APPENDIX

Nomenclature

TABLE A1
LIST OF THE ACRONYMS IN THIS PAPER.

Symbol	Meaning
EPS	electric power system
NGS	natural gas system
IES	integrated energy system
IES-SR	security region of integrated energy system
EPS-SR	security region of electric power system
NGS-SR	security region of natural gas system
ECSR	security region of energy circuit
GT	Gas turbine generator/ Gas generator

TABLE A2
LIST OF THE VARIABLES IN THIS PAPER.

	Symbol	Meaning
Symbols of security region modeling	$\boldsymbol{\Omega}_{\mathrm{SR}}$	generalized symbols for security region
	$\boldsymbol{\Omega}_{\mathrm{EPS\text{-}ECSR}}$	ECSR of EPS
	$\boldsymbol{\Omega}_{\mathrm{NGS\text{-}ECSR}}$	ECSR of NGS
	$\boldsymbol{\Omega}_{\mathrm{IES\text{-}ECSR}}$	ECSR of IES
	$\partial\boldsymbol{\Omega}_{\mathrm{SR}}$	generalized symbols for security boundary
	G_{GTC}	total gas transmission at GTC point
	$\boldsymbol{W}$	operating point
	$\boldsymbol{W}_{\mathrm{s}}$	security operating point
	$\boldsymbol{W}_{\mathrm{b}}$	critical security operating point
	$S_{\mathrm{l},ei}$	power of power load node i
	$S_{\mathrm{l},cm}$	power required to drive compressor m
	$\boldsymbol{I}_{\mathrm{n}}$	vector of node current
	$\boldsymbol{I}_{\mathrm{n}}^{\max}$	vectors of the upper limits of node current
	$\boldsymbol{I}_{\mathrm{n}}^{\min}$	vectors of the lower limits of node current
	$\boldsymbol{I}_{\mathrm{b}}$	vector of branch current
	$\boldsymbol{I}_{\mathrm{b}}^{\max}$	vector of the current-carrying capacity
	$\boldsymbol{U}_{\mathrm{n}}$	vector of node voltage
	$\boldsymbol{U}_{\mathrm{n}}^{\max}$	vectors of the upper limits of node voltage
	$\boldsymbol{U}_{\mathrm{n}}^{\min}$	vectors of the lower limits of node voltage
	$\boldsymbol{K}_{\mathrm{e}}$	transformer ratio matrix
	$G_{\mathrm{l},gj}$	flow rate of natural gas load node j
	$G_{\mathrm{l,GT}n}$	natural gas input by the gas generator n
	$\boldsymbol{G}_{\mathrm{n}}$	vectors of node flow of natural gas
	$\boldsymbol{G}_{\mathrm{n}}^{\max}$	vector of the upper limits of the node gas flow
	$\boldsymbol{G}_{\mathrm{n}}^{\min}$	vector of the lower limits of the node gas flow
	$\boldsymbol{p}_{\mathrm{n}}$	vector of node pressure of natural gas
	$\boldsymbol{p}_{\mathrm{n}}^{\max}$	vector of the upper limits of node pressure
	$\boldsymbol{p}_{\mathrm{n}}^{\min}$	vector of the lower limits of node pressure
	$\boldsymbol{G}_{\mathrm{b}}^{\max}$	vector of pipe capacity
	$\boldsymbol{P}_{\mathrm{n}}$	vector of node pressure
	$\boldsymbol{P}_{\mathrm{n}}^{\max}$	vector of the upper limits of node pressure
	$\boldsymbol{P}_{\mathrm{n}}^{\min}$	vector of the lower limits of node pressure
	$\boldsymbol{F}_{\mathrm{n}}$	vector of energy flow
	$\boldsymbol{F}_{\mathrm{n}}^{\max}$	vector of the upper limits of node flow
	$\boldsymbol{F}_{\mathrm{n}}^{\min}$	vector of the lower limits of node flow
	$\boldsymbol{F}_{\mathrm{b}}^{\max}$	vector of upper limit of allowable flow of branch
	$\boldsymbol{K}$	generalized energy transformation ratio matrix
Symbols of network topology	$\boldsymbol{A}_{\mathrm{g}}$	node-branch incident matrix of NGS
	$\boldsymbol{A}_{\mathrm{g,L}}^{-1}$	Left inverse matrix of $\boldsymbol{A}_{\mathrm{g}}$
	$\boldsymbol{A}_{\mathrm{g+}}$	node-outflow branch incident matrix of NGS
	$\boldsymbol{A}_{\mathrm{g-}}$	node-inflow branch incident matrix of NGS
Symbols of natural gas and pipe parameter	R	constant of natural gas
	T	temperature of natural gas
	v_{b}	basic value of gas flow rate
	Z	compression factor of natural gas
	κ	adiabatic constant of natural gas
	V_{GH}	total calorific value of natural gas
	g	acceleration of gravity
	ρ	density of natural gas
	d_{g}	inner diameter of pipe
	s_{g}	cross-sectional area of pipe
	l_{g}	length of pipe
	λ_{g}	friction coefficient of pipe
	θ_{g}	inclination angle of pipe
Symbols of energy circuit	r_{g}	gas resistance (distributed parameter)
	L_{g}	gas inductance (distributed parameter)
	C_{g}	gas capacitance (distributed parameter)
	k_{g}	controlled pressure source (distributed parameter)
	Z_{gb}	branch impedance
	k_{gb}	controlled gas pressure source
	$Y_{\mathrm{gb},10}$	ground admittance
	$Y_{\mathrm{gb},20}$	ground admittance
	$\boldsymbol{y}_{\mathrm{gb}}$	diagonal matrices of branch admittance (concentrated parameter)
	$\boldsymbol{k}_{\mathrm{gb}}$	diagonal matrices of controlled gas pressure source (concentrated parameter)
	$\boldsymbol{Y}_{\mathrm{g}}$	generalized node admittance matrix of NGS
	$\boldsymbol{Y}_{\mathrm{e}}$	node admittance matrix of EPS
	$\boldsymbol{Y}_{\mathrm{IES}}$	generalized node admittance matrix of IES
Symbols of coupling unit	ψ	efficiency of compressor
	K_{g}	pressure ratio of compressor
	$\boldsymbol{K}_{\mathrm{g}}$	matrix of compressor pressure ratio
	a_{GT}	heat consumption coefficients of gas generator
	b_{GT}	heat consumption coefficients of gas generator
	c_{GT}	heat consumption coefficients of gas generator
	d_{GT}	heat consumption coefficients of gas generator
	e_{GT}	heat consumption coefficients of gas generator
	P_{GT}	active power output by the gas generator
	$P_{\mathrm{GT}}^{\min}$	lower limit of P_{GT}

TABLE A3
GIVEN QUANTITIES AND STATE VARIABLES OF DIFFERENT ENERGY SYSTEMS.

