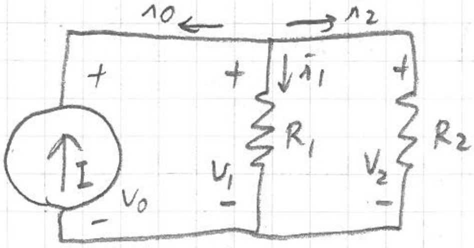
P16

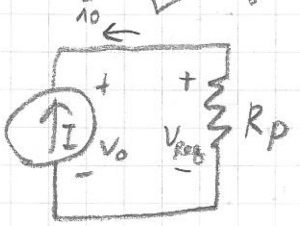
✱ Current Divider



Using the "basic method"

$$\begin{cases} i_0 = -I \\ v_1 = R_1 i_1 \\ v_2 = R_2 i_2 \end{cases} \qquad \begin{cases} i_0 + i_1 + i_2 = 0 \\ v_0 = v_1 = v_2 \end{cases}$$

$\Downarrow$ equivalent



$$\Rightarrow i_1 + i_2 = I$$

$$i_1 + i_2 = \frac{v_1}{R_1} + \frac{v_2}{R_2} = \frac{v_0 R_2 + v_0 R_1}{R_1 R_2}$$

$$= \frac{R_1 + R_2}{R_1 R_2} v_0$$

$$\Rightarrow v_0 = \frac{R_1 R_2}{R_1 + R_2} \cdot I$$

$$v_1 = R_1 i_1 = \frac{R_1 R_2}{R_1 + R_2} I$$

$$\shortparallel$$

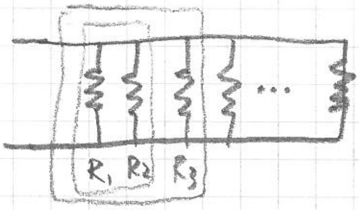
$$v_0 = R_{eq} \cdot I$$

$$\Rightarrow \boxed{R_p = \frac{R_1 R_2}{R_1 + R_2}}$$

$$\Rightarrow \boxed{\begin{aligned} i_1 &= \frac{v_1}{R_1} = \frac{R_2}{R_1 + R_2} I \\ i_2 &= \frac{v_2}{R_2} = \frac{R_1}{R_1 + R_2} I \end{aligned}}$$

$$\Rightarrow \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

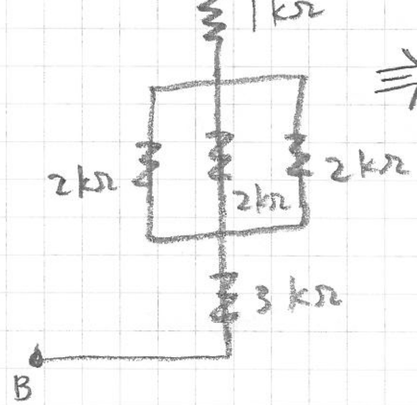
In general, for N resistors connected in parallel,



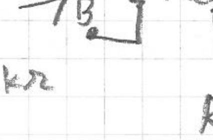
then

$$\frac{1}{R_p} = \sum_{n=1}^{N} \frac{1}{R_n}$$

this can be proved by induction.



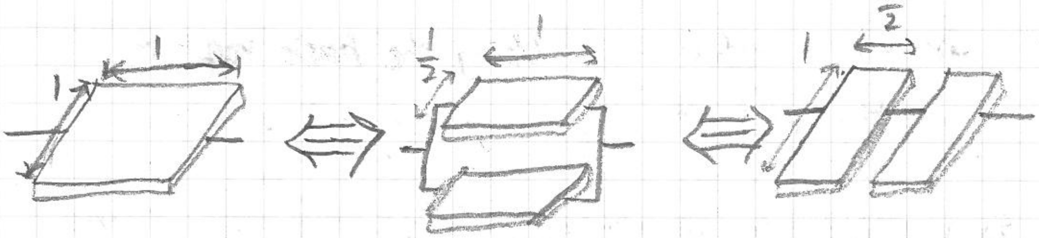
$\Rightarrow$



$$R_{eq} = 1 + \frac{1}{\frac{1}{2} + \frac{1}{2} + \frac{1}{2}} + 3$$

$$= \frac{14}{3} \ \Omega$$

※

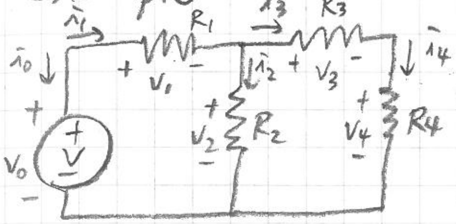


let $R_1 = R_\square$

$$R_2 = \frac{2R_\square \cdot 2R_\square}{2R_\square + 2R_\square} = R_\square = R_1$$

