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Symmetry analysis and rogue wave solutions for the $(2+1)$-dimensional nonlinear Schrödinger equation with variable coefficients

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ABSTRACT

This paper addresses $(2+1)$-dimensional nonlinear Schrödinger equation (NLSE). For the special case, linear Schrödinger equation (LSE), it can be transformed into the same form of equation. On the basis of different gauge constraint, we construct potential symmetries for the LSE. And then, we consider $(2+1)$-dimensional NLSE using Lie symmetry analysis. By means of similarity transformations, we study the $(2+1)$-dimensional NLSE with nonlinearities and potentials depending on time as well as on the spatial coordinates. At last, we present the rouge wave solutions of $(2+1)$-dimensional NLSE.

1. Introduction

The famous nonlinear Schrödinger equation (NLSE) comes up in many science fields. There are various versions of NLSE that are used to explain the complex physical phenomenon. As one of the most key models of nonlinear mathematical physics, they have attracted much attention due to their potential applications. In addition, soliton solutions play a key role in study of NLSE. It is well-known that nonlinear and dispersive effects generate soliton. They are investigated by many authors in different fields, such as in mean-field theory of Bose–Einstein condensates, nonlinear optics and other fields [1–5]. There are a lot of papers that handled the $(1+1)$-dimensional nonlinear Schrödinger equation. In Ref. [6], the authors deal with nonlinear Schrödinger equation with spatially inhomogeneous nonlinearities using Lie group method and canonical transformations. In Ref. [7], the authors again, using Lie group method and canonical transformations, studied quintic nonlinear Schrödinger equations with spatially inhomogeneous nonlinearities. In Ref. [8], the authors, based on the similarity transformations, investigated quintic nonlinear Schrödinger equation with time and space modulated nonlinearities and potentials. The authors in [9] obtained the new classes of

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$(1+1)$-dimensional nonlinear Schrödinger equation with time-dependent potentials, and then, they got the free particle equation via non-local transformations.

Compared with the situation of $(1+1)$-dimensional NLSE, the $(2+1)$-dimensional NLSE has been paid minor treatment. Whether from a mathematics or from a physics point of view, the $(2+1)$-dimensional NLSE or more high dimensional NLSE may provide more backgrounds for real world. Also, there are many methods to deal with the nonlinear evolution equations (NLEEs), such as group method [9–19], generalized multi-symplectic method [20–27] and so on. It is known that the notable symmetry of the infinite dimensional Hamiltonian system, named as the multi-symplectic structure [20–23] was generalized to the non conservative Hamiltonian dynamic system, which is named as generalized multi-symplectic method [24], and was applied in some applied mechanics problems, such as the oscillating problems of the carbon nanotube [25,26] and the competition between dispersion and dissipation in the dynamic system [27]. In addition, there are a lot of papers and excellent books [14,16] that are used to handle NLEEs using group method. The group method provided a systematic route to deal with NLEEs [14,16].

In this paper, we study the following $(2+1)$-dimensional NLSE using group method

$$iu_t + u_{xx} + u_{yy} - g(x,y,t)|u|^2u - V(x,y,t)u = 0, \tag{1}$$

where the first term gives the evolution term, the group velocity dispersion are given by the second and third term, $g(x,y,t)$ represents the coefficient of nonlinear term, and $V(x,y,t)$ is an external potential. Recently, the authors [10] studied the equivalence group of LSE. The authors in paper [11] presented the admissible transformations and normalized classes of NLSE. In Ref. [28], the authors considered the AB and KA soliton solutions, they also consider the mechanism for controlling the obtained localized solutions. They got the bright and dark soliton solutions in 2D graded-index waveguides [29].

The paper is divided in the following manner. In Section 2, we consider the special case, and construct the potential symmetries. In Section 3, we employ Lie group to study (1). In Section 4, we use the similarity transformations to construct explicit rouge wave solutions of the equation. We also give some examples. Conclusions are presented in the last section.