	Given quantities	State variables
Power system	$\boldsymbol{I}_{\mathrm{n}}^{\max}$, $\boldsymbol{I}_{\mathrm{n}}^{\min}$, $\boldsymbol{U}_{\mathrm{n}}^{\max}$, $\boldsymbol{U}_{\mathrm{n}}^{\min}$, $\boldsymbol{S}_{\mathrm{b}}^{\max}$, K_{e}	$\boldsymbol{W}$, $S_{\mathrm{l},ei}$, $S_{\mathrm{l},ci}$, $\boldsymbol{I}_{\mathrm{n}}$, $\boldsymbol{U}_{\mathrm{n}}$, $\boldsymbol{S}_{\mathrm{b}}$, P_{GT}
Natural gas system	$\boldsymbol{G}_{\mathrm{n}}^{\max}$, $\boldsymbol{G}_{\mathrm{n}}^{\min}$, $\boldsymbol{p}_{\mathrm{n}}^{\max}$, $\boldsymbol{p}_{\mathrm{n}}^{\min}$, $\boldsymbol{G}_{\mathrm{b}}^{\max}$, K, η	$\boldsymbol{W}$, $G_{\mathrm{l},gj}$, $G_{\mathrm{l,GT}j}$, $\boldsymbol{G}_{\mathrm{n}}$, $\boldsymbol{p}_{\mathrm{n}}$, $\boldsymbol{G}_{\mathrm{b}}$
Integrated energy system	$\boldsymbol{P}_{\mathrm{n}}^{\max}$, $\boldsymbol{P}_{\mathrm{n}}^{\min}$, $\boldsymbol{F}_{\mathrm{n}}^{\max}$, $\boldsymbol{F}_{\mathrm{n}}^{\min}$, $\boldsymbol{F}_{\mathrm{b}}^{\max}$, $P_{\mathrm{GT}}^{\min}$, a_{GT}, b_{GT}, c_{GT}, d_{GT}, e_{GT}, K, η	$\boldsymbol{W}$, $\boldsymbol{P}_{\mathrm{n}}$, $\boldsymbol{F}_{\mathrm{n}}$, $\boldsymbol{F}_{\mathrm{b}}$

The derivation process of gas circuit and network equation modeling in NGS

Derivation process of steady-state gas circuit of pipe

First, linearize the momentum conservation equation of NGS, the process is as follows.

(1) The natural gas flow in pipelines can be described by the mass conservation equation shown in equation (B1) and the momentum conservation equation shown in equation (B2),

$$\frac{\partial\rho}{\partial t}+\frac{\partial\rho v}{\partial x}=0 \tag{B1}$$

$$\frac{\partial\rho v}{\partial t}+\frac{\partial\rho v^{2}}{\partial x}+\frac{\partial p}{\partial x}+\frac{\lambda_{\mathrm{g}}\rho v^{2}}{2d_{\mathrm{g}}}+\rho g\sin\theta_{\mathrm{g}}=0 \tag{B2}$$

where, v and ρ represent the actual flow rate and pressure of natural gas; ∂t and ∂x represent partial differentials for time and space.

(2) Two commonly approximations in engineering are

introduced for the above momentum conservation equation.

1) Ignore the convection term in equation (B2), as shown in equation (B3):

$$\frac{\partial \rho v^2}{\partial x} = 0 \tag{B3}$$

2) Linearize and approximate the square term of flow velocity in the resistance term of equation (B2), as shown in equation (B4):

$$v^2 = \left(v_b + \Delta v\right)^2 \approx v_b^2 + 2v_b \Delta v = 2v_b v - v_b^2 \tag{B4}$$

where, Δv is the fluctuation of the actual flow rate "v" relative to the basic value of flow rate "v_b" of natural gas.

Then, the resistance term in equation (B2) can be expressed as a linear function of flow velocity, as shown in equation (B5).

$$\frac{\lambda_g \rho v^2}{2d_g} \approx \frac{\lambda_g \rho \left(2v_b v - v_b^2\right)}{2d_g} \tag{B5}$$

Second, establish a gas circuit model of distributed parameter for natural gas pipeline, the process is as follows.

(1) Introduce the state equation of natural gas, as shown in equation (B6),

$$p = RT\rho Z \tag{B6}$$

where, "Z", thus the compression factor of natural gas is generally taken as 1.

(2) Introduce the mass flow rate of natural gas, as shown in equation (B7):

$$G = \rho v s_g \tag{B7}$$

where, G is the mass flow rate of natural gas.

(3) Substitute equations (B3)~(B7) into equations (B1)~(B2), the partial differential equations, which describes the relationship between natural gas flow and pressure of pipelines, can be obtained, as shown in equations (B8) and (B9):

$$s_g \frac{\partial p}{\partial t} + RT \frac{\partial G}{\partial x} = 0 \tag{B8}$$

$$\frac{1}{s_g}\frac{\partial G}{\partial t} + \frac{\partial p}{\partial x} + \frac{\lambda_g v_b p}{2RTd_g} + \frac{pg\sin\theta_g}{RT} = 0 \tag{B9}$$

(4) According to equations (B8) and (B9), the flow difference and pressure difference at both ends of a micro element pipeline with the length of "dx" are expressed as equations (B10) and (B11):

$$\mathrm{d}G = -\frac{s_g}{RT}\cdot\frac{\mathrm{d}p}{\mathrm{d}t}\cdot\mathrm{d}x \tag{B10}$$

$$\mathrm{d}G = -\frac{s_g}{RT}\cdot\frac{\mathrm{d}p}{\mathrm{d}t}\cdot\mathrm{d}x \tag{B11}$$

(5) Make an analogy between the pressure at both ends of natural gas pipeline with the flow through it and the voltage at both ends of electric branch with the current through it. Like the circuit, the distributed parameter model of gas circuit shown in Figure B1, as well as the gas resistance r_g, gas inductance L_g, controlled pressure source k_g and gas capacitance C_g in the model, can be sorted out from the equations (B10) and (B11).

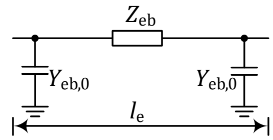

Fig. B1. Distributed parameter model of gas circuit of pipeline

At steady state, by treating the gas inductance as a "short circuit" and the gas capacitance as an "open circuit", the calculation equations for the components of the gas circuit in the form of distributed parameter shown in equations (5)~(8) in section 3.2.2 can be obtained.

$$r_g = \frac{\lambda_g v_b}{s_g d_g} \tag{B12}$$

$$L_g = 0 \tag{B13}$$

$$k_g = \frac{2gd_g \sin\theta_g - \lambda_g v_b^2}{2RTs_g} \tag{B14}$$

$$C_g = 0 \tag{B15}$$

Third, the gas circuit model in centralized parameters of natural gas pipeline is established, the process is as follows.

