12-C4

12.排序

希尔排序

逆序对

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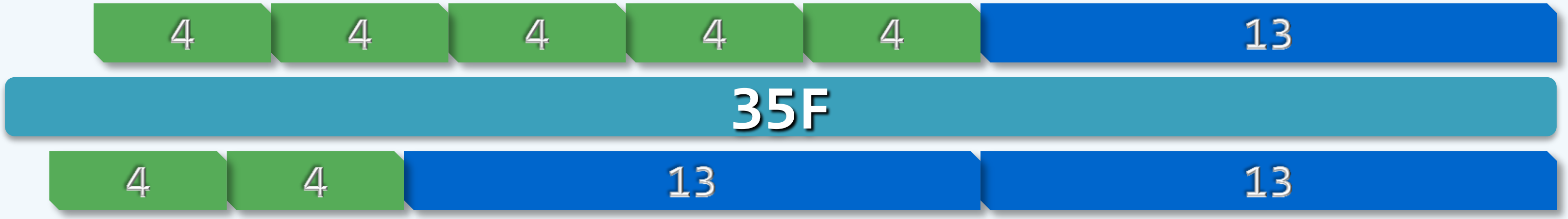
Postage Problem

- The postage for a letter is $\boxed{50F}$, and a postcard $\boxed{35F}$

  But there are only stamps of $\boxed{4F}$ and $\boxed{13F}$ available

- Possible to stamp the letter and the postcard $\boxed{\text{EXACTLY}}$?

- How about other postages?



- For each postage P, determine whether $P \in \{\, n \cdot 4 + m \cdot 13 \mid n, m \in \mathcal{N} \,\}$

Linear Combination

- Let $g, h \in \mathcal{N}$
- For any $n, m \in \mathcal{N}$, $n \cdot g + m \cdot h$ is called a $\boxed{\text{linear combination}}$ of g and h
- Denote $\mathbf{C}(g,h) = \{\, ng + mh \mid n, m \in \mathcal{N} \,\}$

$\mathbf{N}(g,h) = \mathcal{N} \backslash \mathbf{C}(g,h)$ //numbers that are $\boxed{\text{NOT}}$ combinations of g and h

$\mathbf{x}(g,h) = max\{\, \mathbf{N}(g,h) \,\}$ //always exists?

- Theorem: when g and h are $\boxed{\text{relatively prime}}$, we have

$$\mathbf{x}(g,h) = (g-1) \cdot (h-1) - 1 = gh - g - h$$

- e.g. $\mathbf{x}(3,7) = 11$, $\mathbf{x}(4,9) = 23$, $\mathbf{x}(\boxed{4},\boxed{13}) = \boxed{35}$, $\mathbf{x}(5,14) = 51$

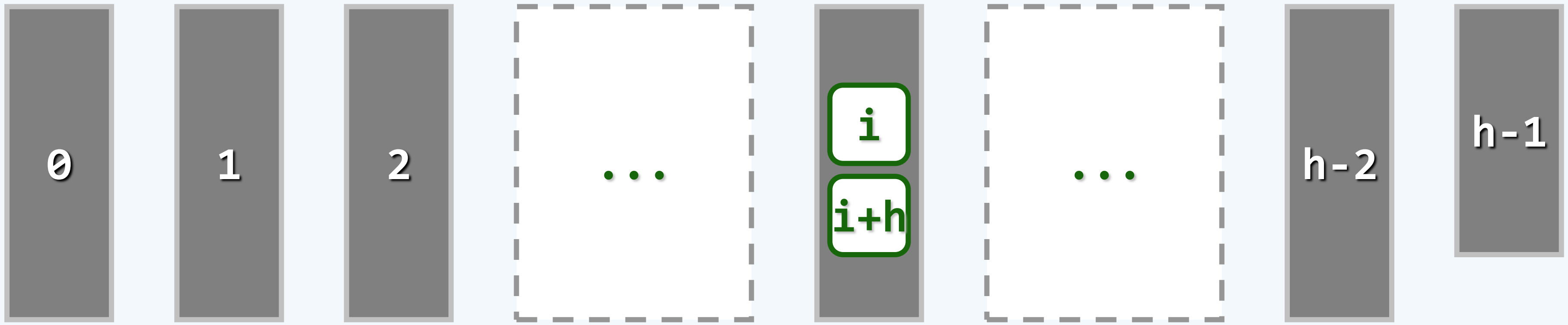
h-sorting & h-ordered

- Let $h \in \mathbb{N}$. A sequence $\boxed{S[0, n)}$ is called $\boxed{\text{h-ordered}}$ if