2. Special case: nonlinearity $g(x,y,t) = 0$.

In this section, we consider the special case, that is to say,

$$iu_t + u_{xx} + u_{yy} - V(x,y,t)u = 0. \tag{2}$$

Consider the following point transformation

$$\tau = T(t), \qquad X = \xi(x,t,u), \qquad Y = \eta(y,t,u), \quad u = U(x,y,t,u). \tag{3}$$

They map (2) into the same form of equation as follows

$$iU_T + U_{\xi\xi} + U_{\eta\eta} - V_1(\xi,\eta,T)U = 0 \tag{4}$$

and we get

$$\begin{aligned} &\xi = a_1(t)x + b_1(t), \qquad \eta = a_1(t)y + c_1(t), \qquad T = \int^t a_1(\mu)^2 d\mu, \\ &U = e^{i\left[\frac{a_{1t}}{4a}(x^2+y^2) + \frac{b_{1t}}{2a}x + \frac{c_{1t}}{2a}y + d_1(t)\right]}u, \end{aligned} \tag{5}$$

and

$$\begin{aligned} V_1(\xi,\eta,T) = \frac{1}{a_1^2}\Bigg[&V(x,y,t) + \frac{2a_{1t}^2 - a_{tt}a_1}{4a_1^2}(x^2+y^2) + \frac{2a_{1t}b_{1t} - a1tb_{1tt}}{2a_1^2}x \\ &+ \frac{2a_{1t}c_{1t} - a1tc_{1tt}}{2a_1^2}y + \frac{b_{1t}^2}{4a_1^2} + \frac{c_{1t}^2}{4a_1^2} + i\frac{a_{1t}}{a_1} - d_{1t}\Bigg], \end{aligned} \tag{6}$$

where $a_1(t), b_1(t), c_1(t)$ and $d_1(t)$ are arbitrary functions of t. For more details of the equivalence group please see [10,11].

2.1. Potential symmetries with gauge constraint

Consider the following operator L and the corresponding adjoint L^* [9,12]

$$\phi Lu - uL^*\phi = D_t(f^1) + D_x(f^2) + D_y(f^3), \tag{7}$$

the operator D means the total derivative operator. One knows that, if

$$L^*\phi = 0, \tag{8}$$

then, one has

$$Lu = 0, \tag{9}$$

in other words,

$$0 = D_t(f^1) + D_x(f^2) + D_y(f^3). \tag{10}$$

Here ϕ is any solution of adjoint equation.

It is clear that L is

$$L = i\frac{\partial}{\partial t} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - V(x, y, t), \tag{11}$$

and the adjoint equation is

$$L^* = -i\frac{\partial}{\partial t} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - V(x, y, t). \tag{12}$$

In this case, one gets

$$(i\phi u)_t + (\phi u_x - \phi_x u)_x + (\phi u_y - \phi_y u)_y = 0. \tag{13}$$

2.1.1. With Lorentz gauge constraint

In order to look for potential symmetries, we need to consider additional gauge constraint [13–15]. We here choose Lorentz gauge. Therefore, one can get the potential system

$$\begin{aligned}
&i\phi u = V2_x - V1_y,\\
&\phi u_x - \phi_x u = V0_y - V2_t,\\
&\phi u_y - \phi_y u = V1_t - V0_x,\\
&V0_t - V1_x - V2_y = 0.
\end{aligned} \tag{14}$$

One can obtain the following results

$$\begin{aligned}
\xi^3 &= \mathbf{c}_1 + t\mathbf{c}_2,\\
\xi^1 &= x\mathbf{c}_2 - y\mathbf{c}_4 + \mathbf{c}_6,\\
\xi^2 &= y\mathbf{c}_2 + x\mathbf{c}_4 + \mathbf{c}_5,\\
\eta_1 &= u\left(\frac{1}{2}\left(-2\mathbf{c}_2 + \mathbf{c}_3 - \mathcal{F}_3\left[t, x, y, \frac{\phi}{u}\right] + \frac{\phi\mathcal{F}_{3\mathfrak{x}_4}}{u}\right) + \mathcal{F}_3\left[t, x, y, \frac{\phi}{u}\right]\right),\\
\eta_2 &= V0\left(-2\mathbf{c}_2 + \mathbf{c}_3\right) + \mathcal{F}_1[x, y, t],
\end{aligned}$$