(1) Substituting equations (B12)~(B15) into equations (B10) and (B11), the flow difference and pressure difference at both ends of a micro element pipeline with the length of "dx" in steady-state can be expressed as equations (B16) and (B17).

$$\mathrm{d}G = -C_g \cdot \mathrm{d}x \cdot \mathrm{d}p/\mathrm{d}t = 0 \tag{B16}$$

$$\begin{aligned}\mathrm{d}p &= -L_g \cdot \mathrm{d}x \cdot \mathrm{d}G/\mathrm{d}t - r_g \cdot \mathrm{d}x \cdot G - k_g \cdot \mathrm{d}x \cdot p \\ &= -r_g \cdot \mathrm{d}x \cdot G - k_g \cdot \mathrm{d}x \cdot p\end{aligned} \tag{B17}$$

(2) Mapping equations (B16) and (B17) to the frequency domain, as well as adding boundary conditions for the flow and pressure of the head end of pipe: $G|_{x=0}=G_0$, $p|_{x=0}=p_0$, the flow rate G_l and pressure p_l of the end of a pipeline with a length of l_g can be obtained, as shown in equations (B18) and (B19).

$$G_l = G_0 \tag{B18}$$

$$p_l = [p_0(\cosh\frac{k_g l_g}{2} - \sinh\frac{k_g l_g}{2}) - G_0\frac{2r_g}{k_g}\sinh\frac{k_g l_g}{2}]e^{-\frac{k_g l_g}{2}} \tag{B19}$$

(3) Express the gas flow and pressure at the beginning and end of the pipeline as a two-port network.

$$\begin{bmatrix} p_l \\ G_l \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} p_0 \\ G_0 \end{bmatrix} \tag{B20}$$

At steady state, the parameters of the two-port network are shown in equations (B21)~(B24).

$$A = (\cosh\frac{k_g l_g}{2} - \sinh\frac{k_g l_g}{2})e^{-\frac{k_g l_g}{2}} \tag{B21}$$

$$B = -2\frac{r_g}{k_g}\cdot\sinh\frac{k_g l_g}{2}\cdot e^{-\frac{k_g l_g}{2}} \tag{B22}$$

$$C = 0 \tag{B23}$$

$$D = 1 \tag{B24}$$

(4) Base on the two-port network shown in equations (B21)~(B24), the concentrated parameter model in the form of π-type equivalent gas circuit shown in Figure B2, as well as the branch impedance Z_{gb}, controlled pressure source k_{gb}, ground

admittance $Y_{gb,10}$ and $Y_{gb,20}$, can be obtained.

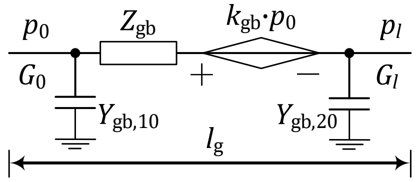

Fig. B2. Concentrated parameter model of gas circuit of pipeline

At steady state, by organizing equations (B21)~(B24), the calculation equation for the components of the π-type gas circuit in the form of concentrated parameter shown in equations (9)~(12) in section 3.2.2 can be obtained.

$$Z_{gb} = -B = 2\frac{r_g}{k_g} \cdot \sinh\frac{k_g l_g}{2} \cdot e^{-\frac{k_g l_g}{2}} \tag{B25}$$

$$k_{gb} = 1 - AD + BC = (\cosh\frac{k_g l_g}{2} - \sinh\frac{k_g l_g}{2})e^{-\frac{k_g l_g}{2}} \tag{B26}$$

$$Y_{gb,10} = (AD - BC - A)/B = 0 \tag{B27}$$

$$Y_{gb,20} = (1 - D)/B = 0 \tag{B28}$$

Derivation process of steady-state network equation

Based on the concentrated parameter model of gas circuit of pipeline, considering the branches characteristics and network topology of NGS, the network equations of NGS shown in equation (13) in section 3.2.2 can be obtained. The specific process is as follows.

First, considering the branch characteristics of the NGS, that is, to model the pipeline branch with a compressor.

(1) For compressor modeling, Ref. [16] equates it to a gas pressure source during modeling, which has the advantage of simplicity and intuition, but the description of actual operation is somewhat ideal. In the field of natural gas, using compressor pressure ratio to simulate pressure changes in pipelines containing compressors is a more common modeling method [24], which is more in line with the operating characteristics of compressors. In the generalized electric circuit theory, a similar modeling method is adopted considering that compressors in NGS and transformers in EPS have similar boosting functions [24]. This paper draws inspiration from Ref. [24], equates the compressor to a gas circuit transformer, as shown in equation (B29):

$$p_{c'} = K_g p_c \tag{B29}$$

where , p_c and $p_{c'}$ are the pressures at the beginning and end of the compressor, K_g is the pressure ratio of compressor.

(2) Based on equation (B29) and the π-type equivalent gas circuit of pipeline in steady-state, the gas circuit of the pipeline branch with a compressor in steady-state can be represented in the form of Figure B3. In the figure, p_f and p_t are the pressures at the beginning and end of pipeline; Z_{gb} and Y_{gb} are the impedance and admittance of pipeline, they use steady-state values of the parameters of π-type equivalent gas circuit; K_{gb} is the controlled gas pressure source of pipeline; G_b and P_b represent the natural gas flow and pressure drop of pipeline.

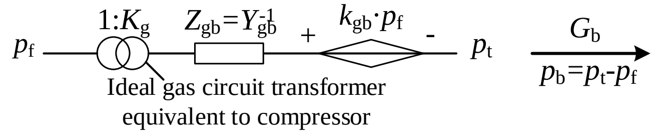

Fig. B3. steady-state gas circuit of pipeline with a compressor

Obviously, the pipeline with compressor shown in Figure B3 satisfies the equation shown in equation (B30).

$$p_f \cdot K_g - G_b \cdot Z_{gb} - k_{gb} \cdot p_f = p_t \tag{B30}$$

(3) By using branch admittance Y_{gb} to represent branch impedance Z_{gb} and organizing equation (B30), the steady-state gas pipeline equation considering branch characteristics can be obtained, as shown in equation (B31).

$$Y_{gb} \cdot (K_g \cdot p_f - k_{gb} \cdot p_f - p_t) = G_b \tag{B31}$$

Further, the matrix form of equation (B31) is as follows,

$$\boldsymbol{Y}_{gb}(\boldsymbol{K}\boldsymbol{p}_f - \boldsymbol{k}_{gb}\boldsymbol{p}_f - \boldsymbol{p}_t) = \boldsymbol{G}_b \tag{B32}$$

where, $\boldsymbol{p}_f$, $\boldsymbol{p}_t$ and $\boldsymbol{G}_b$ are vectors composed of p_f, p_t, and G_b.

Second, based on the branch characteristics, consider the network topology characteristics of NGS further.

(1) To describe the network topology, the node-branch incident matrix $\boldsymbol{A}_g$, node-outflow branch incident matrix $\boldsymbol{A}_{g+}$, and node-inflow branch incident matrix $\boldsymbol{A}_{g-}$ of NGS are introduced. At the same time, it is specified that the outflow direction at the load node is positive.

The meanings and forms of $\boldsymbol{A}_g$, $\boldsymbol{A}_{g+}$ and $\boldsymbol{A}_{g-}$ are as follows.

$\boldsymbol{A}_g$ is a matrix that describes the relationship between nodes and branches, its concept originated from EPS. For a NGS with *m* nodes and *n* pipelines, $\boldsymbol{A}_g$ is represented as a matrix of *m* rows and *n* columns. Use $(\boldsymbol{A}_g)_{i,j}$ to represent the element in the *i*-th row and *j*-th column of Ag. $(\boldsymbol{A}_g)_{i,j}$=0 means that branch *j* is not connected to node *i*; $(\boldsymbol{A}_g)_{i,j}$=1 means that the traffic flow out of node *i* from branch *j*; $(\boldsymbol{A}_g)_{i,j}$=-1 means that traffic flow from branch *j* into node *i*.

$\boldsymbol{A}_{g+}$ is a matrix that describes the relationship between nodes and outflow pipeline branches. $\boldsymbol{A}_{g+}$ only retains non-negative elements of $\boldsymbol{A}_g$, that is, for $(\boldsymbol{A}_{g+})_{i,j}$, if the traffic flow out of node *i* through branch *j*, $(\boldsymbol{A}_{g+})_{i,j}$=1, otherwise $(\boldsymbol{A}_{g+})_{i,j}$=0.

