Accuracy Improvement of the Algebraic Fast Methods for the Volume-Surface Integral Equation

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***Abstract*—The method of moments (MoM) based on volume-surface integral equation (VSIE) has been widely used to analyze the scattering of composite conducting-dielectric materials. Algebraic methods such as the adaptive cross approximation (ACA) and pseudo skeleton approximation (PSA) have been used to decrease the computational complexity of the MoM. Due to the different scattering characteristics of conductor and dielectric, the compression of the impedance matrices based on algebraic methods may lose some key scattering factors, which leads to the reduction of accuracy. A novel way is presented in this paper to enhance the accuracy of the algebraic fast methods for the electromagnetic scattering of the composite conductor-dielectric materials.**

I. Introduction

The electromagnetic scattering analysis of composite conducting-dielectric materials is of great importance in real-world applications. The method of moments (MoM) based on volume-surface integral equation (VSIE) [1] has been widely used to provide accurate solutions to this problem. However, the $O(N^2)$ computational complexity limits its application scope.

Many fast methods have been proposed to overcome the limitation. The fast multipole method (FMM) [2] and its multilevel version, the multilevel FMM (MLFMM) [3] based on the additional theorem of the Green's function are the most well-known fast approaches. However, the FMM and MLFMM are kernel-dependent methods whose formulation and performance require a priori knowledge of the Green's function. What's more, their code implementation is complicated. Algebraic methods such as adaptive cross approximation (ACA) [4] and pseudo skeleton approximation (PSA) [5] reduce the memory usage and accelerate the matrix-vector multiplication (MVP) by compressing the rank-deficient far-field impedance submatrixes, which are kernel-independent and easy to implement.

To solve the VSIE by the MoM, both the equivalent currents on the conductor's surface and in the dielectric body contribute to the impedance matrix elements. The equivalent currents in the dielectric body are related to the dielectric constant and the integral equation for the dielectric body is related to the total field. Thus, there may be several orders of magnitude differences among the impedance elements. When accelerating the MoM by algebraic methods, the strategy of "choosing the index of the maximum element in the current row/column as the index of the next pending column/row" cannot capture the rank of the impedance matrix, which can lead to unacceptable accuracy.

In this paper, a novel way is proposed to enhance the accuracy of the PSA/ACA for the VSIE. By compressing the four coupling parts in the impedance matrix separately, the accuracy of the PSA/ACA improved without additional memory usage and time consumption. Numerical results demonstrate the efficiency of the proposed method.

II. Formulation

A. A Brief Introduction ACA and PSA for VSIE

The ACA algorithm approximates the low-rank $m\times n$ matrix $\boldsymbol{Z}^{m\times n}$ through a product form. Namely,

$$\tilde{\boldsymbol{Z}}^{m\times n} = \boldsymbol{U}^{m\times r}\boldsymbol{V}^{r\times n} \tag{1}$$

where r is the effective rank of the matrix $\boldsymbol{Z}^{m\times n}$, $\boldsymbol{U}^{m\times r}$ and $\boldsymbol{V}^{r\times n}$ are two dense rectangular matrices [4].

Similar to the ACA algorithm, the PSA algorithm approximates the matrix by

$$\tilde{\boldsymbol{Z}}^{m\times n} = \boldsymbol{C}^{m\times r}\hat{\boldsymbol{Z}}^{-1}\boldsymbol{R}^{r\times n} \tag{2}$$

where $\hat{\boldsymbol{Z}}$ is a nonsingular $r\times r$ submatrix in $\boldsymbol{Z}^{m\times n}$, $\boldsymbol{C}^{m\times r}$ is a submatrix containing all the columns in $\hat{\boldsymbol{Z}}$ and $\boldsymbol{C}^{m\times r}$ is a submatrix containing all the rows in $\hat{\boldsymbol{Z}}$ [5].

The VSIE for electromagnetics scattering problem from composite conductor dielectric targets is proposed in [1]. In this paper, the RWG and SWG basis functions are used to discretize the unknown currents. The matrix equation is formed through the Galerkin test method. The resulting matrix equation compressed by the algebraic methods can be expressed as

$$\left(\begin{bmatrix}\boldsymbol{Z}_{SS} & \boldsymbol{Z}_{SV}\\ \boldsymbol{Z}_{VS} & \boldsymbol{Z}_{VV}\end{bmatrix}^{near} + \tilde{\boldsymbol{Z}}^{far}\right)\begin{bmatrix}I_S\\ I_V\end{bmatrix} = \begin{bmatrix}V_S\\ V_V\end{bmatrix} \tag{3}$$

where $\boldsymbol{Z}_{SS}, \boldsymbol{Z}_{SV}, \boldsymbol{Z}_{VS}, \boldsymbol{Z}_{VV}$ are related the self-coupling of the conductor, coupling from the dielectric to the conductor, coupling from the conductor to the dielectric and self-coupling of the dielectric in near-field impedance matrix, $\tilde{\boldsymbol{Z}}$ is the compressed far-field impedance matrix, $\boldsymbol{I}_S$ is the unknown surface current coefficients, $\boldsymbol{I}_V$ is the unknown volume current coefficients, $\boldsymbol{V}_S$ is the surface voltage vector, and $\boldsymbol{V}_V$ is volume voltage vector.