$$
\begin{aligned}
\eta_3 &= -2V1\mathbf{c}_2 + V1\mathbf{c}_3 - V2\mathbf{c}_4 + \int \mathcal{F}_{1t} dx - \int \mathcal{F}_{2y} dx + \mathcal{F}_4[y,t],\\
\eta_4 &= V2\,(-2\mathbf{c}_2 + \mathbf{c}_3) + V1\mathbf{c}_4 + \mathcal{F}_2[x,y,t],\\
\eta_5 &= \frac{1}{2}\left(-2\phi\mathbf{c}_2 + \phi\mathbf{c}_3 - \phi\mathcal{F}_3\left[t,x,y,\frac{\phi}{u}\right] + \frac{\phi^2\mathcal{F}_{3\mathfrak{x}_4}}{u}\right),
\end{aligned}
\tag{15}
$$

in addition we have the constraints as follows

$$
\begin{aligned}
&3iu\phi\mathbf{c}_2 - iu\phi\mathbf{c}_3 + \int \mathcal{F}_{1yt}\, dx - \int \mathcal{F}_{2yy}\, dx + 2iu\eta_5 + iu\phi\mathcal{F}_3\left[t,x,y,\frac{\phi}{u}\right] + \mathcal{F}_{4y} - \mathcal{F}_{2x} = 0,\\
&\int \mathcal{F}_{1tt}\, dx - \int \mathcal{F}_{2yt}\, dx + \mathcal{F}_{4t} - \mathcal{F}_{1x} + u\phi\mathcal{F}_{3y} = 0,\\
&\mathcal{F}_{2t} - \mathcal{F}_{1y} - u\phi\mathcal{F}_{3x} = 0.
\end{aligned}
\tag{16}
$$

2.1.2. With temporal gauge constraint

In this subsection, temporal gauge constraint is considered. One yields the following potential system

$$
\begin{aligned}
i\phi u &= V2_x - V1_y,\\
\phi u_x - \phi_x u &= -V2_t,\\
\phi u_y - \phi_y u &= V1_t.
\end{aligned}
\tag{17}
$$

We get the following results

$$
\begin{aligned}
\xi^3 &= \mathcal{F}_2[t],\\
\xi^1 &= \mathcal{F}_1[x,y],\\
\xi^2 &= \int \mathcal{F}_{1x}\, dy + \mathcal{F}_6[x],\\
\eta_1 &= u\left(\frac{1}{2}\left(\mathcal{F}_5[x,y] - \mathcal{F}_7\left[t,x,y,\frac{\phi}{u}\right] - \mathcal{F}_2' + \mathcal{F}_{1x} + \frac{\phi\mathcal{F}_{7\mathfrak{x}_4}}{u}\right) + \mathcal{F}_7\left[t,x,y,\frac{\phi}{u}\right]\right),\\
\eta_2 &= \mathcal{F}_3[x,y,t] + V1\mathcal{F}_5[x,y] - V2\left(\int \mathcal{F}_{1xx}\, dy + \mathcal{F}_6'\right),\\
\eta_3 &= \mathcal{F}_4[x,y,t] + V2\mathcal{F}_5[x,y] + V1\left(\int \mathcal{F}_{1xx}\, dy + \mathcal{F}_6'\right),\\
\eta_4 &= \frac{1}{2}\left(\phi\mathcal{F}_5[x,y] - \phi\mathcal{F}_7\left[t,x,y,\frac{\phi}{u}\right] - \phi\mathcal{F}_2' + \phi\mathcal{F}_{1x} + \frac{\phi^2\mathcal{F}_{7\mathfrak{x}_4}}{u}\right),
\end{aligned}
\tag{18}
$$

and we get the additional constraints

$$
\begin{aligned}
&\mathcal{F}_6' + \mathcal{F}_{1y} + y\mathcal{F}_{1xx} = 0,\\
&2iu\eta_4 - iu\phi\mathcal{F}_5[x,y] + iu\phi\mathcal{F}_7\left[t,x,y,\frac{\phi}{u}\right] + V1\mathcal{F}_{5y}\\
&\quad + iu\phi\mathcal{F}_{1x} - V2\mathcal{F}_{5x} + V1\mathcal{F}_{1xy} - V2\mathcal{F}_{1xx} + \mathcal{F}_{3y} - \mathcal{F}_{4x} = 0,\\
&\mathcal{F}_{3t} + u\phi\mathcal{F}_{7y} = 0,\\
&\mathcal{F}_{4t} - u\phi\mathcal{F}_{7x} = 0.
\end{aligned}
\tag{19}
$$

