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solution

National Taiwan Normal University  
CSU0007 - Basic Electronics Homework 5

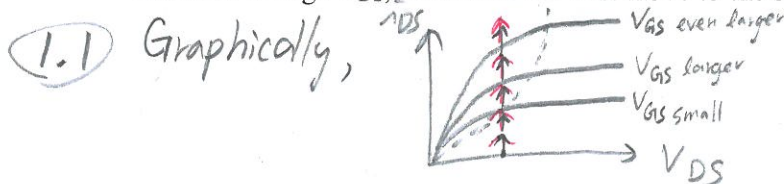
100 points total. Due on 9AM, Monday, 6/8/2020.

Submit your answer via Moodle. Clearly state your analysis to earn full score.

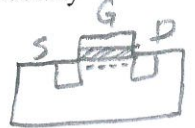
Please tweak your image files to help save some ink! For example, grey-out the background color. Thanks!  
請影像處理您的作業照片以節省列印墨水 (例如把背景顏色調淡)~ 謝謝!

We assume all MOSFETs have  $V_T=1$  V,  $K=1$  mA/V<sup>2</sup>, and we use the SCS model.

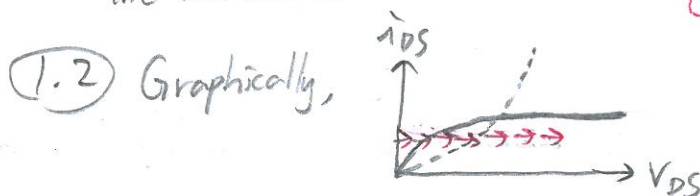
- 1 Explain the following concepts in your own words. You may use figures to illustrate your points.
  - 1.1 (10 points) Suppose that the MOSFET is in the cutoff region. If we fix voltage  $V_{DS}$  (and suppose that  $V_{DS} > 0$ ) and gradually increase voltage  $V_{GS}$ , the MOSFET will move to the other two regions in the following order:  $\rightarrow$  saturation  $\rightarrow$  triode.
  - 1.2 (10 points) Suppose that the MOSFET is in the triode region. If we fix voltage  $V_{GS}$  and gradually increase voltage  $V_{DS}$ , the MOSFET will move to the saturation in the end.



You may also use to explain.



The condition to enter the triode region is  $V_{GS} - V_T \geq V_{DS}$ , which implies the transition.



$\rightarrow$  it's fine to ~~not~~ ignore the "=" sign, because in practice we would not operate a MOSFET at the boundary of regions. The "=" sign is for the purpose of illustration only.



since the channel between S and D became stable.

(As a further study, think of what would happen if  $V_{DS} \gg V_{GS}$ .)

- 2 Consider the MOSFET circuit in Figure 1 and answer the following questions.

- 2.1 (10 points) Suppose  $V_{in}=3$  V. Then  $i_{DS} = ?$
- 2.2 (10 points) Suppose  $V_{in}=3$  V,  $V_{out}=5$  V, and  $R_L=10$  k $\Omega$ . Then  $V_S = ?$

2.1  $V_{GS} = V_{in}$ , and since  $V_{GS} > V_T$  we see that the MOSFET operates in either the saturation region or the triode region (depending on  $V_{DS}$ ). From the  $i_{DS} - V_{DS}$  relation we see that  $i_{DS} \leq \frac{K(V_{GS} - V_T)^2}{2} = 2$  mA

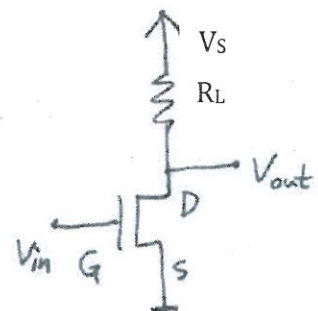


Figure 1

2.2 According to condition  $V_{GS} - V_T \leq V_{DS}$ ,  $V_{out} = 5$  V implies that the MOSFET is operating in the saturation region.

$i_D = \frac{V_S - V_{out}}{R_L}$  and  $i_D = \frac{K(V_{GS} - V_T)^2}{2}$  lead to a conclusion that  $V_S = 25$  V

3 Consider the MOSFET circuit in Figure 2.

3.1 (5 points) Suppose  $V_2 = 4\text{ V}$ . What would be the range of  $V_1$  for the MOSFET to operate in the saturation region?

3.2 (5 points) Suppose that the MOSFET is operating in the triode region. Could we make the MOSFET to move into the saturation region by just increasing  $V_2$ ? If your answer is yes, give the range of  $V_2$ .

