Isogeometric Analysis for the Scattering of NURBS Surfaces with Coinciding Knots

Han Wang, Ruoming Zhang, Yuhao Shen, Ce Ding, Yuechen Zhao, and Hai Lin
The State Key Laboratory of CAD&CG, Zhejiang University, Hangzhou 310058, China
Corresponding author: Hai Lin (e-mail: lin@cad.zju.edu.cn).

***Abstract*—This paper first introduces and addresses the discontinuity problem in isogeometric analysis (IGA) for nonuniform rational B-spline (NURBS) surfaces. Herein, we focus on solving the scattering problem of the perfect electric conductor (PEC) based on the electric field integral equation (EFIE). This discontinuity problem arises from the coinciding knots in the knot vectors of NURBS surfaces, where the NURBS basis functions have a reduced differentiable continuity. The basis functions with reduced continuity may dissatisfy the div-conforming requirements in the EFIE and finally lead to the wrong simulation. To address this problem, this work split the surfaces at the coinciding knots by inserting the knots repeatedly and making the one-to-one correspondence for the two sets of basis functions at the splitting interface. Numerical results demonstrate the validity and accuracy of the proposed method.**

I. Introduction

The analysis of the electromagnetic scattering from the perfect electric conductor (PEC) plays an important role in computational electromagnetics (CEM). Traditional numerical methods such as the method of moments (MoM) [1] solve this problem by transforming the electric field integral equation (EFIE) into a matrix equation. However, the discretization process of these methods requires meshing the objects into triangular elements. Due to the different representations of the engineering design and polygon mesh, the meshing process becomes the bottleneck of the numerical analysis.

Recently, the isogeometric analysis (IGA) [2] has been proposed to unify the model design and numerical analysis. The main idea of IGA is to use the unified basis functions to represent both geometry and unknown physical quantities. The IGA was first proposed as an alternative to the finite element method (FEM) [3]. Recently, it has also been applied in solving integral equations such as EFIE [4]. One challenge of solving the EFIE by the IGA is to ensure the div-conforming requirements of the basis functions. However, due to the different goals of model design and numerical analysis, it cannot always be guaranteed. The most common discontinuity happens on the interfaces of connected nonuniform rational B-spline (NURBS) surface [5], where the one-to-one correspondence [4] can be applied in the fully matching condition. However, that is still not enough to obtain accurate results.

In this paper, the discontinuity problem within a NURBS surface is first introduced and studied. The problem arises from the coinciding knots of the NURBS surface, which will lead to a reduced continuity of basis functions and result in wrong simulation results without special treatment. This paper introduces a method to address this problem. The main idea of this method is to split the surfaces into different parts by inserting the coinciding knots several times. After that, one-to-one correspondence can be applied to ensure continuity among these knots. In this way, the div-conforming requirement is satisfied and an accurate simulation result is achieved. The numerical result demonstrates the seriousness of this problem and the validity of the proposed method.

II. Formulation

In this section, we will first introduce the basic concepts of NURBS modeling. Then the NURBS-based IGA for EFIE is introduced briefly. Finally, the discontinuity within a NURBS surface is introduced and addressed.

A. NURBS Modeling

The NURBS is one of the dominant technologies in the engineering design. The NURBS surface of degree (p, q) in (u, v) direction is defined by:

$$\boldsymbol{S}(u,v)=\sum_{i=1}^{k}\sum_{j=1}^{l}R_{i,j}(u,v)\boldsymbol{P}_{i,j} \tag{1}$$

where $R_{i,j}$ forms the set of NURBS basis functions, and $\boldsymbol{P}_{i,j}$ forms the set of control points.

B. NURBS-Based IGA for EFIE

The EFIE on the PEC is formulated as

$$\hat{\boldsymbol{n}}\times\boldsymbol{E}^{inc}=\hat{\boldsymbol{n}}\times jk\eta\int_S[1+\frac{1}{k^2}\nabla\nabla'\cdot]\boldsymbol{J}(\boldsymbol{r}')Gds',\boldsymbol{r}\in S. \tag{2}$$

where $\hat{\boldsymbol{n}}$ is the normal vector of the surface, $\boldsymbol{E}^{inc}$ is the incident plane wave, k is the wavenumber, η is the free-space intrinsic impedance, $\boldsymbol{J}(\boldsymbol{r}')$ is the equivalent current to be determined, and G is the 3-D Green's function of free space. With the concept of IGA, the unknown current can be represented as the combination of IGA basis parallel to the surface, and thus the EFIE can be transformed into a matrix equation that can be easily solved. The approximated current can be written as

$$\tilde{\boldsymbol{J}}=\sum_{i=1}^{N}a_i\boldsymbol{u}_i \tag{3}$$

with

$$\boldsymbol{u}_i=\iota_F(\hat{\boldsymbol{u}}_i) \tag{4}$$

where $\hat{\boldsymbol{u}}_i$ is the basis of a vector space constructed by NURBS basis, and ι_F is the push-forward transformation (div-conforming transformation is used here), more details can be found in [4] and [6].