3. Lie symmetries

In this section, we consider the general case (2 + 1)-dimensional NLSE with variables using Lie group method. On the basis of Lie group theorem, we get

$$
\xi^3 = \mathcal{F}_2[t],
$$

$$\xi^2 = -x\mathbf{c}_1 + \mathcal{F}_4[t] + \frac{y\mathcal{F}_2'}{2},$$
$$\xi^1 = y\mathbf{c}_1 + \mathcal{F}_3[t] + \frac{x\mathcal{F}_2'}{2},$$
$$\eta = \mathcal{F}_1[x,y,t] + \frac{1}{8}iu\left(-8i\mathcal{F}_5[t] + 4x\mathcal{F}_3' + 4y\mathcal{F}_4' + x^2\mathcal{F}_2'' + y^2\mathcal{F}_2''\right), \tag{20}$$

and $V(x,y,t)$ satisfy the following conditions

$$\begin{aligned}
&-8V[x,y,t]\mathcal{F}_2' + 8i\mathcal{F}_5' + 4i\mathcal{F}_2'' - 4x\mathcal{F}_3'' - 4y\mathcal{F}_4'' - x^2\mathcal{F}_2''' \\
&\quad - y^2\mathcal{F}_2''' - 8\mathcal{F}_2[t]V_{,t} + 8x\mathbf{c}_1V_{,y} - 8\mathcal{F}_4[t]V_{,y} - 4y\mathcal{F}_2'V_{,y} - 8y\mathbf{c}_1V_{,x} - 8\mathcal{F}_3[t]V_{,x} - 4x\mathcal{F}_2'V_{,x} = 0, \\
&-V[x,y,t]\mathcal{F}_1[x,y,t] + i\mathcal{F}_{1t} + \mathcal{F}_{1yy} + \mathcal{F}_{1xx} = 0, \\
&-8V[x,y,t]\mathcal{F}_1[x,y,t] - 8uV[x,y,t]\mathcal{F}_2' + 8iu\mathcal{F}_5' + 4iu\mathcal{F}_2'' - 4ux\mathcal{F}_3'' - 4uy\mathcal{F}_4'' \\
&\quad - ux^2\mathcal{F}_2''' - uy^2\mathcal{F}_2''' - 8u\mathcal{F}_2[t]V_{,t} + 8i\mathcal{F}_{1t} + 8ux\mathbf{c}_1V_{,y} - 8u\mathcal{F}_4[t]V_{,y} - 4uy\mathcal{F}_2'V_{,y} \\
&\quad + 8\mathcal{F}_{1yy} - 8uy\mathbf{c}_1V_{,x} - 8u\mathcal{F}_3[t]V_{,x} - 4ux\mathcal{F}_2'V_{,x} + 8\mathcal{F}_{1xx} = 0.
\end{aligned} \tag{21}$$

4. Similarity transformation of (2 + 1) NLS equation with variable coefficients

In the present section, we consider the general case. First, we use the similarity transformation, to reduce the $(2+1)$ NLS equation with variable coefficients to the stationary cubic nonlinear Schrödinger equation.

Transformation of Eq. (1) to the stationary cubic nonlinear Schrödinger equation

$$EU = -U_{\xi\xi} + G|U|^2U. \tag{22}$$

Here, $U = U(\xi)$ and $\xi = \xi(x,y,t)$ are real functions. The eigenvalue of the nonlinear equation is given by E known as chemical potential. While G is a constant. In this case, we can get the attractive nonlinearity on the condition of $G = -1$, and the repulsive nonlinearity when $G = 1$. In order to get the relation between (22) and (1), we use the following transformation [6–8]

