SYS-6581 Principles of Modeling for Cyber-Physical Systems

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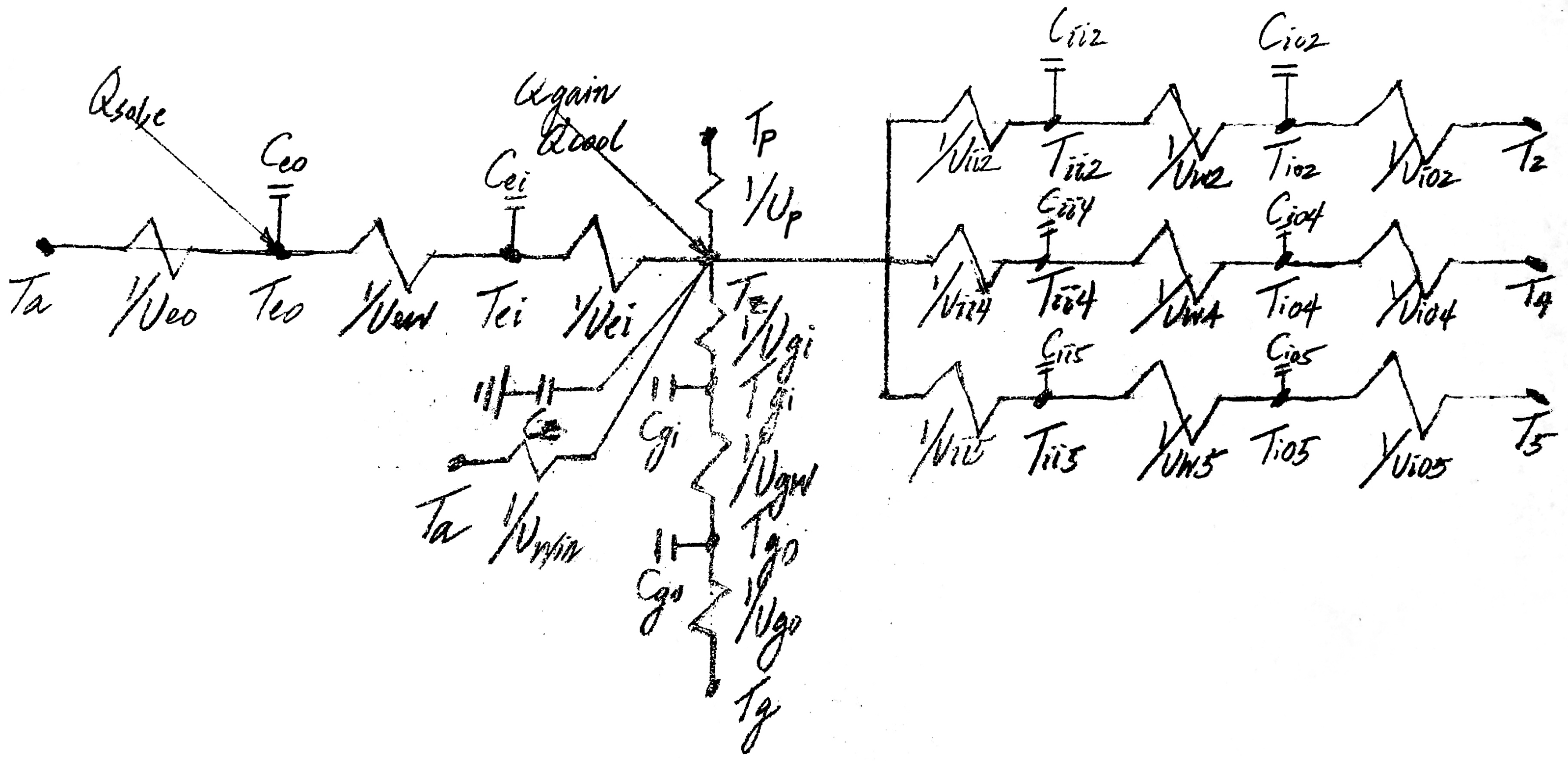
2 Thermal 'RC' Network based State-Space Building Modeling

2.1 Single Zone RC Network

(a) RC Network Model Structure

(b) Labelled:



2.2 Differential Equations for thermal dynamics

(a) Following are the heat balance differential equations

- For the zone

  (i) $C_z \dot{T}_z = U_{ei}(T_{ei}(t) - T_z(t)) + U_p(T_p(t) - T_z(t)) + U_{ii2}(T_{ii2}(t) - T_z(t)) + U_{ii4}(T_{ii4}(t) - T_z(t)) + U_{ii5}(T_{ii5}(t) - T_z(t)) + U_{gi}(T_{gi}(t) - T_z(t)) + U_{win}(T_a(t) - T_z(t)) + Q_{cool}(t) + Q_{gain}(t)$

