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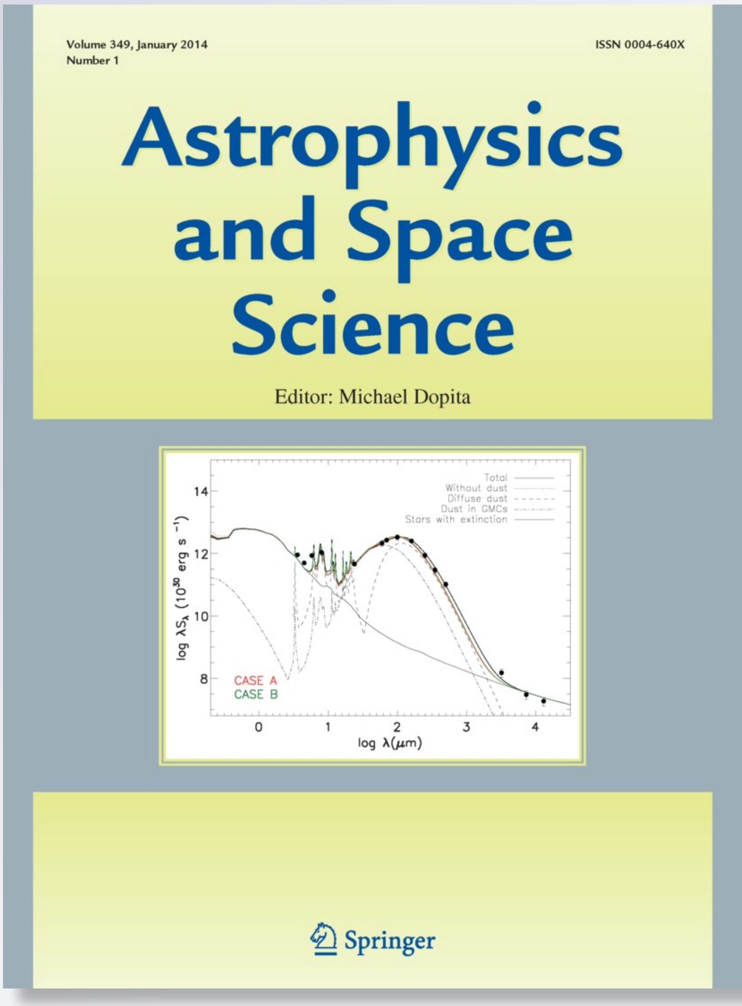
Astrophysics and Space Science
An International Journal of Astronomy, Astrophysics and Space Science

ISSN 0004-640X
Volume 349
Number 1

Astrophys Space Sci (2014) 349:317-327
DOI 10.1007/s10509-013-1659-z



Springer

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ORIGINAL ARTICLE

Solitons and Lie group analysis to an extended quantum Zakharov–Kuznetsov equation

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Received: 26 July 2013 / Accepted: 30 September 2013 / Published online: 16 October 2013

Abstract In this paper, complete geometric symmetry of extended quantum Zakharov–Kuznetsov (QZK) equation are investigated. All of the geometric vector fields for the new extended QZK equation are presented. At the same time, a plethora of exact solutions are obtained by the application of the group theorem. In addition, 1-soliton solution of the extended QZK equation with power law nonlinearity is obtained by the aid of traveling wave hypothesis with the necessary constraints in place for the existence of the soliton.

Keywords Extended quantum Zakharov–Kuznetsov equation · Symmetry groups · Exact solutions · Ansatz method

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1 Introduction

The theory of nonlinear evolution equations (NLEEs) play extremely important roles in several areas of applied physics and mathematical physics (Abdou 2011; Bhrawy et al. 2013; Biswas 2008; Biswas et al. 2011; Bluman and Kumei 1989; Ebadi et al. 2012; Garcia et al. 2005; Ghebache and Tribeche 2013; Ibragimov 1994; Khan and Masood 2008; Lie 1881; Moslem et al. 2007; Olver 1986; Ovsiannikov 1982; Pakzad 2010, 2012; Sabry et al. 2008; Wang et al. 2013; Wazwaz 2008, 2012; Xie et al. 2005; Zakharov and Kuznetsov 1974). Some of the areas where NLEEs are frequently observed are nuclear physics, nonlinear optics, plasma physics, astrophysics, biophysics. Therefore it is imperative to develop the mathematical analysis of these NLEEs in a rigorous manner. There are several mathematical tools that study these NLEEs from an analytic perspective. This paper will employ the classic mathematical analysis that will conduct a rigorous study of the governing equation. This is the Lie symmetry or Lie group analysis. This analysis is a one-time classic immortal tool that is applicable to all types of NLEEs. This integration mechanism extracts several form of important solutions to the equations that are very helpful in all areas of mathematical physics and applied physics.

There are several other integration architectures that are employed to seek solutions to the NLEEs. Some of them are the G'/G-expansion method, Backlund transformation, exp-function approach, simplest equation method and several others. However, as it is said that classic never dies, Lie group analysis is an all-time applicable tool to integrate all forms of NLEEs, such as coupled NLEEs, multi-dimensional NLEEs and several other forms. This paper will apply Lie group analysis to solve an important NLEE, known as the extended QZK equation that arises in the study of astrophysics.

The QZK equation has been found to arise in ion acoustic waves. In 2005 it was used to find the nonlinear coupling between quantum Langmuir waves and quantum ion acoustic waves (Garcia et al. 2005). Garcia et al. discovered a dimensionless quantum parameter that describes the ratio between ion plasma and electron thermal energies. In 2012 the integrability of the QZK equation was studied in the context of dense quantum plasmas (Ebadi et al. 2012). Ebadi et al. found many different solutions including solitons, periodic singular waves, shock waves, and rational functions. In 2013 quantum dust ion acoustic solitary waves were discovered (Ghebache and Tribeche 2013).

2 Governing equation

The extended QZK equation that is going to be considered in this paper is given by Wazwaz (2012)

$$u_t + auu_x + b(u_{xxx} + u_{yyy}) + c(u_{xyy} + u_{xxy}) = 0, \tag{1}$$

where a, b ad c are all constants while $u(x, y, t)$ represents the electrostatic wave potential in plasmas that is a function of the spatial variables x, y and the temporal variable t. The first term in (1) is the temporal evolution term, while the coefficient of a is the nonlinear term and the coefficients of b and c represents the spatial dispersions in multi-dimensions. Equation (1) supports soliton solutions. Solitons are the outcome of a delicate balance between dispersion and nonlinearity.