$\boldsymbol{A}_{g-}$ is a matrix that describes the relationship between nodes and inflow pipeline. $\boldsymbol{A}_{g-}$ only retains non-positive elements of $\boldsymbol{A}_g$, that is, for $(\boldsymbol{A}_{g-})_{i,j}$, if the traffic flow out of node *i* through branch *j*, $(\boldsymbol{A}_{g-})_{i,j}$=1, otherwise $(\boldsymbol{A}_{g+})_{i,j}$=0.

(2) According to Kirchhoff law, the flow and gas pressure of nodes and branches of NGS satisfy the relationship shown in equations (B33)~(B35).

$$-\boldsymbol{A}_g^{T}\boldsymbol{G}_b = \boldsymbol{G}_n \tag{B33}$$

$$Y_{gb,10} = (AD - BC - A)/B = 0 \tag{B34}$$

$$Y_{gb,20} = (1 - D)/B = 0 \tag{B35}$$

(3) Substituting equations (B33)~(B35) into equation (B32), the network equation of NGS will be obtained, as shown in equation (B36).

$$\boldsymbol{G}_n = -\boldsymbol{A}_g\boldsymbol{Y}_{gb}(\boldsymbol{K}_g\boldsymbol{A}_{g+}^{T}\boldsymbol{p}_n - \boldsymbol{k}_{gb}\boldsymbol{A}_{g+}^{T}\boldsymbol{p}_n - \boldsymbol{A}_{g-}^{T}\boldsymbol{p}_n) \tag{B36}$$

(4) Introduce the generalized node admittance matrix of NGS $\boldsymbol{Y}_g$ as follows,

$$\boldsymbol{Y}_g = -\boldsymbol{A}_g\boldsymbol{Y}_{gb}(\boldsymbol{K}_g\boldsymbol{A}_{g+}^{T} - \boldsymbol{k}_{gb}\boldsymbol{A}_{g+}^{T} - \boldsymbol{A}_{g-}^{T}) \tag{B37}$$

then the network equation in the form of equation (B37) can be organized into the form shown in equation (10) in section 3.2.2. Thus, the network equation of NGS, which is consistent with the form of the network equation of EPS is obtained, as shown in equation (B38):

$$\boldsymbol{Y}_{\mathrm{g}}\boldsymbol{p}_{\mathrm{n}}=\boldsymbol{G}_{\mathrm{n}} \tag{B38}$$

Method for setting the basic value of gas flow rate of NGS

The principle for setting the base value of gas flow rate is to be as close as possible to the actual flow rate of the pipeline, because the smaller the fluctuation of the actual flow rate relative to the base value, the smaller the error introduced in theory [18,20].

There are two problems about setting the base value: (1) for different operating points, the corresponding pipeline flow rate varies greatly. If the base value is set to the same value, it will introduce significant errors; (2) for each pipe segment with different pipe diameters, using the same base value will also introduce errors. Therefore, this paper takes into account both of these issues when taking values. The equation for the base value is as follows:

$$\boldsymbol{v}_{\mathrm{b}}=-\boldsymbol{A}_{\mathrm{g,L}}^{-1}\boldsymbol{G}_{\mathrm{n}}\big/\boldsymbol{s}\rho \tag{C1}$$

where, $\boldsymbol{v}_{\mathrm{b}}$ represents the vector of base value of gas flow rate; $\boldsymbol{A}_{\mathrm{g,L}}^{-1}$ represents the Left inverse matrix of $\boldsymbol{A}_{\mathrm{g}}$ [20], which can be obtained by $\boldsymbol{A}_{\mathrm{g}}$ through matrix transformation; $\boldsymbol{G}_{\mathrm{n}}$ represents the vector of the mass flow rate of natural gas injected into each node; $\boldsymbol{s}$ represents the vector of the diameters of each pipe; ρ is the density of natural gas (if volumetric flow is used, ρ will not need to be included).

It needs to be explained further.

(1) The above basic value setting process is mainly aimed at static state. If transient conditions are considered, the base value can be taken as the steady-state value at the initial time [20].

(2) After setting the basic value v_{b}, the network equation of NGS is linear. When it is difficult to ensure the base value is close enough to the actual flow rate, the base value can be corrected through iterative calculation to improve accuracy [20].

Derivation process of ECSR of IES

(1) First, the ECSR of EPS is taken as constraint *s.t.* (D1-1). Second, the ECSR of NGSS is taken as constraint *s.t.* (D1-2). Third, for the coupling units, they are compressor and gas generator in this paper, take the constraints of coupling units as constraint *s.t.* (D1-3). Finally, according to the constraints s.*t.* (D1-1)~*s.t.* (D1-3), the security region model shown in equation (D1) can be obtained:

$$\begin{cases}\boldsymbol{\Omega}_{\text{IES-SR}}=\left\{\boldsymbol{W}_{\mathrm{s}}\,\middle|\,\boldsymbol{h}(\boldsymbol{W}_{\mathrm{s}})=\boldsymbol{0},\boldsymbol{g}(\boldsymbol{W}_{\mathrm{s}})\le\boldsymbol{0}\right\}\\ s.t.(\mathrm{D1}-1)\begin{cases}\boldsymbol{Y}_{\mathrm{e}}\boldsymbol{U}_{\mathrm{n}}=\boldsymbol{I}_{\mathrm{n}}\\ \boldsymbol{U}_{\mathrm{n}}^{\min}\le\boldsymbol{U}_{\mathrm{n}}\le\boldsymbol{U}_{\mathrm{n}}^{\max}\\ \boldsymbol{I}_{\mathrm{n}}^{\min}\le\boldsymbol{I}_{\mathrm{n}}\le\boldsymbol{I}_{\mathrm{n}}^{\max}\\ \boldsymbol{I}_{\mathrm{b}}\le\boldsymbol{I}_{\mathrm{b}}^{\max}\\ \boldsymbol{K}_{\mathrm{e}}^{\min}\le\boldsymbol{K}_{\mathrm{e}}\le\boldsymbol{K}_{\mathrm{e}}^{\max}\end{cases}\\ s.t.(\mathrm{D1}-2)\begin{cases}\boldsymbol{Y}_{\mathrm{g}}\boldsymbol{p}_{\mathrm{n}}=\boldsymbol{G}_{\mathrm{n}}\\ \boldsymbol{p}_{\mathrm{n}}^{\min}\le\boldsymbol{p}_{\mathrm{n}}\le\boldsymbol{p}_{\mathrm{n}}^{\max}\\ \boldsymbol{G}_{\mathrm{n}}^{\min}\le\boldsymbol{G}_{\mathrm{n}}\le\boldsymbol{G}_{\mathrm{n}}^{\max}\\ \boldsymbol{G}_{\mathrm{b}}\le\boldsymbol{G}_{\mathrm{b}}^{\max}\\ \boldsymbol{K}_{\mathrm{g}}^{\min}\le\boldsymbol{K}_{\mathrm{g}}\le\boldsymbol{K}_{\mathrm{g}}^{\max}\end{cases}\\ s.t.(\mathrm{D1}-3)\begin{cases}S_{\mathrm{c}}=\dfrac{151.4653p_0ZTG_c\kappa}{\psi T_0(\kappa-1)}(K_{\mathrm{g}}^{\frac{\kappa}{\kappa-1}}-1)\\ G_{\mathrm{GT}}=\dfrac{1}{\mathrm{V}_{\mathrm{GH}}}(a_{\mathrm{GT}}P_{GT}^2+b_{\mathrm{GT}}P_{GT}+c_{\mathrm{GT}}+|\,d_{\mathrm{GT}}\sin(e_{\mathrm{GT}}(P_{\mathrm{GT}}^{\min}-P_{GT}))\,|)\end{cases}\end{cases} \tag{D1}$$

