# Machine-checked ZKP for NP-relations: Formally Verified Security Proofs and Implementations of MPC-in-the-Head

José Carlos Bacelar Almeida<sup>1</sup>, Manuel Barbosa<sup>1</sup>, Karim Eldefrawy<sup>2</sup>, Stéphane Graham-Lengrand<sup>2</sup>, Hugo Pacheco<sup>1</sup>, and Vitor Pereira<sup>1,2</sup>

<sup>1</sup>University of Porto (FCUP) and INESC TEC <sup>2</sup>SRI International

#### Abstract

MPC-in-the-Head (MitH) is a general framework that allows constructing efficient Zero Knowledge protocols for general NP-relations from secure multiparty computation (MPC) protocols. In this paper we give the first machine-checked implementation of this transformation. We begin with an EasyCrypt formalization of MitH that preserves the modular structure of MitH and can be instantiated with arbitrary MPC protocols that satisfy standard notions of security, which allows us to leverage an existing machine-checked secret-sharing-based MPC protocol development. The resulting concrete ZK protocol is proved secure and correct in EasyCrypt. Using a recently developed code extraction mechanism for EasyCrypt we synthesize a formally verified implementation of the protocol, which we benchmark to get an indication of the overhead associated with our formalization choices and code extraction mechanism.

Distribution Statement "A" (Approved for Public Release, Distribution Unlimited). This material is based upon work supported by DARPA under Contract No. HR001120C0086. Any opinions, findings and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the United States Government or DARPA.

#### Contents

1	Introduction	2					
2 Related work							
3	Preliminaries	5					
	3.1 Basic Primitives	5					
	3.2 Zero-Knowledge	6					
	3.3 Secure Multiparty Computation	7					
	3.4 Background on EasyCrypt						
4	Modular machine-checked proof of MPC-in-the-Head	8					
	4.1 ZK protocols	9					
	4.2 MPC protocols						
	4.3 Basic Primitives	14					
	4.4 Formalizing the MPC-in-the-Head Transformation	15					
5	Verified ZKP implementation supporting general relations	20					
	5.1 Secure arithmetic circuit evaluation	20					
	5.2 PRF-based commitment scheme						
	5.3 MitH-based verified evaluation engine for ZKP	25					
	5.4 Experimental results						
A	Abstract arithmetic gate example	28					

#### 1 Introduction

The MPC-in-the-Head (MitH) paradigm was introduced by Ishai, Kushilevitz, Ostrovsky and Sahai [19] as a new foundational bridge between secure multi-party computation (MPC) and zero-knowledge proof (ZK) protocols. A ZK protocol for an NP relation R(x, w) can be seen as a two party computation where a prover  $\mathcal{P}$  with input (x, w) and a verifier  $\mathcal{V}$  with input x jointly compute the boolean function f(x, w) that accepts the proof if and only if R(x, w) holds. This means that general feasibility results for two-party secure computation—where no honest majority can be assumed and malicious behavior must be considered—translate into feasibility results for ZK protocols for general relations.

However, the MitH paradigm shows that there exists an efficiency advantage in considering MPC protocols for n>2 and using a commit-challenge-response transformation to obtain a ZK protocol. Intuitively,  $\mathcal{P}$  emulates the execution of a secure computation protocol  $\pi$ , where n parties compute  $f(x,\bar{w})$  and commits to the views of all parties—here  $\bar{w}$  denotes a secret sharing of w among the n parties, where any subset of k shares reveals nothing about w.  $\mathcal{V}$  then chooses a (minority) subset of k parties uniformly at random, and  $\mathcal{P}$  opens the corresponding views.  $\mathcal{V}$  accepts the proof if these views are consistent with an honest execution of  $\pi$  by the k parties, where all k parties output 1. The MPC correctness of  $\pi$  and binding guaranteed by the commitment scheme suffice to ensure both completeness and a  $1/(\frac{n}{k})$  soundness error, where the latter can be reduced by repetition. Zero knowledge follows from the MPC security of  $\pi$  in the presence of an honest majority—the k opened views can be simulated without knowing w—as well as the hiding property of the commitment scheme—nothing about w is revealed about the remaining n-k parties' views.

The efficiency gain of MitH stems from two important observations: 1) that  $\pi$  only needs to satisfy a weak notion of security that allows for extremely efficient instantiations and 2) that the round complexity of  $\pi$  has no impact in the final protocol, since  $\pi$  is evaluated "in-the-head". A series of follow-up works [20, 16, 14, 7, 3, 11] demonstrated the efficiency and flexibility of protocol families inspired in the MitH core ideas, by exploring adaptations of this principle to specific MPC protocols and different variants of ZK protocols. One notable takeaway of these works is that MitH offers a high degree of flexibility, as it allows for instantiations that can efficiently handle relations that are more naturally expressed as both arithmetic and boolean circuits.

In this work we explore the elegant simplicity and modularity of the MPC-in-the-Head paradigm to obtain machine-checked security proofs and formally-verified implementations for ZK protocols supporting general relations. To the best of our knowledge this is the first end-to-end machine-checked development of ZK-protocols for all relations in NP. We focused on the MitH variant that can be instantiated with passively secure secret-sharing-based MPC protocols that tolerate two corrupt (i.e., semi-honest) parties. This allows us to build on an existing development that already provides a suitable instantiation for the underlying MPC protocol, secret sharing and commitment schemes. On the other hand, this opens the way for future work formalizing optimizations of the construction that follow along the same path [16, 14].

In more detail, our contributions are as follows:

- We formally specify and verify the foundational results of Ishai et al [19] and create a generic framework that permits the modular specification and formal verification of secret-sharing-based MPC protocols for general circuits, which can then be plugged into a formally-verified MitH generic transformation to produce provably secure ZK protocols for general relations. We give a complete formalization of the general construction and its potential instantiations in EasyCrypt.
- For the MPC protocol, we reuse the formally-verified implementation of the BGW protocol given in [15] instantiated to n=5 parties. For the commitment scheme, we have two instantiations; the first reuses the Pedersen commitment scheme formalized in [15], and the second relies on a more efficient PRF-based construction that we instantiate with HMAC [17]. The machine-checked security proofs for the MPC protocol and commitment scheme directly transpose those found in the literature, which highlights the fact that our framework can be instantiated with standard components.
- For the generic construction, we formalize the security proofs for completeness, soundness and zero-knowledge at two levels: 1) we give a proof of perfect completeness, as well as concrete bounds for the soundness error and for the zero-knowledge simulation strategy when the malicious party (prover and verifier, respectively) is executed once; 2) we then complement these results with formal machine-checked proofs of the meta arguments that require repetition using the EasyCrypt operator logic. In this way we can express in the EasyCrypt logic the repeated independent executions of the same adversary. We chose not to idealize the underlying commitment scheme and formalize the proof as in [19, Theorem 3.2], which makes the proof verification more challenging, but yields a result that can be instantiated with standard commitment schemes.

<sup>&</sup>lt;sup>1</sup>We note that different performance trade-offs can be explored when the underlying MPC protocol offers security against active adversaries; we do not consider these here because the most competitive constructions that follow this path [3, 11] follow a more intricate modular construction to convert the underlying MPC protocol to the resulting ZK protocol based on probabilistically checkable proofs (PCP). Formally verifying these protocol families is also an interesting direction for future work.

• We use the automatic code extraction mechanism for EasyCrypt proposed in [15] to automatically extract a verified implementation of the full Zero-Knowledge protocol produced by the MitH transformation and give preliminary benchmarking results. Our results are promising and open the way for future work where verified implementations of MitH protocols essentially match the performance of non-verified ones.

**Access to development.** Our EasyCrypt formalization of the MitH protocol, as well as its extracted code, can be found at https://github.com/SRI-CSL/high-assurance-crypto/tree/main/high-assurance-zk.

#### 2 Related work

We give an overview of the most relevant work in the field of computer-aided cryptography. This is an active area of research that aims to address the challenges of achieving formally robust, machine-checked, and practically efficient cryptographic protocols, which gave rise to several software developments and formal verification tools [5]. At the design level, tools allow describing the high level specification of the protocol and stating its security properties while managing the complexity of the security proofs via a modular design of the various components. At the implementation level, tools help to establish the functional correctness of (often aggressively optimized) implementations w.r.t. the specification. In our approach, the specification considers the security of abstract cryptographic primitives and representative concrete instantiations; a correct-by-construction implementation for each concrete instantiation is then automatically extracted from the specification. This highlights the modularity and generality of the formalization and grants more flexibility for exploring different instantiations and extracting the respective reference implementations, in contrast to devising and verifying a highly optimized for a particular instantiation.

Outside the realm of computer aided cryptography, cryptographers have devised methodological frameworks for performing security proofs, such as code-based game playing [8]. The motivation for these frameworks is to systematize and the decompose security arguments to minimize the likelihood of errors, even in "pen-and-paper" proofs. The EasyCrypt tool supports a general-purpose relational program logic that is close to code-based game playing techniques. Therefore, EasyCrypt proofs tend to follow the structure of pen-and-paper proofs.

Zero Knowledge protocols are a fast-moving area within cryptography, and many protocols exist both for specific proof goals and for settings that require a flexible solution that can be used for any relation. Only a very small part of this field has been studied from the perspective of computer aided cryptography. The work in [6] was the first to formalize a special class of  $\Sigma^{\phi}$ -protocols in CertiCrypt, a predecessor of EasyCrypt implemented as a Coq library, and to prove the security of general and and or composability theorems for  $\Sigma^{\phi}$ -protocols. The more recent work from [13] restates many of these results for  $\Sigma$ -protocols in CryptHOL. It additionally formalizes abstract and concrete commitment scheme primitives and proves a construction of commitment schemes from  $\Sigma$ -protocols.

The most significant machine checked endeavor for ZK is the work in [4], that developed a full-stack verified framework for developing ZK proofs. The framework encompasses a non-verified optimizing ZK compiler that translates high-level ZK proof goals to C or Java implementations, and a verified compiler that generates a reference implementation. The machine checked effort lies in proving that, for any goal, the reference implementation satisfies the ZK properties and that the optimized implementation has the same observable behavior as the reference implementation. The core of the verified compiler builds on top of the results from [6],

extended with and compositions of  $\Sigma^{GSP}$ -protocols, and generates CertiCrypt proof scripts for automatically proving the equivalence of the two implementations.