  $\boxed{S[\,i\,] \le S[\,i + h\,]}$ holds for $0 \le i < n - h$

- A $\boxed{\text{1-ordered}}$ sequence is sorted

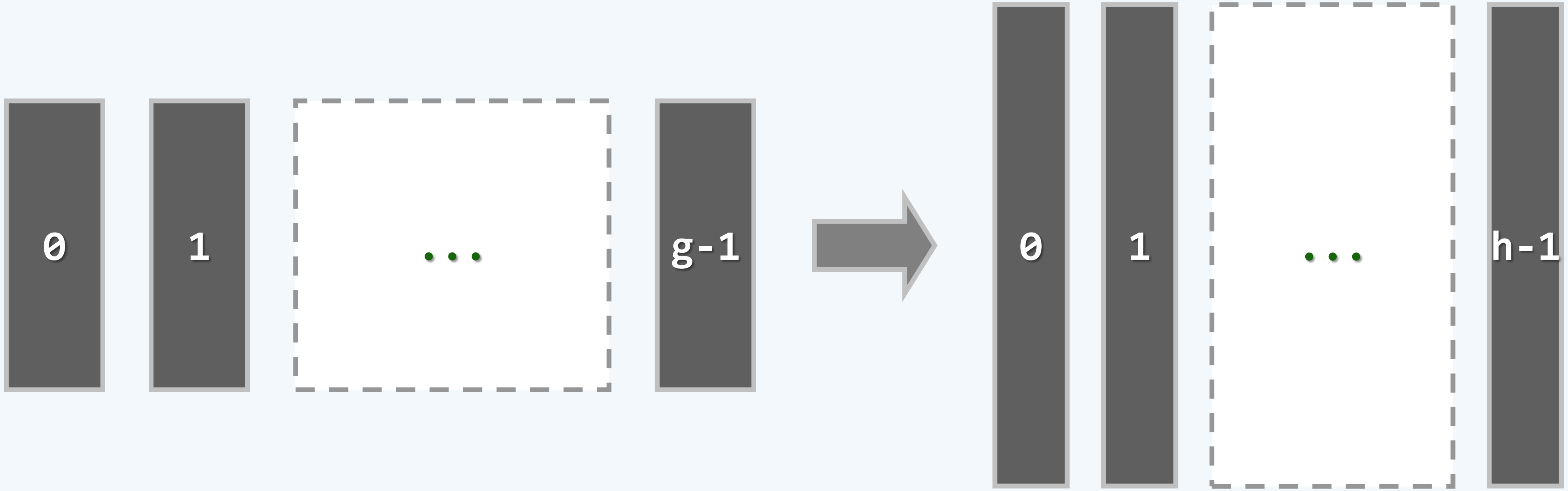
- $\boxed{\text{h-sorting}}$: an h-ordered sequence is obtained by
  - arranging S into a 2D matrix with $\boxed{h}$ columns and
  - sorting each column respectively