It is well known that the celebrated Zakharov and Kuznetsov types of equations have been around for a very long time. Lots of studies have been conducted with these types of equations (Zakharov and Kuznetsov 1974; Moslem et al. 2007; Sabry et al. 2008; Abdou 2011; Khan and Masood 2008; Wazwaz 2008). Zakharov and Kuznetsov (1974) established an equation which is related to nonlinear ion-acoustic waves (IAWs) in magnetized plasma including cold ions and hot isothermal electrons (Zakharov and Kuznetsov 1974; Moslem et al. 2007; Sabry et al. 2008; Abdou 2011; Khan and Masood 2008; Wazwaz 2008; Pakzad 2010, 2012). From both the experimental and theoretical point of view, the quantum plasmas and their new features have gained much attention for many studies. Both plasma experiments and astrophysical observations have indicated that quantum effects play a significant role in the behavior of charged carriers when the de Broglie wavelength of the charged carriers exceeds the Debye wavelength and approaches the Fermi wavelength (Wazwaz 2012; Moslem et al. 2007; Sabry et al. 2008). In dense quantum plasma, the quantum hydrodynamic (QHD) model is the most popular model (Wazwaz 2012). The QHD model is a generalization of the classical fluid model of plasmas where QHD transport equations are expressed with reference to conservation laws for particle momentum and energy (Khan and Masood 2008). A great many of authors have investigated the effect of the magnetic field in various quantum plasma models: for example, Zakharov and Kuznetsov (1974), Moslem et al. (2007), Sabry et al. (2008), Abdou (2011), Khan and Masood (2008) and Wazwaz (2008) and a lot of the references cited therein.

The regular QZK equation has been studied earlier in 2012 and 2013 (Ebadi et al. 2012; Bhrawy et al. 2013). This paper is an extension to these earlier studies, in the sense that the extended version of the QZK equation is being analyzed and that too by the aid of the classic Lie group analysis. The main aim of this paper is to make complete group classifications on the extended quantum Zakharov–Kuznetsov equation and to derive the symmetry reductions and exact solutions by the Lie symmetry analysis method.

The remainder of this paper is organized as follows: In Sect. 2, we perform Lie group classification on the extended quantum Zakharov–Kuznetsov equation, and present all of the geometric vector fields, then the complete symmetry classification of the extended quantum Zakharov–Kuznetsov equation (1) is performed, all of the point symmetries are obtained. In Sect. 3, the symmetry reductions and exact solutions to the extended quantum Zakharov–Kuznetsov equation are investigated. In Sect. 4, 1-soliton solution of the extended QZK equation are obtained by use the traveling wave hypothesis where the QZK equation will be taken up with power law nonlinearity. Finally, the conclusions and some remarks are given in Sect. 6.

3 Group analysis for the extended QZK equation

In this section, we perform group classification on the extended quantum Zakharov–Kuznetsov equation. Then, all of the geometric vector fields of the are provided. In this section, we will perform Lie group method for Eq. (1).

If (1) is invariant under a one parameter Lie group of point transformations

$$\begin{aligned}
t^* &= t + \epsilon\tau(x, y, t, u) + O\left(\epsilon^2\right),\\
x^* &= x + \epsilon\xi(x, y, t, u) + O\left(\epsilon^2\right),\\
y^* &= y + \epsilon\eta(x, y, t, u) + O\left(\epsilon^2\right),\\
u^* &= u + \epsilon\phi(x, y, t, u) + O\left(\epsilon^2\right),
\end{aligned} \tag{2}$$

the vector field of an evolution type of equation is as follows:

$$\begin{aligned}
V = {} & \tau(x, y, t, u)\frac{\partial}{\partial t} + \xi(x, y, t, u)\frac{\partial}{\partial x}\\
& + \eta(x, y, t, u)\frac{\partial}{\partial y} + \phi(x, y, t, u)\frac{\partial}{\partial u},
\end{aligned} \tag{3}$$

where the coefficient functions $\tau(x,y,t,u)$, $\xi(x,y,t,u)$, $\eta(x,y,t,u)$, and $\phi(x,y,t,u)$ of the vector field are to be determined.

If the vector field (3) generates a symmetry of (1), then V must satisfy Lie's symmetry condition

$$\mathrm{pr}^{(3)}\, V(\Delta_1)|_{\Delta_1=0} = 0, \tag{4}$$

where

$$\Delta_1 = u_t + auu_x + b(u_{xxx} + u_{yyy}) + c(u_{xyy} + u_{xxy}) = 0.$$

Applying the third prolongation $\mathrm{pr}^{(3)}\, V$ to the (1), we find the following system of symmetry equations then the invariant condition reads as

$$\begin{aligned}&\phi^t + a\phi u_x + au\phi^x + b\phi^{xxx} + b\phi^{yyy}\\&\quad + c\phi^{xyy} + c\phi^{xxy} = 0,\end{aligned} \tag{5}$$

here

$$\begin{aligned}
\phi^t &= D_t(\phi) - u_x D_t(\xi) - u_y D_t(\eta) - u_t D_t(\tau),\\
\phi^x &= D_x(\phi) - u_x D_x(\xi) - u_y D_x(\eta) - u_t D_x(\tau),\\
\phi^y &= D_y(\phi) - u_x D_y(\xi) - u_y D_y(\eta) - u_t D_y(\tau),\\
\phi^{xx} &= D_x(\phi^x) - u_{xt} D_x(\tau) - u_{xx} D_x(\xi) - u_{xy} D_x(\eta),\\
\phi^{yy} &= D_y(\phi^y) - u_{yt} D_y(\tau) - u_{xy} D_y(\xi) - u_{yy} D_y(\eta),\\
\phi^{xxx} &= D_x(\phi^{xx}) - u_{xxt} D_x(\tau) - u_{xxx} D_x(\xi) - u_{xxy} D_x(\eta),\\
\phi^{yyy} &= D_y(\phi^{yy}) - u_{yyt} D_y(\tau) - u_{yxy} D_y(\xi) - u_{yyy} D_y(\eta),\\
\phi^{xxy} &= D_y(\phi^{xx}) - u_{xxt} D_y(\tau) - u_{xxx} D_y(\xi) - u_{xxy} D_y(\eta),\\
\phi^{xyy} &= D_x(\phi^{yy}) - u_{yyt} D_x(\tau) - u_{yxy} D_x(\xi) - u_{yyy} D_x(\eta),
\end{aligned} \tag{6}$$

where, D_i denotes the total derivative operator and is defined by

$$D_i = \frac{\partial}{\partial x^i} + u_i^p \frac{\partial}{\partial u} + u_{ij}^p \frac{\partial}{\partial u_j} + \cdots \quad i = 1, 2, p = 1, \tag{7}$$

and $(x^1, x^2, x^3) = (t, x, y)$, $(u^1) = (u)$.

