1.
$$f(S_1) = \sum_{e \in E} w_e^-$$
$$f(S_2) = \sum_{e \in E} w_e^+$$

2. For each partition, the endpoints of an edge will be either in one set or in different set, so we can add either w^- or w^+, it's easy to see that
$$OPT \le f(S_1) + f(S_2)$$
So pick the larger one from $f(S_1), f(S_2)$ will give an $\frac{1}{2}$-approximation.

3. You can assign one vertex to any possible cluster, and there will be at most n clusters.
And if two vertices are in the same cluster, you will add $w^+_{(i,j)}$ to the result, if not you will add $w^-_{(i,j)}$.
Each feasible solution corresponds to a valid assignment, and vice versa.
Also, the objective values are the same.

4. The probability is two independent hyperplanes both fail to separate the two vertices, each with a probability $P = 1 - \frac{\theta_{(i,j)}}{\pi}$.
So both two fails with probability:
$$P(X_{ij} = 1) = (1 - \frac{\theta_{(i,j)}}{\pi})^2$$

5.
$$E(X_{ij}) = g(\theta_{(i,j)})$$
$$\begin{aligned} E(f(R)) &= \sum_{(i,j)\in E} \left(w^+_{(x,j)} E(X_{ij}) + w^-_{(i,j)} E(1 - X_{ij}) \right) \\ &= \sum_{(i,j)\in E} \left(w^+_{(i,j)} g(\theta_{(i,j)}) + w^-_{(i,j)} (1 - g(\theta_{(i,j)})) \right) \end{aligned}$$

6. The constraint $v_i \cdot v_j \geq 0$ guarantees that θ_{ij} is in range $[0, \frac{\pi}{2}]$.
   And we know the length of each vector is 1, So $\cos(\theta_{ij}) = v_i \cdot v_j$, and the expectation above will be larger than or equal to

$$\begin{aligned} E(f(R)) &\geq \frac{3}{4} \sum_{(i,j)\in E} \big(w_{ij}^{+} \cos(\theta) + w_{ij}^{-}(1 - \cos(\theta))\big) \\ &= \frac{3}{4} \sum_{(i,j)\in E} \big(w_{ij}^{+} x_i \cdot x_j + w_{ij}^{-}(1 - x_i \cdot x_j)\big) \\ &\geq \frac{3}{4} Z \end{aligned}$$

   Since SDP is a relaxation to the original problem, then it's a $\frac{3}{4}$-approximation.