Name: Solution Student ID: ____________________

National Taiwan Normal University
Department of Computer Science and Information Engineering

CSC0056 - Data Communication
Midterm Exam (Nov. 9, 2020)

Solution attached

Note: Four group questions. 120 points in total. If you scored $x > 100$ points, those (x-100) points will be counted as bonus points, which in the end will be used to boost your semester score.
Exam time: 3 hours (9:10am-12:10pm). Clearly state your derivation to receive full score.

1. (25 points) **Error detection and correction.**
   1a. (10 points) Using the CRC algorithm, with the generator polynomial $g(D)=D^3+D+1$, and suppose that given the data bits represented by a polynomial $s(D)=D^4$. Compute the code word at the data sender.
   1b. (5 points) Following 1a, describe two nonzero error polynomials such that if a data receiver received the sum of the correct code word plus either one of these errors, the receiver would be unable to detect the error. Explain your answer.
   1c. (10 points) Using the following standard generator matrix, what would be the Hamming weight of the resulting linear code?

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

2. (25 points) **Markov chain theory.** A networking server is either busy or idle. If it is currently busy, with 70% probability it will be busy at the next time step; if it is currently idle, with 30% it will be idle at the next time step; but after the server remains idle for two consecutive time steps, at the next time step it will perform some garbage-collection tasks and thus will be busy.
   2a. (5 points) Draw the corresponding discrete-time Markov chain diagram. Label each state.
   2b. (10 points) What would the probability that after 3 time steps the system will be busy, given that the system is busy now?
   2c. (10 points) Compute the stationary distribution for each state. State your answer in fractions.

3. (30 points) **Systems analysis and design.** Consider a message broker that receives 10 independent streams of messages, each with arrival rate 10 messages/second, and transmits those messages to an application. The message lengths for all streams are independent and exponentially distributed.
   3a. (10 points) Using statistical multiplexing, we may model the message broker as M/M/1, where the aggregated arrival rate is 100 messages/second. Suppose the average service time is 5 milliseconds. What would be the average time a message will spend in the broker?
   3b. (10 points) Following 3a, what would be the average number of messages in the broker?
   3c. (10 points) Now, consider this hybrid system design: The broker will choose to use TDM multiplexing if there are less than 3 messages in the broker; otherwise, it will choose to use statistical multiplexing. For TDM multiplexing, we may model the broker as 10 M/M/1s, each with arrival rate 10 messages/second and service rate 20 messages/second. In average, what would be the fraction of time that the broker is using TDM multiplexing? For simplicity, we ignore the overhead of switching between two strategies and assume that the message-in-service will immediate be served by the new strategy.

Name: ______________________ **Student ID:** ______________________

4. (40 points) **Comprehensive understanding.**
   - 4a. (10 points) In the stop-and-wait ARQ, why would the receiver need to acknowledge using the packet sequence number and not just using Ack/Nak to reply to the sender?
   - 4b. (10 points) In the stop-and-wait ARQ, let T be the total time between the transmission of a packet and the reception of its acknowledgement. Explain the pros and cons of using a timer with length less than T for retransmissions.
   - 4c. (10 points) For the M/M/1, state the meaning of the first M and the second M, respectively.
   - 4d. (5 points) In CRC, using generator polynomial g(D)=D+1, all odd numbers of errors can be detected. Explain why.
   - 4e. (5 points) Suppose that the Hamming weight of a linear code is 3, and that a data receiver using the nearest-neighbor rule will always perform the error detection and then follow by the error correction. What is the probability that this approach above will fail to correct error, assuming that there are at most 3 bit errors and that the probability of having 1, 2, and 3 bit errors is 0.03, 0.01, and 0.009, respectively? Explain your answer.