There is nowadays a vast body of MPC protocols and frameworks, some of which have been formally verified using machine-checked tools. CircGen [1] is a verified compiler translates C programs into boolean circuits by extending the CompCert C compiler with an additional backend translation to Boolean circuits. This back-end can then be used to feed to a EasyCrypt machine-checked implementation of Yao's 2-party secure function evaluation protocol. The work in [18] formalizes in EasyCrypt the n-party MPC protocol due to Maurer [21] for the actively secure case. The work in [15] provides verified implementations of pro-actively secure MPC, including an EasyCrypt formalization of the BGW [10] MPC protocol for passive and static active adversaries that we adapt and reuse in this paper.

#### 3 Preliminaries

We give in this section the formal cryptographic definitions that we use in our formalization, which are all standard. We follow [19] closely. We also give a short overview of the EasyCrypt features required to understand the presentation of our development.

#### 3.1 Basic Primitives

The MitH construction requires a very simple form of secret sharing over the inputs of the MPC protocol, although the underlying MPC protocol may internally use additional properties we will not mention here.

We also require standard commitment schemes. Note that, as the MPC protocols and secret sharing protocols we use to instantiate the MitH construction are information theoretically secure, the security of the commitment scheme determines the class of adversaries over the security of the MitH construction. In particular, the standard soundness property only holds if the commitment is statistically hiding; if the commitment scheme is only computationally binding, then we obtain a ZK argument.

**Definition 3.1** (Secret Sharing Scheme). For a secret space W, which we will take to be the same as a share space, a secret sharing scheme for n parties is defined by two ppt algorithms:

- The probabilistic sharing algorithm  $\mathsf{Shr}(w)$  takes a secret  $w \in \mathcal{W}$  and returns a sharing  $\bar{w} \in \mathcal{W}^n$ .
- The deterministic reconstruction algorithm  $\mathsf{USh}(\bar{w})$  takes a sharing  $\bar{w} \in \mathcal{W}^n$  and returns a secret  $w \in \mathcal{W}$ .

A secret sharing scheme should satisfy the following properties:

- CORRECTNESS: For all secrets  $w \in W$  and all sharings  $\bar{w} \in [Shr(w)]$ , we have that  $w = USh(\bar{w})$ . Moreover, USh is a total function over  $W^n$ , which means that all sharings determine a unique secret.
- t-Privacy Any subset of t shares, for  $0 \le t < n$  can be simulated efficiently by sampling from a fixed public distribution. Formally, we require that there exist a simulated share distribution ShS such that, for any subset  $S \subseteq [n]$  such that |S| = t and all  $w \in W$ :

$$\{ [\bar{w}]_{i \in S} : \bar{w} \leftarrow \$ \operatorname{Shr}(w) \} \equiv \{ \hat{w} : \hat{w} \leftarrow \$ \operatorname{ShS} \}$$

**Definition 3.2** (Commitment Scheme). For a message space  $\mathcal{M}$ , a commitment space  $\mathcal{C}$  and an opening space  $\mathcal{K}$ , a commitment scheme is defined by two ppt algorithms:

- The probabilistic commitment algorithm Com(m) takes a secret  $m \in \mathcal{M}$  and returns a pair (c, k) where  $c \in \mathcal{C}$  is a commitment and  $k \in \mathcal{K}$  is an opening.
- The deterministic verification algorithm Ver(m, c, k) takes a message  $m \in \mathcal{M}$ , a commitment  $c \in \mathcal{C}$ , and an opening  $k \in \mathcal{K}$  and it returns 1 or 0 indicating success or failure, respectively.

A commitment scheme should satisfy the following properties:

- CORRECTNESS: For all messages  $m \in \mathcal{M}$ , all commitments  $c \in \mathcal{C}$  and openings  $k \in \mathcal{K}$ , such that  $(c, k) \in [\mathsf{Com}(m)]$ , we have that  $\mathsf{Ver}(m, c, k) = 1$ .
- BINDING: The commitment scheme is computationally (resp. statistically) binding if the probability that the following game returns 1 is bounded by a small  $\epsilon_b$  when  $\mathcal{A}$  is a ppt (resp. a potentially unbounded) algorithm:
  - the binding game first runs an adversary A who outputs a tuple (m, k, m', k', c);
  - the game then runs Ver(m, c, k) and Ver(m', c, k').
  - the game returns 1 if and only if both verifications are successful and  $m \neq m'$ .
- HIDING: The commitment scheme is computationally (resp. statistically) hiding if the left or right indistinguishability advantage of an adversary A in the following game is bounded by a small  $\epsilon_h$  bias, when A is a ppt (resp. a potentially unbounded) algorithm:
  - the hiding game first samples a random coin b and runs adversary A that chooses two messages  $m_0, m_1 \in \mathcal{M}$ ;
  - the game then computes  $(c,k) \leftarrow \$Com(m_b)$ , which it provides to A;
  - eventually A terminates outputting a guess b' and the game outputs 1 if b = b' and 0 otherwise.

The advantage of A is defined in the standard way as  $|p-1/2| \leq \epsilon_h$ , where p is the probability that the hiding game returns 1.

#### 3.2 Zero-Knowledge

A NP-relation R(x, w) is an efficiently decidable and polynomially bounded binary relation, which we see as a boolean function. This implies that, for all (x, w), if R(x, w) = 1, then  $|w| \le p(|x|)$  for some fixed polynomial p.

A ZK protocol for a NP-relation R(x, w) is defined by two ppt interactive algorithms, a prover  $\mathcal{P}$  and a verifier  $\mathcal{V}$ . The prover takes a NP statement x and a corresponding witness w; the verifier is only given the statement x. The prover and the verifier interact—in this paper we consider only three-pass commit-challenge-response protocols—until eventually the verifier outputs 1 or 0 indicating success or failure, respectively. The view of  $\mathcal{V}$  is defined as its input x, its coin tosses and all the messages that it receives.<sup>2</sup>

**Definition 3.3** (**Zero-knowledge proof**). A protocol  $(\mathcal{P}, \mathcal{V})$  is a zero-knowledge proof protocol for the relation R if it satisfies the following requirements:

<sup>&</sup>lt;sup>2</sup>Throughout the paper we will often see interactive protocols as a sequence of calls to *next-message* functions: these functions take the coin tosses of the corresponding party and all received messages and they deterministically define the next message transmitted by the party. In particular, next message functions take partial views and define the next message to be sent. A complete view of a party defines its output, which we see as a particular case of the next-message function output.

- Completeness: In an honest execution, if R(x, w) = 1, then the verifier accepts with probability 1.
- SOUNDNESS: For every malicious and computationally unbounded prover  $\mathcal{P}^*$ , there is a negligible function  $\epsilon(\cdot)$  such that, if R(x,w) = 0 for all  $w \in \{0,1\}^{p(|x|)}$ , then  $\mathcal{P}^*$  can make  $\mathcal{V}$  accept with probability at most  $\epsilon(|x|)$ .
- ZERO-KNOWLEDGE: For any malicious ppt verifier  $\mathcal{V}^*$ , there exists a ppt simulator  $\mathcal{S}^*$ , such that the view of  $\mathcal{V}^*$  when interacting with  $\mathcal{P}$  on inputs (x, w) for which R(x, w) = 1, is computationally indistinguishable from the output of  $\mathcal{S}$  on input x.

We will also consider zero-knowledge protocols that have a constant (non-negligible) soundness error  $\epsilon$ , in which cases the soundness error will be specified.

#### 3.3 Secure Multiparty Computation

The MitH paradigm builds on secure function evaluation protocols that assume synchronous communication over secure point-to-point channels.

Let n be the number of parties, which will be denoted by  $P_1, \ldots, P_n$ . All players share a public input x, and each player  $P_i$  holds a local private input  $w_i$ . We consider protocols that can securely compute a n-input function f that maps the inputs  $((x, w_1), \ldots, (x, w_n))$  to a n-tuple of boolean outputs.

A protocol  $\Pi$  is specified via its next message function. That is,  $\Pi(i, x, w_i, r_i, (m_1, \ldots, m_j))$  returns the set of n messages sent by  $P_i$  in round j+1, given the public input x, its local input  $w_i$ , its random input  $r_i$ , and the messages  $m_1, \ldots, m_j$  it received in the first j rounds. The output of the next message function  $\Pi$  may also indicate that the protocol should terminate, in which case  $\Pi$  returns the local output of  $P_i$ .

The view of  $P_i$ , denoted by  $V_i$ , includes  $w_i$ ,  $r_i$  and the messages received by  $P_i$  during the execution of  $\Pi$ . Note that  $\Pi$  and  $V_i$  fully define the set of messages sent by  $P_i$  and also its output. The following notion of consistent views and relation between local and global consistency are important for the MitH transformation.

**Definition 3.4** (Consistent views). A pair of views  $V_i, V_j$  are consistent (with respect to the protocol  $\Pi$  and some public input x) if the outgoing messages implicit in  $V_i$  are identical to the incoming messages reported in  $V_j$  and vice versa.

**Lemma 3.1** (Local vs. global consistency [19]). Let  $\Pi$  be a n-party protocol as above and x be a public input. Let  $V_1, \ldots, V_n$ ) be an n-tuple of (possibly incorrect) views. Then all pairs of views  $V_i, V_j$  are consistent with respect to  $\Pi$  and x if and only if there exists an honest execution of  $\Pi$  with public input x (and some choice of private inputs  $w_i$  and random inputs  $r_i$ ) in which  $V_i$  is the view of  $P_i$  for every  $1 \le i \le n$ .

We consider security of protocols in the semi-honest model, where we define correctness and privacy as follows.

**Definition 3.5** (Correctness). Protocol  $\Pi$  securely computes function  $f((x, w_1), \ldots, (x, w_n))$  with perfect correctness if, for all inputs  $x, w_1, \ldots, w_n$ , the probability that the output of some player is different from the output of f is 0, where the probability is over the independent choices of the random inputs  $r_1, \ldots, r_n$ .

<sup>&</sup>lt;sup>3</sup>Throughout the paper, when we say that two distributions are computationally (resp. statistically) indistinguishable we mean that, for all ppt (resp. potentially unbounded) distinguishers returning a bit, the probability that the distinguisher returns 1 when fed with a value sampled from either of the distributions changes by a small quantity  $\epsilon$ . For perfect indistinguishability,  $\epsilon$  is 0 and we consider unbounded distinguishers.