Substituting (6) into (5), we obtain the determining equations for the symmetry group. One can get

$$\begin{aligned}
\tau &= 3c_1 t + c_2,\\
\xi &= \frac{c_1}{3} x + c_3 t + c_4,\\
\eta &= \frac{c_1}{3} y + c_5,\\
\phi &= \frac{-2ac_1 u + 3c_3}{3a}.
\end{aligned} \tag{8}$$

Table 1 Commutator table of the Lie algebra of (9)

$[V_i, V_j]$	V_1	V_2	V_3	V_4	V_5
V_1	0	0	0	V_1	0
V_2	0	0	0	V_2	0
V_3	0	0	0	$3V_3$	aV_1
V_4	$-V_1$	$-V_2$	$-3V_3$	0	$2V_5$
V_5	0	0	$-aV_1$	$-2V_5$	0

In this case, the symmetry algebra of (1) is spanned by the five vector fields

$$\begin{aligned}
&V_1 = \frac{\partial}{\partial x}, \qquad V_2 = \frac{\partial}{\partial y}, \qquad V_3 = \frac{\partial}{\partial t},\\
&V_4 = x\frac{\partial}{\partial x} + 3t\frac{\partial}{\partial t} + y\frac{\partial}{\partial y} - 2u\frac{\partial}{\partial u},\\
&V_5 = t\frac{\partial}{\partial x} + \frac{1}{a}\frac{\partial}{\partial u}.
\end{aligned} \tag{9}$$

It easy to check that the symmetry generators found in (9) form a closed Lie algebra whose commutation relations are given in Table 1. The entry in row i and column j representing $[V_i, V_j]$. Here $[V_i, V_j]$ is the commutator for the Lie algebra (Olver 1986) given by

$$[V_i, V_j] = V_i V_j - V_j V_i.$$

It shows that the symmetry generators found in (9) form an five-dimensional Lie algebra. In order to get some exact solutions from the known ones, we should find the Lie symmetry groups from the related symmetries. To get the Lie symmetry group, we should solve the following initial problems

$$\begin{aligned}&\frac{d}{d\varepsilon}(\bar{x}, \bar{y}, \bar{t}, \bar{u}) = \sigma(\bar{x}, \bar{y}, \bar{t}, \bar{u}), (\bar{x}, \bar{y}, \bar{t}, \bar{u})|_{\varepsilon=0}\\&\qquad = (x, y, t, u),\end{aligned} \tag{10}$$

where ε is a parameter and

$$\sigma = \xi u_x + \tau u_t + \eta u_y + \phi u. \tag{11}$$

So we can obtain the Lie symmetry group

$$g : (x, y, t, u) \to (\bar{x}, \bar{y}, \bar{t}, \bar{u}). \tag{12}$$

According to different ξ, τ, η and ϕ, we have the following group

$$\begin{aligned}
&g_1 : (x + \varepsilon, y, t, u),\\
&g_2 : (x, y + \varepsilon, t, u),\\
&g_3 : (x, y, t + \varepsilon, u),\\
&g_4 : (e^{\varepsilon} x, e^{\varepsilon} y, e^{3\varepsilon} t, e^{2\varepsilon} u),\\
&g_5 : (x - at\varepsilon, y, t, u - \varepsilon).
\end{aligned} \tag{13}$$

The symmetry groups g_1, g_2 and g_3 demonstrate the space- and time-invariance of the equation. The well-known scaling symmetry turns up in g_4. We can obtain the corresponding new solutions by applying above groups g_i $(i = 1, \dots, 5)$

$$\begin{aligned} u_1 &= f_1(x-\varepsilon, y, t), \\ u_2 &= f_2(x, y-\varepsilon, t), \\ u_3 &= f_3(x, y, t-\varepsilon), \\ u_4 &= e^{-2\varepsilon} f_4\big(e^{-\varepsilon}x, e^{-\varepsilon}y, e^{-3\varepsilon}t\big), \\ u_5 &= f_5(x+at\varepsilon, y, t) - \varepsilon, \end{aligned} \tag{14}$$

where ε is a arbitrary real number.

If taking the following single-soliton solution (Wazwaz 2012) of (1)

$$u = \frac{24(b+c)k_1^2 \mathrm{e}^{k_1(x+y)-2(b+c)k_1^3 t}}{a(1+\mathrm{e}^{k_1(x+y)-2(b+c)k_1^3 t})^2}, \tag{15}$$

one can obtain new exact solution of Eq. (1) by applying g_5 as follows

$$u = \frac{24(b+c)k_1^2 \mathrm{e}^{k_1(x+at\varepsilon+y)-2(b+c)k_1^3 t}}{a(1+\mathrm{e}^{k_1(x+at\varepsilon+y)-2(b+c)k_1^3 t})^2} - \varepsilon. \tag{16}$$

If taking the following singular soliton, i.e. an explosive pulse (Wazwaz 2012) of (1)

$$u = -\frac{24(b+c)k_1^2 \mathrm{e}^{k_1(x+y)-2(b+c)k_1^3 t}}{a(1-\mathrm{e}^{k_1(x+y)-2(b+c)k_1^3 t})^2}, \tag{17}$$

one can obtain new exact solution of Eq. (1) by applying g_5 as follows

$$u = -\frac{24(b+c)k_1^2 \mathrm{e}^{k_1(x+at\varepsilon+y)-2(b+c)k_1^3 t}}{a(1-\mathrm{e}^{k_1(x+at\varepsilon+y)-2(b+c)k_1^3 t})^2} - \varepsilon. \tag{18}$$

By selecting the arbitrary constants, one can obtain many new solutions.

Remark 1 A lot of new invariant solutions can be found through given solutions (Wazwaz 2012) for the extended quantum Zakharov–Kuznetsov equation. Thus we generalize the corresponding results in Wazwaz (2012).

4 Symmetry reduction and exact solutions to the extended QZK equation

In the preceding section, we obtained the vector fields of (1). Now, we deal with the symmetry reductions and exact solutions to the equations. We will consider the following similarity reductions and group-invariant solutions.