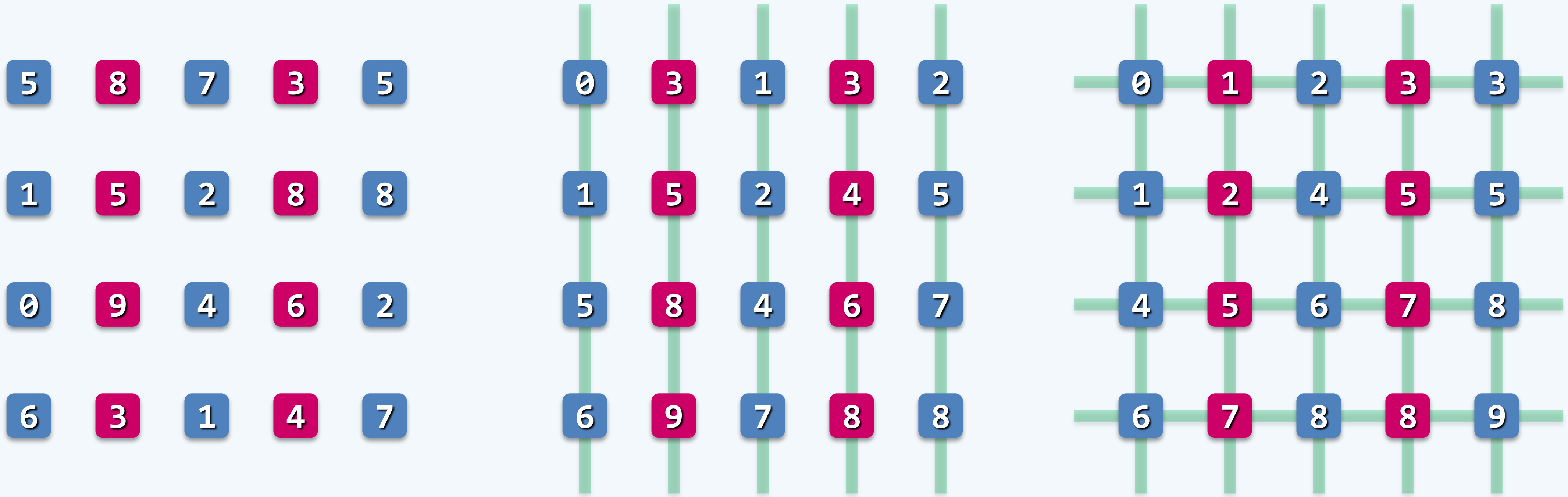
Theorem K

❖ [Knuth, ACP Vol.3 p.90] //习题解析[12-12, 12-13]

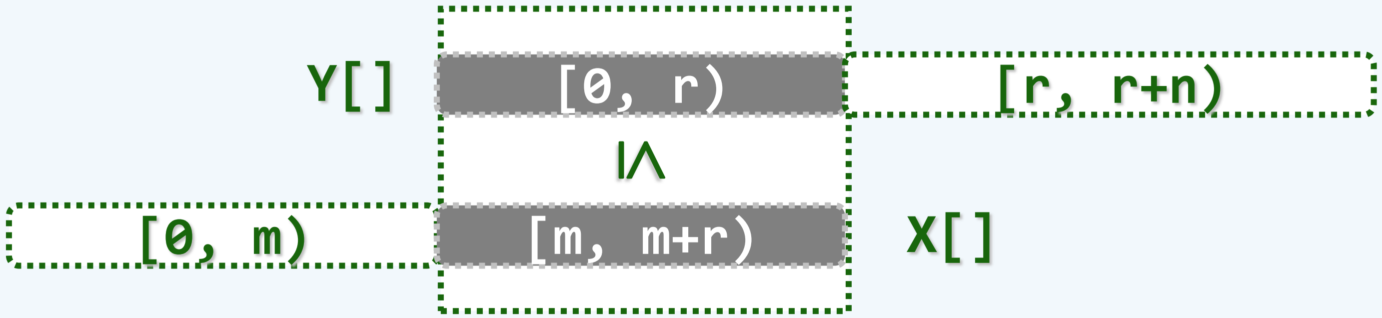
A $\boxed{\text{g-ordered}}$ sequence $\boxed{\text{REMAINS}}$ g-ordered after being $\boxed{\text{h-sorted}}$



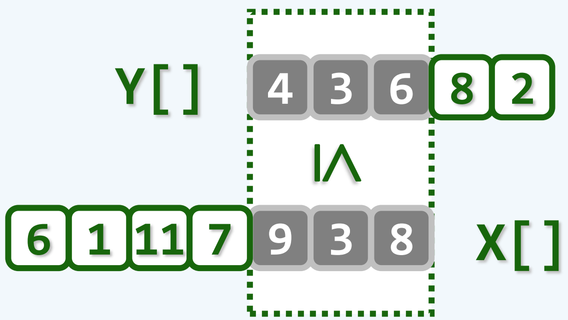
Order Preservation



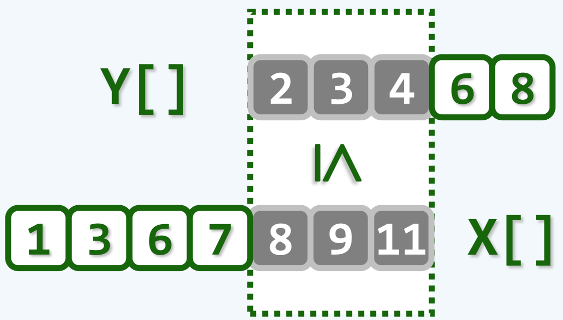
Lemma L



(a)