3.1  $V_1$  is equal to  $V_{GS}$ .  $V_2$  is equal to  $V_{DS}$ .

$$\begin{cases} V_{GS} - V_T \geq 0 \\ V_{GS} - V_T \leq V_{DS} \end{cases} \Rightarrow 1\text{ V} \leq V_1 \leq 5\text{ V}$$

(Again, the equality sign

does not matter 無傷大雅)

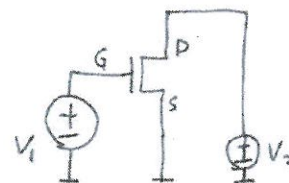
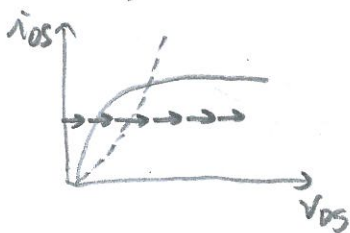


Figure 2

3.2 Yes, also see Question 1.2.



To move from the triode region to the saturation region, we need to have  $V_{DS} > V_{GS} - V_T$ , in other word,  $V_2 > V_1 - V_T$ .

4 (10 points) Consider the MOSFET amplifier in Figure 3. Assume the MOSFET is operating in the saturation region. Suppose  $V_S = 15\text{ V}$ ,  $R_L = 5\text{ k}\Omega$ , and  $V_{in} = 3\text{ V}$ . Compute the gain for this configuration.

the gain =  $\frac{V_{out}}{V_{in}}$  in this case.

$$\text{From } \begin{cases} i_{DS} = \frac{K(V_{in} - V_T)^2}{2} \\ i_{DS} = \frac{V_S - V_{out}}{R_L} \end{cases}$$

$$\text{we have } \frac{1 \times 10^{-3} (3 - 1)^2}{2} = \frac{15 - V_{out}}{5 \times 10^3}$$

$$\Rightarrow V_{out} = 15 - 10 = 5\text{ V}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{5}{3} *$$

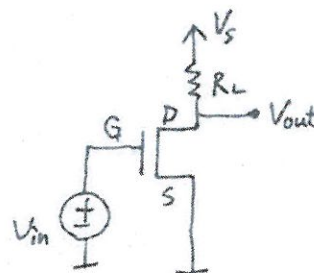


Figure 3



$$1 \quad 1 \quad -15$$

$$(9 - V_{in}) \leq 15 - (9 - V_{in})^2$$

$$\Rightarrow \frac{1 - \sqrt{1+60}}{2} \leq 9 - V_{in} \leq \frac{1 + \sqrt{61}}{2} \uparrow$$

$$9 - \left(\frac{-1 + \sqrt{61}}{2}\right) \leq V_{in} \leq 9 - \left(\frac{-1 - \sqrt{61}}{2}\right)$$

$$15 - \frac{1 \times 10^{-3} (10 - V_{in} - 1)^2}{2 \times 10^3} = 15 - (9 - V_{in})^2$$

5 Consider the MOSFET amplifier in Figure 4. Suppose that the MOSFET is in the saturation region, and  $V_S = 15 \text{ V}$ ,  $R_1 = 5 \text{ k}\Omega$ ,  $R_2 = 10 \text{ k}\Omega$ , and  $R_3 = 2 \text{ k}\Omega$ . (Note: If you've downloaded the earlier version of the lecture note, make sure you've made the following correction: the last equation on page 77 should have a square over the parenthesis.)

5.1 (5 points) If  $V_{in} = 8 \text{ V}$ . Find  $V_{out}$ .

5.2 (5 points) Find the minimum value of  $V_{in}$  for the MOSFET to stay in the saturation region.

5.3 (5 points) If we can change  $R_3$ , what is the maximum value of  $R_3$  for the MOSFET to stay in the saturation region?

5.4 (5 points) What is the maximum  $V_{in}$  that will make the gain larger than or equal to 1?

5.1  $V_{GS} = (\text{the branch voltage across } R_2) - V_{in}$

$$= \left(15 \times \frac{10}{5+10}\right) - V_{in} = 10 - V_{in}$$

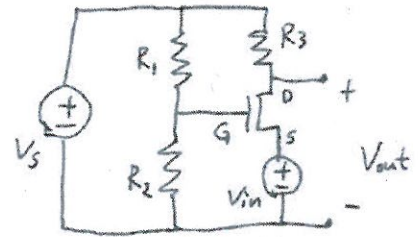


Figure 4

$$V_{out} = V_S - i_{DS} \cdot R_3 \quad (\text{from } i_{DS} = \frac{V_S - V_{out}}{R_3})$$

$$= V_S - \frac{K(V_{GS} - V_T)^2}{2} \cdot R_3 \Rightarrow V_{out} = 15 - (9 - V_{in})^2$$

$$V_{in} = 8 \text{ V} \Rightarrow V_{out} = 14 \quad \#$$

5.2  $\begin{cases} V_{GS} - V_T \geq 0 \\ V_{GS} - V_T \leq V_{DS} \end{cases} \Rightarrow \begin{cases} 10 - V_{in} - 1 \geq 0 \\ 10 - V_{in} - 1 \leq V_{out} - V_{in} = 15 - (9 - V_{in})^2 - V_{in} \end{cases} \Rightarrow \begin{cases} V_{in} \leq 9 \\ 5.59 \leq V_{in} \leq 11.45 \end{cases}$