$$u = \rho(x,y,t)e^{i\varphi(x,y,t)}U(\xi(x,y,t)). \tag{23}$$

Putting them (23) into (1), one can get

$$u_t = \left(\rho_tU + i\rho\varphi_tU + \rho U_\xi\xi_t\right)e^{i\varphi}, \tag{24}$$
$$u_{xx} = \left(\rho_{xx}U + 2\rho_x\xi_xU_\xi + 2i\rho_x\varphi_xU + i\rho\varphi_{xx}U + 2i\rho\varphi_x\xi_xU_\xi + \rho\xi_x^2U_{\xi\xi} + \rho\xi_{xx}U_\xi + i^2\rho\varphi_x^2U\right)e^{i\varphi}, \tag{25}$$
$$u_{yy} = \left(\rho_{yy}U + 2\rho_y\xi_yU_\xi + 2i\rho_y\varphi_yU + i\rho\varphi_{yy}U + 2i\rho\varphi_y\xi_yU_\xi + \rho\xi_y^2U_{\xi\xi} + \rho\xi_{yy}U_\xi + i^2\rho\varphi_y^2U\right)e^{i\varphi}, \tag{26}$$

and then substitute them into (1), and separate real and imaginary part, one can get

$$i\rho_tU + i\rho\xi_tU_\xi + i\varphi_{xx}\rho U + 2i\rho_x\varphi_xU + 2i\varphi_x\xi_x\rho U_\xi + i\varphi_{yy}\rho U + 2i\varphi_y\rho_yU + 2i\varphi_y\xi_y\rho U_\xi = 0, \tag{27}$$
$$\begin{aligned}
&-\varphi_t\rho U + \rho_{xx}U + 2\rho_x\xi_xU_\xi + \rho\xi_x^2U_{\xi\xi} + \rho\xi_{xx}U_\xi - \varphi_x^2\rho U \\
&\quad + \rho_{yy}U + 2\rho_y\xi_yU_\xi + \rho\xi_y^2U_{\xi\xi} + \rho\xi_{yy}U_\xi - \varphi_y^2\rho U - V\rho U - g\rho^3|U|^2U = 0.
\end{aligned} \tag{28}$$

Now, letting the coefficients of U and its different derivatives equal to zero, one can get

$$\rho_t + \rho\varphi_{xx} + 2\varphi_x\rho_x + \rho\varphi_{yy} + 2\varphi_y\rho_y = 0, \tag{29}$$
$$\rho\xi_t + 2\rho\varphi_x\xi_x + 2\rho\varphi_y\xi_y = 0, \tag{30}$$
$$\rho_{xx} - \rho\varphi_t - \rho\varphi_x^2 - \rho\varphi_y^2 - V\rho - E\rho(\xi_x^2 + \xi_y^2) = 0, \tag{31}$$
$$2\rho_x\xi_x + \rho\xi_{xx} + 2\rho_y\xi_y + \rho\xi_{yy} = 0, \tag{32}$$
$$\rho G(\xi_x^2 + \xi_y^2) - g\rho^3 = 0. \tag{33}$$

Solving (31) and (33), one can get

$$V(x,y,t) = \frac{\rho_{xx} - \rho\varphi_t - \rho\varphi_x^2 - \rho\varphi_y^2 - E\rho(\xi_x^2 + \xi_y^2)}{\rho},$$
$$g(x,y,t) = \frac{G(\xi_x^2 + \xi_y^2)}{\rho^2}. \tag{34}$$

Now, consider other equations, one yields,

$$\varphi = -\frac{p_t}{8p}x^2 - \frac{p_t}{8p}y^2 - \frac{p_t}{4p}xy - \frac{q_t}{4p}x - \frac{q_t}{4p}y + c, \tag{35}$$
$$\rho^2 = \frac{p}{F'(Q)}, \quad Q = px + py + q, \tag{36}$$
$$\xi = F(px + py + q), \tag{37}$$

where p, q and c are arbitrary functions of t.

This way, if we select the appropriate parameters $p(t), q(t), F(px + py + q)$, the pairs $g(t,x,y), V(t,x,y)$ can be derived. Consequently, we can use them to exhibit many interesting results.

It is generally known that (22) has many solutions. It is easy to get

$$\xi - \xi_0 = \pm \int_{U_0}^{U} \frac{d\mu}{\sqrt{2\left(c_0 + \frac{1}{2}E\mu^2 + \frac{1}{4}G\mu^4\right)}}. \tag{38}$$

Here ξ_0 and c_0 are integration constants.