4.1 Symmetry reductions to the extended quantum Zakharov–Kuznetsov equation

(I) V_1

For the generator V_1, we have

$$u = f(\tau, \eta), \tag{19}$$

where $\eta = y, \tau = t$ are the group-invariant. Substituting (19) into (1), we reduce it to the following PDE

$$f_\tau + b f_{\eta\eta\eta} = 0. \tag{20}$$

(II) V_2

For the generator V_2, we have

$$u = f(\tau, \xi), \tag{21}$$

where $\xi = x, \tau = t$ are the group-invariant. Substituting (21) into (1), we reduce it to the following PDE

$$f_\tau + a f f_\xi + b f_{\xi\xi\xi} = 0. \tag{22}$$

It is interesting to note that (22) is just the famous KdV equation.

(III) V_3

For the generator V_3, we have

$$u = f(\eta, \xi), \tag{23}$$

where $\xi = x, \eta = y$ are the group-invariant. Substituting (21) into (1), we reduce it to the following PDE

$$a f f_\xi + b f_{\xi\xi\xi} + b f_{\eta\eta\eta} + c f_{\xi\eta\eta} + c f_{\xi\xi\eta} = 0. \tag{24}$$

(IV) V_4

For the generator V_4, we get

$$u = f(\xi, \eta) t^{-\frac{2}{3}}, \tag{25}$$

where $\xi = x t^{-\frac{1}{3}}, \eta = y t^{-\frac{1}{3}}$ are the group-invariant. Substituting (25) into (1), one can obtain

$$\begin{aligned} &-\xi f_\xi - \eta f_\eta - 2f + 3a f f_\xi + 3b f_{\xi\xi\xi} + 3b f_{\eta\eta\eta} \\ &\quad + 3c f_{\xi\xi\eta} + 3c f_{\xi\eta\eta} = 0. \end{aligned} \tag{26}$$

(V) V_5

In this case, one can obtain

$$u = f(\xi, \eta) + \frac{x}{at}, \tag{27}$$

where $\xi = t, \eta = y$ are the group-invariant. Substituting (27) into (1), one can obtain

$$\xi f_\xi + f + b\xi f_{\eta\eta\eta} = 0. \tag{28}$$

It is should been noted that all above reduced equation are $(1+1)$-dimensional PDEs. It is also difficult to get solutions of them. In order to get their solutions. Once again, by using Lie symmetry method. For the sake of simplicity, in what follows, we only consider (24) in details.

4.2 Symmetry reductions and exact solutions of (24)

Now, we deal with (24), if (24) is invariant under a one parameter Lie group of point transformations

$$\begin{aligned} \xi^* &= \xi + \epsilon\xi_1(\xi,\eta,f) + O(\epsilon^2), \\ \eta^* &= \eta + \epsilon\xi_2(\xi,\eta,f) + O(\epsilon^2), \\ f^* &= f + \epsilon\xi_3(\xi,\eta,f) + O(\epsilon^2), \end{aligned} \tag{29}$$

the vector field of an evolution type of equation is as follows:

$$V = \xi_1(x,t,u)\frac{\partial}{\partial\xi} + \xi_2(x,t,u)\frac{\partial}{\partial\eta} + \xi_3(x,t,u)\frac{\partial}{\partial f}, \tag{30}$$

where the coefficient functions $\xi_1(x,yt,u)$, $\xi_2(x,t,u)$ and $\xi_3(x,t,u)$ of the vector field are to be determined.

If the vector field (30) generates a symmetry of (24), then V must satisfy Lie's symmetry condition

$$\mathrm{pr}^{(3)}\, V(\Delta_1)|_{\Delta_1=0} = 0, \tag{31}$$

where $\Delta_1 = Aff_\xi + Bf_{\xi\xi\xi} + Bf_{\eta\eta\eta} + Cf_{\xi\eta\eta} + Cf_{\xi\xi\eta} = 0$.

As we have given in previous process, we obtain the determining equations for the symmetry group. One can get

$$\xi_1 = c_1\xi + c_3, \xi_2 = c_1\eta + c_2, \xi_3 = -2c_1 f, \tag{32}$$

the symmetry algebra of (24) is spanned by the three vector fields

$$\begin{aligned} V_1 &= \frac{\partial}{\partial\eta}, \qquad V_2 = \frac{\partial}{\partial\xi}, \\ V_3 &= \xi\frac{\partial}{\partial\xi} + \eta\frac{\partial}{\partial\eta} - 2f\frac{\partial}{\partial f}. \end{aligned} \tag{33}$$

(i) V_1

For the generator V_1, we have

$$f = g(p), \tag{34}$$

where $p=\xi$ is the group-invariant. Substituting (34) into (24), we reduce it to the following ODE

$$agg' + bg''' = 0. \tag{35}$$

If we integrate (35), then one can get

$$\frac{1}{2}ag^2 + bg'' + c = 0, \tag{36}$$

here c is the integration constant.

(ii) V_2

For the generator V_2, we have

$$u = g(p), \tag{37}$$

where $p=\eta$ is the group-invariant. Substituting (36) into (24), we reduce it to the following ODE

$$bg''' = 0. \tag{38}$$

Substituting (36) into (23), one can get the exact solutions of (1)

$$u = C_1y^2 + C_2y + C_3. \tag{39}$$

(iii) V_3

For the generator V_3, we have

$$f = g(p)\eta^{-2}, \tag{40}$$

where $p=\xi\eta^{-1}$ is the group-invariant. Substituting (39) into (24), we can get

$$\begin{aligned} &agg_p + bg_{ppp} + b(-p^3g_{ppp} - 12p^2g_{pp} \\ &\quad - 36pg_{pp} - 24g) + c(p^2g_{ppp} + 8pg_{pp} + 12g_p) \\ &\quad + c(-pg_{ppp} - 4g_{pp}) = 0. \end{aligned} \tag{41}$$

(iv) $V_1 + \lambda V_2$

For the generator $V_1 + \lambda V_2$, we get

$$f = g(p), \tag{42}$$

where $p = \xi - \lambda\eta$ is the group-invariant. Substituting (41) into (24), we can obtain

$$agg' + bg''' - b\lambda^3g''' + c\lambda^2g''' - c\lambda g''' = 0. \tag{43}$$

If we integrate (43), then one can get

$$\frac{1}{2}ag^2 + (b - b\lambda^3 + c\lambda^2 - c\lambda)g'' + c_1 = 0, \tag{44}$$

here c_1 is the integration constant.

It is should been noted that if $\lambda \neq 0$, then (44) is the traveling wave solution to the equation, where $\lambda \neq 0$ denotes the wave speed. In particular, if $\lambda = 0$, then we get (35). Now, we consider the traveling wave solutions of the (44).