(2) Combine the node admittance matrix of EPS "$\boldsymbol{Y}_{\mathrm{e}}$" and the node admittance matrix of NGS "$\boldsymbol{Y}_{\mathrm{g}}$" to form the node admittance matrix of IES "$\boldsymbol{Y}_{\mathrm{IES}}$" in blocks. Combine voltage and gas pressure to form the pressure vector of IES "$\boldsymbol{P}_{\mathrm{n}}$". Combine current and gas flow to form the flow vector of IES "$\boldsymbol{F}_{\mathrm{n}}$". Combine the transformer ratio matrix "$\boldsymbol{K}_{\mathrm{e}}$" and the compressor pressure ratio matrix "$\boldsymbol{K}_{\mathrm{g}}$" to form a generalized energy ratio matrix "$\boldsymbol{K}$".

(3) By combining *s.t.* (D1-1) and *s.t.* (D1-2) in equation (D1) to form *s.t.* (D2-1) in equation (D2), taking *s.t.* (D1-1) in equation (D1) as *s.t.* (D2-2) in equation (D2), the ECSR of IES shown in equation (D2) can be obtained:

$$
\begin{cases}
\boldsymbol{\Omega}_{\text{IES-SR}} = \left\{ \boldsymbol{W}_{\text{s}} \,\middle|\, \boldsymbol{h}(\boldsymbol{W}_{\text{s}}) = \boldsymbol{0}, \boldsymbol{g}(\boldsymbol{W}_{\text{s}}) \le \boldsymbol{0} \right\} \\
s.t.(\text{D2}-1) \begin{cases}
\boldsymbol{Y}_{\text{IES}} \boldsymbol{P}_{\text{n}} = \boldsymbol{F}_{\text{n}} \\
\boldsymbol{P}_{\text{n}}^{\min} \le \boldsymbol{P}_{\text{n}} \le \boldsymbol{P}_{\text{n}}^{\max} \\
\boldsymbol{F}_{\text{n}}^{\min} \le \boldsymbol{F}_{\text{n}} \le \boldsymbol{F}_{\text{n}}^{\max} \\
\boldsymbol{F}_{\text{b}} \le \boldsymbol{F}_{\text{b}}^{\max} \\
\boldsymbol{K}^{\min} \le \boldsymbol{K}_{\text{e}} \le \boldsymbol{K}^{\max}
\end{cases} \\
s.t.(\text{D2}-2) \begin{cases}
S_{\text{c}} = \dfrac{151.4653 p_0 Z T G_c \kappa}{\psi T_0 (\kappa - 1)} \left(K_{\text{g}}^{\frac{\kappa}{\kappa - 1}} - 1 \right) \\
G_{\text{GT}} = \dfrac{1}{\text{V}_{\text{GH}}} \left(a_{\text{GT}} P_{GT}^2 + b_{\text{GT}} P_{GT} + c_{\text{GT}} + | d_{\text{GT}} \sin(e_{\text{GT}} (P_{\text{GT}}^{\min} - P_{GT})) | \right)
\end{cases}
\end{cases}
\tag{D2}
$$

Structure and detailed parameters of the cases

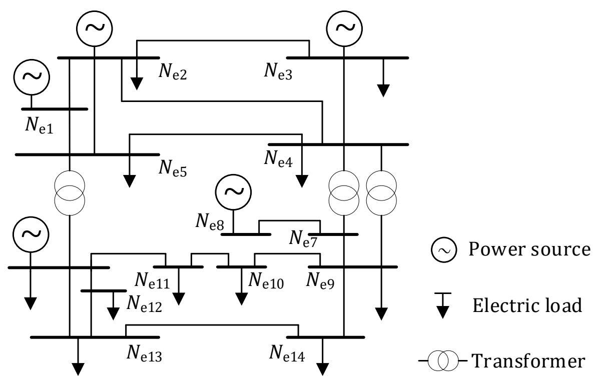

Fig. E1 Structure of IEEE 14-bus system

TABLE E1
BUS PARAMETERS OF IEEE 14-BUS EPS.

Bus	Bus type	Bus Voltage range (p.u.)	Generator active power range (MW)	Generator reactive power range (MVar)	Load apparent power range (MVA)	Load power factor
N_{e1}	slack bus	1.06	[0, 250]	[-10, 10]	-	-
N_{e2}	PV bus	1.045	[0, 50]	[-20, 20]	[20, 30]	0.95
N_{e3}	PV bus	1.01	[0, 20]	[0, 20]	[85, 100]	0.95
N_{e4}	PQ bus	[0.969, 1.069]	-	-	[40, 55]	0.90
N_{e5}	PQ bus	[0.97, 1.07]	-	-	[0, 10]	0.90
N_{e6}	PV bus	1.07	0	[-6, 24]	[0, 15]	0.95
N_{e7}	PQ bus	[1.012, 1.112]	-	-	-	-
N_{e8}	PV bus	1.09	0	[-6, 24]	-	-
N_{e9}	PQ bus	[1.006, 1.106]	-	-	[25, 40]	0.95
N_{e10}	PQ bus	[1.001, 1.101]	-	-	[0, 15]	0.95
N_{e11}	PQ bus	[1.007, 1.107]	-	-	[0, 5]	0.95
N_{e12}	PQ bus	[1.005, 1.105]	-	-	[0, 10]	0.95
N_{e13}	PQ bus	[1, 1.1]	-	-	[0, 15]	0.90
N_{e14}	PQ bus	[0.986, 1.086]	-	-	[10, 20]	0.90

TABLE E2
BUS PARAMETERS OF IEEE 14-BUS EPS.