**Definition 3.6** (t-Privacy). We say that  $\pi$  securely computes f with perfect t-privacy, for  $1 \leq t \leq n$ , if there is a ppt simulator S such that, for any inputs  $x, w_1, \ldots, w_n$  and every set of corrupted parties  $T \subseteq [n]$ , where  $|T| \leq t$ , the joint view  $(V_{i_1}, \ldots, V_{i_k})$  of the parties in  $T = \{i_1, \ldots, i_k\}$  is computationally indistinguishable from  $S(T, x, [w_i]_{i \in T}, [f(x, w_1, \ldots, w_n)_i]_{i \in T})$ .

**Remark.** For the concrete protocols we consider to instantiate the MitH transformation,  $\bar{w} = (w_1, \ldots, w_n)$  will be a secret sharing of the witness that is input to the ZK protocol. Furthermore, function f will be the boolean function that computes R(x, w), where w is the secret that results from the reconstruction of  $\bar{w}$ , and outputs the result to all parties.

#### 3.4 Background on EasyCrypt

EasyCrypt is an interactive proof assistant tailored for the verification of the security of cryptographic constructions in the computational model. EasyCrypt adopts the code-based approach, in which primitives, security goals and hardness assumptions are expressed as probabilistic programs. The EasyCrypt proof system incorporates an ambient logic—for first-order and higher-order logic reasoning through a set of interactive proof tactics—and a probabilistic relational Hoare logic—for reasoning about programs written in an imperative C-like language, as well as establishing equality results between two program executions. Moreover, EasyCrypt uses formal tools from program verification, particularly SMT provers, that can be used in cooperation with the ambient logic to discard proof goals.

EasyCrypt supports a high degree of modular reasoning via its theory system. In short, a theory is a collection of types and operators that can later be refined or extended with more data structures. Alternatively, it can be seen as an abstract class that fixes the interface that *objects* (i.e., concrete instantiations of that theory) must follow. EasyCrypt also provides a cloning mechanism that allows theories to refine the types and operators of more abstract theories, or to combine the elements of existing ones.

Another relevant EasyCrypt specificity that we widely use in our formalization is the ability to establish a *module type* (i.e., an abstract interface), that defines the signature of procedures that an EasyCrypt module implementing that type are expected to follow. Module types are very useful because the EasyCrypt proof system permits the universal quantification over all possible implementations of the module type. Moreover, they can also be used as parameters to other modules, in order to define generic constructions based on abstract primitives.

The EasyCrypt development, as well as the SMT solvers used in cooperation with the Easy-Crypt proof system are part of our trusted computing base (TCB). The EasyCrypt code extraction tool has been recently developed in [15] is also part of the TCB. This tool allows to extract OCaml executable code from an EasyCrypt proof script where code is defined via functional operators.

### 4 Modular machine-checked proof of MPC-in-the-Head

Our EasyCrypt development defines an abstract and modular infrastructure that follows the general MitH transformation, and can be instantiated with different concrete components. Figure 1 depicts the relation between these different components in our formalization, and includes abstractions for the syntax and security of ZK protocols, secret sharing schemes, MPC protocols and commitment schemes, as introduced in Section 3. Our syntax for MPC protocols fixes an abstract notion of a circuit, which is used to formalize the semantics of evaluating a functionality. On top of these abstractions we specify the MitH transformation and prove that

it can be used to construct ZK protocols for NP-relations expressed as circuit satisfiability. Finally, we formalized concrete instantiations of each component, which are then put together to obtain fully concrete instantiations of ZK protocols via the general MitH transformation. Our concrete instantiations can handle proof goals expressed using *arithmetic* circuits, as we rely on the BGW [9] protocol under the hood. However, our framework is general enough to allow for different instantiations, including MPC protocols that rely on different circuit representations. In what follows, we will provide a more detailed view of our formalization, resorting directly to snippets of EasyCrypt code.

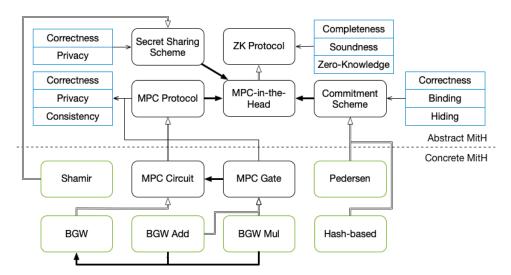


Figure 1: Bird's eye view of the relation between different parts of our formalization. The horizontal dotted line separates abstract MitH components (above) from our proof-of-concept concrete instantiations (below). Rounded rectangles represent cryptographic constructions. Rounded green rectangles represent concrete instantiations of the respective cryptographic primitives. Blue rectangles depict security definitions. Arrows  $\rightarrow$  represent security definitions applied to the connected cryptographic constructions. Double arrows  $\Rightarrow$  entail that a component instantiates a more abstract component, and in particular provides concrete proofs for its security definitions. Finally, if two boxes are connected via a bold arrow  $\Rightarrow$ , then one is a sub-component of the other.

#### 4.1 ZK protocols

**Syntax.** The ZKProtocol theory fixes the syntax of ZK protocols in EasyCrypt, i.e., the types and operators that must be defined by a concrete ZK protocol. We fix the commit-challenge-response three pass protocol structure, since this is all we need for the MitH transformation.

```
theory ZKProtocol.

type witness_t.

type statement_t.

op relation: witness_t → statement_t → bool.

op language(x: statement_t) = ∃ w, relation w x = true.

type prover_input_t = witness_t * statement_t.

type verifier_input_t = statement_t.

type prover_state_t.

type verifier_state_t.

type prover_rand_t.
```

```
type verifier\_rand\_t.
  type prover\_output\_t = unit.
  type verifier\_output\_t = bool.
  type commitment_t.
  type challenge_t.
  type response_t.
  op commit : prover\_rand\_t \rightarrow prover\_input\_t \rightarrow prover\_state\_t * commitment\_t.
  op challenge: verifier\_rand\_t \rightarrow verifier\_input\_t \rightarrow commitment\_t \rightarrow verifier\_state\_t * challenge\_t.
  op response : prover\_state\_t \rightarrow challenge\_t \rightarrow response\_t.
  op check : verifier\_state\_t \rightarrow response\_t \rightarrow bool.
  type trace_t = commitment_t * challenge_t * response_t.
  op protocol (r: prover_rand_t * verifier_rand_t) (x: prover_input_t * verifier_input_t) : trace_t * (
       prover\_output\_t * verifier\_output\_t) =
    let (r_p, r_v) = r in let (x_p, x_v) = x in
    let (st_p,c) = commit r_p x_p in
    let (st_v,ch) = challenge r_v x_v c in
    let r = response st_p ch in
    let b = check st_v r in ((c, ch, r), ((),b)).
end ZKProtocol.
```

Types that are undefined at this level must be specified by each protocol. This is the case, for example, for the type of witnesses and statements, but not for the outputs of the prover and verifier, which are hardwired in the syntax to be the singleton type and a boolean value, respectively. Each protocol is associated with a relation, which at this level is modeled as an abstract boolean function. Finally, the theory also defines what it means to honestly execute the protocol via the protocol operator.

**Completeness.** The completeness game is parameterized by a randomness generator algorithm R. Each protocol needs to define what it means to sample the random coins of the prover and verifier using this mechanism.<sup>4</sup> The EasyCrypt formalization is a direct translation of the completeness requirement given in Section 3.

```
module Completeness(R : Rand_t) = {
    proc main(w : witness_t, x : statement_t) : bool = {
        var r_p, r_v, tr,y;
        (r_p, r_v) < @ R.gen();
        (tr,y) ← protocol (r_p, r_v) ((w,x), x);
        return (snd y);
    }
}.</pre>
```

A completeness claim in EasyCrypt can be written as:

```
\forall w x, relation w x \Rightarrow Pr [Compleneness(R).main(w,x) : res ] = 1%r.
```

<sup>&</sup>lt;sup>4</sup>This pattern will appear in the remaining games, so we will omit further descriptions of randomness generation modules.

**Soundness.** For the soundness definition, we need to quantify over potentially malicious provers. In EasyCrypt this is done by defining a module type, i.e., the interface that the adversary exposes Module type <code>MProver\_t</code> specifies this interface.

```
module type MProver_t = {
    proc commitment (x: statement_t) : commitment_t
    proc response(x : statement_t, c : commitment_t, ch : challenge_t) : response_t
}.
```

The soundness property can now be expressed as a game which is parameterized by an attacker of this type. This allows us to quantify universally over malicious provers.

```
module Soundness(RV : RandV_t, MP : MProver_t) = {
    proc main(x : statement_t) : bool = {
        var r_v, c, ch,st_r, resp;
        r_v < @ RV.gen();
        c < @ MP.commitment(x);
        (st_r, ch) ← challenge r_v x c;
        resp < @ MP.response(x, c, ch);
        return (!language x ∧ check st_r resp);
    }
}.
```

A soundness claim in EasyCrypt can be written as:

```
\forall x MP, Pr [Soundness(RV,MP).main(x) : res] \leq epsilon.
```

**Zero Knowledge.** We formalize the zero knowledge property using the following experiment in which a (malicious) verifier interacts with an honest prover or with the simulator. We will see below how the evaluator module E is defined for either case.

```
module ZKGame (D : Distinguisher_t) (RP : RandP_t) (E : Evaluator_t) (MV : MVerifier_t) = {
    proc main(w : witness_t, x : statement_t) : bool option = {
        var rp, b, check, tr;
        rp <@ RP.gen();
        (check,tr) <@ E.eval(w,x,rp);
        b <@ D.guess(w,x,check,tr);
        return b;
    }
}.</pre>
```

For both cases we need to define the type of malicious verifier modules  $MVerifier_t$ , which we omit for brevity. To define the evaluator module for the simulated view, we first specify a type for the simulator which acts like a prover.

```
module type Simulator_t = {
    proc commitment(x : statement_t) : commitment_t option
    proc response(x : statement_t, ch : challenge_t) : response_t option
}.
```

Note however that, unlike a prover, the simulator response may signal an abort (option means that the procedure may sometimes fail to produce a result), often triggered by a mismatch between the received challenge and some initial guess. Of course, we shall bound the probability of such simulation failures as it will have an immediate impact on the Zero Knowledge property.