By balancing the highest order partial derivative term and the nonlinear term in (44), we get the value of m,

$$(g(\xi))^{m+2} = (g(\xi))^{2m}, \tag{45}$$

so one can get $m=2$. In this case, we get

$$g(\xi) = a_0 + a_1\varphi + a_2\varphi^2, \tag{46}$$

where a_0, a_1 and a_2 are constants to be determined, $\varphi(\xi)$ satisfies

$$\varphi' = A + B\varphi + C\varphi^2. \tag{47}$$

Substituting the ansatz (46) along with (47) into (44), collecting coefficients of monomials of φ with the aid of Maple, then setting each coefficients to zero, we can deduce

$$\begin{aligned}
&A = 1/8\frac{C(12a_0a_2 - {a_1}^2)}{{a_2}^2}, \qquad B = \frac{Ca_1}{a_2},\\
&C = C, \qquad a = -12\frac{C^2(b\lambda^3 - c\lambda^2 + c\lambda - b)}{a_2},\\
&b = b, \qquad c = c, \qquad \lambda = \lambda,\\
&a_0 = a_0, \qquad a_1 = a_1, \qquad a_2 = a_2, \qquad\qquad (48)\\
&c_1 = \frac{3C^2}{32{a_2}^{-3}}\big(16b\lambda^3{a_0}^2{a_2}^2 - 8b\lambda^3a_0{a_1}^2a_2 + b\lambda^3{a_1}^4\\
&\quad - 16c\lambda^2{a_0}^2{a_2}^2 + 8c\lambda^2a_0{a_1}^2a_2 - c\lambda^2{a_1}^4\\
&\quad + 16c\lambda{a_0}^2{a_2}^2 - 8c\lambda a_0{a_1}^2a_2 + c\lambda{a_1}^4\\
&\quad - 16b{a_0}^2{a_2}^2 + 8ba_0{a_1}^2a_2 - b{a_{1d}}^4\big).
\end{aligned}$$

In view of (47) has a lot of fundamental solutions (twenty seven solutions) [13], one can find a number of exact travelling wave solutions for (1) by using (42) and (23), which are listed some special solutions as follows.

Family 1 : When $B^2 - 4AC > 0$ and $BC \neq 0$ (or $AC \neq 0$),

$$\begin{aligned}
u(x,y,t) &= a_0 - a_1\frac{1}{2C}\left[B + \sqrt{B^2-4AC}\right.\\
&\quad\left.\times \tanh\left(\frac{\sqrt{B^2-4AC}}{2}p\right)\right]\\
&\quad + a_2\left(-\frac{1}{2C}\left[B + \sqrt{B^2-4AC}\right.\right.\\
&\quad\left.\left.\times \tanh\left(\frac{\sqrt{B^2-4AC}}{2}p\right)\right]\right)^2.
\end{aligned} \tag{49}$$

$$\begin{aligned}
u(x,y,t) &= a_0 - a_1\frac{1}{2C}\left[B + \sqrt{B^2-4AC}\right.\\
&\quad\left.\times \coth\left(\frac{\sqrt{B^2-4AC}}{2}p\right)\right]\\
&\quad + a_2\left(-\frac{1}{2C}\left[B + \sqrt{B^2-4AC}\right.\right.\\
&\quad\left.\left.\times \coth\left(\frac{\sqrt{B^2-4AC}}{2}p\right)\right]\right)^2.
\end{aligned} \tag{50}$$

$$\begin{aligned}
&u(x,y,t)\\
&= a_0 - a_1\frac{1}{2C}\big[B + \sqrt{B^2-4AC}\big(\tanh\big(\sqrt{B^2-4AC}\,p\big)\\
&\quad \pm i\,\mathrm{sech}\big(\sqrt{B^2-4AC}\,p\big)\big)\big]\\
&\quad + a_2\bigg(-\frac{1}{2C}\big[B + \sqrt{B^2-4AC}\big(\tanh\big(\sqrt{B^2-4AC}\,p\big)\\
&\quad \pm i\,\mathrm{sech}\big(\sqrt{B^2-4AC}\,p\big)\big)\big]\bigg)^2.
\end{aligned} \tag{51}$$

$$\begin{aligned}
&u(x,y,t)\\
&= a_0 - a_1\frac{1}{2C}\big[B + \sqrt{B^2-4AC}\big(\coth\big(\sqrt{B^2-4AC}\,p\big)\\
&\quad \pm i\,\mathrm{csch}\big(\sqrt{B^2-4AC}\,p\big)\big)\big]\\
&\quad + a_2\bigg(-\frac{1}{2C}\big[B + \sqrt{B^2-4AC}\big(\coth\big(\sqrt{B^2-4AC}\,p\big)\\
&\quad \pm i\,\mathrm{csch}\big(\sqrt{B^2-4AC}\,p\big)\big)\big]\bigg)^2.
\end{aligned} \tag{52}$$

$$\begin{aligned}
&u(x,y,t)\\
&= a_0 - a_1\frac{1}{4C}\left[2B + \sqrt{B^2-4AC}\right.\\
&\quad \times\left(\tanh\left(\frac{\sqrt{B^2-4AC}}{4}p\right)\right.\\
&\quad \left.\left.+ \coth\left(\frac{\sqrt{B^2-4AC}}{4}p\right)\right)\right]\\
&\quad + a_2\left(-\frac{1}{4C}\left[2B + \sqrt{B^2-4AC}\right.\right.\\
&\quad \times\left(\tanh\left(\frac{\sqrt{B^2-4AC}}{4}p\right)\right.\\
&\quad \left.\left.\left.+ \coth\left(\frac{\sqrt{B^2-4AC}}{4}p\right)\right)\right]\right)^2.
\end{aligned} \tag{53}$$

$$\begin{aligned}
&u(x,y,t)\\
&= a_0 + a_1\frac{1}{2C}\left[-B + \frac{\sqrt{(E^2+F^2)(B^2-4AC)}}{E\sinh(\sqrt{B^2-4AC}\,p) + F}\right.\\
&\quad \left.- \frac{E\sqrt{B^2-4AC}\cosh(\sqrt{B^2-4AC}\,p)}{E\sinh(\sqrt{B^2-4AC}\,p) + F}\right]\\
&\quad + a_2\left(\frac{1}{2C}\left[-B + \frac{\sqrt{(E^2+F^2)(B^2-4AC)}}{E\sinh(\sqrt{B^2-4AC}\,p) + F}\right.\right.\\
&\quad \left.\left.- \frac{E\sqrt{B^2-4AC}\cosh(\sqrt{B^2-4AC}\,p)}{E\sinh(\sqrt{B^2-4AC}\,p) + F}\right]\right)^2.
\end{aligned} \tag{54}$$