$\Rightarrow \text{minimum } V_{in} \approx 5.59 \quad \#$

5.3 following 5.1, from  $V_{out} = V_S - \frac{K(V_{GS} - V_T)^2}{2} \cdot R_3$

$$\Rightarrow V_{DS} = V_{out} - V_{in} = 15 - \frac{1 \times 10^{-3} (9 - V_{in})^2}{2} \cdot R_3 - V_{in}$$

the condition is  $V_{DS} \geq V_{GS} - V_T$

$$\Rightarrow 15 - \frac{1}{2} \times 10^{-3} (9 - V_{in})^2 \cdot R_3 - V_{in} \geq (10 - V_{in}) - 1$$

$$\text{if } V_{in} = 8 \text{ V} \Rightarrow 15 - \frac{1}{2} \times 10^{-3} \cdot R_3 \geq 9 \Rightarrow R_3 \leq 12 \text{ k}\Omega \quad \#$$

5.4  $V_{out} = f(V_{in}) = 15 - (9 - V_{in})^2 \Rightarrow \frac{V_{out}}{V_{in}} \geq 1 \Rightarrow \frac{15 - (9 - V_{in})^2}{V_{in}} \geq 1 \Rightarrow 6 \leq V_{in} \leq 11$

$f'(V_{in}) = 2(9 - V_{in})$

$f'(V_{in}) \geq 1 \Rightarrow 2(9 - V_{in}) \geq 1 \Rightarrow V_{in} \leq 8.5 \text{ V} \quad \#$

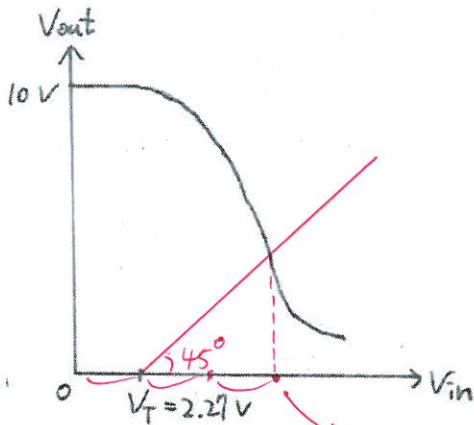
~~gain = 切线斜率~~  
~~= - 大微分~~

BUT if  $V_{in} > 9$   
the MOSFET will enter the cut off region, and  
 $i_{DS} = 0$ ,  $V_{out} = V_S = 15$

Therefore, for  $\frac{V_{out}}{V_{in}} \geq 1$ , the maximum  $V_{in} = 15 \text{ V} \quad \#$

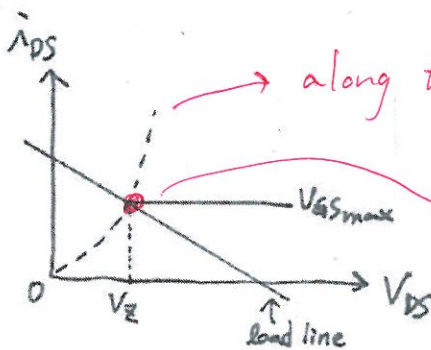


- 7 Apply the graphical analysis for the following two questions. Consider the MOSFET amplifier in Figure 5.
- 7.1 (5 points) Suppose the following plot of  $V_{out}$ - $V_{in}$  relation is accurate. To estimate the maximum  $V_{in}$  for the MOSFET to stay in the saturation region, which one of the following three estimation is better than the other two, and why? (A) 4.2 V; (B) 6.8; (C) 8.1.



around  $3 \times 2.27 \approx 6.8 \text{ V}$  #

- 7.2 (5 points) Following Question 7.1, suppose  $V_{GSmax}$  in the following plot is our estimation of the maximum  $V_{in}$ . Determine the value of  $V_Z$ .



along this dotted curve we have  $V_{DS} = V_{GS} - V_T$   
(page 70 in the note)

at this intersection,

$$V_Z = V_{DS} = V_{GSmax} - V_T$$

$$\approx 6.8 - 2.27 = 4.53 \text{ V}$$

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