Branch	From	To	Resistance (Ω)	Reactance (Ω)	Susceptance (s)	Transformer ratio	Capacity (MVA)
b_{e1}	N_{e1}	N_{e2}	23.07	70.43	4.43×10^{-5}	-	200
b_{e2}	N_{e1}	N_{e5}	64.31	265.47	4.13×10^{-5}	-	100
b_{e3}	N_{e2}	N_{e3}	55.93	235.63	3.68×10^{-5}	-	100
b_{e4}	N_{e2}	N_{e4}	69.17	209.86	2.86×10^{-5}	-	75
b_{e5}	N_{e2}	N_{e5}	67.78	206.96	2.91×10^{-5}	-	50
b_{e6}	N_{e3}	N_{e4}	79.76	203.57	1.08×10^{-5}	-	50
b_{e7}	N_{e4}	N_{e5}	15.89	50.12	0	-	75
b_{e8}	N_{e4}	N_{e7}	0	248.91	0	0.978	50
b_{e9}	N_{e4}	N_{e9}	0	661.99	0	0.969	25

b_{e10}	N_{e5}	N_{e6}	0	299.97	0	0.932	75
b_{e11}	N_{e6}	N_{e11}	113.05	236.74	0	-	25
b_{e12}	N_{e6}	N_{e12}	146.29	304.48	0	-	25
b_{e13}	N_{e6}	N_{e13}	78.74	155.05	0	-	25
b_{e14}	N_{e7}	N_{e8}	0	209.66	0	-	25
b_{e15}	N_{e7}	N_{e9}	0	130.94	0	-	50
b_{e16}	N_{e9}	N_{e10}	37.86	100.58	0	-	10
b_{e17}	N_{e9}	N_{e14}	151.29	321.82	0	-	25
b_{e18}	N_{e10}	N_{e11}	97.66	228.61	0	-	10
b_{e19}	N_{e12}	N_{e13}	262.95	237.91	0	-	5
b_{e20}	N_{e13}	N_{e14}	203.45	414.23	0	-	10

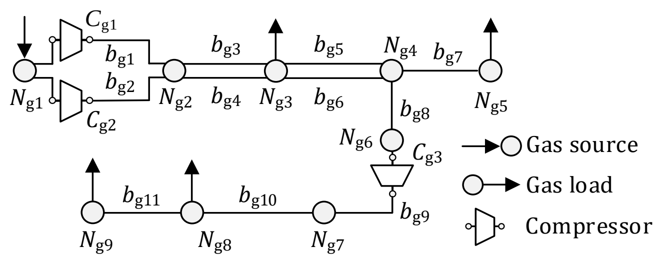

Fig. E2 Structure of southeastern Belgium pipeline system

TABLE E3
NODE PARAMETERS OF SOUTHEASTERN BELGIUM PIPELINE SYSTEM.

Node	Town	Node type	Pressure range (MPa)	Flow range (m^3/s)
N_{g1}	Voeren	Source	[5.5, 6.5]	[-500, 0]
N_{g2}	Berneau	-	[0, 6.5]	-
N_{g3}	Liege	Load	[3, 6.5]	[69, 129]
N_{g4}	Warnand-Dreye	-	[0, 6.5]	-
N_{g5}	Namur	Load	[0, 6.5]	[126, 191]
N_{g6}	Wanze	-	[0, 6.5]	-
N_{g7}	Sinsin	-	[0, 6.3]	-
N_{g8}	Arlon	Load	[0, 6.5]	[0, 30]
N_{g9}	Petange	Load	[2.5, 6.5]	[0, 60]

TABLE E4
PIPE PARAMETERS OF SOUTHEASTERN BELGIUM PIPELINE SYSTEM.

Pipe	From	To	Diameter (10^{-3}m)	Length (km)	Capacity (m^3/s)
b_{g1}	N_{g1}	N_{g2}	890	5	231.5
b_{g2}	N_{g1}	N_{g2}	395.5	5	115.7
b_{g3}	N_{g2}	N_{g3}	890	20	231.5
b_{g4}	N_{g2}	N_{g3}	395.5	20	115.7
b_{g5}	N_{g3}	N_{g4}	890	25	231.5
b_{g6}	N_{g3}	N_{g4}	395.5	25	115.7
b_{g7}	N_{g4}	N_{g5}	890	42	231.5
b_{g8}	N_{g4}	N_{g6}	315.5	10.5	115.7
b_{g9}	N_{g6}	N_{g7}	315.5	26	115.7
b_{g10}	N_{g7}	N_{g8}	315.5	98	115.7
b_{g11}	N_{g8}	N_{g9}	315.5	6	115.7

TABLE E5
COMPRESSOR PARAMETERS OF SOUTHEASTERN BELGIUM PIPELINE SYSTEM.

Compressor	Intake node	Pressure ratio range
C_{g1}	N_{g1}	[1, 1.2]
C_{g2}	N_{g1}	[1, 1.2]
C_{g3}	N_{g6}	[1, 1.2]

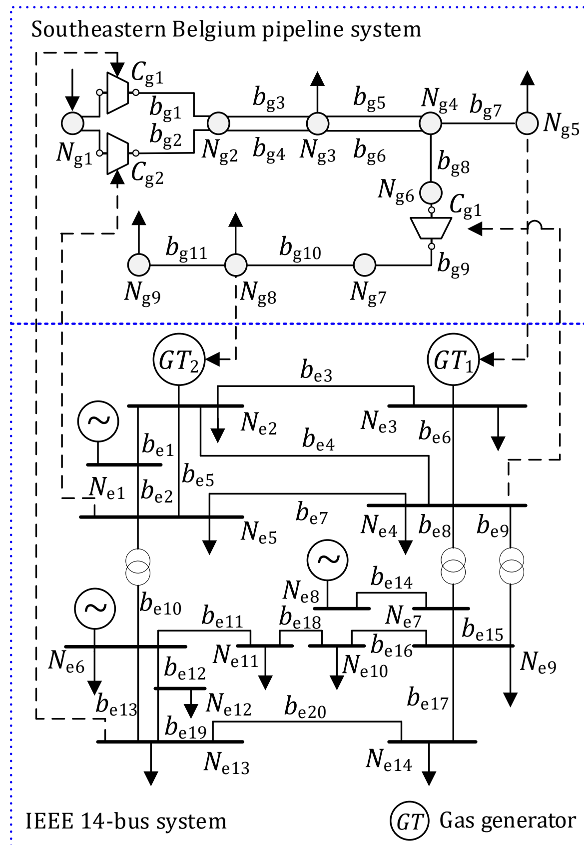

Fig. E3 Structure of 23 node system of IES

TABLE E6
COMPRESSOR PARAMETERS OF 23 NODE SYSTEM OF IES.

Compressor	Intake node	Source for driving	Efficiency	Pressure ratio range
C_{g1}	N_{g1}	N_{e4}	0.8	[1, 1.2]
C_{g2}	N_{g1}	N_{e5}	0.8	[1, 1.2]
C_{g3}	N_{g6}	N_{e13}	0.8	[1, 1.2]

TABLE E7
GAS GENERATOR PARAMETERS OF 23-NODE SYSTEM OF IES.

Genera-tor	Intake node	Heat consumption coefficient					Output lower limit (MW)
		a_{GT}	b_{GT}	c_{GT}	d_{GT}	e_{GT}	
GT_1	N_{g5}	0.01	4	150	15	0.5	0
GT_2	N_{g8}	0.01	4	150	15	0.5	0

Comparison of the security region models of IES between the proposed method and existing method

TABLE F1
COMPARISON OF THE SECURITY REGION MODELS OF IES BETWEEN THE PROPOSED METHOD AND EXISTING METHOD.

