☆ Signals, Systems, and Computing

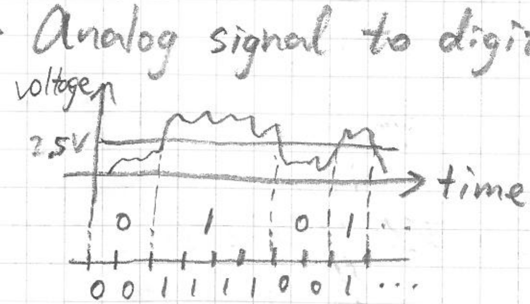
- In computing systems, information (and energy) is stored and transferred in terms of <u>signals</u>, which are <u>currents</u> or <u>voltages</u> as a function of time.

A circuit (or a system of circuits), as we study in this course, is used to
① carry signals
② transform signals from one to another

Computing is also a transformation of signals; though the signals are digital are the transformation is described at a higher layer of the abstraction (Figure 1.1 in the textbook).

Example: We use a smart phone to record this lecture (voice signal → currents and voltages), and the recording is stored in the phone, transferred via USB to your laptop, uploaded to a cloud drive, downloaded by your friend who cannot make it to the class, and finally played by your friend's speakers/headphone (currents and voltages → voice signals).

- Analog signal to digital signal via discretization:



P10 Section 1.8 in the textbook mentioned sinusoidal signals and the <u>root mean square</u> value. For your interest, the $\sqrt{2}$ ratio between the amplitude of a sinusoidal signal and its rms value comes from 三角函數2倍角轉換.

Example: let signal $i(t) = I_m \cos(\omega t)$

$$i_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t) \cdot dt}$$

$$= \sqrt{\frac{1}{T}\int_0^T (I_m^2 \cos^2 \omega t)\, dt}$$

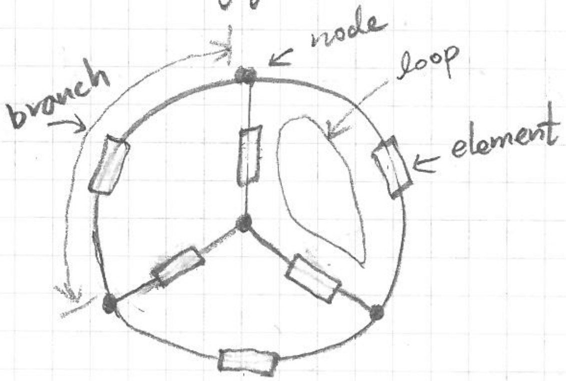
$$= \sqrt{\frac{I_m^2}{2T}\int_0^T (1+\cos 2\omega t) dt}$$

$$= \frac{I_m}{\sqrt{2}}\sqrt{\frac{1}{T}\int_0^T (1+\cos 2\omega t)\, dt}$$

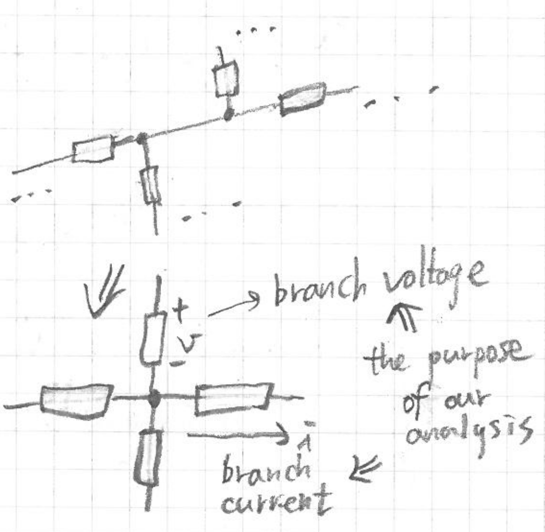
$$= \frac{I_m}{\sqrt{2}} \quad ※$$

☆ Resistive Networks and How to Analyze Them

- Terminology



ideal wire: no resistance



- Kirchhoff's Laws $\begin{cases} \text{KCL} \\ \text{KVL} \end{cases}$

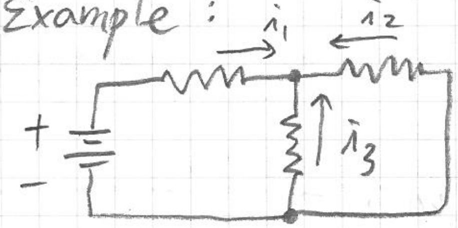
following LMD, Kirchhoff's laws are simplifications of Maxwell's Equations (see Appendix A.2 in the textbook)

$\rightarrow$ KCL and KVL are extremely useful tools to help us analyze a circuit!

KCL : Kirchhoff's current law

The algebraic sum of all branch currents flowing into any node must be zero.

