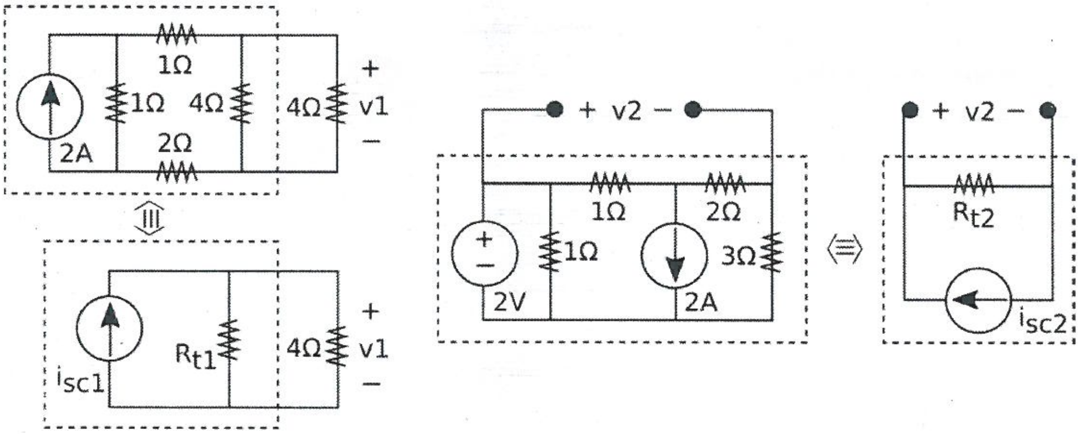
Solution attached

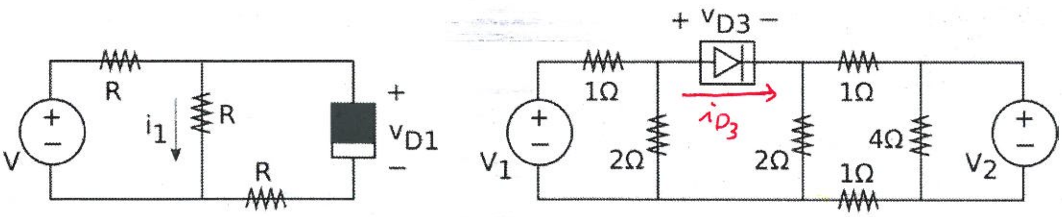
CSU0007 Basic Electronics, Homework 3

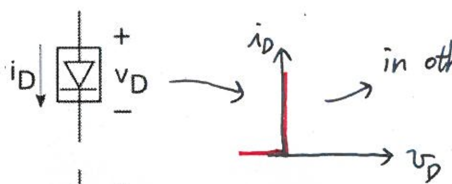
- **IMPORTANT!!** Submit your work via Moodle **before 9AM, Nov 10th, 2020.** We will have a review session on Nov 10th in class. The midterm exam will be on Nov 13th in class.
- Four questions in total. Please clearly label your answer for each question, and clearly state your calculation steps.

1. (42 points) Use Norton's Theorem to find $\{i_{sc1}, R_{t1}, v_1\}$ and $\{i_{sc2}, R_{t2}, v_2\}$. 7 points for each variable.



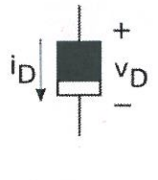
2. (40 points) For the following nonlinear circuits,
   1. (20 points) use the analytical method to determine v_{D_1} and i_1 (the method is covered in class and is described in Section 4.2 in the textbook);
   2. (20 points) if $0 < 2V_2 < V_1$, what would be the value of i_{D3}? Use the piecewise analysis method.





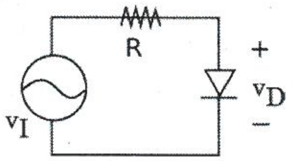
in other word, $v_D \geq 0 \Rightarrow$ short circuit

$v_D < 0 \Rightarrow$ open circuit



$$i_D = \begin{cases} K \cdot v_D^2 & \text{for } v_D > 0 \\ 0 & \text{otherwise} \end{cases}$$

3. (10 points) Use the graphical analysis method to explain the following property: Let $v_{I1} > 0$ and $v_{I2} > 0$, and such that v_{I1} caused v_{D1} and v_{I2} caused v_{D2}. Then we have $\Delta v_I > \Delta v_D$ where $\Delta v_I = |v_{I1} - v_{I2}|$ and $\Delta v_D = |v_{D1} - v_{D2}|$. Illustrate and use your own word to explain why $\Delta v_I > \Delta v_D$.



4. (8 points) In class we've talked about an approach to determine R_{TH} in Thevenin's Theorem by using a potentiometer (i.e., a variable resistor 可變電阻). Now, with the additional help of Norton's Theorem, we may determine R_{TH} without using any potentiometer. Think about it and describe an approach to determine R_{TH} by only using a multimeter (i.e., a volt-ohm-milliammeter 三用電表).