The evaluator module that animates the interaction between the simulator and the malicious verifier is defined as follows.

```
module (IdealEvaluator(S : Simulator_t) : Evaluator_t) (MV : MVerifier_t) = {
  proc eval(w : witness\_t, x : statement\_t, rp : prover\_rand\_t) : bool option = \{
       var c, ch, oresp, resp, acceptance, ret, oc;
       ret \leftarrow None:
       oc < @ S.gen\_commitment(x);
       if (oc \neq None) {
         c \leftarrow oget oc:
         ch < @MV.gen\_challenge(x, c);
         oresp < @ S.gen_response(x, ch);
         if (oresp \neq None) {
           resp \leftarrow oget oresp;
           acceptance \leftarrow check \ x \ c \ ch \ resp;
           ret \leftarrow Some (acceptance, (oget oc, ch, oget oresp));
       }
      return ret;
  }
}.
```

For a real protocol execution, the ZKGame shall be parametrized by the following evaluator module that encodes the normal protocol execution, and never fails.

```
module (RealEvaluator : Evaluator_t) (MV : MVerifier_t) = {
    proc eval(w : witness_t, x : statement_t, rp : prover_rand_t) = {
        var st_p, c, ch, resp,b;
        (st_p, c) \( \cdot \) commit rp (w,x);
        ch <@ MV.gen_challenge(x, c);
        resp \( \cdot \) response st_p ch;
        acceptance \( \cdot \) check x c ch resp;
        return Some (acceptance, (c,ch,resp));
    }
}.</pre>
```

A zero-knowledge claim, for a concrete simulator S in EasyCrypt can be written as:

```
\forall w x (D <: Distinguisher_t) (MV <: MVerifier_t), relation w x \Rightarrow | ZKGame(RP,RealEvaluator,MV).main(w,x) : res ] - ZKGame(RP,IdealEvaluator(S),MV)).main(w,x) : res ] | \leq epsilon
```

#### 4.2 MPC protocols

Our formalization of MPC protocols shares many similarities with the ZKProtocol. The main difference, in addition to considering an arbitrary number of parties, is that we need to consider protocols that are parametric on an abstract type for circuits (i.e., a way to represent n-input to n-output computations) and an abstract operator that defines what it means to evaluate an arbitrary circuit. The Protocol theory is similar to that used in [15].

```
theory MPCProtocol.

const n : \{int \mid 2 \le n\} as n_pos.

const t : \{int \mid 0 \le t < n\} as t_pos.

type pid_-t.

type pid_-t list.

type circuit_-t.
```

```
type sinput_{-}t.
  type input_{-}t = pinput_{-}t * sinput_{-}t.
  type output_t.
  op f: circuit_t \to (pid_t * input_t) \ list \to (pid_t * output_t) \ list
  type rand_t.
  type msgs_t.
  type in_messages_t.
  type out_messages_t.
  \mathbf{op} \ \mathrm{get\_messages\_from} : \mathit{pid\_t} \to \mathit{in\_messages\_t} \to \mathit{msgs\_t}.
  op get_messages_to : pid_t \rightarrow out_messages_t \rightarrow msgs_t.
  type trace_t = in_messages_t.
  type view_t = input_t * rand_t * in_messages_t.
  type views_t = (pid_t * view_t) list.
  op protocol : circuit_t \to (pid_t * rand_t) list \to (pid_t * input_t) list \to
                                                       (pid_{-}t * trace_{-}t) list * (pid_{-}t * output_{-}t) list.
  op out_messages : circuit_t \to pid_t \to view_t \to out_messages_t.
  op local_output : circuit_t \to pid_t \to view_t \to output_t.
end MPCProtocol.
```

Party inputs  $input_t$  can be defined as having a public and a secret part, and these should not be interpreted as a single input wire to the circuit. Indeed, it is the circuit evaluation operator f that defines the semantics of evaluating a circuit on given inputs. The operators that define the behavior of parties are out\_messages and local\_output, which match the next-message function approach introduced in Section 3. Finally, the protocol operator is used to define the global protocol evaluation, which allows for some flexibility in defining the message scheduling.

Given these definitions we can capture the notion of pairwise view consistency as the following EasyCrypt predicate, where pinput is a simple operator that extracts the public input from the view; the get\_messages\_from, get\_messages\_to and in\_messages operators simply project views and lists of output messages to the relevant party identifiers for comparison.

```
op consistent_views (c : circuit_t) (xp : pinput_t) (vi vj : view_t) (i j : pid_t) = let (xi, ri, ti) = vi in let (xj, rj, tj) = vj in let outj = out_messages c j vj in valid_rand c ri \land valid_rand c rj \land xi.'1 = xp \land xj.'1 = xp \land valid_circuit_trace c ti \land valid_circuit_trace c tj \land get_messages_from j ti = get_messages_to i tj \land get_messages_from i tj = get_messages_to j ti.
```

The correctness of MPC protocols is formalized analogously to completeness for ZK protocols, so we omit some details. The t-privacy property is formalized using the same approach used for the zero-knowledge property, the main difference being that the simulator must now construct t views to be fed to a distinguisher. We give here only the details of the real view evaluator and the ideal view evaluator for a fixed simulator S with given module type. In the real-world, the extract\_views operator returns the views of the corrupt parties. In the ideal world, the extract\_inputs operator extracts the full inputs of corrupt parties. This is sufficient

because we assume throughout that the public inputs to all parties are identical, consistently with Section 3.

```
module type Simulator_t = \{
  proc simulate(c : circuit_t, xs : (pid_t * input_t) list, rs : (pid_t * rand_t) list, cr : pid_t list, ys :
       ProtocolFunctionality.output_{-}t): views_{-}t
}.
module RealEvaluator = {
  proc eval(c: circuit_t, xs: (pid_t * input_t) list, cr: pid_t list, rs: (pid_t * rand_t) list) = {
      var tr,y,vsc;
    (tr, y) \leftarrow protocol c xs rs;
    vsc \leftarrow extract\_views cr tr;
    return vsc;
}.
module IdealEvaluator (S : Simulator_{-}t) = {
  proc eval(c: circuit_t, xs: (pid_t * input_t) list, cr: pid_t list, rs: (pid_t * rand_t) list) = {
    var ins.vsc:
    ins \leftarrow extract\_inputs \ cr \ xs;
    y \leftarrow f c xs;
    vsc \leftarrow S. simulate(c, xsc, rs, cr, y);
    return vsc;
```

A t-privacy claim for a n-party protocol can be written in EasyCrypt as follows.

```
 \begin{tabular}{ll} $\forall c \ x \ aux, \\ & | \ Pr \ [ \ PrivGame(D,R,RealEvaluator).main(c,x,aux) : res \ ] \ - \\ & Pr \ [ \ PrivGame(D,R,IdealEvaluator(S)).main(c,x,aux) : res \ ] \ | \ \le \ epsilon \end{tabular}
```

#### 4.3 Basic Primitives

**Secret Sharing.** The EasyCrypt formalization of a secret sharing scheme is straightforward. The syntax is defined as follows.

```
theory SecretSharingScheme.

const n: \{int \mid 2 \leq n\} as n_pos.

const t: \{int \mid 0 \leq t < n\} as t_pos.

type pid\_t.

op pid_set: pid\_t list.

type secret\_t.

type share\_t.

type share\_t = (pid\_t*share\_t) list.

type rand\_t.

op share: rand\_t \rightarrow secret\_t \rightarrow (pid\_t*share\_t) list.

op public_encoding: secret\_t \rightarrow (pid\_t*share\_t) list.

op pub_reconstruct: pid\_t \rightarrow share\_t \rightarrow secret\_t.

op reconstruct: (pid\_t*share\_t) list \rightarrow secret\_t.

end SecretSharingScheme.
```

The notion of t-privacy for a fixed share simulator S is defined by the following real and ideal evaluator.

```
module type Simulator_t = \{
  proc simulate(aux : aux_t, cr : pid_t list) : shares_t
}.
module RealEvaluator = {
  proc share(aux : aux_t, r : rand_t, s : secret_t, cr : pid_t list) : shares_t = \{
    var ss, ssc;
    ss \leftarrow share r s;
    ssc \leftarrow map  (fun pid \Rightarrow (pid, oget (assoc ss pid))) cr;
    return ssc;
}.
module IdealEvaluator (S : Simulator_{-}t) = {
  proc share(aux : aux_t, r : rand_t, s : secret_t, cr : pid_t list) : shares_t = \{
    var ssc;
    ssc \leftarrow S. simulate(aux,cr);
    return ssc;
}.
```

A t-privacy claim for an n-party secret sharing scheme can be written in EasyCrypt as follows.

**Commitment schemes** We close this section with a brief description of our abstract formalization of commitment schemes, which is essentially the same as that used in [15].

```
theory CommitmentScheme.

type msg\_t.

type rand\_t.

type commitment\_t.

type commitment\_t.

type commit\_info\_t = commitment\_t * opening\_string\_t.

op commit: rand\_t \to msg\_t \to commit\_info\_t.

op verify: msg\_t \to commit\_info\_t \to bool.

end CommitmentScheme.
```

The EasyCrypt definition of the correctness, binding and hiding properties exactly match the definitions given in Section 3 and the follow the style of the definitions presented in this section. We omit them for brevity.

#### 4.4 Formalizing the MPC-in-the-Head Transformation

Our formalization of MPC-in-the-Head relies only on the previous high-level abstractions and can be instantiated with any MPC protocol, commitment scheme and secret sharing scheme that meet the given syntax, correctness and security requirements. We followed the modular structure of [19] in our formalization, so the MPCInTheHead theory below relies on sub-theories for the underlying components. Our result can be instantiated with any MPC protocol that

supports an arbitrary number of parties, but here we fix n = 5, as this allows us to explicitly unfold the hybrid arguments that appear in the proof, which reduces proof complexity.

```
theory MPCInTheHead.
  type witness_t.
  type instance_t.
  clone import SecretSharingScheme as SS with
    type secret_t \leftarrow witness_t, op n = 5, op t = 2.
  clone import MPCProtocol as MPC with
    op n = SS.n, op t = SS.t,
    type pid_t = SS.pid_t, op pid_set = SS.pid_set,
    type pinput_t = instance_t,
    type sinput_{-}t = share_{-}t,
    type output_{-}t = bool.
  clone import CommitmentScheme as CS with type msg_{-}t = view_{-}t.
  op relc : circuit_t.
  type statement_t = statement_instance_t.
  axiom good_circuit (x : statement_instance_t) w :
        valid_circuit relc \land
        \forall ss ss'.
           \exists r r', ss = share w r \Rightarrow ss' = share w r' \Rightarrow
            f relc (mkseq (fun i \Rightarrow (x,(nth witness ss i).'2)) SS.n) =
                f relc (mkseq (fun i \Rightarrow (x,(nth witness ss' i).'2)) SS.n).
  op relation(w : witness_t ,x : statement_instance_t) =
     \exists (ss : (pid_t * share_t) \ list),
         (w = SS.unshare ss \wedge
           let ins = mkseq (fun i \Rightarrow (x,(nth witness ss i).'2)) SS.n in
            let outs = f relc ins in
                all (fun o \Rightarrow o) outs).
```

Note that the relation that is associated with the ZK protocol is defined as the acceptance by all parties of the circuit computed by the MPC protocol. We bind the secret type of the secret sharing scheme to that of witnesses and the input types of the MPC protocol to statements and secret shares, as expected. Intuitively, it suffices that the prover is able to find a sharing of w such that the MPC circuit accepts for the relation to hold; naturally, we restrict our attention to circuits that are oblivious of which particular sharing is chosen by the prover (axiom good\_circuit). However, the notion of circuit, witness and statement are still abstract. The output type of the MPC protocol parties is set to bool and commitments are restricted to operate over protocol views. Apart from these refinements, the other type definitions remain abstract and can be instantiated arbitrarily.

