

# Solving the Scattering of Combined Conductor-Dielectric Bodies Based on EPA

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**Abstract**—This paper introduces a new method for analyzing the scattering from combined conductor-dielectric bodies based on equivalent principle algorithm (EPA). The object is decomposed into several different parts. In each part, it can be an inhomogeneous dielectric or a perfect electric conductor (PEC) of arbitrary shape. Equivalent surfaces (ES) are built for the interactions between each parts. And the current translation from the object to ES can be accelerated by the transformed multilevel fast multipole algorithm (MLFMA). Numerical result demonstrates the efficiency and accuracy of the proposed method.

## I. INTRODUCTION

The analysis of electromagnetic scattering from combined conductor-dielectric bodies has attracted many attentions in the computational electromagnetics (CEM). An efficient method to solve this problem is the method of moments (MoM) based on the volume-surface integral equation (VSIE) [1], [2]. In VSIE, the interactions between the perfect electric conductor (PEC) and dielectric are considered directly through the integral between the PEC surface and dielectric volume, which leads to high computation complexity and large memory usage.

The equivalent principle algorithm (EPA) has been widely used in electromagnetic solvers to improve computation efficiency [3]–[5]. In those methods, the whole solution domain is decomposed into several subdomains by equivalence surfaces (ES). Interactions between the subdomains are considered through ES which have less unknowns. Thus, the computation complexity is decreased efficiently. However, EPA was only applied to the scattering problem of pure PEC or dielectric.

In this paper, a new method based on EPA is proposed to solve the scattering from conductor-dielectric bodies. The combined conductor-dielectric object is decomposed into isolated parts with equivalent surfaces. Based on Huygens' Principle, the equivalent electric currents and magnetic currents are created on the ES. And the transformed MLFMA is introduced to accelerate the translation from the currents on object to currents on ES. Then the interactions between subdomains are replaced by the interactions between ES. The matrix equations built by ES has a better convergence performance compared to MoM based on VSIE. Thus, we can gain a more efficient solution with nearly the same accuracy. Moreover, many CEM methods such as FEM, PO can be easily plugged in for the computation in each part.

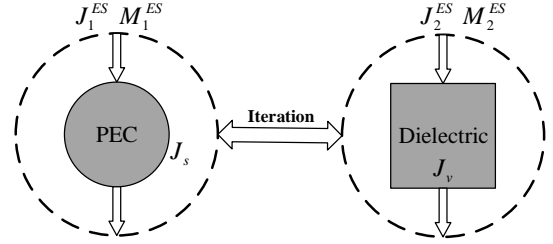


Fig. 1. Example of the domain decomposition and interactions among a PEC sphere and a dielectric cube. The arrows on the top represent the outside-in process, and the arrows at the bottom represent the inside-out process.

## II. FORMULATION

For an object which consists of several well separated conductor or dielectric parts, we can build ES for each part and mesh them independently, as illustrated in Fig. 1. The PEC surface and ES are discretized by triangle pairs and the dielectric volume is discretized by tetrahedral pairs.

The solving process consists of three procedures: outside-in process, inner-solving process and inside-out process. The interactions between different parts are considered through the iteration process. Details are introduced in the following subsections.

### A. Outside-In Process

In this paper, the currents on ES are discretized by RWG basis functions. Given the electric and magnetic currents RWG basis weights on the ES, the electromagnetic fields to the object can be gained using the formulations below:

$$\mathbf{E} = \eta \mathbf{L}(\mathbf{J}) - \mathbf{K}(\mathbf{M}) \quad (1)$$

$$\mathbf{H} = 1/\eta \mathbf{L}(\mathbf{M}) + \mathbf{K}(\mathbf{J}) \quad (2)$$

where

$$\mathbf{L}(\mathbf{X}) = -jk \int \mathbf{X} + \frac{1}{k^2} \nabla(\nabla' \cdot \mathbf{X}) G d\tau' \quad (3)$$

$$\mathbf{K}(\mathbf{X}) = - \int \mathbf{X} \times \nabla G d\tau' \quad (4)$$

### B. Inner-Solving Process

The incident field of each part is generated by the currents on the ES. Different methods can be used to calculate the currents on the object, such as PO and MoM. In this implementation, the solver for the PEC part is MoM based on surface integral equation, and the solver for the dielectric part is MoM based on volume integral equation.