Therefore, we can get the exact rogue wave solution of (1) by using following results:

$$u = \rho(x,y,t)e^{i\varphi(x,y,t)}U(\xi(x,y,t)), \tag{39}$$

where

$$\rho = \sqrt{\frac{p}{F'(Q)}}, \quad Q = px + py + q, \tag{40}$$

and

$$\varphi = -\frac{p_t}{8p}x^2 - \frac{p_t}{8p}y^2 - \frac{p_t}{4p}xy - \frac{q_t}{4p}x - \frac{q_t}{4p}y + c. \tag{41}$$

In addition,

$$V(x,y,t) = \frac{\rho_{xx} - \rho\varphi_t - \rho\varphi_x^2 - \rho\varphi_y^2 - 2E\frac{p^4}{\rho^3}}{\rho},$$
$$g(x,y,t) = \frac{2Gp^3}{\rho^6}. \tag{42}$$

It is clear that (22) has a bright soliton solution,

$$U = \frac{1}{\cosh(\kappa * \xi)}, \tag{43}$$

where $E = -\kappa^2$ and $G = -2\kappa^2$. And has a dark soliton solution,

$$U = \frac{\sqrt{GE}\tanh\left(\frac{\sqrt{2E}}{2}\xi\right)}{G}, \tag{44}$$

where E, G are positive.

In next section, we will deal with different nonlinearity G and potential $V(t,x,y)$.

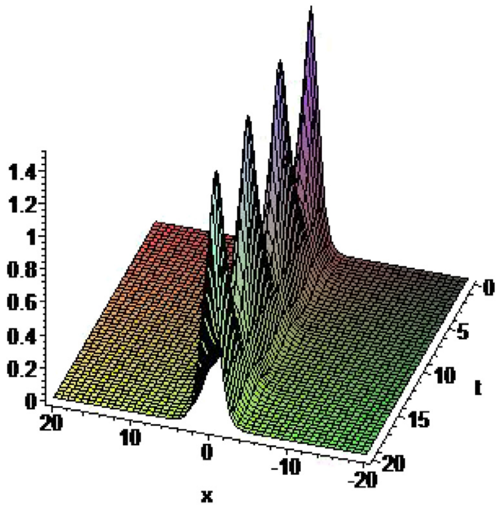

Fig. 1. Plot of $|u|^2$ with $a = \frac{1}{2}, \omega = 1, y = 1, \kappa = 1$.

4.0.3. Example 1

First, we choose $\rho(x, y, t) = \sqrt{p}$, for this case, we get $g(x, y, t) = 2G$, and let $\xi = F(Q) = Q = p(x+y)+q$. And we have

$$V(x, y, t) = A(x^2 + y^2) + B(x + y) + 2Axy - \left(c_t + \frac{q_t}{8p^2} + 2Ep^2\right), \tag{45}$$

where $A = \frac{p_{tt}p-2p_t^2}{8p^2}$, $B = \frac{q_{tt}p-2p_tq_t}{4p^2}$. For this case, if we choose p, q and c as different functions of t, we can get abundant solutions.

Now, we let

$$p = 1 + a\cos(\omega t), \qquad q = c = 0, \tag{46}$$

where $|a| < 1$ and $\omega \in R$. For this case, the coefficients of solution (39) with regard to the trigonometric function of t, so they are breathers solutions. And, we get the exact solutions of (1) as follows

$$u(x, y, t) = \sqrt{1 + a\cos(\omega t)}e^{-i\left(\frac{\omega\sin(\omega t)}{8(1+a\cos(\omega t))}(x^2+y^2)+\frac{\omega\sin(\omega t)}{4(1+a\cos(\omega t))(xy)}\right)}\frac{1}{\cosh(\kappa * \xi)}, \tag{47}$$

and

$$u(x, y, t) = \sqrt{1 + a\cos(\omega t)}e^{-i\left(\frac{\omega\sin(\omega t)}{8(1+a\cos(\omega t))}(x^2+y^2)+\frac{\omega\sin(\omega t)}{4(1+a\cos(\omega t))(xy)}\right)}\frac{\sqrt{GE}\tanh\left(\frac{\sqrt{2E}}{2}\xi\right)}{G}. \tag{48}$$

They are depicted in Figs. 1 and 2.

