we gradually increase V_{DS}.

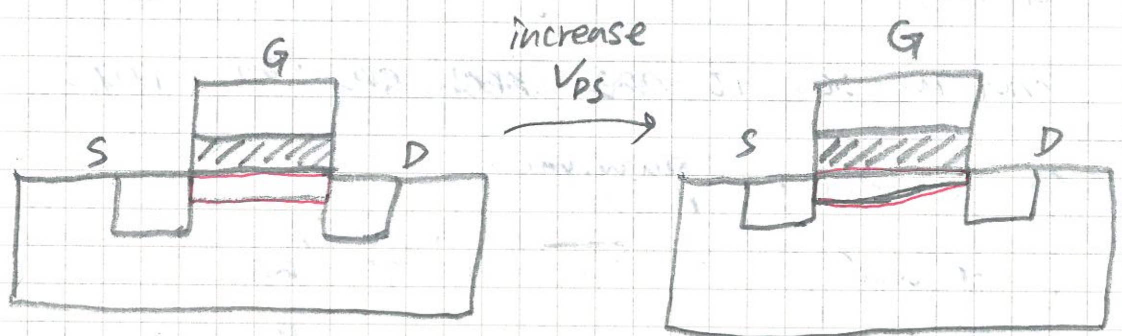




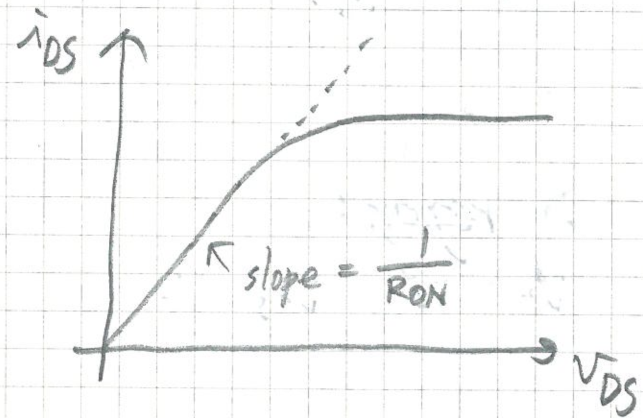
As we increase V_{DS}, the difference in electrical potentials between G and D decreases.

電位差

This in turn will reduce the amount of free electrons near D, esentially shrinking the thickness of the conductible channel near D:



Therefore we will see a bending of the curve on the $i_{DS}-V_{DS}$ plot:



電阻截面積 ↓

⇒ 電阻值 ↑

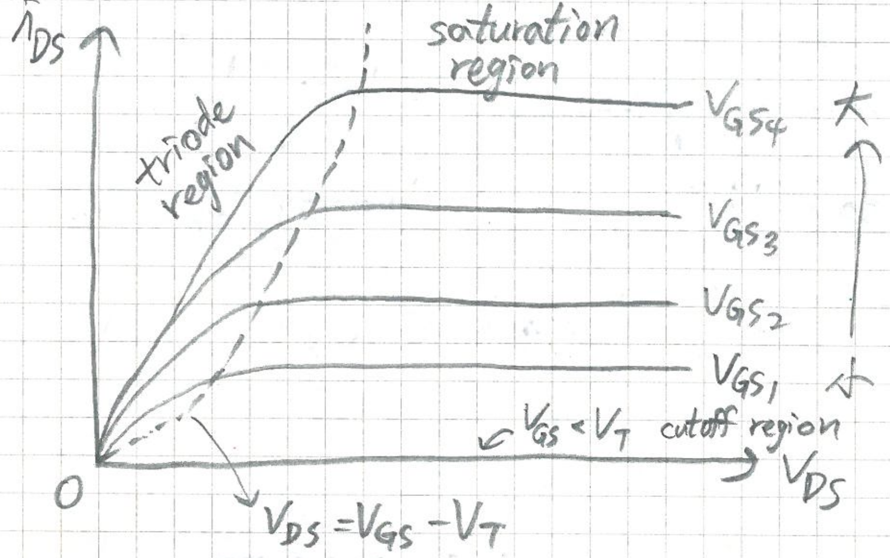
⇒ slope ↓

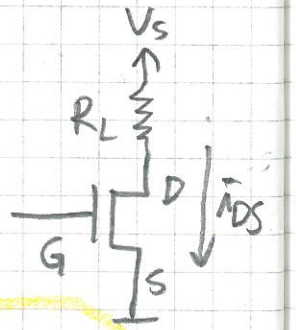
☆ The SCS model of a MOSFET

switch-current source

Compared to the S model and the SR model, the SCS model is a more accurate MOSFET model (closer to the real physical characteristic).