Summary of results, from the required textbook for this course:

Notation

p_n = Steady-state probability of having n customers in the system

λ = Arrival rate (inverse of average interarrival time)

μ = Service rate (inverse of average service time)

N = Average number of customers in the system

N_Q = Average number of customers waiting in queue

T = Average customer time in the system

W = Average customer waiting time in queue (does not include service time)

$\overline{X}$ = Average service time

Little's Theorem

$$N = \lambda T$$

$$N_Q = \lambda W$$

Poisson distribution with parameter *m*

$$p_n = \frac{e^{-m} m^n}{n!}, \quad n = 0, 1, \ldots$$

$$\text{Mean} = \text{Variance} = m$$

Exponential distribution with parameter λ

$$P\{\tau \le s\} = 1 - e^{-\lambda s}, \qquad s \ge 0$$

$$\text{Density: } p(\tau) = \lambda e^{-\lambda \tau}$$

$$\text{Mean} = \frac{1}{\lambda}$$

$$\text{Variance} = \frac{1}{\lambda^2}$$

,me: ______________________ Student ID: ______________________

Summary of $M/M/1$ system results

1. Utilization factor (proportion of time the server is busy)

$$\rho = \frac{\lambda}{\mu}$$

2. Probability of n customers in the system

$$p_n = \rho^n(1-\rho), \qquad n = 0, 1, \ldots$$

3. Average number of customers in the system

$$N = \frac{\rho}{1-\rho}$$

4. Average customer time in the system

$$T = \frac{\rho}{\lambda(1-\rho)}$$

5. Average number of customers in queue

$$N_Q = \frac{\rho^2}{1-\rho}$$

6. Average waiting time in queue of a customer

$$W = \frac{\rho}{\mu-\lambda}$$

Summary of $M/M/m$ system results

1. Ratio of arrival rate to maximal system service rate

$$\rho = \frac{\lambda}{m\mu}$$

2. Probability of n customers in the system

$$p_0 = \left[\sum_{k=0}^{m-1} \frac{(m\rho)^k}{k!} + \frac{(m\rho)^m}{m!(1-\rho)}\right]^{-1}, \qquad n = 0$$

$$p_n = \begin{cases} p_0 \dfrac{(m\rho)^n}{n!}, & n \le m \\ p_0 \dfrac{m^m \rho^n}{m!}, & n > m \end{cases}$$

3. Probability that an arriving customer has to wait in queue (m customers or more in the system)

$$P_Q = \frac{p_0(m\rho)^m}{m!(1-\rho)}$$ (Erlang C Formula)

4. Average customer time in the system

$$T = \frac{1}{\mu} + \frac{\rho P_Q}{\lambda(1-\rho)}$$

5. Average number of customers in the system

$$N = m\rho + \frac{\rho P_Q}{1-\rho}$$

Name: ______________________ **Student ID:** ______________________

①

(1a) $g(D)=D^3+D+1 \Rightarrow (1011)_2$

$S(D)=D^4 \Rightarrow (10000)_2$

```
          10111
     ___________
1011 ) 10000000
       1011
       ----
        0110
        0000
        ----
         1100
         1011
         ----
          1110
          1011
          ----
           1010
           1011
           ----
              1
```

$\Rightarrow$ code word $= \underline{(10000001)_2}$

i.e., $\underline{D^7+1}$ #

(1b) CRC will assume no error if the remainder polynomial of the long division is $(0)_2$.

Therefore, two such code words are

① $e(D)=g(D)=\underline{D^3+D+1 \Rightarrow (1011)_2}$ #

② $e(D)=g(D)\cdot(D+1)=(D^3+D+1)(D+1)$

$\underline{=D^4+D^3+D^2+1 \Rightarrow (11101)_2}$ #

```
 1011
  1011
 -----
 11101
```

(1c)

