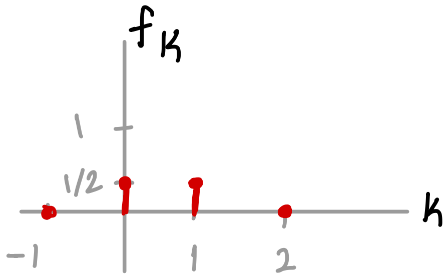
2

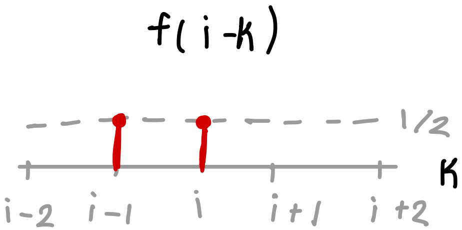
$f = (f_i)_{i \in \mathbb{Z}}$

$$f_i = \begin{cases} \frac{1}{2} & i \in \{0, 1\}, \\ 0 & \text{otherwise} \end{cases}$$

$$(f * f)_i = \sum_{k \in \mathbb{Z}} f_k f_{(i-k)}$$



$k \in \{0, 1\} \rightarrow \frac{1}{2}$



$k \in \{i-1, i\} \rightarrow \frac{1}{2}$

$k = 0 \rightarrow i = 1, \ i = 0$

$k = 1 \rightarrow i = 2, \ i = 1$

$$\sum_{k=i-1}^{i} f_k \cdot f(i-k) \qquad k \in \{i-1, i\}$$

$$\hookrightarrow f_{(i-1)} \cdot f_1 + f_i \cdot f_0$$

$i=0 \qquad f_{-1} \cdot f_1 + f_0 \cdot f_0 = 0 + \frac{1}{4} = \frac{1}{4}$

$i=1 \qquad f_0 \cdot f_1 + f_1 \cdot f_0 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$i=2 \qquad f_1 \cdot f_1 + f_2 \cdot f_0 = \frac{1}{4} + 0 = \frac{1}{4}$

$$(f * f)_i = \left[\dots 0, \frac{1}{4}, \frac{1}{2}, \frac{1}{4}, 0 \dots\right]$$

$$(f * f)_i = \begin{cases} 1/4 & i \in \{0, 2\}, \\ 1/2 & i = 1, \\ 0 & \text{otherwise} \end{cases}$$

$$\hookrightarrow y_i$$

$$(y * f)_i = \sum_{k \in \mathbb{Z}} f_k \cdot y_{(i-k)}$$

$$k \in \{0, 1\}$$

$$i-k \in \{0, 1, 2\}$$

$$k \in \{i, i-1, i-2\}$$

$$(y * f)_i = f_i \cdot y_0 + f_{i-1} \cdot y_1 + f_{i-2} \cdot y_2$$

$$i=0 \rightarrow f_0 \cdot y_0 + \cancel{f_{-1} \cdot y_1} + \cancel{f_{-2} \cdot y_2} = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$$

$$i=1 \rightarrow f_1 \cdot y_0 + f_0 \cdot y_1 + \cancel{f_{-1} \cdot y_2} = \frac{1}{2} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{2}$$

$$= \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$$

(non-zero results from now on)

$$i=2 \rightarrow f_1 \cdot y_1 + f_0 \cdot y_2 = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{4} = \frac{3}{8}$$

$$i=3 \rightarrow f_1 \cdot y_2 = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$$

$$(f * f * f)_i = \begin{cases} 1/8 & i \in \{0, 3\}, \\ 3/8 & i \in \{1, 2\}, \\ 0 & \text{otherwise} \end{cases}$$

$\hookrightarrow z_i$

$$(f * z)_i = \sum_{k \in \mathbb{Z}} f_k \, z_{(i-k)}$$

$i - k \in \{0, 1, 2, 3\}$

$k \in \{i, i-1, i-2, i-3\}$

$$(f * z)_i = f_i z_0 + f_{i-1} z_1 + f_{i-2} z_2 + f_{i-3} z_3$$

$i = 0 \rightarrow f_0 z_0 = \frac{1}{2} \cdot \frac{1}{8} = \frac{1}{16}$

$i = 1 \rightarrow f_1 z_0 + f_0 z_1 = \frac{1}{2} \cdot \frac{1}{8} + \frac{1}{2} \cdot \frac{3}{8} = \frac{4}{16} = \frac{1}{4}$

$i = 2 \rightarrow f_1 z_1 + f_0 z_2 = \frac{1}{2} \cdot \frac{3}{8} + \frac{1}{2} \cdot \frac{3}{8} = \frac{6}{16} = \frac{3}{8}$

$i = 3 \rightarrow f_1 z_2 + f_0 z_3 = \frac{1}{2} \cdot \frac{3}{8} + \frac{1}{2} \cdot \frac{1}{8} = \frac{1}{4}$

$i = 4 \rightarrow f_1 z_3 = \frac{1}{2} \cdot \frac{1}{8} = \frac{1}{16}$

$$(f * f * f * f)_i = \begin{cases} 1/16 & i \in \{0, 4\}, \\ 1/4 & i \in \{1, 3\}, \\ 3/8 & i = 2, \\ 0 & \text{otherwise} \end{cases}$$