$$u(x,y,t) = a_0 + a_1\frac{1}{2C}\left[-B - \frac{\sqrt{(F^2-E^2)(B^2-4AC)}}{E\cosh(\sqrt{B^2-4AC}\,p)+F} + \frac{E\sqrt{B^2-4AC}\sinh(\sqrt{B^2-4AC}\,p)}{E\cosh(\sqrt{B^2-4AC}\,p)+F}\right] + a_2\left(\frac{1}{2C}\left[-B - \frac{\sqrt{(F^2-E^2)(B^2-4AC)}}{E\cosh(\sqrt{B^2-4AC}\,p)+F} + \frac{E\sqrt{B^2-4AC}\sinh(\sqrt{B^2-4AC}\,p)}{E\cosh(\sqrt{B^2-4AC}\,p)+F}\right]\right)^2. \tag{55}$$

where E and F are two non-zero real constants and satisfies $F^2-E^2>0$.

$$u(x,y,t) = a_0 + a_1\left(2A\cosh\left(\frac{\sqrt{B^2-4AC}}{2}p\right)\right) \div \left[\sqrt{B^2-4AC}\sinh\left(\frac{\sqrt{B^2-4AC}}{2}p\right) - B\cosh\left(\frac{\sqrt{B^2-4AC}}{2}p\right)\right] + a_2\left(2A\cosh\left(\frac{\sqrt{B^2-4AC}}{2}p\right)\right)^2 \div \left[\sqrt{B^2-4AC}\sinh\left(\frac{\sqrt{B^2-4AC}}{2}p\right) - B\cosh\left(\frac{\sqrt{B^2-4AC}}{2}p\right)\right]^2. \tag{56}$$

$$u(x,y,t) = a_0 + a_1\left(-2A\sinh\left(\frac{\sqrt{B^2-4AC}}{2}p\right)\right) \div \left[-\sqrt{B^2-4AC}\cosh\left(\frac{\sqrt{B^2-4AC}}{2}p\right) + B\sinh\left(\frac{\sqrt{B^2-4AC}}{2}p\right)\right] + a_2\left(-2A\sinh\left(\frac{\sqrt{B^2-4AC}}{2}p\right)\right)^2 \div \left[-\sqrt{B^2-4AC}\cosh\left(\frac{\sqrt{B^2-4AC}}{2}p\right) + B\sinh\left(\frac{\sqrt{B^2-4AC}}{2}p\right)\right]^2. \tag{57}$$

$$u(x,y,t) = a_0 + a_1\big(2A\cosh\big(\sqrt{B^2-4AC}\,p\big)\big) \div \big[\sqrt{B^2-4AC}\sinh\big(\sqrt{B^2-4AC}\,p\big) - B\cosh\big(\sqrt{B^2-4ACp}\big) \pm i\sqrt{B^2-4ACp}\big] + a_2\big(2A\cosh\big(\sqrt{B^2-4AC}\,p\big)\big)^2 \div \big[\sqrt{B^2-4AC}\sinh\big(\sqrt{B^2-4AC}\,p\big) - B\cosh\big(\sqrt{B^2-4ACp}\big) \pm i\sqrt{B^2-4ACp}\big]^2. \tag{58}$$

$$u(x,y,t) = a_0 + a_1\big(2A\sinh\big(\sqrt{B^2-4AC}\,p\big)\big) \div \big[\sqrt{B^2-4AC}\cosh\big(\sqrt{B^2-4AC}\,p\big) - B\sinh\big(\sqrt{B^2-4ACp}\big) \pm \sqrt{B^2-4ACp}\big] + a_2\big(2A\sinh\big(\sqrt{B^2-4AC}\,p\big)\big)^2 \div \big[\sqrt{B^2-4AC}\cosh\big(\sqrt{B^2-4AC}\,p\big) - B\sinh\big(\sqrt{B^2-4ACp}\big) \pm \sqrt{B^2-4ACp}\big]^2. \tag{59}$$

$$u(x,y,t) = a_0 + a_1\left(4A\sinh\left(\frac{\sqrt{B^2-4AC}}{4}p\right) \times \cosh\left(\frac{\sqrt{B^2-4AC}}{4}p\right)\right) \div \left[-2B\sinh\left(\frac{\sqrt{B^2-4AC}}{4}p\right)\cosh\left(\frac{\sqrt{B^2-4AC}}{4}p\right) + 2\sqrt{B^2-4AC}\cosh^2\left(\frac{\sqrt{B^2-4AC}}{4}p\right) - \sqrt{B^2-4AC}\right] + a_2\left(4A\sinh\left(\frac{\sqrt{B^2-4AC}}{4}p\right) \times \cosh\left(\frac{\sqrt{B^2-4AC}}{4}p\right)\right)^2 \div \left[-2B\sinh\left(\frac{\sqrt{B^2-4AC}}{4}p\right)\cosh\left(\frac{\sqrt{B^2-4AC}}{4}p\right) + 2\sqrt{B^2-4AC}\cosh^2\left(\frac{\sqrt{B^2-4AC}}{4}p\right) - \sqrt{B^2-4AC}\right]^2. \tag{60}$$

Family 2 : When $B^2-4AC<0$ and $BC\neq 0$ (or $AC\neq 0$),

$$u(x,y,t)=a_0+a_1\frac{1}{2C}\Big[-B+\sqrt{4AC-B^2}\times\tan\Big(\frac{\sqrt{4AC-B^2}}{2}p\Big)\Big]+a_2\Big(\frac{1}{2C}\Big[-B+\sqrt{4AC-B^2}\times\tan\Big(\frac{\sqrt{4AC-B^2}}{2}p\Big)\Big]\Big)^2. \tag{61}$$

$$u(x,y,t)=a_0-a_1\frac{1}{2C}\Big[B+\sqrt{4AC-B^2}\times\cot\Big(\frac{\sqrt{4AC-B^2}}{2}p\Big)\Big]+a_2\Big(-\frac{1}{2C}\Big[B+\sqrt{4AC-B^2}\times\cot\Big(\frac{\sqrt{4AC-B^2}}{2}p\Big)\Big]\Big)^2. \tag{62}$$