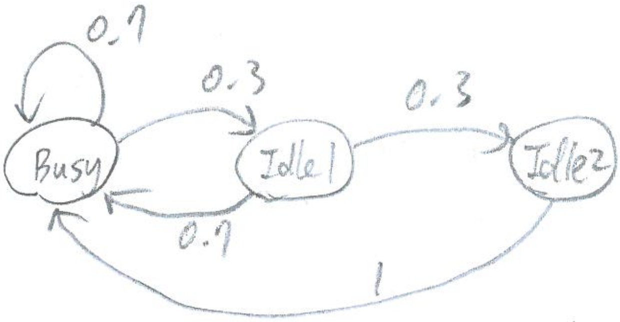
$$\left\{\begin{matrix} 000 \\ 001 \\ 010 \\ 011 \\ 100 \\ 101 \\ 110 \\ 111 \end{matrix}\right. \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \rightarrow \left.\begin{matrix} 00000 \\ 00101 \\ 01010 \\ 01111 \\ 10011 \\ 10110 \\ 11001 \\ 11101 \end{matrix}\right\}$$

$\therefore$ The Hamming weight of this linear code is 2

#

me: ____________________ Student ID: ____________________

(2a)



(2b)

0.49
0.21

$$\mathbb{P} = \begin{array}{c|ccc} \text{from} \backslash \text{to} & B & I_1 & I_2 \\ \hline B & 0.7 & 0.3 & 0 \\ I1 & 0.7 & 0 & 0.3 \\ I2 & 1 & 0 & 0 \end{array}$$

$$\mathbb{P}^2 = \begin{bmatrix} 0.7 & 0.3 & 0 \\ 0.7 & 0 & 0.3 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.7 & 0.3 & 0 \\ 0.7 & 0 & 0.3 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.7 & 0.21 & 0.09 \\ & \cdots & \end{bmatrix}$$

237
0.49
0.147
0.09

$$\mathbb{P}^3 = \begin{bmatrix} 0.7 & 0.21 & 0.09 \\ & \cdots & \end{bmatrix} \begin{bmatrix} 0.7 & 0.3 & 0 \\ 0.7 & 0 & 0.3 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0.727 & \cdots \\ & \cdots \end{bmatrix}$$

☆ Note that $\mathbb{P}^3 = \mathbb{P}^2 \cdot \mathbb{P}$
$= \mathbb{P} \cdot \mathbb{P}^2$

$\therefore \mathbb{P}^3_{BB} = 0.727$ #

(2c)

$$\begin{cases} \pi_B = \pi_B \times 0.7 + \pi_{I1} \times 0.7 + \pi_{I2} \times 1 \\ \pi_{I1} = \pi_B \times 0.3 \\ \pi_{I2} = \pi_{I1} \times 0.3 = \pi_B \times 0.09 \\ \pi_B + \pi_{I1} + \pi_{I2} = 1 \end{cases}$$

$$\Rightarrow (\pi_B, \pi_{I1}, \pi_{I2}) = \left(\frac{1}{1.39}, \frac{0.3}{1.39}, \frac{0.09}{1.39} \right)$$ #

Name: ______________________ Student ID: ______________________

(3a) $\lambda = 100$ messages/second

$$\mu = \frac{1 \text{ message}}{5 \times 10^{-3} \text{ second}} = 200 \text{ messages/second}$$

$$\Rightarrow T = \frac{1}{\mu - \lambda} = \frac{1}{200-100} = \frac{1}{100} \text{ seconds} = 10 \text{ ms}$$

#

(3b)

$$\rho = \frac{\lambda}{\mu} = \frac{100}{200} = 0.5$$

$$N = \frac{\rho}{1-\rho} = \frac{0.5}{0.5} = 1$$

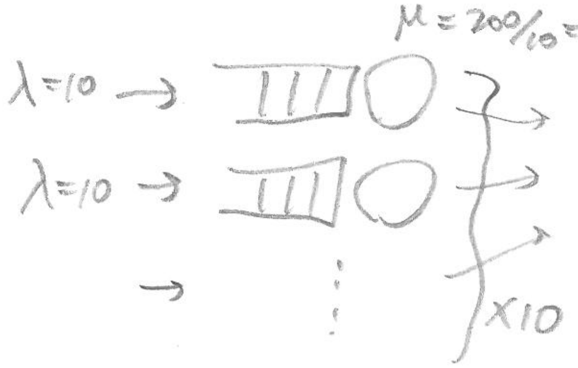
#

(3c) The motivation of this design is that

① TDM multiplexing incurrs less jitter;

② Statistical multiplexing incurrs less delay.