(b)

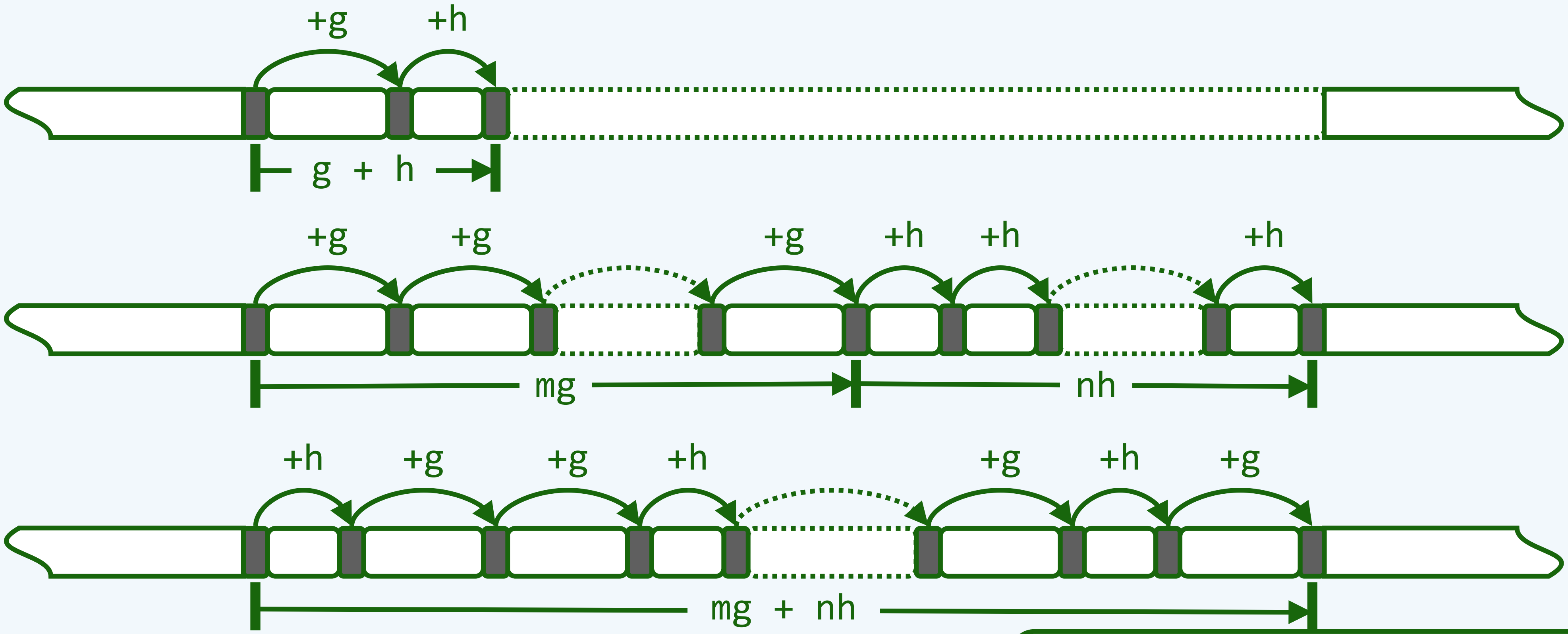


Linear Combination

❖ A sequence that is both $\boxed{g}$-ordered and $\boxed{h}$-ordered

is called $\boxed{(g,\ h)}$-ordered, which must be both

$\boxed{(g + h)}$-ordered and $\boxed{(mg + nh)}$-ordered for any $m, n \in N$



Inversion

- Let S[0, n) be a $\boxed{\text{(g, h)}}$-ordered sequence, where g & h are $\boxed{\text{relatively prime}}$
- Then for all elements S[i] and S[j], we have

$$j - i \ \geq \ \mathbf{x}(g,h) + 1 \ = \ (g-1)\cdot(h-1) \qquad only \ \ if \qquad S[i] \leq S[j]$$

- This implies that to the $\boxed{\text{RIGHT}}$ of each element,

  only the next $\mathbf{x}(g,h)$ elements could be smaller

- There would be no more than $n \cdot \mathbf{x}(g,h)$ inversions altogether