In the SCS model, a MOSFET can operate with three very different behaviors, and we say it has three "operational regions".

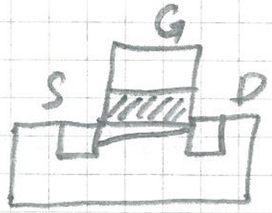




Conditions for each region:

- cutoff : $V_{GS} < V_T$, i.e., $V_{GS} - V_T < 0$
- saturation : $0 \leq V_{GS} - V_T \leq V_{DS}$
- triode : $V_{DS} < V_{GS} - V_T$

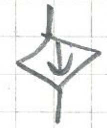
In the saturation region, current i_{DS} would stay the same as we keep increasing V_{DS}, because the channel between source and drain has become stable :

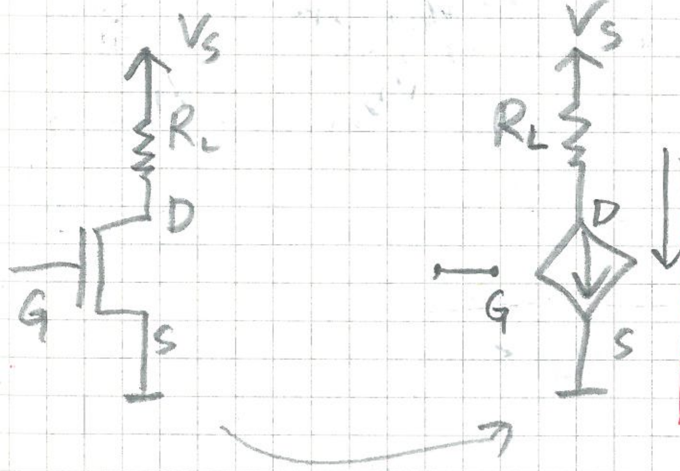


(review P_{58}, P_{66-67})

Therefore, we may consider such a behavior as if there is a "current source" (P_{13})

But i_{DS} would depend on voltage V_{GS}, and thus we say it is like a "voltage-controlled current source". symbol :





$$i_{DS} = f(V_{GS}) = \frac{K(V_{GS}-V_T)^2}{2}$$

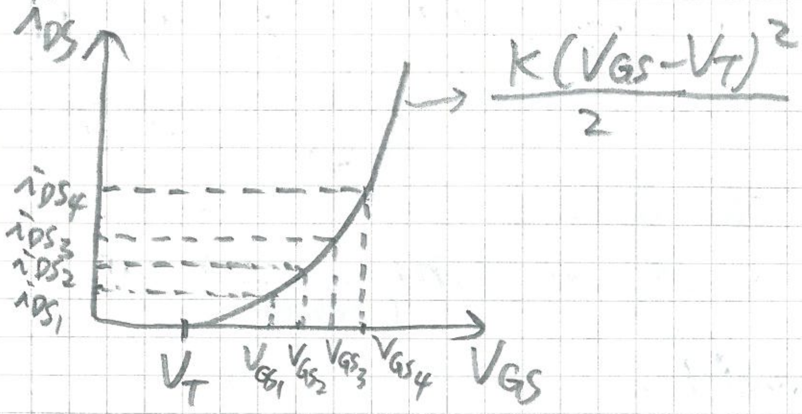
(according to physics)

