Monte Carlo Simulation

October 19, 2018

In this homework we are going to build Monte Carlo based simulation for pricing options. In particular, we will need to price the following two options

1. The current market conditions are: the price of IBM is 152.35\$, the volatility σ is 0.01 per day, and $r = 0.0001$ per day. What is the price of a call option that ends 252 days into the future and the strike price is 165\$.
2. The current market conditions are: the price of IBM is 152.35\$, the volatility σ is 0.01 per day, and $r = 0.0001$ per day. What is the price of an asian call option that ends 252 days into the future and the strike price is 164\$. Note an asian call option will pay the maximum between zero and the average price during the 252 days minus the strike price.

This calculations needs to be done where the stopping criteria is with probability 96% the estimation error is less than 0.1\$.
In our class we discuss some interfaces that we are going to use to create our simulator. Here are few instructions and guidlines as you design your solution.

1. You will need to create random samples from standard Gaussian distribution for creating the stock path. You should use the Box-Muller algorithm to create these samples. See https://en.wikipedia.org/wiki/Box
2. Recall how to work the criteria for ending the simulation. As a first step you need to find the X such that a standard Gaussian distribution is, in our case, 96%. Of course you can not look at a table. However, the following formula can be used to create a "approximate" table and you can search in that "approximate" table for the constant. Assume that you need to solve the following equation: $Q(x_p) = p$ where $Q(\cdot)$ is the CDF of the standard Gaussian random variable. In this case, x_p is approximately: $x_p \approx t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_r t^2 + d_3 t^3}$
   $t = \sqrt{\ln \frac{1}{p^2}}$
   $c_0 = 2.515517$ $d_1 = 1.432788$
   $c_1 = .802853$ $d_2 = 0.189269$
   $c_2 = 0.010328$ $d_3 = 0.001308$
3. You should use in your simulation 252 days and use our interface for describing the data points. Assume the first day is 1/1/2019 and that Saturday/Sunday are regular days. Essentially, you can make 252 consecutive days.
4. You will need to implement for each of the option the payout interface.
5. Eventually you will have a main which will have the following steps. Configure a stock path generation, configure an option, pass them to an engine that will run experiments and average the results. The engine stop making experiments based on the stoping criteria. You will need to report the average. Don't worry about discounting to present value, the average is sufficient.
6. Last, but not least, from the lecture you saw that the stopping criteria depends on knowing σ. However, σ is unknown. You will need to warmup the simulation. You will run 10,000 times will used the estimate for σ from this 10,000 runs. You can use the stat collector to estimate σ from these data.

Please ask question, and if necessary, I will provide additional clarification as needed.