第一章练习题答案(部分)

一. 填空

1. 11,奇. 2. $i=5,k=3$. 3. n^2, $\dfrac{n(n-1)}{2}$. 4. $n=4k+1$或$4k+4(k\geq 0)$. 5. 奇 偶.

6. $\dfrac{n(n-1)}{2}-k$. 7. 正, 正. 8. $-a_{14}a_{23}a_{31}a_{42}$. 9.否. 10. $(-1)^n c$.

11. 3. 12. -4. 13. $2,-12$. 14. 0. 15. $\sum_{i=1}^{4}\sum_{j=1}^{4}A_{ij}=-4$.

16. $a=-24$. 17. $A_{31}=8$. 18. $a=-7$. 19. 3. 20. -1.

21. $\lambda=1$或$\lambda=-1$. 22. $a=1$或2. 23. $\begin{cases} x_1=-(b_1a_{22}-b_2a_{12}) \\ x_2=b_2a_{11}-b_1a_{21} \end{cases}$.

24. $\pm1,\pm3$, $1,2,-2$, 25. $0,1,2,\cdots,n-2$.

二 计算题

1. 计算行列式

2) $$\begin{vmatrix} 1 & \cdots & 1 & 1 \\ 0 & \cdots & 2 & 1 \\ \vdots & & \vdots & \vdots \\ n & \cdots & 0 & 1 \end{vmatrix}=\begin{vmatrix} 1 & \cdots & 1 & 1-\sum\limits_{k=2}^{n}\dfrac{1}{k} \\ 0 & \cdots & 2 & 0 \\ \vdots & & \vdots & \vdots \\ n & \cdots & 0 & 0 \end{vmatrix}=(-1)^{\frac{n(n-1)}{2}}\left(1-\sum_{k=2}^{n}\frac{1}{k}\right)n!,$$

$$\begin{vmatrix} 1 & 1 & \cdots & 1 & 2-n \\ 1 & 1 & \cdots & 2-n & 1 \\ \vdots & \vdots & & \vdots & \vdots \\ 2-n & 1 & \cdots & 1 & 1 \end{vmatrix}_n=\begin{vmatrix} 1 & 1 & \cdots & 1 & 2-n \\ 0 & 0 & \cdots & 1-n & n-1 \\ \vdots & \vdots & & \vdots & \vdots \\ 1-n & 0 & \cdots & 0 & n-1 \end{vmatrix}_n=\begin{vmatrix} 1 & 1 & \cdots & 1 & 1 \\ 0 & 0 & \cdots & 1-n & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 1-n & 0 & \cdots & 0 & 0 \end{vmatrix}_n=(-1)^{\frac{n(n-1)}{2}}(1-n)^{n-1}$$

4) $$\begin{vmatrix} a+x_1 & a & a & \cdots & a \\ a & a+x_2 & a & \cdots & a \\ a & a & a+x_3 & \cdots & a \\ \vdots & \vdots & \vdots & & \vdots \\ a & a & a & \cdots & a+x_n \end{vmatrix}=\prod_{i=1}^{n}x_i\begin{vmatrix} 1+\dfrac{a}{x_1} & \dfrac{a}{x_2} & \dfrac{a}{x_3} & \cdots & \dfrac{a}{x_n} \\ -1 & 1 & 0 & \cdots & 0 \\ -1 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -1 & 0 & 0 & \cdots & 1 \end{vmatrix}=\prod_{i=1}^{n}x_i\left(1+\sum_{i=1}^{n}\frac{a}{x_i}\right),$$

6) $$D=\begin{vmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^n \\ 1 & x_2 & x_2^2 & \cdots & x_2^n \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^n \\ 2-2 & -2 & -2 & \cdots & -2 \end{vmatrix}=\begin{vmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^n \\ 1 & x_2 & x_2^2 & \cdots & x_2^n \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^n \\ 2 & 0 & 0 & \cdots & 0 \end{vmatrix}+\begin{vmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^n \\ 1 & x_2 & x_2^2 & \cdots & x_2^n \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^n \\ -2 & -2 & -2 & \cdots & -2 \end{vmatrix}$$

$$=(-1)^n2\left(\prod_{i=1}^{n}x_i\right)V+(-2)V\left(\prod_{i=1}^{n}(1-x_i)\right)=2V\left[(-1)^n\left(\prod_{i=1}^{n}x_i\right)-\left(\prod_{i=1}^{n}(1-x_i)\right)\right]$$