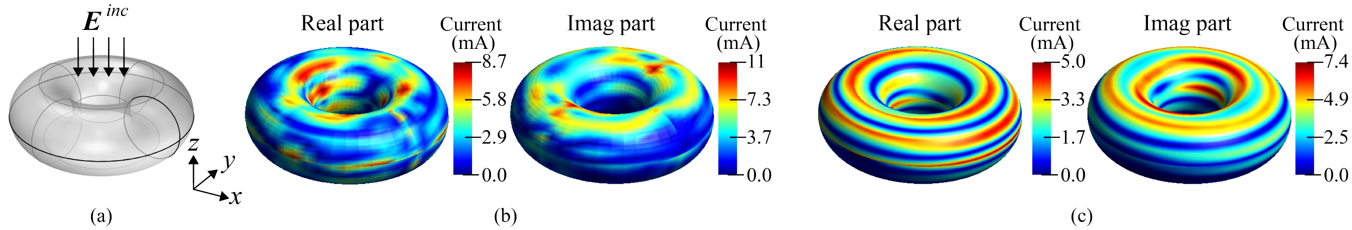

Fig. 1. (a) The torus model. The structural lines indicate the knots of the NURBS surface, and the darker lines are the boundaries of the surface. (b) The current distribution obtained by the trivial IGA. (c) The current distribution obtained by the proposed method.

C. NURBS Surface with Coinciding Knots

To ensure the accuracy of the solution, the IGA basis functions used to solve the EFIE should be differentiable on the object's surface due to the div-conforming requirement. However, it is well known that the NURBS basis is $p-k$ ($q-k$) times differentiable in the u (v) direction where k is the multiplicity of the knot. Thus, the IGA basis functions may be C^0 continuous, resulting in the wrong simulation result.

To address this problem, this paper inserts the coinciding knots for $p-k+1$ times at first. In this way, the original surface is split into several non-overlapping surfaces with connected interfaces, and geometric mapping along these interfaces is the same. Then the continuity among the interfaces can be ensured by one-to-one correspondence [4], which is, specifically, linking the related basis functions across the interfaces at the interface with the same index in the matrix system as illustrated in Fig. 2.

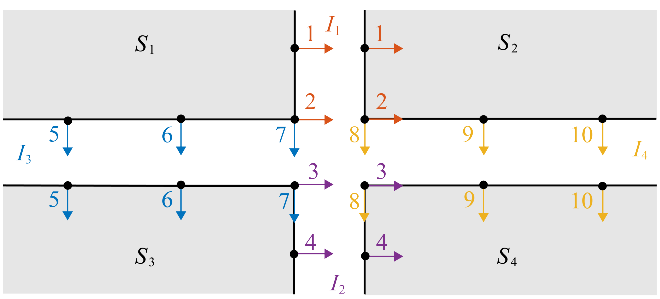

Fig. 2. A NURBS surface S is split into 4 smaller surfaces S_1, S_2, S_3, S_4, and the one-to-one correspondence is applied to the basis functions on the interfaces I_1, I_2, I_3, I_4. Different colors indicate different one-to-one correspondence groups.

III. Numerical Result

To demonstrate the validity and efficiency of the proposed method, we studied the scattering problem on a torus. The torus is defined by a NURBS surface with the same knot vector in the u and v direction, written as $\{0, 0, 0, 0.25, 0.25, 0.5, 0.5, 0.75, 0.75, 1.0, 1.0, 1.0\}$. The incident plane wave is excited opposite the z direction with the frequency 500 MHz. The current distribution on the surfaces and the RCS profiles computed by the MoM, the trivial IGA, and the proposed method are presented in Fig. 1 and Fig. 3, respectively. We can see the trivial IGA obtains wrong current distribution and RCS profile, while the RCS profile obtained by the proposed method agrees well with the MoM. We can conclude that the proposed method addresses the discontinuity problem of IGA for the NURBS surfaces with coinciding knots.

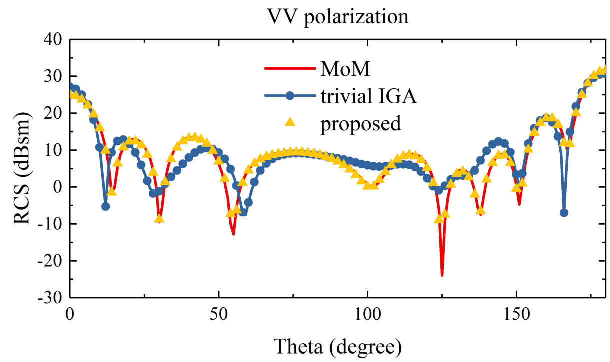

Fig. 3. The RCS profiles of the torus model obtained by the MoM, the trivial IGA, and the proposed method.

IV. Conclusion

In this paper, the discontinuity problem in IGA for the scattering of NURBS surfaces is first introduced and studied. This problem arises from the coinciding knots in the knot vectors of NURBS surfaces, which will lead to reduced differentiable continuity. In the context of EFIE, the reduced differential continuity may dissatisfy the div-conforming requirement, and results in a wrong simulation result. This paper solves this problem by splitting the surfaces at the coinciding knots and ensuring continuity through the one-to-one correspondence. In addition, the accuracy of the proposed method is verified by the numerical results.

Acknowledgement

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