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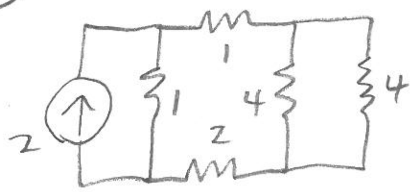
PART I



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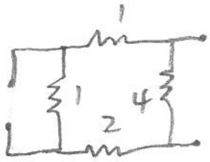
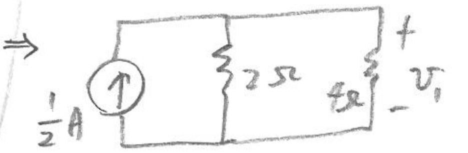


$R_{t_1} = \frac{4 \times (1+1+2)}{4+(1+1+2)} = 2\,\Omega$ ※

from

$i_{sc1} = 2 \times \frac{1}{1+(1+2)} = \frac{1}{2}\,A$ ※

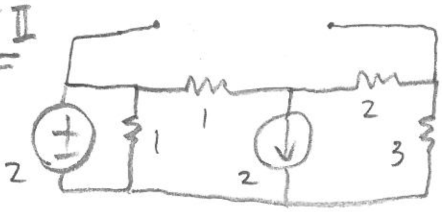
$\Rightarrow$



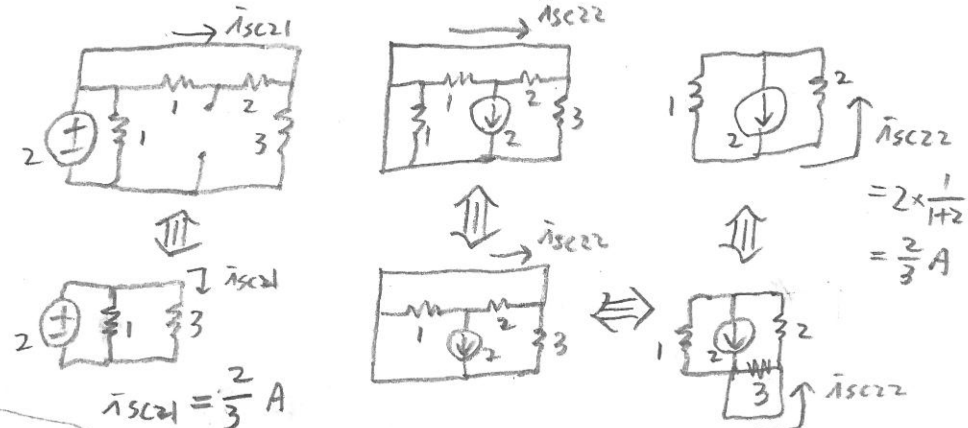
$v_1 = \left(\frac{1}{2} \times \frac{2}{4+2}\right) \times 4$

$= \frac{2}{3}\,V$ ※

PART II

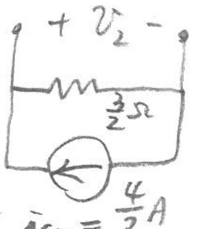


$i_{sc2} = i_{sc21} + i_{sc22}$ by superposition



$i_{sc22} = 2 \times \frac{1}{1+2} = \frac{2}{3}\,A$

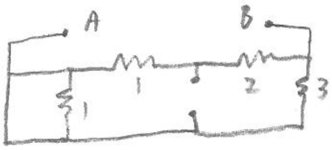
$i_{sc21} = \frac{2}{3}\,A$



$v_2 = \frac{4}{3} \times \frac{3}{2} = 2\,V$ ※

$i_{sc2} = \frac{4}{3}\,A$

from



There are at least two ways to analyze this:

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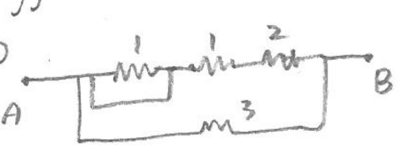

short circuit has resistance = 0

$\Rightarrow R_{t_2} = \frac{3 \times \left(\frac{1 \times 0}{1+0} + 1 + 2\right)}{3 + \left(\frac{1 \times 0}{1+0} + 1 + 2\right)} = \frac{3}{2}\,\Omega$ ※

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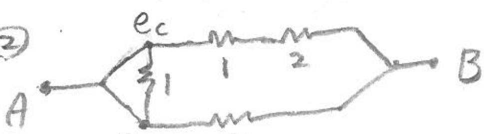
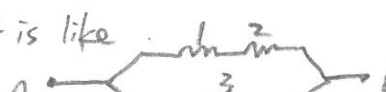


since node voltage $e_c = e_d$

there will be no current flowing through 1 if we attached A, B with a voltage source.