7) $\begin{vmatrix} a & -1 & 0 & \cdots & 0 & 0 \\ ax & a & -1 & \cdots & 0 & 0 \\ ax^2 & ax & a & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ ax^{n-2} & ax^{n-3} & ax^{n-4} & \cdots & a & -1 \\ ax^{n-1} & ax^{n-2} & ax^{n-3} & \cdots & ax & a \end{vmatrix} = \begin{vmatrix} a & -1 & 0 & \cdots & 0 & 0 \\ 0 & a+x & -1 & \cdots & 0 & 0 \\ 0 & 0 & a+x & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & a+x & -1 \\ 0 & 0 & 0 & \cdots & 0 & a+x \end{vmatrix} = a(a+x)^{n-1}$

8) $\begin{vmatrix} a_1 & -1 & 0 & \cdots & 0 & 0 \\ a_2 & x & -1 & \cdots & 0 & 0 \\ a_3 & 0 & x & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ a_{n-1} & 0 & 0 & \cdots & x & -1 \\ a_n & 0 & 0 & \cdots & 0 & x \end{vmatrix} = \begin{vmatrix} a_1 & -1 & 0 & \cdots & 0 & 0 \\ a_2+a_1x & 0 & -1 & \cdots & 0 & 0 \\ a_3+a_2x+a_1x^2 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ a_{n-1}+a_{n-2}x+\cdots & 0 & 0 & \cdots & 0 & -1 \\ a_n+a_{n-1}x+a_{n-2}x^2+\cdots+a_1x^{n-1} & 0 & 0 & \cdots & 0 & 0 \end{vmatrix}$

$= a_n + a_{n-1}x + a_{n-2}x^2 + \cdots + a_1x^{n-1}$

9) $\begin{vmatrix} 1 & 2 & 3 & \cdots & n \\ 1 & x+1 & 3 & \cdots & n \\ 1 & 2 & x+1 & \cdots & n \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & 2 & 3 & \cdots & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 & \cdots & n \\ 0 & x-1 & 0 & \cdots & 0 \\ 0 & 0 & x-2 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & x-n+1 \end{vmatrix} = \ldots,$

11) $\begin{cases} 1, n=1 \\ -2, n=2 \\ 0, n\ge 3 \end{cases}.$

三 证明题

1.

$\begin{vmatrix} 1 & 2 & 3 & 4 & \cdots & n \\ 1 & 1 & 2 & 3 & \cdots & n-1 \\ 1 & x & 1 & 2 & \cdots & n-2 \\ 1 & x & x & 1 & \cdots & n-3 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 1 & x & x & x & \cdots & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 & 4 & \cdots & n \\ 0 & -1 & -1 & -1 & \cdots & -1 \\ 0 & x-1 & -1 & -1 & \cdots & -1 \\ 0 & 0 & x-1 & -1 & \cdots & -1 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -1 \end{vmatrix} = \begin{vmatrix} -1 & -1 & -1 & -1 & \cdots & -1 \\ x-1 & -1 & -1 & -1 & \cdots & -1 \\ 0 & x-1 & -1 & -1 & \cdots & -1 \\ 0 & 0 & x-1 & -1 & \cdots & -1 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -1 \end{vmatrix}_{n-1}$

$= \begin{vmatrix} -x & 0 & 0 & 0 & \cdots & 0 \\ x-1 & -x & 0 & 0 & \cdots & 0 \\ 0 & x-1 & -x & 0 & \cdots & 0 \\ 0 & 0 & x-1 & -x & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -1 \end{vmatrix}_{n-1} = (-1)^{n+1}x^{n-2} \quad (x \neq 0)$

2　按照中间两行展开