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二、计算  $n$  阶行列式  $D_n = \begin{vmatrix} 2 & 0 & 1 & \cdots & 1 & 1 \\ 1 & 2 & 0 & \cdots & 1 & 1 \\ 1 & 1 & 2 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & 1 & 1 & \cdots & 0 & 1 \\ 1 & 1 & 1 & \cdots & 2 & 0 \\ 1 & 1 & 1 & \cdots & 1 & 2 \end{vmatrix}$ .

解法一  $D_n = \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ 0 & 2 & 0 & \cdots & 1 & 1 \\ 0 & 1 & 2 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 1 & 1 & \cdots & 2 & 0 \\ 0 & 1 & 1 & \cdots & 1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ -1 & 1 & -1 & \cdots & 0 & 0 \\ -1 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ -1 & 0 & 0 & \cdots & 1 & -1 \\ -1 & 0 & 0 & \cdots & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1-n & 2 & 1 & \cdots & 1 & 1 \\ -n & 1 & 0 & \cdots & 0 & 0 \\ -(n-1) & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ -2 & 0 & 0 & \cdots & 1 & 0 \\ -1 & 0 & 0 & \cdots & 0 & 1 \end{vmatrix}$

$$= (1-n+2n+(n-1)+\cdots+2+1)$$

$$= 1 + \frac{n(n+1)}{2}$$

法二  $D_n = \begin{vmatrix} 1 & -1 & 0 & \cdots & 0 & -1 \\ 0 & 1 & -1 & \cdots & 0 & -1 \\ 0 & 0 & 1 & \cdots & 0 & -1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -1 & -1 \\ 0 & 0 & 0 & \cdots & 1 & -2 \\ 1 & 1 & 1 & \cdots & 1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & \cdots & 0 & -1 \\ 0 & 1 & 0 & \cdots & 0 & -1 \\ 0 & 0 & 1 & \cdots & 0 & -1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & -1 \\ 0 & 0 & 0 & \cdots & 1 & -2 \\ 1 & 2 & 3 & \cdots & n-1 & 2 \end{vmatrix}$

$$= \begin{vmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 1 & 2 & 3 & \cdots & n-1 & 2 \end{vmatrix}$$

$$= \frac{n(n+1)}{2} + 1$$

$$2 + 1 + 2 + \cdots + 2(n-1)$$

三、计算  $n$  阶行列式  $D_n = \begin{vmatrix} 0 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 0 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 0 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & 1 & 1 & \cdots & 0 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 0 \end{vmatrix}$ .

解 法一  $D_n = \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ 0 & 0 & 1 & \cdots & 1 & 1 \\ 0 & 1 & 0 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 1 & 1 & \cdots & 0 & 1 \\ 0 & 1 & 1 & \cdots & 1 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ -1 & -1 & 0 & \cdots & 0 & 0 \\ -1 & 0 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ -1 & 0 & 0 & \cdots & -1 & 0 \\ -1 & 0 & 0 & \cdots & 0 & -1 \end{vmatrix} = (1-n)(-1)^n = (-1)^n (1-n)$

法二  $D_n = \begin{vmatrix} -1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 1 & \cdots & 0 & 0 \\ 0 & 0 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -1 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 0 \end{vmatrix} = \begin{vmatrix} -1 & 0 & & & & \\ & -1 & 0 & & & \\ & & -1 & & & \\ & & & \ddots & & \\ & & & & -1 & 0 \\ 1 & 2 & 3 & \cdots & n-1 & n-1 \end{vmatrix} = (-1)^{n-1} (n-1)$

四、利用克拉默法则求解线性方程组  $\begin{cases} x_1 + 2x_2 - x_3 = 2, \\ x_1 - 2x_2 + 2x_3 = 3, \\ 2x_1 - x_2 + x_3 = 3. \end{cases}$

解  $D = \begin{vmatrix} 1 & 2 & -1 \\ 1 & -2 & 2 \\ 2 & -1 & 1 \end{vmatrix} = 3$

$D_1 = \begin{vmatrix} 2 & 2 & -1 \\ 3 & -2 & 2 \\ 3 & -1 & 1 \end{vmatrix} = 3, \quad D_2 = \begin{vmatrix} 1 & 2 & -1 \\ 1 & 3 & 2 \\ 2 & 3 & 1 \end{vmatrix} = 6, \quad D_3 = \begin{vmatrix} 1 & 2 & 2 \\ 1 & -2 & 3 \\ 2 & -1 & 3 \end{vmatrix} = 9$

$x_1 = \frac{D_1}{D} = 1, \quad x_2 = \frac{D_2}{D} = 2, \quad x_3 = \frac{D_3}{D} = 3.$

五、 $n$  阶行列式  $D$  中每个数  $a_{ij}$  分别用  $2^{i-j}$  乘所得的行列式记为  $D_1$ ，求行列式  $D_1$  的值。

解  $D_1 = \begin{vmatrix} a_{11} & 2^{-1}a_{12} & 2^{-2}a_{13} & \cdots & 2^{1-n}a_{1n} \\ 2a_{21} & a_{22} & 2^{-1}a_{23} & \cdots & 2^{-n}a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ 2^{n-1}a_{n1} & 2^{n-2}a_{n2} & 2^{n-3}a_{n3} & \cdots & a_{nn} \end{vmatrix}$   
 $= 2^{1+2+\cdots+n} \begin{vmatrix} a_{11} & 2^{-1}a_{12} & 2^{-2}a_{13} & \cdots & 2^{1-n}a_{1n} \\ a_{21} & 2^{-1}a_{22} & 2^{-2}a_{23} & \cdots & 2^{1-n}a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & 2^{-1}a_{n2} & 2^{-2}a_{n3} & \cdots & 2^{1-n}a_{nn} \end{vmatrix}$   
 $= 2^{1+2+\cdots+n} 2^{-1-2-\cdots-n} D$   
 $= D$