- For the exterior wall

(i) $C_{eo}\dot{T}_{eo}(t) = U_{eo}(T_a(t) - T_{eo}(t)) + U_{ew}(T_{ei}(t) - T_{eo}(t)) + Q_{sol,e}(t)$
(ii) $C_{ei}\dot{T}_{ei}(t) = U_{ew}(T_{eo}(t) - T_{ei}(t)) + U_{ei}(T_z(t) - T_{ei}(t))$

- For the floor

(i) $C_{go}\dot{T}_{go}(t) = U_{go}(T_g(t) - T_{go}(t)) + U_{gw}(T_{gw}(t) - T_{go}(t))$
(ii) $C_{gw}\dot{T}_{gw}(t) = U_{gw}(T_{go}(t) - T_{gw}(t)) + U_{gi}(T_z(t) - T_{gw}(t))$

- For the interior walls

(i) $C_{io2}\dot{T}_{io2}(t) = U_{io2}(T_2(t) - T_{io2}(t)) + U_{w2}(T_{ii2}(t) - T_{io2}(t))$
(ii) $C_{ii2}\dot{T}_{ii2}(t) = U_{w2}(T_{io2}(t) - T_{ii2}(t)) + U_{ii2}(T_z(t) - T_{ii2}(t))$
(iii) $C_{io4}\dot{T}_{io4}(t) = U_{io4}(T_4(t) - T_{io4}(t)) + U_{w4}(T_{ii2}(t) - T_{io4}(t))$
(iv) $C_{ii4}\dot{T}_{ii4}(t) = U_{w4}(T_{io4}(t) - T_{ii4}(t)) + U_{ii2}(T_z(t) - T_{ii4}(t))$
(v) $C_{io5}\dot{T}_{io5}(t) = U_{io5}(T_5(t) - T_{io5}(t)) + U_{w5}(T_{ii5}(t) - T_{io5}(t))$
(vi) $C_{ii5}\dot{T}_{ii5}(t) = U_{w5}(T_{io5}(t) - T_{ii5}(t)) + U_{ii5}(T_z(t) - T_{ii5}(t))$

(b) There are 11 ODE equations. The number of ODE correspond with the number of capacitors in the system.

(c) List all the thermal resistance/conductance and capacitance parameters:
$R_{eo}, C_{eo}, R_{ew}, C_{ei}, R_{ei}, C_z, R_{win}, R_p, R_{io2}, C_{io2},$
$R_{w2}, C_{ii2}, R_{ii2}, R_{io4}, C_{io4}, R_{uw4}, C_{ii4}, R_{ii4}, R_{io5}, C_{io5},$
$R_{uw5}, R_{ii5}, C_{ii5}, C_{go}, R_{go}, R_{gw}, C_{gi}, R_{gi}$

2.3 States, inputs, and outputs

(a) There exists 11 state variables for the system, which is the same as the answer to problem P2-2.b. State variables: $T_z, T_{eo}, T_{ei}, T_{gi}, T_{go}, T_{io2}, T_{i2}, T_{io4}, T_{i4}, T_{io5}, T_{i5}$

(b) Disturbances: $T_a, T_g, T_2, T_4, T_5, T_p.Q_{sol}, Q_{gain}$

(c) Control inputs: Q_{cool}

(d) Output variables: T_z

2.4 State space model

(a) (i) state vector $x(t) = [T_z, T_{eo}, T_{ei}, T_{go}, T_{gi}, T_{io2}, T_{ii2}, T_{io4}, T_{iia4}, T_{io5}, T_{ii5}]^T$
(ii) input vector $u(t) = [T_a, T_g, T_2, T_4, T_5, T_p, Q_{sol}, Q_{gain}, Q_{cool}]^T$
(iii) output vector $y(t) = [T_z]^T$

(b) Show all elements of the A,B,C,D Matrix:
$\dot{x}(t) = Ax(t) + Bu(t) =$

.