Note that K is a coefficient ; unit : mA/V^2

if $0 \leq V_{GS} - V_T \leq V_{DS}$

→ in saturation region

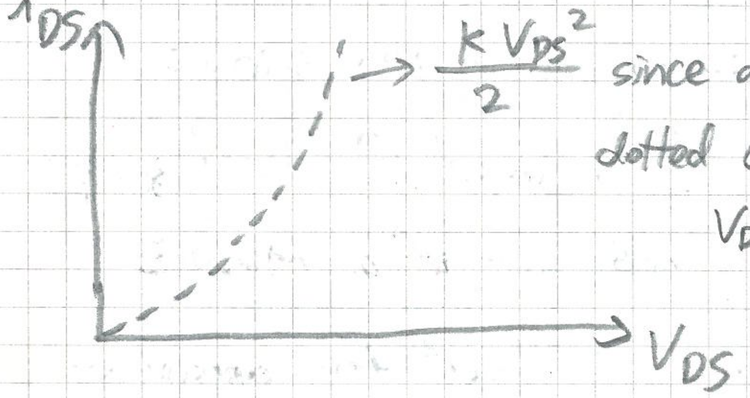
Plotting the $i_{DS} - V_{GS}$ relation, we see:



Therefore, if $|V_{GS_1} - V_{GS_2}| = |V_{GS_2} - V_{GS_3}|$
$= |V_{GS_3} - V_{GS_4}|$

we will have

$|i_{DS_1} - i_{DS_2}| < |i_{DS_2} - i_{DS_3}| < |i_{DS_3} - i_{DS_4}|$



$$\Rightarrow \boxed{i_{DS} = \begin{cases} 0 & \text{for } V_{GS} < V_T \\ \frac{K(V_{GS}-V_T)^2}{2} & \text{for } 0 \le V_{GS} - V_T \le V_{DS} \end{cases}}$$

From the aspect of the SCS model, the R_{ON} in the SR model can be thought of as a piecewise approximation of the $i_{DS} - V_{DS}$ relation in the triode region in the SCS model: (P39) (Section 4.4 in textbook)

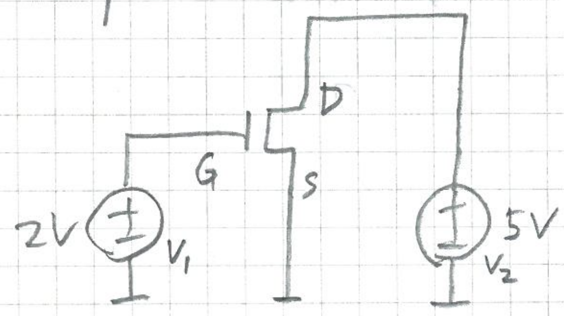
We may summarize in this way:

The S model is good for analyzing the ON/OFF behavior of a MOSFET.

The SR model is good for approximating the MOSFET's behavior in the triode region.

Let's work on two examples to get ourselves familiar with the SCS model and its analysis:

Example 1.



Assuming
$V_T = 1\,V$
$K = 1\,mA/V^2$

① $i_{DS} = ?$ since $V_{GS} = 2V > V_T$
and $V_{DS} = 5V > V_{GS} - V_T = 1$,
we see the MOSFET is operating in the saturation region $\Rightarrow i_{DS} = \frac{K(V_{GS}-V_T)^2}{2} = 0.5\ mA$

② If we keep V_1 and decrease V_2, at what condition of V_2 will the MOSFET enter the triode region?

$\rightarrow$ As long as $V_{DS} \geq V_{GS} - V_T$ the MOSFET will stay in the saturation region

$\rightarrow$ if $V_2 = V_{DS} < V_{GS} - V_T = 1\,V$, the MOSFET will enter the triode region.

③ If we keep V_2, what would be the range of V_1 for the MOSFET to stay in the saturation region?

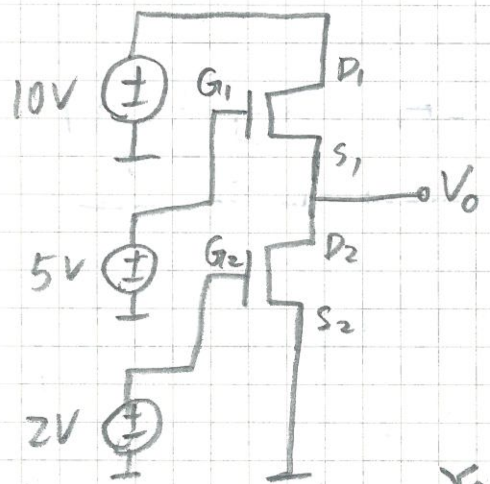
$\rightarrow V_{DS} \geq V_{GS} - V_T$

$\rightarrow 5 \geq V_{GS} - 1 \rightarrow V_{GS} \leq 6$

$\rightarrow V_1 = V_{GS} \leq 6$ ※

Example 2.