$$u(x,y,t)=a_0+a_1\frac{1}{2C}\big[-B+\sqrt{4AC-B^2}\times\big(\tan\big(\sqrt{4AC-B^2}p\big)\pm\sec\big(\sqrt{4AC-B^2}p\big)\big)\big]+a_2\Big(\frac{1}{2C}\big[-B+\sqrt{4AC-B^2}\times\big(\tan\big(\sqrt{4AC-B^2}p\big)\times\pm\sec\big(\sqrt{4AC-B^2}p\big)\big)\big]\Big)^2. \tag{63}$$

$$u(x,y,t)=a_0-a_1\frac{1}{2C}\big[B+\sqrt{4AC-B^2}\times\big(\cot\big(\sqrt{4AC-B^2}p\big)\pm\csc\big(\sqrt{4AC-B^2}p\big)\big)\big]+a_2\Big(-\frac{1}{2C}\big[B+\sqrt{4AC-B^2}\times\big(\cot\big(\sqrt{4AC-B^2}p\big)\pm\csc\big(\sqrt{4AC-B^2}p\big)\big)\big]\Big)^2. \tag{64}$$

$$u(x,y,t)=a_0-a_1\frac{1}{4C}\Big[-2B+\sqrt{4AC-B^2}\times\Big(\tan\Big(\frac{\sqrt{4AC-B^2}}{4}p\Big)-\cot\Big(\frac{\sqrt{4AC-B^2}}{4}p\Big)\Big)\Big]+a_2\Big(-\frac{1}{4C}\Big[-2B+\sqrt{4AC-B^2}\times\Big(\tan\Big(\frac{\sqrt{4AC-B^2}}{4}p\Big)-\cot\Big(\frac{\sqrt{4AC-B^2}}{4}p\Big)\Big)\Big]\Big)^2. \tag{65}$$

$$u(x,y,t)=a_0+a_1\frac{1}{2C}\Big[-B+\frac{\pm\sqrt{(F^2-E^2)(4AC-B^2)}}{E\sin(\sqrt{4AC-B^2}p)+F}-\frac{E\sqrt{4AC-B^2}\cos(\sqrt{4AC-B^2}p)}{E\sin(\sqrt{4AC-B^2}p)+F}\Big]+a_2\Big(\frac{1}{2C}\Big[-B+\frac{\pm\sqrt{(F^2-E^2)(4AC-B^2)}}{E\sin(\sqrt{4AC-B^2}p)+F}-\frac{E\sqrt{4AC-B^2}\cos(\sqrt{4AC-B^2}p)}{E\sin(\sqrt{4AC-B^2}p)+F}\Big]\Big)^2. \tag{66}$$

$$u(x,y,t)=a_0+a_1\frac{1}{2C}\Big[-B+\frac{\pm\sqrt{(F^2-E^2)(4AC-B^2)}}{E\cos(\sqrt{4AC-B^2}p)+F}+\frac{E\sqrt{4AC-B^2}\sinh(\sqrt{4AC-B^2}p)}{E\cos(\sqrt{4AC-B^2}p)+F}\Big]+a_2\Big(\frac{1}{2C}\Big[-B+\frac{\pm\sqrt{(F^2-E^2)(4AC-B^2)}}{E\cos(\sqrt{4AC-B^2}p)+F}+\frac{E\sqrt{4AC-B^2}\sinh(\sqrt{4AC-B^2}p)}{E\cos(\sqrt{4AC-B^2}p)+F}\Big]\Big)^2, \tag{67}$$

where E and F are two non-zero real constants and satisfies $F^2-E^2>0$.

$$u(x,y,t)=a_0+a_1\Big(-2A\cos\Big(\frac{\sqrt{4AC-B^2}}{2}p\Big)\Big)\div\Big[\sqrt{4AC-B^2}\sin\Big(\frac{\sqrt{4AC-B^2}}{2}p\Big)+B\cos\Big(\frac{\sqrt{4AC-B^2}}{2}p\Big)\Big]+a_2\Big(-2A\cos\Big(\frac{\sqrt{4AC-B^2}}{2}p\Big)\Big)^2\div\Big[\sqrt{4AC-B^2}\sin\Big(\frac{\sqrt{4AC-B^2}}{2}p\Big)$$

$$+B\cos\left(\frac{\sqrt{4AC-B^2}}{2}p\right)\bigg]^2. \tag{68}$$

$$\begin{aligned}u(x,y,t)&=a_0+a_1\left(2A\sin\left(\frac{\sqrt{4AC-B^2}}{2}p\right)\right)\\&\div\left[\sqrt{4AC-B^2}\cos\left(\frac{\sqrt{4AC-B^2}}{2}p\right)\right.\\&\left.-B\sin\left(\frac{\sqrt{4AC-B^2}}{2}p\right)\right]\\&+a_2\left(2A\sin\left(\frac{\sqrt{4AC-B^2}}{2}p\right)\right)^2\\&\div\left[\sqrt{4AC-B^2}\cos\left(\frac{\sqrt{4AC-B^2}}{2}p\right)\right.\\&\left.-B\sin\left(\frac{\sqrt{4AC-B^2}}{2}p\right)\right]^2.\end{aligned} \tag{69}$$

$$\begin{aligned}&u(x,y,t)\\&=a_0+a_1\left(-2A\cos\left(\sqrt{4AC-B^2}p\right)\right)\\&\quad\div\left[\sqrt{4AC-B^2}\sin\left(\sqrt{4AC-B^2}p\right)\right.\\&\quad\left.+B\cos\left(\sqrt{4AC-B^2p}\right)\pm i\sqrt{4AC-B^2p}\right]\\&\quad+a_2\left(-2A\cos\left(\sqrt{4AC-B^2}p\right)\right)^2\\&\quad\div\left[\sqrt{4AC-B^2}\sin\left(\sqrt{4AC-B^2}p\right)\right.\\&\quad\left.+B\cos\left(\sqrt{4AC-B^2p}\right)\pm i\sqrt{4AC-B^2p}\right]^2.\end{aligned} \tag{70}$$

$$\begin{aligned}&u(x,y,t)\\&=a_0+a_1\left(2A\sin\left(\sqrt{4AC-B^2}p\right)\right)\\&\quad\div\left[\sqrt{4AC-B^2}\cos\left(\sqrt{4AC-B^2}p\right)\right.\\&\quad\left.-B\sin\left(\sqrt{4AC-B^2p}\right)\pm\sqrt{4AC-B^2p}\right]\\&\quad+a_2\left(2A\sin\left(\sqrt{4AC-B^2}p\right)\right)\\&\quad\div\left[\sqrt{4AC-B^2}\cos\left(\sqrt{4AC-B^2}p\right)\right.\\&\quad\left.-B\sin\left(\sqrt{4AC-B^2p}\right)\pm\sqrt{4AC-B^2p}\right]^2.\end{aligned} \tag{71}$$