Using TDM multiplexing, the system can be modeled as:

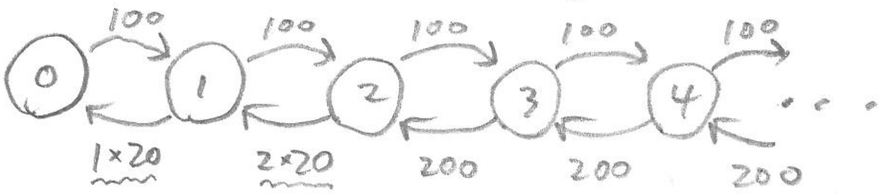


Using statistical multiplexing, the system can be modeled as:

Continue to the next page →

ame: ______________________ **Student ID:** ______________________

3C cont. Therefore, the corresponding Markov chain is as follows, where we use "the total # of messages in the system" to differentiate states:



for $n < 3$, we have

$$\begin{cases} P_0 \times 100 = P_1 \times 20 \\ P_1 \times 100 + P_1 \times 20 = P_0 \times 100 + P_2 \times 40 \\ P_2 \times 100 + P_2 \times 40 = P_1 \times 100 + P_3 \times 200 \end{cases}$$

$$\Rightarrow P_1 = 5P_0,\ P_2 = \frac{10}{4}P_1 = \frac{50}{4}P_0,\ P_3 = \frac{1}{2}P_2$$

for $n \geq 3$, we have

$$P_3 \times 100 + P_3 \times 200 = P_2 \times 100 + P_4 \times 200$$

$$P_4 \times 100 + P_4 \times 200 = P_3 \times 100 + P_5 \times 200$$

$$\cdots$$

$$\Rightarrow P_n = P_{n-1} \times \frac{1}{2} \text{ for } n \geq 3$$

Now, from $\sum_{n=0}^{\infty} P_n = 1$, we have

$$P_0 + P_1 + P_2 + \sum_{n=3}^{\infty} (2^{-1})^{n-2} \left(\frac{50}{4}\right) P_0 = 1$$

$$\Rightarrow P_0 + 5P_0 + \frac{50}{4}P_0 + \frac{50}{4}P_0 \times \left(\frac{\frac{1}{2}}{1-\frac{1}{2}}\right) = 1$$

$$\Rightarrow P_0 = \frac{1}{31},\ P_1 = \frac{5}{31},\ P_2 = \frac{50}{4} \times \frac{1}{31}$$

$$\text{TDM\%} = P_0 + P_1 + P_2 = \frac{37}{62} \approx \underline{60\%}$$

\#

Name: ____________________ Student ID: ____________________

4a See lecture04.pdf, page 8.

4b see lecture04.pdf, pages 10~12.

4c see the note 1019.pdf, page 4; also, textbook pages 162~163.

4d $g(D) = D+1 \Rightarrow (11)_2$

The key is to show that if an error polynomial $e(D)$ has an odd number of nonzero terms, then $g(D)$ cannot divide $e(D)$. (Hence the remainder $\neq (0)_2$)

To show this, it is sufficient to show that any multiple of $(11)_2$ will have even numbers of 1s in the result. Consider shifting $(11)_2$ around and adding the result:

$$\begin{array}{r} 11 \\ +_2\ 011 \\ \hline 101 \end{array} \qquad \begin{array}{r} 11 \\ 0011 \\ +_2\ 0011 \\ \hline 11101 \end{array} \qquad \cdots$$

#

4e Ans: 0.019 #

For any 1-bit error, the nearest-neighbor rule can catch it in this case;

for any 3-bit error, it cannot be detected neither be corrected because it had convert one code word into another code word;

for any 2-bit error, it cannot be corrected since the nearest-neighbor rule will make the wrong decision.

☆ This shows you that sometimes after error detection we will not attempt to do error correction; we will ask for retransmission instead :)