The protocol itself instantiates the ZKProtocol theory and fixes the necessary types accordingly: the prover's commitment message is a tuple of commitments corresponding to the views of the five MPC protocol parties; the challenge returned by the verifier is a pair of party identifiers, and the response returned by the prover is a pair of opening strings for the selected views.

```
type commitment_t = (pid_t * CS.commitment_t) list.

type challenge_t = pid_t * pid_t.

type response_t = (MPC.view_t * CS.opening_string_t) * (MPC.view_t * CS.opening_string_t).
```

We omit the details of the types of the randomness taken by prover and verifier. In the case of the prover this includes all the randomness required for secret sharing, emulating the MPC protocol and generating the commitments. The case of the verifier is simpler, since its randomness includes only the choice of party identifiers for the challenge.

We give a short snippet of the commitment generation operator, i.e., the first stage of the prover in the MitH construction, where the prover emulates the MPC protocol execution and commits to the view of party P1. The full code replicates the same process for all parties.

The challenge and response steps are simpler to formalize. The challenge is simply a random sampling of a pair of identifiers, which translates to copying random values from the randomness input. The response selects the views and opening strings corresponding to the selected parties, which are kept as internal state by the prover using the <code>get\_party\_committed\_view</code> operator. We give the <code>EasyCrypt</code> code below.

Finally the verifier checks the response as follows.

```
op check (xv : verifier_input_t) (cs : commitment_t) (rv : verifier_rand_t) (r : response_t) : bool =
let (c,x) = xv in
let (i,j) = rv in

let (vosi, vosj) = r in
let (vi, osi) = vosi in
let (vj, osj) = vosj in
```

```
let (xi,ri,tri) = vi in
let (xj,rj,trj) = vj in

let ci = get_party_commitment i cs in
let cj = get_party_commitment j cs in

CS.verify vi (ci,osi) \( \Lambda \) CS.verify vj (cj,osj) \( \Lambda \)
MPC.consistent_views c x vi vj i j \( \Lambda \)
MPC.local_output c i (xi,ri,tri) \( \Lambda \) MPC.local_output c j (xj,rj,trj).
```

The check operator verifies three conditions: 1) that the commitment openings are valid wrt to the provided views; 2) the provided views are consistent with each other (using operator consistent\_views); and 3) that the local output reported by the selected parties is *true*, which implies that the MPC protocol execution for these parties reported that the relation between statement and witness holds.

Completeness Our completeness theorem states that the MitH construction has perfect completeness assuming perfect correctness for the underlying components. Formally, in EasyCrypt we prove that for all valid randomness samplers  $\mathtt{R}$ , all statements  $\mathtt{x}$  and all witnesses  $\mathtt{w}$ , the following holds.

```
lemma completeness w x:

relation w x \Rightarrow Pr [ Compleneness(R).main(w,x) : res ] = 1%r.
```

The proof intuition is as follows. The good\_circuit restriction imposes that the circuit that defines the relation is well behaved, in the sense that, for all sharings  $\bar{w}, \bar{w}' \in [\mathsf{Share}(w)]$ , the circuit outputs the same consistent values for all parties. Then, if the MPC protocol is correct, it will correctly compute the relation of the ZK proof system and every two views will be pairwise consistent (by Lemma 3.1), Since this is an honest execution, the commitments are well constructed and the openings will be valid, as per the correctness property of the commitment scheme.

**Soundness** Our MitH implementation is sound for a soundness error of  $1 - 1/\binom{n}{2} + \epsilon$  in a single execution. Here n is the number of parties in the MPC protocol and  $\epsilon$  is bounded using the binding property of the commitment scheme. The statement in EasyCrypt is as follows:

```
lemma soundness xc (MP <: MProver_t) : !language (snd xc) ⇒ Pr [ Soundness(RV,MP).main(x) : res ] ≤ 2%r * Pr [Binding(A).main() : res ] + (1 - 1/\binom{n}{2})
```

The proof is done by a sequence of two game hops. In the first hop, we specify a bad event that checks if the dishonest prover opened a commitment for the first view requested in the challenge that is not the originally committed one and reduce this bad event to the binding property of the commitment scheme. The second hop repeats the reduction, but for the second opened view. Finally we, prove that the probability that the prover cheats in the resulting game is 0 bounded by  $1/\binom{n}{2}$ . Intuitively, if the verifier accepts then both views are consistent with each other and they both return 1. However, we know that no sharing of w would cause the circuit to make all parties return 1 in an honest execution. Here we rely on Lemma 3.1, which we axiomatize in EasyCrypt as follows.

```
axiom local_global_consistency (c : circuit_t) (xp : pinput_t list) (vv : view_t list) : (\forall i j, consistent_views c xp (get_view i vv) (get_view j vv) i j) \Leftrightarrow
```

```
(\exists xs rs, let (tr,y) = eval\_protocol c rs (xp,xs) in 
 <math>\forall i, ((pinput i xp, sintput j xs), get\_rand i rs, get\_trace i tr) = get\_view i vv get\_view i.
```

This states that there must exist a pair of views that fails the checks performed by the verifier. The soundness result follows from the fact that the (honest) verifier chooses two views at random for opening, so the probability of hitting one problematic pair of views is at least  $1/\binom{n}{2}$ 

**Zero-knowledge** For the Zero-Knowledge property we formalize a simulation strategy that guesses the challenge the verifier will produce by generating it uniformly at random. It then runs the MPC simulator to generate the two views that will be opened and fixes the other ones to an arbitrary value. Note here that the outputs of the simulated views can be programmed to 1. It completes these views by sampling two random shares using the secret sharing scheme's share simulator. Finally, it commits to all views to get a simulated first round message. When computing the response, this simulation strategy fails if the challenge guess was wrong. Otherwise it returns the simulated views.

The proof that this is a good simulation strategy is established as two independent results. The first result shows that a single execution of our simulation strategy is good if the simulator's guess is successful. The second result is a meta-theorem described at the end of this section.

To prove the fist result, we first define a modified real game that initially samples a challenge uniformly at random rand aborts if the verifier's challenge does not equal the randomly sampled one. We then prove, using a sequence of hops, that a single run of our simulation strategy is indistinguishable from this modified real game for any distinguisher that only observes it if the challenge guess is successful. This is stated as follows, where RealEvaluatorMod denotes the modified real world.

```
 \begin{array}{l} \textbf{lemma} \  \, \text{zero\_knowledge} \  \, \text{w} \  \, \text{x} \  \, (D <: \textit{Distinguisher\_t}) \  \, (MV <: \textit{MVerifier\_t}) : \\ \text{valid\_circuit} \  \, (\text{fst} \  \, \text{xc}) \Rightarrow \\ | \  \, \text{Pr} \  \, [ \  \, \text{ZKGame}(D,RP,RealEvaluatorMod(MV)).game(w,x) : res} \  \, ] \  \, - \\ \text{Pr} \  \, [ \  \, \text{ZKGame}(D,RP,IdealEvaluator(MV,S(S_{MPC}))).game(w,x) : res} \  \, ] \  \, | \\ \text{Smr} \cdot | \  \, \text{Pr} \  \, [ \  \, \text{Hiding}(B(D)).game(true) : res} \  \, ] \  \, - \\ \text{Pr} \  \, [ \  \, \text{Hiding}(B(D)).game(true) : res} \  \, ] \  \, - \\ \text{Pr} \  \, [ \  \, \text{PrivGame}[(D'(D),R_{MPC},RealEvaluator).main(fst \  \, x,w,cr) : res} \  \, ] \  \, - \\ \text{Pr} \  \, [ \  \, \text{PrivGame}[(D'(D),R_{MPC},IdealEvaluator(S_{MPC})).main(fst \  \, x,w,cr) : res} \  \, ] \  \, - \\ \text{Pr} \  \, [ \  \, \text{SSGame} \  \, (C(D),R_{SS},RealEvaluator).main(aux,x) : res} \  \, ] \  \, - \\ \text{Pr} \  \, [ \  \, \text{SSGame} \  \, (C(D),R_{SS},IdealEvaluator(S)).main(aux,x) : res} \  \, ] \  \, - \\ \text{Pr} \  \, [ \  \, \text{SSGame} \  \, (C(D),R_{SS},IdealEvaluator(S)).main(aux,x) : res} \  \, ] \  \, - \\ \text{Pr} \  \, [ \  \, \text{SSGame} \  \, (C(D),R_{SS},IdealEvaluator(S)).main(aux,x) : res} \  \, ] \  \, - \\ \text{Pr} \  \, [ \  \, \text{SSGame} \  \, (C(D),R_{SS},IdealEvaluator(S)).main(aux,x) : res} \  \, ] \  \, - \\ \text{Pr} \  \, [ \  \, \text{SSGame} \  \, (C(D),R_{SS},IdealEvaluator(S)).main(aux,x) : res} \  \, ] \  \, - \\ \text{Pr} \  \, [ \  \, \text{SSGame} \  \, (C(D),R_{SS},IdealEvaluator(S)).main(aux,x) : res} \  \, ] \  \, - \\ \text{Pr} \  \, [ \  \, \text{SSGame} \  \, (C(D),R_{SS},IdealEvaluator(S)).main(aux,x) : res} \  \, ] \  \, - \\ \text{Pr} \  \, [ \  \, \text{SSGame} \  \, (C(D),R_{SS},IdealEvaluator(S)).main(aux,x) : res} \  \, ] \  \, - \\ \text{Pr} \  \, [ \  \, \text{SSGame} \  \, (C(D),R_{SS},IdealEvaluator(S)).main(aux,x) : res} \  \, ] \  \, - \\ \text{Pr} \  \, [ \  \, \text{SSGame} \  \, (C(D),R_{SS},IdealEvaluator(S)).main(aux,x) : res} \  \, ] \  \, - \\ \text{Pr} \  \, [ \  \, \text{SSGame} \  \, (C(D),R_{SS},IdealEvaluator(S)).main(aux,x) : res} \  \, ] \  \, - \\ \text{Pr} \  \, [ \  \, \text{SSGame} \  \, (C(D),R_{SS},IdealEvaluator(S)).main(aux,x) : res} \  \, ] \  \, - \\ \text{Pr} \  \, [ \  \,
```