$$R_3 = \frac{1}{2}R_\square + \frac{1}{2}R_\square = R_\square = R_1$$

Example:



Determine v_1, v_2, v_3, v_4 and i_1, i_2, i_3, i_4.

Way ①: we may use the basic method

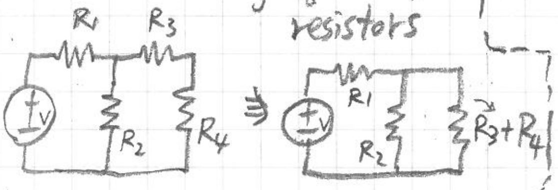
2° element law ...

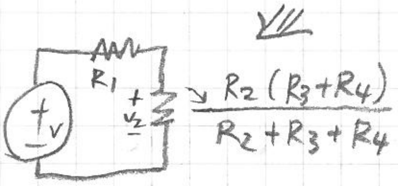
3° KCL & KVL ...

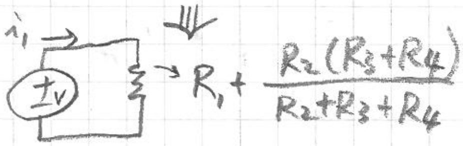
4° 解聯立方程式

give it a try yourself here:

Way ②: transformation using equivalent resistors







$$i_1 = V \Big/ \left(R_1 + \frac{R_2(R_3+R_4)}{R_2+R_3+R_4}\right)$$

$v_1 = i_1 R_1 =$ ___

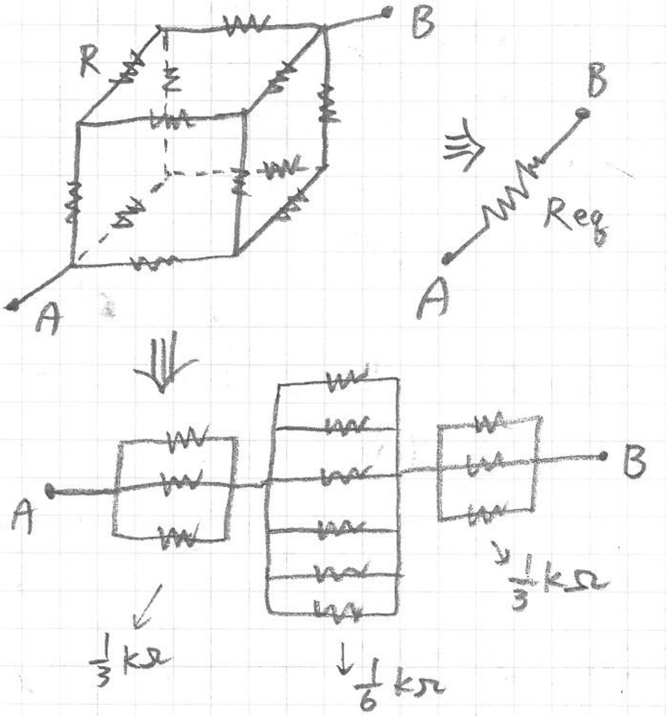
$v_2 =$ ___ (voltage divider)

$i_2 = v_2 / R_2$, $i_3 = v_2 / (R_3+R_4)$

$v_3 = i_3 R_3$, $v_4 = i_3 R_4$

Sometime we may leverage symmetry to greatly reduce the complexity of analysis:

Example: assuming all resistors are the same on a cube, with resistance $R = 1\,k\Omega$, determine R_{eq}