Assuming $V_T = 1\ V$
$k = 4\ mA/V^2$



If $V_o = 3$, are both MOSFETs operating in the saturation region?

Yes, since $V_{G_1S_1} - V_T$
$= (5-3) - 1 = 1 > 0$
and $\underbrace{V_{G_1S_1} - V_T}_{=1} \leq V_{D_1S_1} = 10-3 = 7$

And $V_{G_2S_2} - V_T = 2 - 1 = 1 > 0$
and $\underbrace{V_{G_2S_2} - V_T}_{=2-1} \leq \underbrace{V_{D_2S_2}}_{=3}$

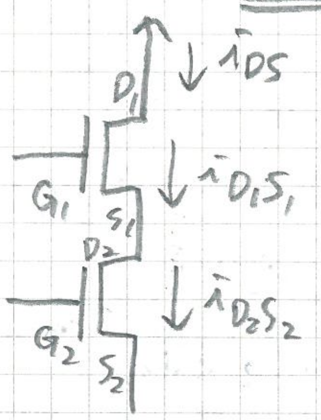
And $V_{G_1S_1} = V_{G_2S_2}$ correctly implies $i_{D_1S_1} = i_{D_2S_2}$ ※

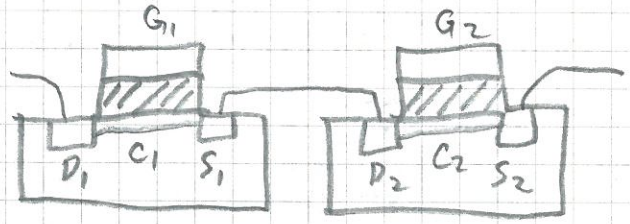
(Note on 2020/6/1 3PM :

I apologize for the confusion regarding whether $V_0 = 2$ in Example 2 makes sense or not.

After some more thoughts, I think what I said this morning is wrong.

It is <u>wrong</u> that $i_{DS} = i_{D_1S_1} + i_{D_2S_2}$





In physics, $i_{D_1S_1}$ is constrained by channel C_1, and $i_{D_2S_2}$ is constrained by channel C_2. It should be that

$$i_{DS} = i_{D_1S_1} = i_{D_2S_2}.$$

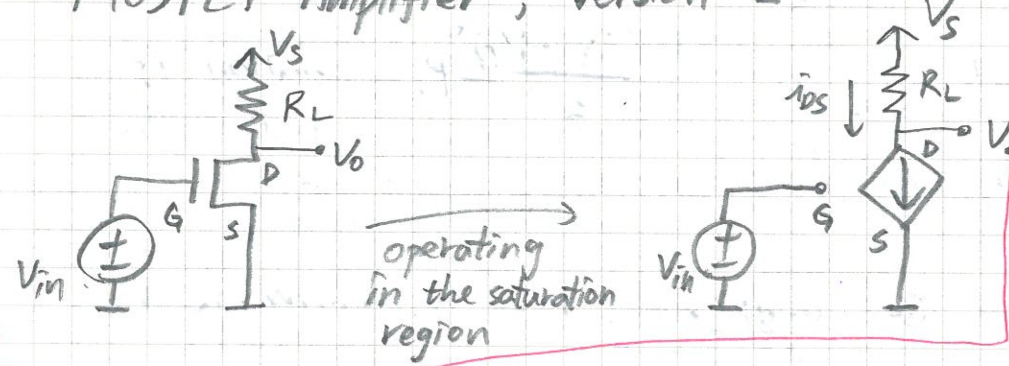
And therefore $V_0 = 2$ does NOT make sense.)

Now let's see how we may leverage a MOSFET operating in the saturation region for some useful purposes.

It turns out that in the saturation region a MOSFET may be used to amplify a signal. Amplifiers are used in many real-world applications and systems. Headphones and speakers are two examples.