We now detail the game hopping proof. In the first three hops, we replace the view of every party different from i and j, where (i,j) randomly sampled challenge, by the default EasyCrypt value witness. We can construct adversary B that attacks the hiding property of the commitment scheme and interpolates between the two experiments as follows. Adversary B executes the commitment round in the exact same way as the protocol does in the real world. It then selects one party (different from i and j) and adopts as its query the view of that party and the witness value. When B finishes the execution of the MitH protocol, it adopts as its decision bit the one given by the zero-knowledge distinguisher. These three hops lead to a term in the final statement that is bounded by 3 times the advantage of B against the commitment scheme. Next, we replace the execution of the MPC protocol by its simulator. Because, by assumption, the MPC protocol achieves 2-privacy, it is possible to bound the difference between the two games as the advantage of a distinguisher against the privacy of the MPC protocol. Finally, in the last hop, we replace the value of the witness w by an arbitrary value and reduce the transition to the t-privacy of the secret-sharing scheme. The proof is complete by showing that this final game is identical to the ideal game when instantiated with our simulator.

Meta Theorems We have proved in EasyCrypt three meta-theorems that allow us to justify why our soundness and ZK results imply the standard soundness properties (wich negligible soundness error) and ZK property. The first meta-result shows that n repetitions of an experiment that returns 1 with small probability  $\epsilon$ , and checking that all attempts succeeded has an overall probability of success bounded by  $\epsilon^n$ . The second meta-result considers an algorithm such as our simulator that is good enough when its challenge guess is correct. It then shows that rejection sampling of such a simulator permits obtaining a computationally indistinguishable simulation that only fails with negligible probability. Finally, the third meta theorem shows that any protocol that follows the next message syntax satisfies the global consistency versus pairwise consistency property.

### 5 Verified ZKP implementation supporting general relations

We now describe how the abstract MitH construction was instantiated to derive a concrete formalization of a MitH protocol based on the Shamir's secret sharing scheme, the BGW protocol [9] and the Pedersen commitment scheme. We leveraged prior results in the verification of MPC protocols [15], including a formalization of these lower level components and a code extraction tool that automatically synthesizes OCaml code from EasyCrypt specifications. As a result, we obtain a verified implementation of our MitH instantiation, with only small adaptations to the original formalization and extraction tool. To demonstrate the modularity of our development, we also replace the Pedersen commitment scheme by a more efficient PRF-based construction. We conclude the section with a preliminary performance analysis of our MitH verified implementation.

#### 5.1 Secure arithmetic circuit evaluation

As a first step, we refine the notion of MPC protocol to the concrete case of secure arithmetic circuit evaluation where parties evaluate addition, multiplication and scalar multiplication gates sequentially. The EasyCrypt definition of an arithmetic circuit is depicted below.

```
theory ArithmeticCircuit.
type wire_t = t.
type gid_t = int.
type topology_t = int * int * int.

type gates_t = [
    | PInput of int
    | SInput of int
    | Constant of int & t
    | Addition of int & gates_t & gates_t
    | Multiplication of int & gates_t & gates_t
    | SMultiplication of int & gates_t & gates_t
].
type circuit_t = topology_t * gates_t.
end ArithmeticCircuit.
```

The circuit is defined over wires, which are elements of a finite field t. The topology (type  $topology_t$ ) is a pair of integers that fixes the number of public input wires, the number of secret input wires and the number of gates to the circuit. Intuitively, when n parties securely evaluate such a circuit, all of them will receive the values of the public input wires in the clear, which is consistent with our assumption in Section 4.2 that all parties receive the same public

input. The  $gates_t$  permits specifying an arbitrary arithmetic circuit using a tree structure. This is less efficient that a graph structure, but it reduces the verification effort.

From this point on, different protocols for secure arithmetic circuit evaluation can be obtained by instantiating the three gate-level secure computation sub-protocols. Indeed, our refinement of the MPC protocol formalization is parametric on the specification of these gates.<sup>5</sup> However, we make concrete the number of parties n=5, as this is all we need for the concrete setting where we use the BGW protocol instantiation to instantiate the MitH construction. Note that we define the trace of messages to follow the same structure as that of the circuit, which again simplifies proofs.

```
theory Arithmetic Protocol.
  clone import AddGate.
  clone import MulGate.
  clone import SMulGate.
  op n = 5. op t = 2.
  op pid_{-}t = [ | P1 | P2 | P3 | P4 | P5 ].
  type pinput_{-}t = t \ list.
  type sinput_{-}t = t \ list.
  type output_t = t.
  type randg_t = [
     AddRand of AddGate.rand_t
     MulRand of MulGate.rand_t
     SMulRand of SMulGate. rand_t
 ].
  type rand_{-}t = (gid_{-}t * randg_{-}t) list.
  type msgs_{-}t = [
     PInputM of wid_t
     SInputM of wid_t
     ConstantM of qid_t \& t
     AdditionM of qid_t & AdditionGate.Gate.msqs_t & msqs_t & msqs_t
     MultiplicationM of gid_t & MultiplicationGate.msqs_t & msqs_t & msqs_t
     SMultiplicationM of gid_t & SMultiplicationGate.msgs_t & msgs_t & msgs_t
```

At this level of abstraction, we can define the secure evaluation of a circuit using a secretsharing based protocol which captures the BGW protocol as a particular case. The remaining details are fixed by instantiating the secret sharing scheme and the arithmetic gates. Note below that the  $input_t$  type for secret inputs is defined as a share of an initial input value and the. Moreover, the output of the protocol is computed by explicitly taking the partial outputs produced by each party and using them to reconstruct an unshared output value that is then fixed as the output for all parties as a consequence of a public opening. This choice of design comes from a security relaxation that is made at the circuit evaluation level, prior to the output phase. To obtain a t-privacy arithmetic protocol, it is still necessary to compose the protocol provided here with a gate that securely implements a share randomization functionality. This compositional property is studied in [12, 2] and its formalization was adapted from [15].

```
op eval_circuit (gg : gates_t) (r : rand_t list) (x : input_t list) : trace_t * output_t = ...

with gg = SInput w ⇒

let ys = map (fun pid ⇒ let sec = nth witness (sinput pid xs) w in (pid, sec)) pid_set in

let tr = SInputT w in (tr, ys)

with gg = Addition gid wl wr ⇒
```

<sup>&</sup>lt;sup>5</sup>We show an example of such an arithmetic gate specification in Appendix A.

```
let ra = map (fun pid \Rightarrow (pid, as\_gate\_rand\_addition (oget (assoc (oget (assoc rs pid)) gid))))
         pid_set in
    let (tl, vwl) = eval\_gates wl rs xs in
    let (tr, vwr) = eval\_gates wr rs xs in
    let gxs = map (fun pid \Rightarrow (pid, ((), (oget (assoc vwl pid), oget (assoc vwr pid))))) pid_set in
    let (gtr, gys) = AdditionGate.Gate.gate ra gxs in
    let gtrs = AdditionT gid gtr tl tr in (gtrs, gys)
  with gg = Multiplication gid wl wr \Rightarrow ...
  with gg = SMultiplication gid wl wr <math>\Rightarrow ...
op protocol (c : circuit_t) (r : rand_t list) (x : input_t list) : trace_t * output_s_t list =
  let cc = snd c in
  let c_out = eval\_circuit cc r x in
  let tr' = unzip1 c_out in
  let y' = unzip2 c_out in
  let y = map (fun y \Rightarrow oget (reconstruct y)) y' in
  let tr = (y',tr') in (tr, (y,y,y,y,y)).
```

We end the discussion of our arithmetic protocol specification with the refinement of the local\_output and out\_messages operators. Similarly to the protocol evaluation function, these two operators are defined based on the operators with the same names defined by the lower-level arithmetic gate theories.

```
op local_output_gates (pid: pid_t) (tr: trace_t) (x: pinput_t * sinput_t) (r: rand_t): output_t =
  with im = PInputLT w \Rightarrow ...
  with im = SInputLT w \Rightarrow nth witness (snd x) w
  with im = ConstantLT gid c \Rightarrow ConstantGate.local\_output pid ((c,()), (), [])
  with im = AdditionLT gid tr trl trr \Rightarrow
    let vl = local\_output\_gates pid x r trl in
    let vr = local\_output\_gates pid x r trr in
    let ra = as_gate_rand_addition (oget (assoc r gid)) in
    AdditionGate.local_output pid (((),(vl,vr)), ra, tr)
  with tr = MultiplicationLT gid ta tl tr \Rightarrow ...
  with tr = SMultiplicationLT gid ta tl tr \Rightarrow ...
op local_output (c : circuit_t) (pid : pid_t) (v : view_t) : output_t =
  let (topo, gg) = c in let (x,r,tr) = v in
  if valid_circuit_trace c tr then local_output_gates pid x r tr else witness.
op gate_out_messages (pid: pid_t) (t: gate_local_trace_t) (x: input_t) (r: rand_t): gate_local_trace_t =
  with im = PInputLT w \Rightarrow PInputLT w
  with im = SInputLT w \Rightarrow SInputLT w
  with im = ConstantLT gid c \Rightarrow ConstantLT gid c
  with im = AdditionLT gid tr trl trr \Rightarrow
    let vl = local\_output\_gates pid x r trl in
    let vr = local_output_gates pid x r trr in
    let owl = out_messages_gates pid x r trl in
    let owr = out_messages_gates pid x r trr in
    let ra = as_gate_rand_addition (oget (assoc r gid)) in
    AdditionLT gid (AdditionGate.out_messages pid (((),(vl,vr)), ra, tr)) owl owr
  with t = MultiplicationLT gid ta tl tr \Rightarrow ...
  with t = \text{SMultiplicationLT gid ta tl tr} \Rightarrow \dots
op out_messages (c : circuit_t) (pid : pid_t) (v : view_t) : out_messages_t =
  let (topo, gg) = c in let (x,r,tr) = v in
```

