12-C5

12.排序

希尔排序

PS序列

邓俊辉

deng@tsinghua.edu.cn

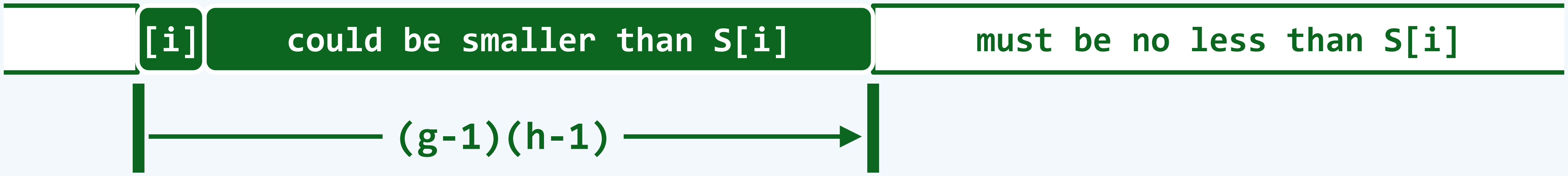
d-Sorting an $\mathcal{O}$(d)-Ordered Sequence in $\mathcal{O}$(dn) Time

❖ If $\boxed{g}$ and $\boxed{h}$ are relatively prime and are both in $\boxed{\mathcal{O}(d)}$

we can d-sort the sequence in $\boxed{\mathcal{O}(dn)}$ time ...

- re-arrange the sequence as a 2D matrix with $\boxed{d}$ columns
- each element is swapped with $\mathcal{O}((g-1)\cdot(h-1)/d) = \mathcal{O}(d)$ elements

❖ Since this holds for all elements, $\mathcal{O}$(dn) steps are enough



PS Sequence

- Papernov & Stasevic, 1965 //also called Hibbard's sequence

$$\mathcal{H}_{PS} = \mathcal{H}_{Shell} - 1 = \{ 2^k - 1 \mid k \in \mathcal{N} \} = \{ 1, 3, 7, 15, 31, 63, 127, 255, \ldots \}$$

- Different items **may not** be relatively prime, e.g. $h_{2k} = h_k \cdot (h_k + 2)$

  But **adjacent** items **must** be, since $h_{k+1} - 2 \cdot h_k \equiv 1$

- Shellsort with $\mathcal{H}_{ps}$ needs
  - $\mathcal{O}(logn)$ outer iterations and
  - $\mathcal{O}(n^{3/2})$ time to sorts a sequence of length n //Why ...

t < k

❖ Let $\boxed{h_t}$ be the h closest to $\boxed{\sqrt{n}}$ and hence $h_t \approx \sqrt{n} = \Theta(n^{1/2})$

1) Consider those iterations for $\{ h_k \mid t < k \} = \{ \overleftarrow{h_{t+1}, h_{t+2}, \dots, h_m} \}$

$\because$ there would be $\mathcal{O}(n/h_k)$ elements in each of the h_k columns

$\therefore$ we can $\boxed{\text{insertionsort}}$ each column in $\mathcal{O}((n/h_k)^2)$ time

$\therefore$ each h_k-sorting costs $\mathcal{O}(n^2/h_k)$ time

$\therefore$ all these iterations cost time of

$$\mathcal{O}(2 \times n^2/h_t) = \mathcal{O}(n^{3/2})$$

k ≤ t

$h_k \leq h_t$

t < k

$h_t < h_k$

k = t

$h_k = h_t$

k ≤ t

2) Consider those iterations for $\{\ h_k \mid k \le t\ \} = \{\ \overleftarrow{h_1, h_2, \ldots, h_t}\ \}$

∵ $\boxed{h_{k+1}}$ and $\boxed{h_{k+2}}$ are relatively prime and are both in $\mathcal{O}(\boxed{h_k})$

∴ each h_k-sorting costs $\mathcal{O}(n \times h_k)$ time

∴ all these iterations cost $\mathcal{O}(n \times 2 \cdot h_t) = \mathcal{O}(n^{3/2})$ time

❖ This upper bound is $\boxed{\text{TIGHT}}$

❖ How about the average cases?

- $\mathcal{O}(n^{5/4})$ based on simulations
- but not proved yet

k ≤ t

$h_k \le h_t$

t < k

$h_t < h_k$

k = t

$h_k = h_t$