Security region model based on typical existing method [7]	Energy circuit security region model based on the proposed method
$\begin{cases} \boldsymbol{\Omega}_{\text{IES-SR}} = \{\boldsymbol{W}_{\text{s}} \mid \boldsymbol{h}(\boldsymbol{W}_{\text{s}}) = \boldsymbol{0}, \boldsymbol{g}(\boldsymbol{W}_{\text{s}}) \le \boldsymbol{0}\} \\ s.t.(1) \begin{cases} P_i = \lvert V_i \rvert \sum_{j=1}^{N} \lvert V_j \rvert \left(G_{ij} \cos\theta_{ij} + B_{ij} \cos\theta_{ij} \right) \\ Q_i = \lvert V_i \rvert \sum_{j=1}^{N} \lvert V_j \rvert \left(G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij} \right) \\ \boldsymbol{U}_{\text{n}}^{\min} \le \boldsymbol{U}_{\text{n}} \le \boldsymbol{U}_{\text{n}}^{\max} \\ \boldsymbol{P}_{\text{n}}^{\min} \le \boldsymbol{P}_{\text{n}} \le \boldsymbol{P}_{\text{n}}^{\max} \\ \boldsymbol{Q}_{\text{n}}^{\min} \le \boldsymbol{Q}_{\text{n}} \le \boldsymbol{Q}_{\text{n}}^{\max} \\ \boldsymbol{S}_{\text{b}} \le \boldsymbol{S}_{\text{b}}^{\max} \\ \boldsymbol{K}_{\text{e}}^{\min} \le \boldsymbol{K}_{\text{e}} \le \boldsymbol{K}_{\text{e}}^{\max} \end{cases} \\ s.t.(2) \begin{cases} -\boldsymbol{A}_{\text{g, L}}^{-1} \boldsymbol{G}_{\text{n}} = \boldsymbol{\eta}_{\text{g}} \boldsymbol{\sigma}_{\text{g}} \sqrt{\left(\boldsymbol{K}_{\text{g}}^{2} \boldsymbol{A}_{\text{g+}}^{\text{T}} \cdot\times \boldsymbol{A}_{\text{g}}^{\text{T}} \right) \boldsymbol{p}_{\text{n}}^{2}} \\ \boldsymbol{p}_{\text{n}}^{\min} \le \boldsymbol{p}_{\text{n}} \le \boldsymbol{p}_{\text{n}}^{\max} \\ \boldsymbol{G}_{\text{n}}^{\min} \le \boldsymbol{G}_{\text{n}} \le \boldsymbol{G}_{\text{n}}^{\max} \\ \boldsymbol{G}_{\text{b}} \le \boldsymbol{G}_{\text{b}}^{\max} \\ \boldsymbol{K}_{\text{g}}^{\min} \le \boldsymbol{K}_{\text{g}} \le \boldsymbol{K}_{\text{g}}^{\max} \end{cases} \\ s.t.(3) \begin{cases} \boldsymbol{g}_{\text{m}} = f\left(\boldsymbol{h}_{\text{m}}\right) \\ \boldsymbol{g}_{\text{m}} \subseteq \boldsymbol{F}_{\text{n}}, \boldsymbol{h}_{\text{m}} \subseteq \boldsymbol{F}_{\text{n}} \end{cases} \end{cases}$	$\begin{cases} \boldsymbol{\Omega}_{\text{IES-ECSR}} = \{\boldsymbol{W}_{\text{s}} \mid \boldsymbol{h}(\boldsymbol{W}_{\text{s}}) = \boldsymbol{0}, \boldsymbol{g}(\boldsymbol{W}_{\text{s}}) \le \boldsymbol{0}\} \\ s.t.(1) \begin{cases} \boldsymbol{Y}_{\text{IES}} \boldsymbol{P}_{\text{n}} = \boldsymbol{F}_{\text{n}} \\ \boldsymbol{P}_{\text{n}}^{\min} \le \boldsymbol{P}_{\text{n}} \le \boldsymbol{P}_{\text{n}}^{\max} \\ \boldsymbol{F}_{\text{n}}^{\min} \le \boldsymbol{F}_{\text{n}} \le \boldsymbol{F}_{\text{n}}^{\max} \\ \boldsymbol{F}_{\text{b}} \le \boldsymbol{F}_{\text{b}}^{\max} \\ \boldsymbol{K}^{\min} \le \boldsymbol{K}_{\text{IES}} \le \boldsymbol{K}^{\max} \end{cases} \\ s.t.(2) \begin{cases} \boldsymbol{g}_{\text{m}} = f\left(\boldsymbol{h}_{\text{m}}\right) \\ \boldsymbol{g}_{\text{m}} \subseteq \boldsymbol{F}_{\text{n}}, \boldsymbol{h}_{\text{m}} \subseteq \boldsymbol{F}_{\text{n}} \end{cases} \end{cases}$

Explanations of the main formula

TABLE G1
EXPLANATIONS OF THE MAIN FORMULAS

Formula number	Formula	Explanation
(1)	$\boldsymbol{W} = [S_{\text{l,e1}}, \ldots, S_{\text{l},ei}, \ldots, S_{\text{l},em}, G_{\text{l,g1}}, \ldots, G_{\text{l,g}\,j}, \ldots, G_{\text{l,g}\,n}]$	operating point
(2)	$\boldsymbol{W} = [S_{\text{l,e1}}, \ldots, S_{\text{l},ei}, \ldots, S_{\text{l,c1}}, \ldots, S_{\text{l},cm}, \ldots \; G_{\text{l,g1}}, \ldots, G_{\text{l,g}\,j}, \ldots, G_{\text{l,GT1}}, \ldots, G_{\text{l,GT}n}, \ldots]$	operating point with compressor and gas generator as coupling units
(3)	$\boldsymbol{\Omega}_{\text{SR}} = \{\boldsymbol{W}_{\text{s}} \mid \boldsymbol{h}(\boldsymbol{W}_{\text{s}}) = \boldsymbol{0}, \; \boldsymbol{g}(\boldsymbol{W}_{\text{s}}) \le \boldsymbol{0}\}$	security region
(4)	$\boldsymbol{Y}_{\text{e}} \boldsymbol{U}_{\text{n}} = \boldsymbol{I}_{\text{n}}$	electric power network equation
(5)	$r_{\text{g}} = \dfrac{\lambda_{\text{g}} v_{\text{b}}}{s_{\text{g}} d_{\text{g}}}$	distributed parameter of gas resistance in steady state (characterizing the frictional effect of pipelines on natural gas flow in steady state)
(6)	$L_{\text{g}} = 0$	distributed parameter of gas inductance (characterizing the inertia of natural gas flow in steady state)
(7)	$k_{\text{g}} = \dfrac{2gd_{\text{g}} \sin\theta_{\text{g}} - \lambda_{\text{g}} v_{\text{b}}^{2}}{2RTs_{\text{g}}}$	distributed parameter of controlled gas pressure source (characterizing the influence of pipeline inclination angle and flow velocity changes on pipeline friction in steady state)
(8)	$C_{\text{g}} = 0$	distributed parameter of gas capacitance (characterizing the natural gas pipeline storage effect in steady state)
(9)	$Z_{\text{gb}} = 2\dfrac{r_{\text{g}}}{k_{\text{g}}} \cdot \sinh\dfrac{k_{\text{g}} l_{\text{g}}}{2} \cdot e^{-\frac{k_{\text{g}} l_{\text{g}}}{2}}$	branch impedance of pipe
(10)	$k_{\text{gb}} = (\cosh\dfrac{k_{\text{g}} l_{\text{g}}}{2} - \sinh\dfrac{k_{\text{g}} l_{\text{g}}}{2}) e^{-\frac{k_{\text{g}} l_{\text{g}}}{2}}$	controlled gas pressure source of pipe
(11)	$Y_{\text{gb,10}} = 0$	ground admittance of pipe
(12)	$Y_{\text{gb,20}} = 0$	ground admittance of pipe
(13)	$\boldsymbol{Y}_{\text{g}} \boldsymbol{p}_{\text{n}} = \boldsymbol{G}_{\text{n}}$	natural gas network equation
(14)	$\boldsymbol{Y}_{\text{g}} = -\boldsymbol{A}_{\text{g}} \boldsymbol{y}_{\text{gb}} (\boldsymbol{K}_{\text{g}} \boldsymbol{A}_{\text{g+}}^{\text{T}} - \boldsymbol{k}_{\text{gb}} \boldsymbol{A}_{\text{g+}}^{\text{T}} - \boldsymbol{A}_{\text{g-}}^{\text{T}})$	generalized node admittance matrix of natural gas system
(15)	$\begin{cases} \boldsymbol{\Omega}_{\text{EPS-ECSR}} = \{\boldsymbol{W}_{\text{s}} \mid \boldsymbol{h}(\boldsymbol{W}_{\text{s}}) = \boldsymbol{0}, \boldsymbol{g}(\boldsymbol{W}_{\text{s}}) \le \boldsymbol{0}\} \\ s.t. \begin{cases} s.t.(17-1) \;\; \boldsymbol{Y}_{\text{e}} \boldsymbol{U}_{\text{n}} = \boldsymbol{I}_{\text{n}} \\ s.t.(17-2) \;\; \boldsymbol{I}_{\text{n}}^{\min} \le \boldsymbol{I}_{\text{n}} \le \boldsymbol{I}_{\text{n}}^{\max} \\ s.t.(17-3) \;\; \boldsymbol{U}_{\text{n}}^{\min} \le \boldsymbol{U}_{\text{n}} \le \boldsymbol{U}_{\text{n}}^{\max} \\ s.t.(17-4) \;\; \boldsymbol{I}_{\text{b}} \le \boldsymbol{I}_{\text{b}}^{\max} \\ s.t.(17-5) \;\; \boldsymbol{K}_{\text{e}}^{\min} \le \boldsymbol{K}_{\text{e}} \le \boldsymbol{K}_{\text{e}}^{\max} \end{cases} \end{cases}$	Energy circuit security region of electric power system (EPS-ECSR)