$$\begin{aligned}u(x,y,t)&=a_0+a_1\left(4A\sin\left(\frac{\sqrt{4AC-B^2}}{4}p\right)\right.\\&\left.\times\cos\left(\frac{\sqrt{4AC-B^2}}{4}p\right)\right)\\&\div\left[-2B\sin\left(\frac{\sqrt{4AC-B^2}}{4}p\right)\right.\\&\times\cos\left(\frac{\sqrt{4AC-B^2}}{4}p\right)\\&+2\sqrt{4AC-B^2}\cos^2\left(\frac{\sqrt{4AC-B^2}}{4}p\right)\\&\left.-\sqrt{4AC-B^2}\right]\\&+a_2\left(4A\sin\left(\frac{\sqrt{4AC-B^2}}{4}p\right)\right.\\&\left.\times\cos\left(\frac{\sqrt{4AC-B^2}}{4}p\right)\right)^2\\&\div\left[-2B\sin\left(\frac{\sqrt{4AC-B^2}}{4}p\right)\right.\\&\times\cos\left(\frac{\sqrt{4AC-B^2}}{4}p\right)\\&+2\sqrt{4AC-B^2}\cos^2\left(\frac{\sqrt{4AC-B^2}}{4}p\right)\\&\left.-\sqrt{4AC-B^2}\right]^2.\end{aligned} \tag{72}$$

Family 3 : When $A=0$ and $BC\neq 0$,

$$\begin{aligned}u(x,y,t)&=a_0+a_1\left(\frac{-Bd}{B(d+\cosh(Bp)-\sinh(Bp))}\right)\\&+a_2\left(\left(\frac{-Bd}{B(d+\cosh(Bp)-\sinh(Bp))}\right)\right)^2.\end{aligned} \tag{73}$$

where d is an arbitrary constant.

Family 4 : When $A=B=0$ and $C\neq 0$,

$$u(x,y,t)=a_0+a_1\left(\frac{-1}{B(p)+k}\right)+a_2\left(\frac{-1}{B(p)+k}\right)^2, \tag{74}$$

where k is an arbitrary constant.

Remark 2 For brevity, we only give the solution of (24). In fact, we can get the solutions of others reduced equation. Here we do not list them for simplicity.

Remark 3 It is should been noted that (26) and (41) are higher-order nonlinear or nonautonomous DEs. In general, we can not get the exact solutions of them. The power series method (Wang et al. 2013), however, is very effective for higher-order nonlinear or nonautonomous DEs. We have omitted them here. To the best of our knowledge, the solutions obtained in this paper have not been reported in previous literature. Thus, these solutions are new solutions of (1). And above all, these power series play an important role in the investigation of physical phenomena and other natural phenomenon.

5 Traveling wave solution

This section will reconsider the extended QZK equation with power law nonlinearity in order to study this equation from a generalized setting. Therefore Eq. (1), is now rewritten as

$$u_t + au^n u_x + b(u_{xxx} + u_{yyy}) + c(u_{xyy} + u_{xxy}) = 0, \quad (75)$$

where the parameter n dictates the power law nonlinearity. The imposed restriction is that $n > 0$. The traveling wave hypothesis will be employed to integrate this Eq. (75). In order to get started, the initial hypothesis is taken to be

$$u(x, y, t) = g(B_1 x + B_2 y - vt) = g(s), \quad (76)$$

where

$$s = B_1 x + B_2 y - vt, \quad (77)$$

and B_j for $j = 1, 2$ are the direction ratios while v is the speed of the soliton. The functional form g represents the wave of permanent form. Substituting (76) into (75) implies

$$0 = vg' - aB_1 g^n g' - \left\{b\left(B_1^3 + B_2^3\right) - cB_1 B_2 (B_1 + B_2)\right\}g''', \quad (78)$$

where the notations $g' = dg/ds$, $g'' = d^2g/ds^2$ and so on is employed. Integrating (78) once with respect to s and taking the integration constant to be zero, since the search is for a soliton solution, leads to the 1-soliton solution of the extended QZK equation as

$$0 = vg - \frac{aB_1 g^{n+1}}{n+1} - \left\{b\left(B_1^3 + B_2^3\right) - cB_1 B_2 (B_1 + B_2)\right\}g''. \quad (79)$$

Now, multiplying both sides by g' and integrating once again, leads to

$$0 = vg^2 - \frac{2aB_1 g^{n+2}}{(n+1)(n+2)} - \left\{b\left(B_1^3 + B_2^3\right) - cB_1 B_2 (B_1 + B_2)\right\}\left(g'\right)^2, \quad (80)$$

after choosing the integration constant to be zero again. From (80), separating variables and integrating leads to

$$u(x, y, t) = g(B_1 x + B_2 y - vt) = A \operatorname{sech}^{\frac{2}{n}}\left[B(B_1 x + B_2 y - vt)\right], \quad (81)$$

where the amplitude A is given by

$$A = \left[\frac{(n+1)(n+2)v}{2aB_1}\right]^{\frac{1}{n}}, \quad (82)$$

and the inverse width (B) is

$$B = \frac{n}{2}\sqrt{\frac{v}{b(B_1^3 + B_2^3) + cB_1 B_2 (B_1 + B_2)}}. \quad (83)$$

These relations introduces the constraint relations

$$avB_1 > 0, \quad (84)$$

for even values of n and

$$v\left\{b\left(B_1^3 + B_2^3\right) + cB_1 B_2 (B_1 + B_2)\right\} > 0, \quad (85)$$

for the solitons to exist.

6 Conclusions

In this paper, we have considered a new extended QZK equation. All of the geometric vector fields of the equation are presented for the first time in the literature. Based on the symmetry reductions, some exact solutions are investigated in detail, which include hyperbolic function solutions, trigonometric function solutions and so on. Some of them are obtained for the first time which generalizes the previous results. In addition, the 1-soliton solution was also obtained by the traveling wave hypothesis. All of these results are very concrete and consequently stand on a strong footing to launch further and deeper investigation into this equation, possibly in presence of perturbation terms.

Acknowledgement The project is supported by the National Natural Science Foundation of China (NNSFC) (Grant No. 11171022).

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