**if** valid\_circuit\_trace c tr **then** out\_messages\_gates pid x r tr **else** witness. **end** ArithmeticProtocol.

The BGW protocol is obtained by instantiating the secret sharing scheme with Shamir's secret sharing and the low-level arithmetic gate protocols (including the randomization output gate refresh) proved secure in [15]. We briefly describe how correctness and security of the full protocol are proved in EasyCrypt. Correctness is proved by induction on the structure of the circuit and relying on the correctness of the low-level gates at each inductive step. The 2-privacy of the protocol is provide by instantiating the secure composition theorem, assuming that the low-level arithmetic gates are t-pre-output-privacy and that the refresh gate is t-privacy.

```
 \begin{array}{l} \textbf{lemma} \ privacy\_arithmetic \ (D <: \textit{Distinguisher\_t}) \ c \ x \ aux \ , \\ valid\_secret \ x \Rightarrow \\ | \ Pr \ [ \ PrivGame(D,R,RealEvaluator).main(c,x,aux) : res \ ] \ - \\ Pr \ [ \ PrivGame(D,R,IdealEvaluator(S)).main(c,x,aux) : res \ ] \ | \le \\ | \ Pr \ [ \ PreOutPrivGame^{ArithProt}(D_{ArithProt}(D),R_{ArithProt},RealEvaluator).main(ArithProt,x,aux) : res \ ] \\ - \\ Pr \ [ \ PreOutPrivGame^{ArithProt}(D_{ArithProt}(D),R_{ArithProt},IdealEvaluator(S_{ArithProt})).main(ArithProt,x,aux) : res \ ] \ | \\ Pr \ [ \ PreOutPrivGame^{Refresh}(D_{Refresh}(D),R_{Refresh},RealEvaluator).main(Refresh,x,aux) : res \ ] \ - \\ Pr \ [ \ PrivGame^{Refresh}(D_{Refresh}(D),R_{Refresh},IdealEvaluator(S_{Refresh})).main(Refresh,x,aux) : res \ ] \ | \\ Pr \ [ \ PrivGame^{Refresh}(D_{Refresh}(D),R_{Refresh},IdealEvaluator(S_{Refresh})).main(Refresh,x,aux) : res \ ] \ | \\ Pr \ [ \ PrivGame^{Refresh}(D_{Refresh}(D),R_{Refresh},IdealEvaluator(S_{Refresh})).main(Refresh,x,aux) : res \ ] \ | \\ Pr \ [ \ PrivGame^{Refresh}(D_{Refresh}(D),R_{Refresh},IdealEvaluator(S_{Refresh})).main(Refresh,x,aux) : res \ ] \ | \\ Pr \ [ \ PrivGame^{Refresh}(D_{Refresh}(D),R_{Refresh},IdealEvaluator(S_{Refresh})).main(Refresh,x,aux) : res \ ] \ | \\ Pr \ [ \ PrivGame^{Refresh}(D_{Refresh}(D),R_{Refresh},IdealEvaluator(S_{Refresh})).main(Refresh,x,aux) : res \ ] \ | \\ Pr \ [ \ PrivGame^{Refresh}(D_{Refresh}(D),R_{Refresh},IdealEvaluator(S_{Refresh})).main(Refresh,x,aux) : res \ ] \ | \\ Pr \ [ \ PrivGame^{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{Refresh}(D_{
```

where PreOutPrivacy represents the t-privacy relaxed security definition used at the protocol evaluation level.

Proof. The proof is done in two hops, where the executions of the low-level t-pre-output-privacy secure arithmetic gate evaluation protocol and the t-privacy refresh gate are replaced by their corresponding simulators. For example, it is possible to define real evaluator RealEvaluator1 that replaces every execution of the arithmetic protocol by simulator S<sub>ArithProt</sub> and bounding the difference to the real world by building an adversary against ArithProt. When the execution of the composed protocol is replaced by the respective simulator, we will be in the ideal world and it is possible to trace back the difference between RealEvaluator and IdealEvaluator to be the sum of the advantages against the arithmetic protocol and of the refresh gate.

#### 5.2 PRF-based commitment scheme

Our first realization of the MitH construction considered the Pedersen commitment scheme in the computation of commitments for the party views resulting from the protocol execution. This particular commitment scheme works just for one finite field element, which means that producing a commitment to a view—a large list of finite field elements—implies producing a commitment to every value in the view. For this reason, we formalized the (collision-resistant) PRF-based construction given in [17], which we then instantiate with HMAC. To this end, we first formalize the underlying PRF primitive as a keyed function as follows.

```
theory CRPRF.

type input.

type output.

type key.

op f : \text{key} \rightarrow \text{input} \rightarrow \text{output}.

end CRPRF.
```

Using a PRF that follows this syntax, it is possible to build a commitment scheme by defining the commit algorithm to be an evaluation of the PRF and the verifying algorithm to be the comparison between the received commitment and the one that the verifying party is able to locally compute once the PRF key is revealed.

```
theory CRPRFCommitment.
  clone include CRPRF.
  op commit(k,m) = (f k m,k).
  op verify(m : input, ci : output * key) = ci.'1 = f ci.'2 m.
  clone import CommitmentScheme with
    type msg_t = input,
    type rand_t = key,
    op valid_rand = fun _ ⇒ true,
    type opening_string_t = key,
    type commitment_t = output,
    op commit = commit,
    op verify = verify.
```

Our binding theorem below states that our PRF-based commitment scheme achieves binding if the underlying PRF is collision resistant.

```
lemma prf_commitment_binding (A <: Adversary\_t):
Pr [ Binding(A).main() : res ] \leq Pr [ CR_{PRF}(B(A)).main() : res ]
```

where Pr[ CR().main(): res ] represents the probability of finding a collision under PRF.

*Proof.* The proof is a classical reduction to the collision resistance property. Based on adversary A, we are able to construct adversary B that works as follows. To produce its query, B invokes the query generation adversary from A and adopts its query as its own. The collision resistance game will then execute in the exact same way as the binding game, since in both experiences the challenger will check if the two queried values are different and if they produce the same commitment.

Finally, we prove that our PRF-based commitment scheme achieves hiding.

```
 \begin{array}{l} \textbf{lemma} \ prf\_commitment\_hiding} \ (A <: Adversary\_t) : \\ | \ Pr \ [ \ Hiding(A).game(true) : res \ ] - Pr \ [ \ Hiding(A).game(false) : !res \ ] \ | \\ | \ 1/2 + | \ Pr \ [ \ PRF(B(A)).game(true) : res \ ] - Pr \ [ \ PRF(B(A)).game(false) : !res \ ] \ | \\ \end{array}
```

where | Pr[PRF(B(A)).game(true): res] - Pr[PRF(B(A)).game(false): !res] | represents the advantage against the underlying PRF assumption.

*Proof.* The proof is a classical reduction to the PRF assumption. Based on adversary A, we are able to construct adversary B against the PRF assumption that works as follows. To produce its query, B invokes the query generation method from adversary A and adopts the obtained query as its own. To make its guess, B will simply adopt the decision bit from A as its own. Note that, in the process, B has access to an oracle that computes the outputs of the PRF.

Finally, because of the PRF assumption, we can replace the output of f by a random output value, giving the adversary a 1 / 2 probability on winning the game.

#### 5.3 MitH-based verified evaluation engine for ZKP

We have obtained a verified implementation of our MitH EasyCrypt implementation via the EasyCrypt extraction tool developed in [15], that was refined to be used in the particular context of this work. This general-purpose tool is able to synthesize correct-by-construction executable code from an EasyCrypt proof script.

We have applied the extraction tool at two different levels. First, we applied the extraction tool against a concrete EasyCrypt specification of MitH, where commitments were computed using the Pedersen commitment scheme. Nevertheless, this extraction strategy loses the modular infrastructure that surrounds the proof and relies on a commitment scheme that entails a significant performance penalty (a performance analysis can be consulted in Table 1). Therefore, after specifying the efficient commitment scheme based on a PRF, we applied a different code extraction approach to it: instead of performing a *flat* code synthesis that disregarded the modular structure, we manually extracted the modular structure and automatically extracted the concrete components, that were later matched against the modular structure. This methodology allowed us to a modular implementation of MitH which is re-usable and more efficient because it does not rely on group operations to compute the required commitments.

A graphical description of these two extraction pathways can be found in Figure 2, where the former extraction path is depicted on top and the latter is depicted on the bottom.

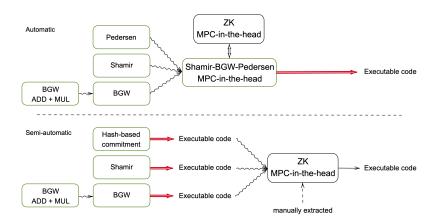


Figure 2: Code extraction pathways. The red arrow  $(\Rightarrow)$  denotes executable code obtained via the automated extraction mechanism.

#### 5.4 Experimental results

We performed a preliminary performance analysis of our executable OCaml MitH implementation obtained from EasyCrypt, that we summarize in Table 1. The table contains not only the current performance analysis of the MitH implementation, but also of the individual cryptographic primitives that we developed, with different field sizes. The benchmarking was carried out in a commodity 13-inch 2016 MacBook Pro, with a dual-core Intel Core i5 processor clocked at 2.9 GHz, 16 GB RAM, 256 KB L2 cache and 4 MB L3 cache.

As expected, our performance results are intrinsically related to the field size that is used for the field operations. Moreover, we refer the reader to the last two rows of the performance table, where is possible to compare the MitH-based ZK implementation that uses the Pedersen commitment scheme (A) and the implementation that uses a PRF-based commitment scheme (SA), instantiated with the SHA256 hash function. Naturally, there are significant efficiency

Table 1: Performance analysis of automatically extracted executable code for MitH-based ZKP (times in ms).