5. Conclusions

In this paper, we have used similarity transformations to study the cubic nonlinear $(2 + 1)$-dimensional Schrödinger equation with variables, i.e. time and space (x and y space) modulated nonlinearities and potentials. First, we considered the special case, that is, the linear $(2+1)$-dimensional Schrödinger equation with variables. We employed the equivalence group and found that this equation is related to another equation with the same form. From the view of linear operators and its adjoint operators, we found that

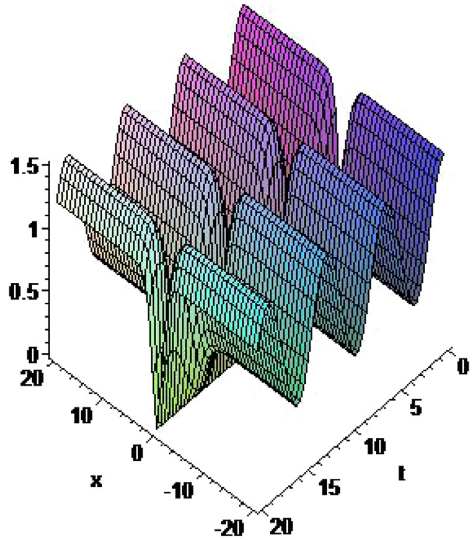

Fig. 2. Plot of $|u|^2$ with $a = \frac{1}{2}, \omega = 1, y = 1, G = E = 1$.

potential symmetries with both Lorentz and temporal gauge constraints. Classical Lie group method is also employed to find the point symmetries. And then, this equation is transformed to the stationary cubic nonlinear Schrödinger equation. Based on the relation between the $(2+1)$-dimensional Schrödinger equation with variables and the stationary cubic nonlinear Schrödinger equation, we constructed the rogue wave solutions for the original equation.

It should be noted that, in this paper, we only considered the symmetries and some exact solutions. Therefore, there are some issues that need to be pursued further, such as conservation laws and its invariantizations [30,31]. It also should be emphasized that the obtained results can be served as by other tools, such as Painlevé test, Darboux transformation, Bäcklund transformation and so on. The method used here, of course, can also be extended to study higher-dimensional systems and their versions with variables. It may help to promote further research on these topics and, also help to better study the behavior of nonlinear waves and other complicated phenomenon.

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References

[1] J. Denschlag, et al., Generating solitons by phase engineering of a Bose–Einstein condensate, Science 287 (2000) 97–101.
[2] B. Eiermann, et al., Bright Bose–Einstein gap solitons of atoms with repulsive interaction, Phys. Rev. Lett. 92 (2004) 230401.
[3] V.E. Zakharov, A.B. Shabat, Exact theory of two-dimensional self-focusing and one-dimensional self-modulation of waves in nonlinear media, Sov. Phys.—JETP 34 (1972) 62–65.
[4] G.I. Stegeman, M. Segev, Optical spatial solitons and their interactions: Universality and diversity, Science 286 (1999) 1518–1523.
[5] K.E. Strecker, G.B. Partridge, A.G. Truscott, R.G. Hulet, Formation and propagation of matter-wave soliton train, Nature 417 (2002) 150–153.
[6] J. Belmonte-Beitia, V.M. Perez-Garcia, V. Vekslerchik, P.J. Torres, Lie symmetries and solitons in nonlinear systems with spatially inhomogeneous nonlinearities, Phys. Rev. Lett. 98 (2007) 064102.