### C. Inside-Out Process

Based on Huygens' equivalence principle, the scattered field from a scatterer can be replaced by the field radiated by equivalent sources on a surface which encloses the scatterer. Thus, the currents on the object can be translated to the electric and magnetic currents on the ES using the formulations below:

$$\mathbf{J}_{1,2}^{ES} = \hat{n} \times \mathbf{K}(\mathbf{J}_{s,v}) \quad (5)$$

$$\mathbf{M}_{1,2}^{ES} = \eta \mathbf{L}(\mathbf{J}_{s,v}) \times \hat{n} \quad (6)$$

In this paper, the transformed MLFMA is proposed to accelerate the translation process. The radiation function is defined on the object and the receiving function is defined on the ES. To gain the currents RWG basis weights [5], the receiving patterns are transformed as below:

$$\mathbf{R}^E(\hat{k}) = -\hat{k} \times \hat{v} \frac{e^{-jk\hat{k} \cdot \mathbf{r}_{ra}}}{4\pi} \quad (7)$$

$$\mathbf{R}^H(\hat{k}) = \hat{v} \frac{e^{-jk\hat{k} \cdot \mathbf{r}_{ra}}}{4\pi} \quad (8)$$

where

$$\hat{v} = 0.5 \times (\hat{m}_1 \times \hat{n}_1 + \hat{m}_2 \times \hat{n}_2). \quad (9)$$

$\hat{n}$  is the normal vector of the face,  $\hat{m}$  is the normal vector of the edge on the face. And the radiation function and translation function are the same with those in MLFMA.

### D. Iteration Process

The interactions between ES are considered through the iteration process. At the first iteration, the currents on ES only come from the incident plane wave. After the first iteration, the currents on ES are translated from the scattered field of object in each part, which will be treated as the incident fields to ES of other parts in the next iteration.

## III. NUMERICAL RESULTS

To demonstrate the accuracy and efficiency, bi-static RCS of the model shown in Fig. 1 is computed by the proposed EPA method and VSIE method. MLFMA is used to accelerate the inner solving process for each part and the transformed MLFMA is used to accelerate the currents translation from object to ES.

Fig. 2 shows that the result of the new method agrees well with the VSIE. Table I shows that the proposed EPA method consumes fewer memory and has lower computation complexity than VSIE. The proposed method has a good performance on convergence, only two iterations are needed to get a reasonable result.

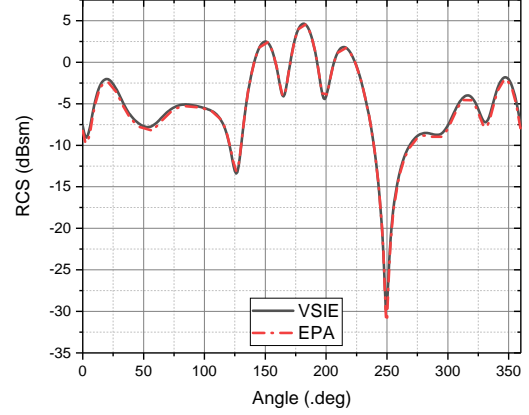


Fig. 2. RCS patterns of a PEC ball and dielectric cube with incident plane wave model. The number of unknowns on the ball is 252, and the number of unknowns on the dielectric cube is 13698. The incident plane wave with frequency 300 MHz satisfies  $\theta = 0^\circ$  and  $\phi = 0^\circ$ , V-V polarization. The RCS with  $\theta$  varies from  $0^\circ$  to  $360^\circ$  and  $\phi = 0^\circ$  is calculated.

TABLE I  
CALCULATION EFFICIENCY

Methods	Memory Usage	Computation Time
VSIE	6292.85MB	2711.00s
EPA	2344.23MB	704.00s

## IV. CONCLUSION

In this paper, a new method is proposed to analyze the scattering from the combined conductor-dielectric bodies based on EPA. ES are built for PEC and dielectric parts separately and interactions between different parts are considered through ES. Thus, many CEM methods can be plugged in to solve the scattering of PEC and dielectric independently. Furthermore, the transformed MLFMA can be used to accelerate the currents translation from the object to ES. Numerical results demonstrate the method can provide reasonable accuracy and efficiency improvement.

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