$\rightarrow$ so, equivalently, it is like



$R_{t_2} = \frac{3}{2}\,\Omega$

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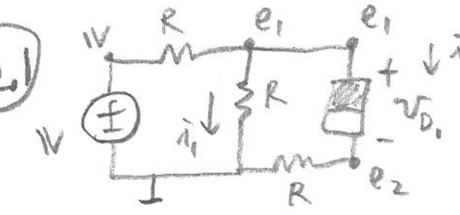
2.1



$$\begin{cases} \frac{1V - e_1}{R} = \frac{e_1}{R} + i_{D_1} \\ e_1 - e_2 = v_{D_1} \\ i_{D_1} = \frac{e_2}{R} \\ i_{D_1} = 1K\, v_{D_1}^2 \end{cases} \Rightarrow \begin{cases} 1V - 2e_1 = R \cdot 1K \cdot v_{D_1}^2 \\ e_1 = v_{D_1} + e_2 \\ e_2 = R\, 1K \cdot v_{D_1}^2 \end{cases}$$

$$\Rightarrow 3R\,1K\, v_{D_1}^2 + 2v_{D_1} - 1V = 0$$

$$\Rightarrow v_{D_1} = \frac{-2 + \sqrt{4 + 12\,1K\,1V\,R}}{6R\,1K} \quad \#$$

equation 1

first of all, observe that v_{D_1} must be larger than zero.

$$i_1 = \frac{e_1}{R} = \frac{v_{D_1} + e_2}{R} = \frac{v_{D_1} + R\,1K\, v_{D_1}^2}{R}$$

$$= \frac{-2 + \sqrt{4 + 12\,1K\,1V\,R}}{6\,1K\,R^2} + 1K\left(\frac{-2 + \sqrt{4 + 12\,1K\,1V\,R}}{6R\,1K}\right)^2$$

$$= \frac{-2 + \sqrt{4 + 12\,1K\,1V\,R}}{6\,1K\,R^2} + \frac{4 + (4 + 12\,1K\,1V\,R) - 4\sqrt{4 + 12\,1K\,1V\,R}}{36\,1K\,R^2}$$

$$= -\frac{1}{3}\frac{1}{1K\,R^2} + \frac{2}{9}\frac{1}{1K\,R^2} + \frac{1V}{3R} + \frac{1}{18}\sqrt{4 + 12\,1K\,1V\,R}$$

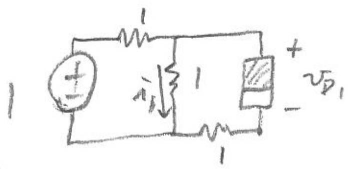
$$\Rightarrow i_1 = -\frac{1}{9}\frac{1}{1K\,R^2} + \frac{1V}{3R} + \frac{1}{18}\sqrt{4 + 12\,1K\,1V\,R} \quad \#$$

$\rightarrow$ equation 2

A way to verify our derivation is to plug in some good numbers and analyze the circuit with those numbers.

For example, let $1K = 1\ \mathrm{mA/V^2}$, $1V = 1\ \mathrm{V}$, $R = 1\ \mathrm{k\Omega}$ we have $v_{D_1} = \frac{-2 + \sqrt{4 + 12}}{6} = \frac{1}{3}\ \mathrm{V}$

$$\Rightarrow i_{D_1} = \frac{1}{9}\ \mathrm{mA}$$



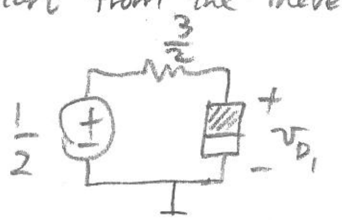
$$i_1 = \left(\frac{1}{3} + \left(\frac{1}{9} \times 1\mathrm{k\Omega}\right)\right) / 1\mathrm{k\Omega} = \frac{4}{9}\ \mathrm{mA}$$

and $i_1 = -\frac{1}{9}\left(\frac{1}{1 \times 1^2}\right) + \frac{1}{3 \cdot 1} + \frac{1}{18}\sqrt{4 + 12 \cdot 1 \cdot 1 \cdot 1}$

$$= -\frac{1}{9} + \frac{1}{3} + \frac{2}{9} = \frac{4}{9}\ \mathrm{mA}$$

which gives us some assurance that equation 2 is the correct result given that equation 1 is correct.

To see that equation 1 is correct (and for the sake of practice), we can start from the Thévenin's equivalence:



and use the node analysis

$$\begin{cases} \frac{3}{2} i_D + v_{D_1} = \frac{1}{2} \\ i_D = v_{D_1}^2 \end{cases} \Rightarrow v_{D_1} = \frac{1}{3}\ \mathrm{V},$$ which is the same as we plugged those numbers into equation 1.