		Random generation	Share	Reconstruct	Protocol	Commit	Verify
	Shamir Secret Share	0.008	0.010	0.002	-	-	-
	BGW Addition	-	-	-	0.001	-	-
Field: 256 bits	BGW Scalar Multiplication	-	-	-	0.002	-	-
	BGW Multiplication	0.028	-	-	0.053	-	-
	Pedersen Commitment	0.002	-	-	-	0.033	0.033
	SHA256-based Commitment	-	-	-	-	0.003	0.002
	Shamir Secret Share	0.006	0.009	0.001	-	-	-
	BGW Addition	-	-	-	0.002	-	-
Field: 1024 bits	BGW Scalar Multiplication	-	-	-	0.004	-	-
	BGW Multiplication	0.022	-	-	0.052	-	-
	Pedersen Commitment	0.002	-	-	-	1.093	1.090
	SHA256-based Commitment	-	-	-	-	0.004	0.004
							<u> </u>
Field value: 101	A MitH (7 gates, 2 MUL)	0.233	-	-	2.692	1.687	0.434
Field value: 97	A MitH (11 gates, 3 MUL)	0.396	-	-	3.946	1.962	0.520
Field value: 101	SA MitH (7 gates, 2 MUL)	0.137	-	-	0.925	0.051	0.014
Field value: 97	SA MitH (11 gates, 3 MUL)	0.175	-	-	1.166	0.077	0.029

gains when using the PRF-based commitment to the detriment of Pedersen commitment scheme, mainly because the latter heavily relies on group operations, whereas the former is simply an application of an (efficient) hash function. Finally, a rough comparison with results for other implementations of the same primitives and of MitH-based ZK allows us to conclude that the performance penalty induced by our formal approach is not prohibitive and that real-world applications are within reach of the implementations automatically generated by our approach. Furthermore, additional optimization effort can lead to significant performance gains, e.g., by resorting faster MPC protocols that do not require polynomial interpolation (such as the one proposed in [21]) or by deploying highly optimized low-level implementations of the cryptographic primitives that we developed in this project, such as the ones given by the Jasmin language.

#### References

- [1] José Bacelar Almeida, Manuel Barbosa, Gilles Barthe, François Dupressoir, Benjamin Grégoire, Vincent Laporte, and Vitor Pereira. A fast and verified software stack for secure function evaluation. In *Proceedings of the 2017 ACM SIGSAC Conference on Computer and Communications Security*, pages 1989–2006, 2017.
- [2] José Bacelar Almeida, Manuel Barbosa, Gilles Barthe, Hugo Pacheco, Vitor Pereira, and Bernardo Portela. Enforcing ideal-world leakage bounds in real-world secret sharing mpc frameworks. In 2018 IEEE 31st Computer Security Foundations Symposium (CSF), pages 132–146. IEEE, 2018.
- [3] Scott Ames, Carmit Hazay, Yuval Ishai, and Muthuramakrishnan Venkitasubramaniam. Ligero: Lightweight sublinear arguments without a trusted setup. In Bhavani M. Thuraisingham, David Evans, Tal Malkin, and Dongyan Xu, editors, *Proceedings of the 2017 ACM SIGSAC Conference on Computer and Communications Security, CCS 2017, Dallas, TX, USA, October 30 November 03, 2017*, pages 2087–2104. ACM, 2017.

- [4] José Bacelar Almeida, Manuel Barbosa, Endre Bangerter, Gilles Barthe, Stephan Krenn, and Santiago Zanella Béguelin. Full proof cryptography: verifiable compilation of efficient zero-knowledge protocols. In *Proceedings of the 2012 ACM conference on Computer and communications security*, pages 488–500, 2012.
- [5] Manuel Barbosa, Gilles Barthe, Karthikeyan Bhargavan, Bruno Blanchet, Cas Cremers, Kevin Liao, and Bryan Parno. Sok: Computer-aided cryptography. *IEEE Security and Privacy*, 2021.
- [6] Gilles Barthe, Daniel Hedin, Santiago Zanella Béguelin, Benjamin Grégoire, and Sylvain Heraud. A machine-checked formalization of sigma-protocols. In 2010 23rd IEEE Computer Security Foundations Symposium, pages 246–260. IEEE, 2010.
- [7] Carsten Baum and Ariel Nof. Concretely-efficient zero-knowledge arguments for arithmetic circuits and their application to lattice-based cryptography. In *IACR International Conference on Public-Key Cryptography*, pages 495–526. Springer, 2020.
- [8] Mihir Bellare and Phillip Rogaway. The security of triple encryption and a framework for code-based game-playing proofs. In *Annual International Conference on the Theory and Applications of Cryptographic Techniques*, pages 409–426. Springer, 2006.
- [9] Michael Ben-Or, Shafi Goldwasser, and Avi Wigderson. Completeness theorems for non-cryptographic fault-tolerant distributed computation. In *Proceedings of the 20th Annual Symposium on Theory of Computing*, pages 1–10. ACM, 1988.
- [10] Michael Ben-Or, Shafi Goldwasser, and Avi Wigderson. Completeness theorems for noncryptographic fault-tolerant distributed computation. In Providing Sound Foundations for Cryptography: On the Work of Shafi Goldwasser and Silvio Micali, pages 351–371. 2019.
- [11] Rishabh Bhadauria, Zhiyong Fang, Carmit Hazay, Muthuramakrishnan Venkitasubramaniam, Tiancheng Xie, and Yupeng Zhang. Ligero++: A new optimized sublinear iop. In *Proceedings of the 2020 ACM SIGSAC Conference on Computer and Communications Security*, pages 2025–2038. Association for Computing Machinery, 2020.
- [12] Dan Bogdanov, Sven Laur, and Jan Willemson. Sharemind: A framework for fast privacy-preserving computations. In *Proceedings of the 13th European Symposium on Research in Computer Security*, pages 192–206. Springer, 2008.
- [13] David Butler, Andreas Lochbihler, David Aspinall, and Adrià Gascón. Formalising ς-protocols and commitment schemes using CryptHOL. Journal of Automated Reasoning, pages 1–47, 2020.
- [14] Melissa Chase, David Derler, Steven Goldfeder, Claudio Orlandi, Sebastian Ramacher, Christian Rechberger, Daniel Slamanig, and Greg Zaverucha. Post-quantum zero-knowledge and signatures from symmetric-key primitives. In Bhavani M. Thuraisingham, David Evans, Tal Malkin, and Dongyan Xu, editors, Proceedings of the 2017 ACM SIGSAC Conference on Computer and Communications Security, CCS 2017, Dallas, TX, USA, October 30 November 03, 2017, pages 1825–1842. ACM, 2017.
- [15] Karim Eldefrawy and Vitor Pereira. A high-assurance evaluator for machine-checked secure multiparty computation. In Lorenzo Cavallaro, Johannes Kinder, XiaoFeng Wang, and Jonathan Katz, editors, Proceedings of the 2019 ACM SIGSAC Conference on Computer and Communications Security, CCS 2019, London, UK, November 11-15, 2019, pages 851–868. ACM, 2019.

- [16] Irene Giacomelli, Jesper Madsen, and Claudio Orlandi. Zkboo: Faster zero-knowledge for boolean circuits. In Thorsten Holz and Stefan Savage, editors, 25th USENIX Security Symposium, USENIX Security 16, Austin, TX, USA, August 10-12, 2016, pages 1069–1083. USENIX Association, 2016.
- [17] Paul Grubbs, Jiahui Lu, and Thomas Ristenpart. Message franking via committing authenticated encryption. In Jonathan Katz and Hovav Shacham, editors, Advances in Cryptology CRYPTO 2017 37th Annual International Cryptology Conference, Santa Barbara, CA, USA, August 20-24, 2017, Proceedings, Part III, volume 10403 of Lecture Notes in Computer Science, pages 66–97. Springer, 2017.
- [18] Helene Haagh, Aleksandr Karbyshev, Sabine Oechsner, Bas Spitters, and Pierre-Yves Strub. Computer-aided proofs for multiparty computation with active security. In 2018 IEEE 31st Computer Security Foundations Symposium (CSF), pages 119–131. IEEE, 2018.
- [19] Yuval Ishai, Eyal Kushilevitz, Rafail Ostrovsky, and Amit Sahai. Zero-knowledge from secure multiparty computation. In David S. Johnson and Uriel Feige, editors, *Proceedings of the 39th Annual ACM Symposium on Theory of Computing, San Diego, California, USA, June 11-13, 2007*, pages 21–30. ACM, 2007.
- [20] Jonathan Katz, Vladimir Kolesnikov, and Xiao Wang. Improved non-interactive zero knowledge with applications to post-quantum signatures. In David Lie, Mohammad Mannan, Michael Backes, and XiaoFeng Wang, editors, *Proceedings of the 2018 ACM SIGSAC Conference on Computer and Communications Security, CCS 2018, Toronto, ON, Canada, October 15-19, 2018*, pages 525–537. ACM, 2018.
- [21] Ueli Maurer. Secure multi-party computation made simple. Discrete Applied Mathematics, 154(2):370–381, 2006.

## A Abstract arithmetic gate example

To show how arithmetic gates were formalized, we will provide, as example, a walkthrough of how addition gates are abstracted as sub-protocols in our formalization.

The addition gate works over elements of a finite field. Every party enters the gate evaluation protocol with two input values, i.e., every party holds a share of two values. The functionality will be given the reconstruction of those two secrets from where the input shares come from. All parties end the gate evaluation with one share of the output value.

Because we are specifying a gate for a concrete operation, we can already refine the functionality to be the addition operation over the finite field applied to the field elements reconstructed from the local inputs of the parties.

```
theory AdditionGate.
  op n : int.
  type pid_t = int.

type pinput_t.
  type sinput_t = t * t.
  type input_t = pinput_t * sinput_t.
  type output_t = t.
  op f (xs : input_t list) : output_t list = map (+) xs.

type rand_t.
  type nextmsg_t.
```

```
type view_{-}t = input_{-}t * rand_{-}t * nextmsg_{-}t \ list.

op out_messages : pid_{-}t \rightarrow view_{-}t \rightarrow (nextmsg_{-}t \ list) option.

op local_output : pid_{-}t \rightarrow view_{-}t \rightarrow output_{-}t \ option.

type trace_{-}t = view_{-}t \ list.

op gate : input_{-}t \ list \rightarrow rand_{-}t \ list \rightarrow trace_{-}t * output_{-}t \ list.
```

#### end AdditionGate.

We purposely leave the randomness type underspecified to capture different instantiations of addition gates that may or may not require some random tape to securely perform the required computations.

Note that all methods related with the actual gate evaluation are left underspecified, meaning that we do not provide an actual realization of an addition gate. Indeed, the purpose of this gate specification was not to provide any concrete realization of an addition gate but to lay the foundations for the future definition of secure addition gate instantiations.