Problem 3:

$$\hat{g}_p = \frac{1}{\sqrt{2M}} \sum_{m=0}^{2M-1} g_m \exp\left\{-i2\pi p \frac{m}{2M}\right\}$$

$$= \frac{1}{\sqrt{2M}} \sum_{m=0}^{M-1} f_m \exp\left\{-i2\pi p \frac{m}{2M}\right\} + \frac{1}{\sqrt{2M}} \sum_{m=M}^{2M-1} f_{2M-1-m} \exp\left\{-i2\pi p \frac{m}{2M}\right\}$$

$$\hat{g}_{p-\frac{1}{2}} = e^{-i2\pi p \frac{1}{2}} \hat{g}_p$$

$$= \frac{1}{\sqrt{2M}} \left[\sum_{m=0}^{M-1} f_m \exp\left\{-i2\pi p \frac{(m+\frac{1}{2})}{2M}\right\} + \sum_{m=M}^{2M-1} f_{2M-1-m} \exp\left\{-i2\pi p \frac{(m+\frac{1}{2})}{2M}\right\} \right]$$

let $\hat{h}_p = \sum_{m=M}^{2M-1} f_{2M-1-m} \exp\left\{-i2\pi p \frac{(m+\frac{1}{2})}{2M}\right\}$

$$\overset{n=2M-1-m}{=\!=\!=} \sum_{n=M-1}^{0} f_n \exp\left\{-i2\pi p \frac{(2M-n-\frac{1}{2})}{2M}\right\}$$

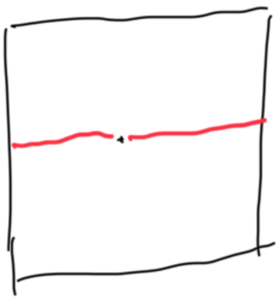
$$\overset{m=n}{=\!=\!=} \sum_{m=0}^{M-1} f_m \exp\left\{i2\pi p \frac{m+\frac{1}{2}}{2M}\right\}$$

Thus $\hat{g}_{p-\frac{1}{2}} = \frac{1}{\sqrt{2M}} \left[\sum_{m=0}^{M-1} f_m \exp\left\{-i2\pi p \frac{m+\frac{1}{2}}{2M}\right\} + \sum_{m=0}^{M-1} f_m \exp\left\{i2\pi p \frac{m+\frac{1}{2}}{2M}\right\} \right]$

$$= \frac{1}{\sqrt{2M}} \sum_{m=0}^{M-1} f_m 2\cos \frac{\pi(2m+1)p}{2M}$$

$$= \sqrt{\frac{2}{M}} \sum_{m=0}^{M-1} f_m \cos \frac{\pi(2m+1)p}{2M}$$

$$\widehat{DFT}(f_m)_p = \begin{cases} \hat{g}_{p-\frac{1}{2}}, & p > 0 \\ \frac{1}{\sqrt{2}} \hat{g}_{-\frac{1}{2}} & p = 0 \end{cases}$$

(a) 

We say that the width of the line is height in the code, which allows us to adapt the strength of the filter.

Cause the stripes in the image are vertical.
We chose a filter that is a line at the center of the image except the center point (256.256).
This filter maps any pixel it contains to zero.
The point (256.256) in the spatial domain is the average of the whole image. So it can not be set to zero.
The vertical stripes are points on the line, they should be removed.

(b) The line artefacts can not be totally removed.
Cause the image itself contains similar pattern to the artefacts. If the artefacts are fully removed, that means the image itself is noise.