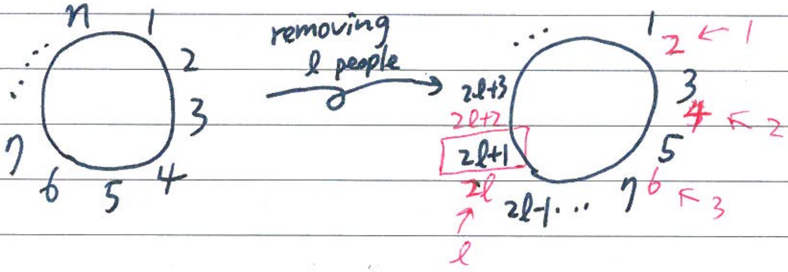
A supplementary note for ~~the~~ Section 1.3:

① An alternative approach to see $J(n) = 2l+1$:
In class, we've shown that $J(2^m) = 1$
and here's the remaining part.
Since we defined $n = 2^m + l$, after removing l people we have $n' = 2^m$, and therefore $J(n') = 1$. The last step we need is to find the mapping of people's ID for the n'-people case back to people's ID for the n-people case.



A needed lemma: $l < \frac{n}{2}$

Proof: $2^m \le n < 2^{m+1}$

$\Rightarrow l = n - 2^m < 2^{m+1} - 2^m = 2^m = n - l$

$\Rightarrow 2l < n$

$\Rightarrow l < \frac{n}{2}$ ※

$(n = 2^m + l$ and $0 \le l < 2^m)$

② Another way to make sense that

$$\begin{cases} A(n) = 2^m \\ B(n) = 2^m - 1 - l \\ C(n) = l \end{cases}$$

for $f(n) = A(n)\alpha + B(n)\beta + C(n)\gamma$ to be a closed-form solution of the recurrence

$$\begin{cases} f(1) = \alpha \\ f(2n) = 2f(n) + \beta & \text{for } n \ge 1 \\ f(2n+1) = 2f(n) + \gamma & \text{for } n \ge 1 \end{cases}$$ is as follows:

$$f(n) = \underbrace{2(2(2 \cdots}_{=2^m} f(1) + (\beta \text{ or } \gamma)) + (\beta \text{ or } \gamma)) + \cdots)$$

and since $f(1) = \alpha \Rightarrow A(n) = 2^m$

Now, consider in terms of shifting bits to the left.

If n is even $\Rightarrow (f(n))_2$ is equal to "$(f(\frac{n}{2}))_2$ appends a 0"

If n is odd $\Rightarrow (f(n))_2$ is equal to "$(f(\lfloor \frac{n}{2} \rfloor))_2$ appends a 1"

$n = (1\, b_{m-1} b_{m-2} \dots b_1 b_0)_2$

$l = (b_{m-1} b_{m-2} \dots b_1 b_0)_2 \longrightarrow$ each 1 in position b_n implies $2^n \cdot \gamma$

Therefore, $C(n) = l$

Finally, since $B(n) + C(n) = 2^m - 1$, we have $B(n) = 2^m - 1 - l$

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