[7] J. Belmonte-Beitia, G.F. Calvo, Exact solutions for the quintic nonlinear Schrödinger equation with time and space modulated nonlinearities and potentials, Phys. Lett. A 373 (2009) 448–453.
[8] J. Belmonte-Beitia, Exact solutions for the nonlinear Schrödinger equation with inhomogeneous nonlinearity, Chaos Solitons Fractals 41 (2009) 1005–1009.
[9] G. Bluman, V. Shtelen, New classes of Schrödinger equations equivalent to the free particle equation through non-local transformations, J. Phys. A 29 (1996) 4473–4480.
[10] R.O. Popovych, M. Kunzinger, H. Eshraghi, Admissible transformations and normalized classes of nonlinear Schrödinger equations, Acta Appl. Math. 109 (2010) 315–359.
[11] C. Kurujyibwami, Equivalence groupoid for $(1 + 2)$-dimensional linear Schrödinger equations with complex potentials, J. Phys.: Conf. Ser. 621 (2015) 012008.
[12] G. Bluman, S. Kumei, Symmetries and Differential Equations, Springer, New York, 1989.
[13] S. Anco, G. Bluman, Nonlocal symmetries and nonlocal conservation laws of Maxwell's equations, J. Math. Phys. 38 (1997) 3508–3532.
[14] G. Bluman, A. Cheviakov, S. Anco, Applications of Symmetry Methods to Partial Differential Equations, Springer, New York, 2010.
[15] A. Cheviakov, G. Bluman, Multidimensional partial differential equations systems: Generating new systems via conservation laws, potentials, gauges, subsystems, J. Math. Phys. 51 (2010) 103521.
[16] P.J. Olver, Application of Lie Group to Differential Equation, Springer, New York, 1986.
[17] G. Wang, A.H. Kara, Nonlocal symmetry analysis, explicit solutions and conservation laws for the fourth-order Burgers' equation, Chaos Solitons Fractals 81 (2015) 290–298.
[18] G. Wang, K. Fakhar, Lie symmetry analysis, nonlinear self-adjointness and conservation laws to an extended $(2 + 1)$-dimensional Zakharov-Kuznetsov-Burgers equation, Comput. & Fluids 119 (2015) 143–148.
[19] G. Wang, A.H. Kara, K. Fakhar, Nonlocal symmetry analysis and conservation laws for the third-order Burgers' equation, Nonlinear Dynam. (2015) http://dx.doi.org/10.1007/s11071-015-2480-8.
[20] T.J. Bridges, Multi-symplectic structures and wave propagation, Math. Proc. Cambridge Philos. Soc. 121 (1997) 147–190.
[21] S. Reich, Multi-symplectic Runge–Kutta collocation methods for Hamiltonian wave equations, J. Comput. Phys. 157 (2000) 473–499.
[22] W. Hu, Z. Deng, Y. Zhang, Multi-symplectic method for peakon-antipeakon collision of quasi-Degasperis-Procesi equation, Comput. Phys. Comm. 185 (2014) 2020–2028.
[23] W. Hu, Z. Deng, Y. Qin, Multi-symplectic method to simulate soliton resonance of $(2+1)$-dimensional Boussinesq equation, J. Geom. Mech. 5 (2013) 295–31.
[24] W. Hu, Z. Deng, S. Han, W. Zhang, Generalized multi-symplectic integrators for a class of Hamiltonian nonlinear wave PDEs, J. Comput. Phys. 235 (2013) 394–406.
[25] W. Hu, Z. Deng, B. Wang, H. Ouyang, Chaos in an embedded single-walled carbon nanotube, Nonlinear Dynam. 72 (2013) 389–398.
[26] W. Hu, Z. Deng, Chaos in embedded fluid-conveying single-walled carbon nanotube under transverse harmonic load series, Nonlinear Dynam. 79 (2015) 325–333.
[27] W. Hu, Z. Deng, Competition between geometric dispersion and viscous dissipation in wave propagation of KdV-Burgers equation, J. Vib. Control 21 (2015) 2937–2945.
[28] C.Q. Dai, W.H. Huang, Controllable mechanism of breathers in the $(2 + 1)$-dimensional nonlinear Schrödinger equation with different forms of distributed transverse diffraction, Phys. Lett. A 378 (2014) 1113–1118.
[29] C.Q. Dai, Y.Y. Wang, J.F. Zhang, Exact spatial similaritons and rogons in 2d graded-index waveguides, Opt. Lett. 35 (2010) 2651–2653.
[30] A. Bihlo, G. Bluman, Conservative parameterization schemes, J. Math. Phys. 54 (2013) 083101.
[31] A. Bihlo, R.O. Popovych, Invariant discretization schemes for the shallow-water equations, SIAM J. Sci. Comput. 34 (2012) B810–B839.