Example :



$$\sum_{n=1}^{3} i_n = 0$$

$$\sum_{n=1}^{3} (-i_n) = 0$$

In general, for integers N, M, we have

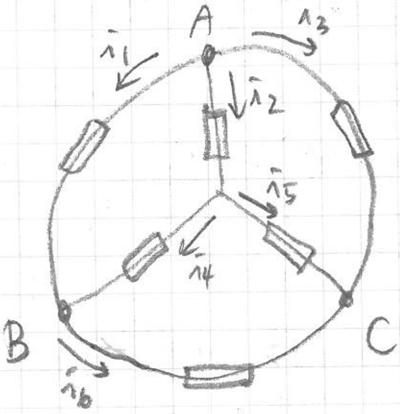
$$\sum_{n=1}^{N} i_n = 0 \Rightarrow \sum_{n=1}^{M} i_n + \sum_{n=M+1}^{N} i_n = 0$$

$$\Rightarrow \sum_{n=1}^{M} i_n = \sum_{n=M+1}^{N} (-i_n)$$

$\Rightarrow$ Sum of total current flowing into a node = sum of total current flowing out from a node.

$i_1 = i_2$ ideal wire

Example:



Using KCL

node A: $0 = -i_1 - i_2 - i_3$

B: $0 = i_1 + i_4 - i_6$

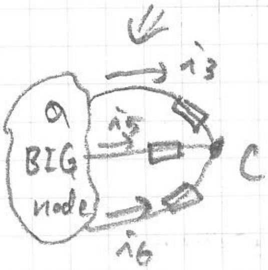
C: $0 = i_2 - i_4 - i_5$

D: $0 = i_3 + i_5 + i_6$

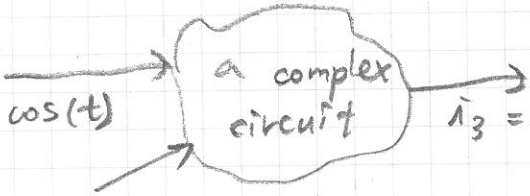
Suppose $i_1 = 1$, $i_3 = 3 \Rightarrow i_2 = -4$

In addition, suppose $i_5 = -2$

$\Rightarrow i_4 = -2$, $i_6 = -1$ #



In general, for N KCL statements, only N-1 of them are independent. Therefore, we need N-1 known values to solve the circuit.



$i_1 = 3\cos(t)$

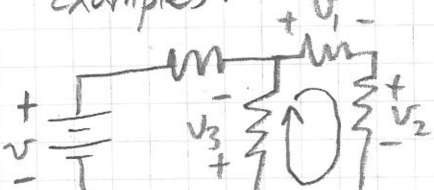
$i_2 = 6\cos(t)$

$i_3 = i_1 + i_2 = 9\cos(t)$

KVL: Kirchhoff's voltage law

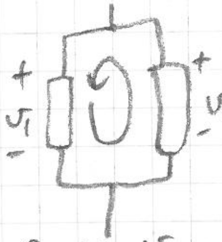
The algebraic sum of the branch voltages around any closed path in a network must be zero.

Examples:



$$\sum_{n=1}^{3} v_n = 0$$

並聯



$0 = v_1 - v_2$

$\Rightarrow v_1 = v_2$

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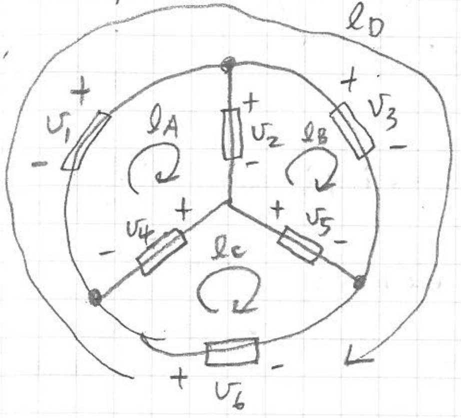
$0 = -v + v_1 + v_2$

$\Rightarrow v = v_1 + v_2$

Example:



Using KVL

loop l_A: $0 = -v_1 + v_2 + v_4$

l_B: $0 = -v_2 + v_3 - v_5$

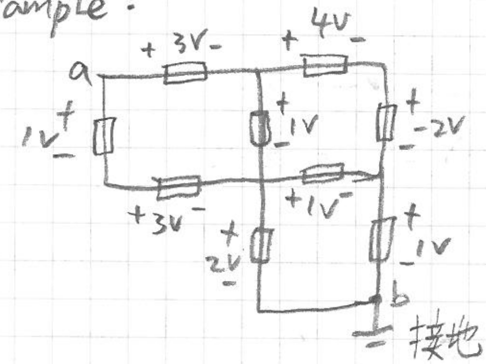
l_C: $0 = -v_4 + v_5 - v_6$

l_D: $0 = -v_1 + v_3 - v_6$

Suppose that $v_1 = 1$, $v_3 = 3$, $v_2 = 2$

then $v_4 = -1$, $v_5 = 1$, $v_6 = 2$ ✱

Example:



v_{ab} can be determined using KVL

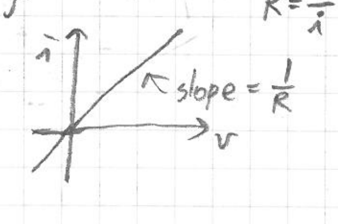
(which loop(s) would you pick?)