$$
\begin{bmatrix} \dot{T_z} \\ \dot{T_{eo}} \\ \dot{T_{ei}} \\ \dot{T_{go}} \\ \dot{T_{gi}} \\ \dot{T_{io2}} \\ \dot{T_{ii2}} \\ \dot{T_{io4}} \\ \dot{T_{ii4}} \\ \dot{T_{io5}} \\ \dot{T_{ii5}} \end{bmatrix} =
$$

$$
\begin{bmatrix}
\frac{-U_{ei}-U_p-U_{ii2}-U_{ii4}-U_{ii5}-U_{gi}-U_{win}}{C_z} & 0 & \frac{U_{ei}}{C_z} & 0 & \frac{U_{gi}}{C_z} & 0 & \frac{U_{ii2}}{C_z} & 0 & \frac{U_{ii4}}{C_z} & 0 & \frac{U_{ii5}}{C_z} \\
0 & \frac{-U_{eo}-U_{ew}}{C_{eo}} & \frac{U_{ew}}{C_{eo}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{U_{ei}}{C_{ei}} & \frac{U_{ew}}{C_{ei}} & \frac{-U_{ew}-U_{ei}}{C_{ei}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{-U_{go}-U_{gw}}{C_{go}} & \frac{U_{gw}}{C_{go}} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{U_{gi}}{C_{gi}} & 0 & 0 & \frac{U_{gw}}{C_{gi}} & \frac{-U_{gw}-U_{gi}}{C_{gi}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{-U_{io2}-U_{w2}}{C_{io2}} & \frac{U_{w2}}{C_{io2}} & 0 & 0 & 0 & 0 \\
\frac{U_{ii2}}{C_{ii2}} & 0 & 0 & 0 & 0 & \frac{U_{w2}}{C_{ii2}} & \frac{-U_{w2}-U_{ii2}}{C_{ii2}} & 0 & 0 & 0 & 0 \\
\vdots & & & & & & & & & & \\
 & & & & & & & & & &
\end{bmatrix}
$$

$$
\cdot \begin{bmatrix} T_z \\ T_{eo} \\ T_{ei} \\ T_{go} \\ T_{gi} \\ T_{io2} \\ T_{ii2} \\ T_{io4} \\ T_{ii4} \\ T_{io5} \\ T_{ii5} \end{bmatrix} +
\begin{bmatrix}
\frac{U_{win}}{C_z} & 0 & 0 & 0 & 0 & \frac{U_p}{C_z} & 0 & \frac{1}{C_z} & \frac{1}{C_z} \\
\frac{U_{eo}}{C_{eo}} & 0 & 0 & 0 & 0 & 0 & \frac{1}{C_{eo}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{U_{go}}{C_{go}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{U_{io2}}{C_{io2}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{U_{io4}}{C_{io4}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{U_{io5}}{C_{io5}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\cdot \begin{bmatrix} T_a \\ T_g \\ T_2 \\ T_4 \\ T_5 \\ T_p \\ \dot{Q}_{sol} \\ \dot{Q}_{cool} \\ \dot{Q}_{gain} \end{bmatrix}
$$

The matrices depend on the RC parameters non-linearly. Due to the size of the matrix, I abbreviated the rest of the interior walls: $T_{io4}, T_{ii4}, T_{io5}, T_{ii5}$. However, I want to note here that they share a similar pattern with shown T_{ii2}, T_{io2}, where T_{iin} is connected to T_z via a $\frac{U_{iin}}{C_{iin}}$ relationship, and share the $\frac{-U_{ion}-U_{wn}}{C_{ion}}$ and $\frac{U_{wn}}{C_{ion}}$ for the T_{ion} and $\frac{U_{wn}}{C_{iin}}$ and $\frac{-U_{wn}-U_{iin}}{C_{iin}}$ for the T_{iin} relationship, for $n = \{4, 5\}$, respectively.

$\dot{y}(t) = Cx(t) + Du(t) =$

$$T_Z(t) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_z \\ T_{eo} \\ T_{ei} \\ T_{go} \\ T_{gi} \\ T_{io2} \\ T_{ii2} \\ T_{io4} \\ T_{ii4} \\ T_{io5} \\ T_{ii5} \end{bmatrix} + [0] \begin{bmatrix} T_a \\ T_g \\ T_2 \\ T_4 \\ T_5 \\ T_p \\ \dot{Q}_{sol} \\ \dot{Q}_{cool} \\ \dot{Q}_{gain} \end{bmatrix}$$

(c) Summarizing the state space model

(i) Model order for the state-space model: 11

(ii) Number of input variables for the state-space model: 9

(iii) Number of outputs for the state-space model: 1

(iv) Total parameters present in the state-space model: 28

(v) Dimensions of A,B,C,D in the state space model:

- A: 11 by 11
- B: 11 by 9
- C: 1 by 11
- D: 1 by 9 (Zero matrix)