Example 7.1 in the textbook

- MOSFET Amplifier, Version 1



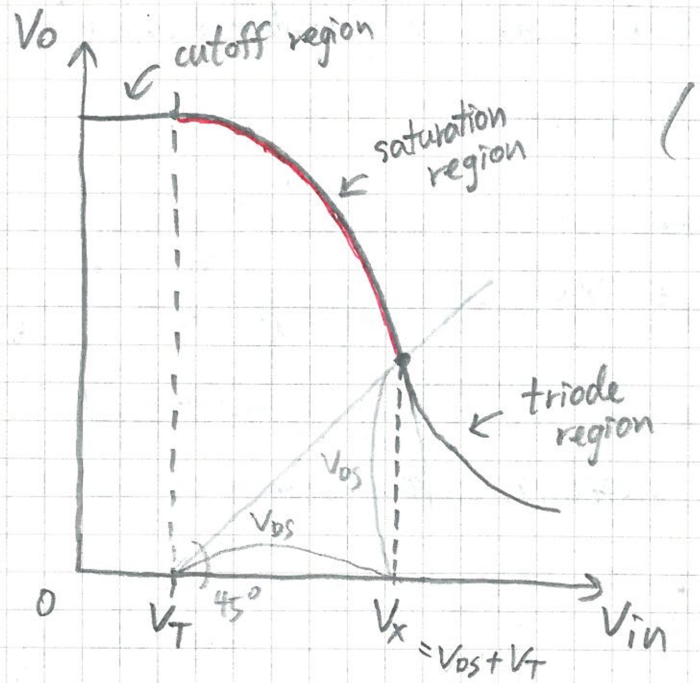
The voltage-controlled current source  gives

$$i_{DS} = \frac{K(V_{GS}-V_T)^2}{2} \quad (\text{see P69})$$

and in addition, by Ohm's law $i_{DS} = \dfrac{V_S - V_0}{R_L}$

$$\Rightarrow \left\{ \begin{array}{l} \dfrac{K(V_{GS}-V_T)^2}{2} = \dfrac{V_S - V_0}{R_L} \\ \text{and that } V_{GS} = V_{in} \end{array} \right\} \Rightarrow \boxed{V_0 = V_S - K \cdot \frac{(V_{in}-V_T)^2}{2} \cdot R_L}$$

following the equation on P75, we have



(compare this to the plots on P57 and P65)

The $V_o = V_s - \frac{K(V_{in}-V_T)^2}{2} R_L$ relation is → a parabola
only valid when $V_T \le V_{in} \le V_x$.

Later we will study how to determine V_x. (P81)

The slope $\frac{V_o}{V_{in}}$ is also the ratio between Vin and Vo. We see that in some part of the saturation region the magnitude of the slope is greater than one. Therefore we may amplify Vin by $\frac{V_o}{V_{in}}$ times and output the result as Vo.

The "gain" of the amplifier is defined to be $\frac{V_o}{V_{in}}$.

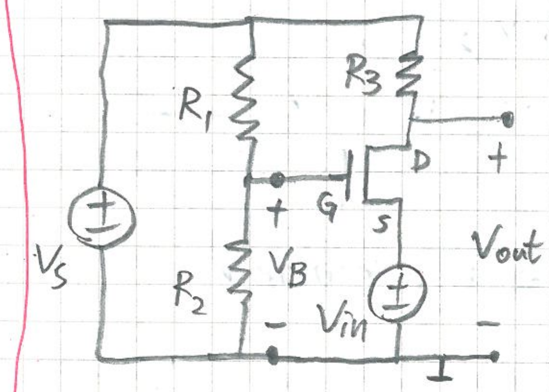
Using this MOSFET amplifier, however, the output will be inverted, which might not be what we want:

$$\begin{cases} V_{in} \downarrow \Rightarrow V_o \uparrow \\ V_{in} \uparrow \Rightarrow V_o \downarrow \end{cases}$$

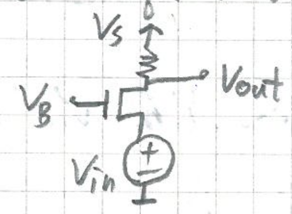
(can be verified by the sign of the slope in the V_o-V_{in} plot.)