$v_{ab} = 6$ V.

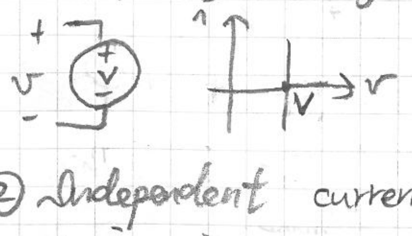
> In general, we model individual elements based on LMD assumptions and we analyze a circuit of elements using KCL and KVL.

☆ Two more basic elements and their i-v characteristics

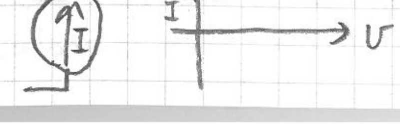
for linear resistor it obeys Ohm's law $R = \frac{v}{i}$



① Independent voltage source



② Independent current source



☆ Basic Method to Analyze A Circuit

four steps

see Page 8

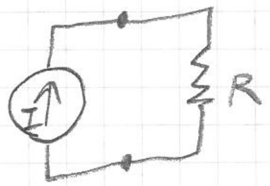
1° define branch current and voltage <u>consistently</u>

2° apply element laws for each elements (e.g., Ohm's law for linear resistors)

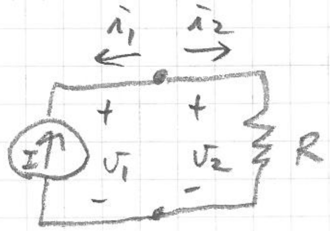
3° apply KCL and KVL

4° jointly solve the equations obtained from 2° and 3°.

Example:



1°



2° $i_1 = -I$, $v_2 = i_2 \cdot R$

3° KCL $\Rightarrow i_1 + i_2 = 0$

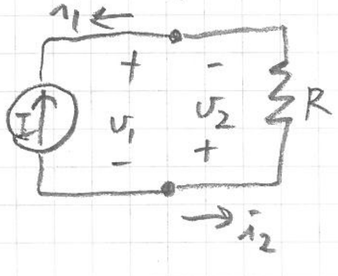
KVL $\Rightarrow -v_1 + v_2 = 0$

4° $i_2 = -i_1 = I$

$v_2 = i_2 R = IR$

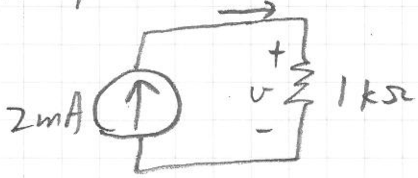
$v_1 = v_2 = IR$ ※

if at step 1° we define v_2 inversely, we must also define i_2 inversely, to be consistent. Applying the basic method you will find $i_2 = -I$ and $v_2 = -IR$. This may seem to be strange. But if we recall that both i_2 and v_2 have a reversed direction, then it makes sense.



Alternatively, we may solve a circuit by considering "Energy conservation".

Example :



power out from the source :

$$P_{out} = 2mA \times v$$

power into the resistor :

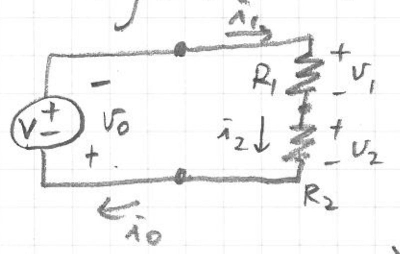
$$P_{in} = i \times v = \frac{v^2}{R} = \frac{v^2}{1k\Omega}$$

$$P_{out} = P_{in} \Rightarrow 2\times10^{-3} \cdot v = \frac{v^2}{1\times10^{3}}$$

$$\Rightarrow v = 2\ V \ast$$

(Example 2.14 in the textbook has a typo saying $v = 0.5\ V$)

☆ Voltage Divider



we may analyze it using the basic method :

$$\begin{cases} v_0 = V \\ v_1 = R_1 i_1 \\ v_2 = R_2 i_2 \end{cases} \quad \begin{cases} i_0 = i_1 \\ i_1 = i_2 \end{cases} \text{KCL} \quad v_0 + v_1 + v_2 = 0 \quad \text{KVL}$$

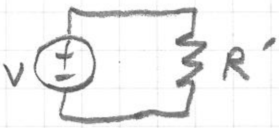
$$\Rightarrow \boxed{v_2 = \frac{R_2}{R_1 + R_2} V}$$

Further, from $i_2 = \frac{v_2}{R_2}$ we see $i = \frac{1}{R_1 + R_2} V$

in other word, $V = i \times (R_1 + R_2)$.

$\Rightarrow$ We may replace R_1 and R_2 by an equivalent resistor $R' = R_1 + R_2$

and the circuit is equivalent as



This lead to a general planar linear resistor analysis such as that in Example 2.21 in the textbook.