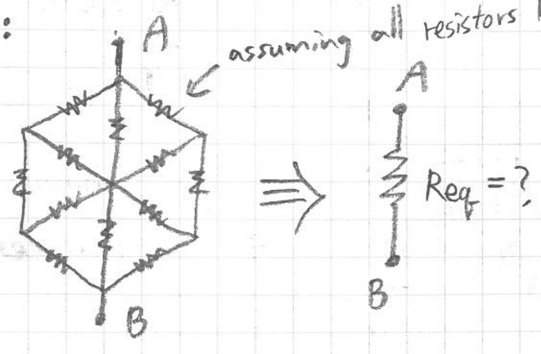


(see example 2.24 in the textbook)

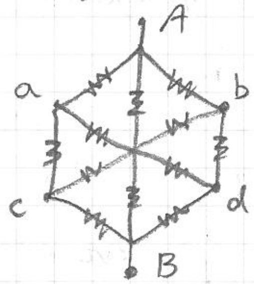
$$R_{eq} = \frac{1}{3} + \frac{1}{6} + \frac{1}{3} = \frac{5}{6}\,k\Omega \quad *$$

Often, we still need to apply the basic method after reducing a circuit by using equivalent resistors and symmetry !!

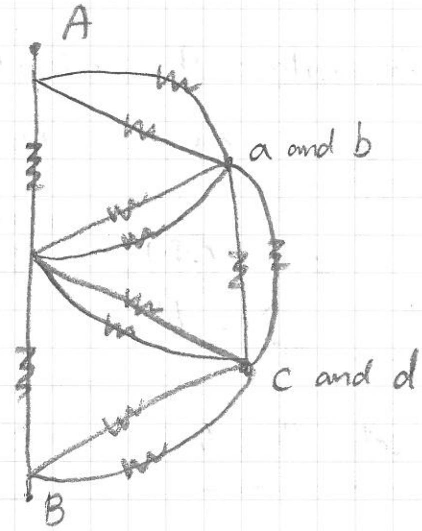
Example: assuming all resistors have 1Ω resistance

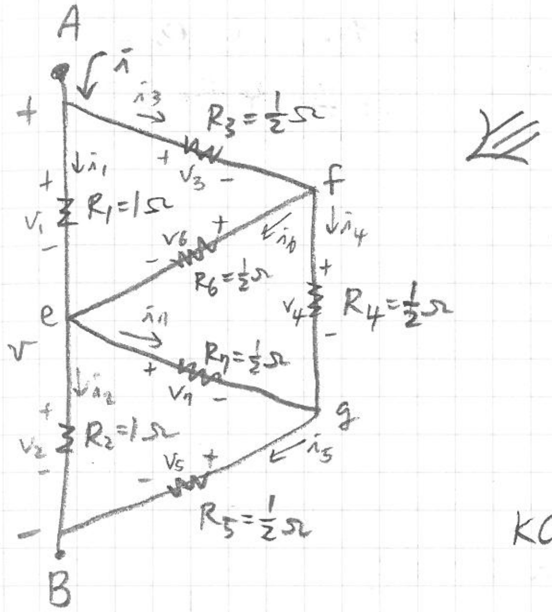


following P18,



$\Rightarrow$





element laws:

$$\begin{cases} v_1 = i_1,\ v_2 = i_2,\ v_3 = \frac{1}{2} i_3 \\ v_4 = \frac{1}{2} i_4,\ v_5 = \frac{1}{2} i_5,\ v_6 = \frac{1}{2} i_6 \\ v_7 = \frac{1}{2} i_7 \end{cases}$$

KCL:

$$\begin{cases} ① & i = i_1 + i_3 = i_2 + i_5 & \text{(node A \& B)} \\ ② & i_1 + i_6 = i_2 + i_7 & \text{(node e)} \\ ③ & i_3 = i_6 + i_4 & \text{(node f)} \\ ④ & i_5 = i_7 + i_4 & \text{(node g)} \end{cases}$$

KVL:

$$\begin{cases} ⑤ & v_1 + v_2 = v_3 + v_4 + v_5 \\ ⑥ & v_1 = v_3 + v_6 \\ ⑦ & v_2 = v_7 + v_5 \end{cases}$$

$$Req = \frac{v}{i} = \frac{v_1 + v_2}{i_1 + i_3} = \frac{i_1 + i_2}{i_1 + i_3}$$

therefore，我們可將所有電壓值代換為電流值，去解電流的聯立方程式!!