Let's study an alternative design ↓ example 7.12 in the textbook

- MOSFET Amplifier, Version 2



In this configuration, we use one voltage source to provide voltages to the gate and the drain. This is equivalent to:



$$V_{GS} = V_B - V_{in}$$
$$= \left(V_s \times \frac{R_2}{R_1+R_2}\right) - V_{in}$$

In saturation region, we have

$$i_{DS} = \frac{K(V_{GS}-V_T)^2}{2}$$

and by Ohm's law

$$i_{DS} = \frac{V_s - V_{out}}{R_3}$$

Therefore, we have

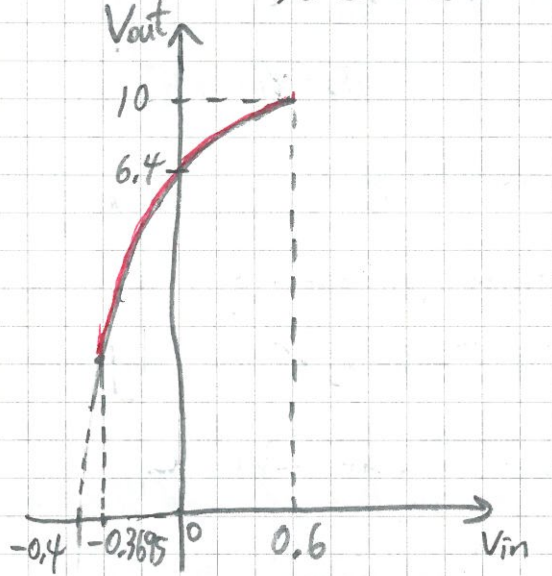
$$\frac{K(V_{GS}-V_T)^2}{2} = \frac{V_s - V_{out}}{R_3}$$

$$\Rightarrow \frac{K\left(V_s \times \frac{R_2}{R_1+R_2} - V_{in} - V_T\right)^2}{2} = \frac{V_s - V_{out}}{R_3}$$

following the equation on P77, suppose

that $\begin{cases} V_S = 10\ V \\ R_1 = 84\ k\Omega,\ R_2 = 16\ k\Omega,\ R_3 = 20\ k\Omega \\ V_T = 1\ V,\ K = 1\ mA/V^2 \end{cases}$

then $V_B = 1.6$ and $V_{out} = 10 - 10(0.6 - V_{in})^2$



To operate in the saturation region, both of the following conditions must be satisified:

$$\begin{cases} V_{GS} \geq V_T & —① \\ V_{GS} - V_T \leq V_{DS} & —② \end{cases}$$

from ①, $1.6 - V_{in} \geq 1 \Rightarrow V_{in} \leq 0.6\ V$

from ②, $(1.6 - V_{in}) - 1 \leq V_{out} - V_{in}$

$\Rightarrow V_{out} \geq 0.6\ V$

$\Rightarrow 10 - 10(0.6 - V_{in})^2 \geq 0.6$

$\Rightarrow -0.3695\ V \leq V_{in} \leq 1.5695\ V$

$-0.3695\ V \leq V_{in} \leq 0.6\ V$

Thus we see that it is an amplifier ($\frac{V_{out}}{V_{in}} > 1$)

and $\begin{cases} V_{in} \uparrow \Rightarrow V_{out} \uparrow \\ V_{in} \downarrow \Rightarrow V_{out} \downarrow \end{cases}$ ※

For the purpose of signal amplification, and for the SCS model in general, we would want to make sure that the MOSFET operates in the saturation region. Given the circuit's parameters, in order to determine whether the MOSFET will operate in the saturation region (or to determine the valid range of values of parameters), we may use the conditions listed on P68 or use graphical analysis.

For example, for the MOSFET amplifier on P75, suppose $V_S = 5V$, $R_L = 1\ k\Omega$, $V_T = 0.8\ V$, $V_{in} = 2.5\ V$, $K = 0.5\ mA/V^2$ is the MOSFET in the saturation region?

Answer: $V_{GS} = V_{in} = 2.5\ V$

$$V_{DS} = V_o = V_S - K\frac{(V_{in} - V_T)^2}{2}R_L = 4.28\ V$$

and we see that both $\begin{cases} V_{GS} \geq V_T \\ V_{DS} \geq V_{GS} - V_T \end{cases}$

Therefore, it is indeed in the saturation region. #