(16)	$\begin{cases} \boldsymbol{\Omega}_{\text{NGS-ECSR}} = \left\{ \boldsymbol{W}_{\text{s}} \middle\vert \boldsymbol{h}(\boldsymbol{W}_{\text{s}}) = \mathbf{0}, \boldsymbol{g}(\boldsymbol{W}_{\text{s}}) \le \mathbf{0} \right\} \\ s.t. \begin{cases} s.t.(18-1) \ \ \boldsymbol{Y}_{\text{g}} \boldsymbol{p}_{\text{n}} = \boldsymbol{G}_{\text{n}} \\ s.t.(18-2) \ \ \boldsymbol{G}_{\text{n}}^{\min} \le \boldsymbol{G}_{\text{n}} \le \boldsymbol{G}_{\text{n}}^{\max} \\ s.t.(18-3) \ \ \boldsymbol{p}_{\text{n}}^{\min} \le \boldsymbol{p}_{\text{n}} \le \boldsymbol{p}_{\text{n}}^{\max} \\ s.t.(18-4) \ \ \boldsymbol{-} \boldsymbol{A}_{\text{g, L}}^{-1} \boldsymbol{G}_{\text{n}} \le \boldsymbol{G}_{\text{b}}^{\max} \\ s.t.(18-5) \ \ \boldsymbol{K}_{\text{g}}^{\min} \le \boldsymbol{K}_{\text{g}} \le \boldsymbol{K}_{\text{g}}^{\max} \end{cases} \end{cases}$	Energy circuit security region of natural gas system (EPS-ECSR)
(17)	$\begin{cases} \boldsymbol{\Omega}_{\text{IES-SR}} = \left\{ \boldsymbol{W}_{\text{s}} \middle\vert \boldsymbol{h}(\boldsymbol{W}_{\text{s}}) = \mathbf{0}, \boldsymbol{g}(\boldsymbol{W}_{\text{s}}) \le \mathbf{0} \right\} \\ s.t.(\text{R}1-1)\text{:} \\ \begin{cases} \boldsymbol{Y}_{\text{IES}} \boldsymbol{P}_{\text{n}} = \boldsymbol{F}_{\text{n}} \\ \boldsymbol{P}_{\text{n}}^{\min} \le \boldsymbol{P}_{\text{n}} \le \boldsymbol{P}_{\text{n}}^{\max} \\ \boldsymbol{F}_{\text{n}}^{\min} \le \boldsymbol{F}_{\text{n}} \le \boldsymbol{F}_{\text{n}}^{\max} \\ \boldsymbol{F}_{\text{b}} \le \boldsymbol{F}_{\text{b}}^{\max} \\ \boldsymbol{K}^{\min} \le \boldsymbol{K} \le \boldsymbol{K}^{\max} \end{cases} \\ s.t.(\text{R}1-2)\text{:} \\ \boldsymbol{g}_{\text{m}} = f\left(\boldsymbol{h}_{\text{m}}\right) \\ \boldsymbol{g}_{\text{m}} \subseteq \boldsymbol{F}_{\text{n}}, \boldsymbol{h}_{\text{m}} \subseteq \boldsymbol{F}_{\text{n}} \end{cases}$	Energy circuit security region of integrated energy system (IES-ECSR)
(18)	$\begin{cases} \boldsymbol{\Omega}_{\text{IES-ECSR}} = \left\{ \boldsymbol{W}_{\text{s}} \middle\vert \boldsymbol{h}(\boldsymbol{W}_{\text{s}}) = \mathbf{0}, \boldsymbol{g}(\boldsymbol{W}_{\text{s}}) \le \mathbf{0} \right\} \\ s.t.(19-1)\text{:} \\ \begin{cases} \boldsymbol{Y}_{\text{IES}} \boldsymbol{P}_{\text{n}} = \boldsymbol{F}_{\text{n}} \\ \boldsymbol{P}_{\text{n}}^{\min} \le \boldsymbol{P}_{\text{n}} \le \boldsymbol{P}_{\text{n}}^{\max} \\ \boldsymbol{F}_{\text{n}}^{\min} \le \boldsymbol{F}_{\text{n}} \le \boldsymbol{F}_{\text{n}}^{\max} \\ \boldsymbol{F}_{\text{b}} \le \boldsymbol{F}_{\text{b}}^{\max} \\ \boldsymbol{K}^{\min} \le \boldsymbol{K} \le \boldsymbol{K}^{\max} \end{cases} \\ s.t.(19-2)\text{:} \\ \begin{cases} S_{\text{c}} = \dfrac{151.4653 \text{p}_0 Z T G_c \kappa}{\psi \text{T}_0 (\kappa - 1)} (K_{\text{g}}^{\frac{\kappa}{\kappa-1}} - 1) \\ G_{\text{GT}} = \dfrac{1}{\text{V}_{\text{GH}}} (a_{\text{GT}} P_{GT}^2 + b_{\text{GT}} P_{GT} + c_{\text{GT}} + \vert d_{\text{GT}} \sin(e_{\text{GT}} (P_{\text{GT}}^{\min} - P_{GT})) \vert) \end{cases} \end{cases}$	IES-ECSR with compressor and gas generator as coupling units