$$\begin{array}{c} ① \\ ② \\ ③ \\ ④ \\ ⑤ \\ ⑥ \\ ⑦ \end{array} \begin{bmatrix} 1 & -1 & 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & -1 \\ 2 & 2 & -1 & -1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

note: equation ④ is dependent,

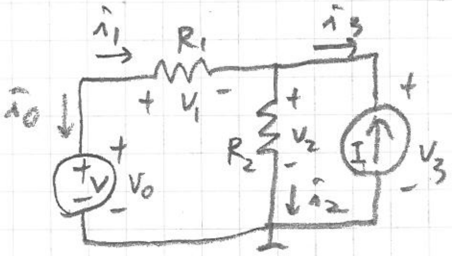
使用高斯消去法 可得 $\begin{cases} i_1 = 2 i_7 & i_3 = 3 i_7 \\ i_2 = 2 i_7 \end{cases}$

or www.mathtools.com Matrix Calculator

$$\therefore Req = \frac{i_1 + i_2}{i_1 + i_3} = \frac{2+2}{2+3} = \frac{4}{5} \Omega \ ※$$

Calculation is a necessary part in engineering!

P_{20} Example : A circuit with two independent source



Determine i_2.

element law $\begin{cases} v_0 = V \\ v_1 = R_1 i_1 \\ v_2 = R_2 i_2 \\ i_3 = -I \end{cases}$

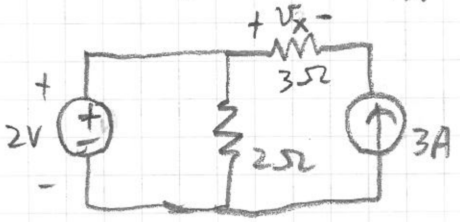
KCL $\begin{cases} i_0 = -i_1 \\ i_1 = i_2 + i_3 \end{cases}$

KVL $\begin{cases} v_0 = v_1 + v_2 \\ v_2 = v_3 \end{cases}$

then solve these linear equations (see textbook P95).

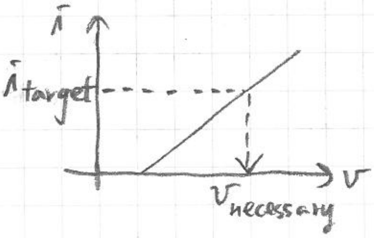
$$i_2 = \frac{R_1}{R_1+R_2} I + \frac{1}{R_1+R_2} V \quad ※$$

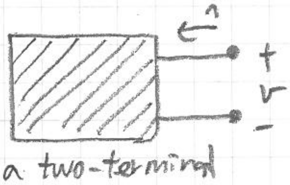
Exercise: $v_x = ?$



(ans: $v_x = -11V$)

☆☆ The I-V characteristic of a circuit

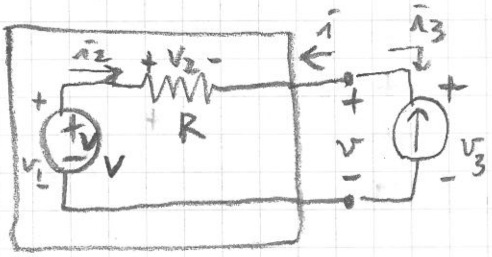




a two-terminal device

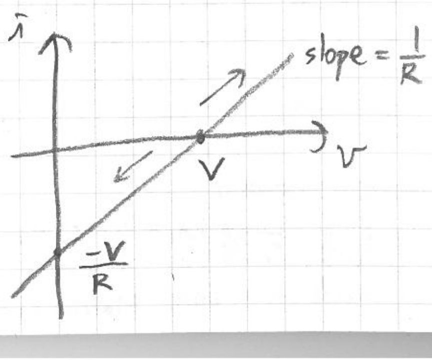
我們可藉由給定v量測i (or vice versa) 來繪製 I-V relation.

If we know the device's internals, we may also determine its I-V relation:



using the basic method, we have

$$v = V + iR \Rightarrow i = \frac{1}{R} v - \frac{V}{R}$$



example usage:

預測电流流向

$\begin{cases} v \geq V \rightarrow i \geq 0 \\ v < V \rightarrow i < 0 \end{cases}$ ⇐