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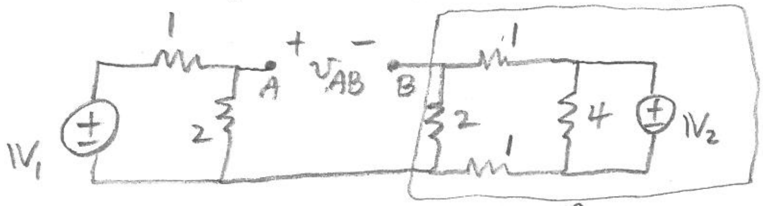
(2.2) First of all, we need to determine

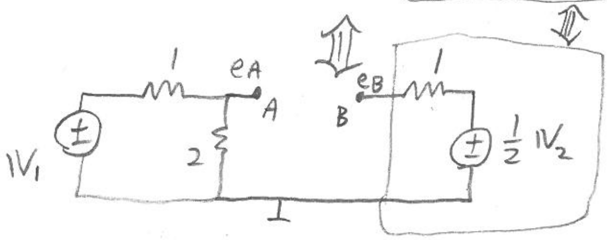
whether we can replace by

or we can replace by

We analyze this by first taking away :



→ we may use Thévenin Theorem to simplify this part.



Then we may use either the node analysis method or superposition to obtain that $v_{AB} = \frac{2}{3}V_1 - \frac{1}{2}V_2$.

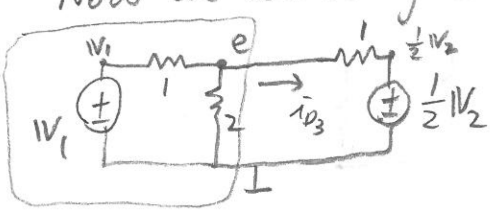
Given condition $0 < \underbrace{2V_2 < V_1}$, we have $v_{AB} = \frac{2}{3}V_1 - \frac{1}{2}V_2$

$\Rightarrow V_1 > 2V_2$ $\qquad > \frac{4}{3}V_2 - \frac{1}{2}V_2 > 0$

Thus, we now see we should consider as

because the i-v characteristic of the diode told us so.

Now we are ready to compute i_{D_3}!



$$\begin{cases} \frac{V_1 - e}{1} = \frac{e-0}{2} + \frac{e - \frac{1}{2}V_2}{1} \\ i_{D_3} = \frac{e - \frac{1}{2}V_2}{1} \end{cases}$$

$$\Rightarrow \begin{cases} 2V_1 - 2e = e + 2e - V_2 \\ i_{D_3} = e - \frac{1}{2}V_2 \end{cases}$$

$$\Rightarrow \begin{cases} e = \frac{1}{5}(2V_1 + V_2) \\ i_{D_3} = \frac{2}{5}V_1 - \frac{3}{10}V_2 \end{cases}$$

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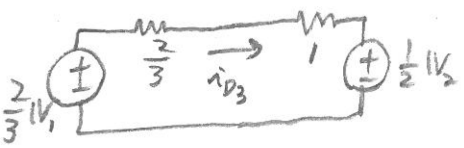
alternatively, you may transform the circuit further:



$$i_{D_3} = \frac{\frac{2}{3}V_1 - \frac{1}{2}V_2}{\frac{2}{3}+1} = \frac{2}{5}V_1 - \frac{3}{10}V_2$$

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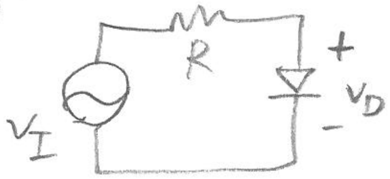
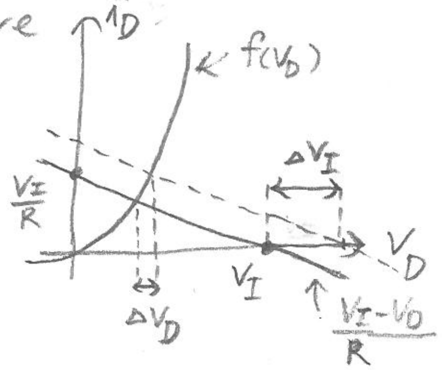


let $f(V_D)$ be the branch current flowing through (i.e., i_D)

By KCL we have $\frac{V_I - V_D}{R} = f(V_D)$

Therefore we have



$\Delta V_I > \Delta V_D$

because the tangent slope of $f(V_D)$ for all $V_D > 0$ is larger than zero.

4 Refer to page 33 of the lecture note.

Because $R_{TH} = \frac{V_{open\text{-}circuit}}{I_{short\text{-}circuit}}$, it is sufficient to just measure $V_{open\text{-}circuit}$ and $I_{short\text{-}circuit}$:)