B. Improved ACA/PSA for VSIE

Due to the different scattering characteristics, there are several orders of magnitude differences among the different coupling parts in the impedance matrix. However, both the ACA and the PSA use the "choosing the index of the maximum element in the current row/column as the index of the next pending column/row" rule. Thus, the compressed matrix may lose the key factors in VSIE, which leads to the reduction of accuracy.

A novel way is proposed to overcome the problem by decomposing the four coupling parts in the impedance matrix separately, which leads to a new compressed impedance matrix

$$\begin{bmatrix} \boldsymbol{Z}_{SS}^{near} + \tilde{\boldsymbol{Z}}_{SS}^{far} & \boldsymbol{Z}_{SV}^{near} + \tilde{\boldsymbol{Z}}_{SV}^{far} \\ \boldsymbol{Z}_{VS}^{near} + \tilde{\boldsymbol{Z}}_{VS}^{far} & \boldsymbol{Z}_{VV}^{near} + \tilde{\boldsymbol{Z}}_{VV}^{far} \end{bmatrix} \tag{4}$$

where $\tilde{\boldsymbol{Z}}_{SS}^{far}$, $\tilde{\boldsymbol{Z}}_{SV}^{far}$, $\tilde{\boldsymbol{Z}}_{VS}^{far}$, $\tilde{\boldsymbol{Z}}_{VV}^{far}$ are the far-field submatrixes compressed by (1) or (2). The rank approximation formulation of the proposed method is the same as the traditional method. In this way, the accuracy can be improved without additional memory usage and time consumption.

III. Numerical Results

To demonstrate the validity of the proposed method, the bi-static RCS with VV polarization of a conducting ball coated with dielectric is computed. The radius of the inside conducting ball is 0.2 m and the radius of the outside coat is 0.3 m. The dielectric constant of the dielectric is $5.0 - 0.5j$. The traditional MoM is set as the benchmark and the error is defined as $|R - R_{MoM}|$, where R is the RCS value calculated by ACA, improved ACA, PSA or improved PSA and R_{MoM} is the RCS value calculated by MoM. The error of ACA and the improved ACA is presented in Fig.1 and the error of PSA and the improved PSA is presented in Fig.2. Table I illustrates the mean and standard deviation of the error in the four different methods. From the result, we can conclude the improved ACA/PSA method can greatly improve the accuracy of the traditional ACA/PSA for the VSIE.

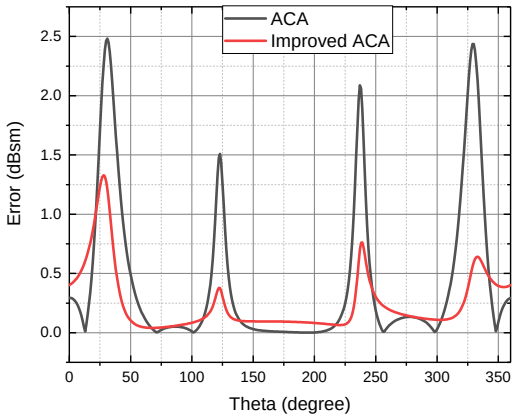

Fig. 1. The error of the ACA and the improved ACA.

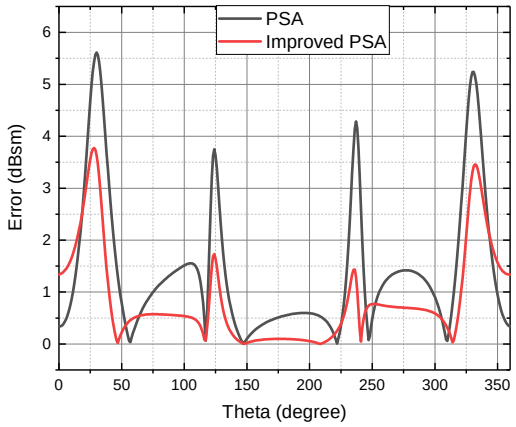

Fig. 2. The error of the PSA and the improved PSA.

TABLE I
MEAN AND STANDARD DEVIATION OF THE ERROR IN FOUR METHODS

Method	ACA	Improved ACA	PSA	Improved PSA
Mean	0.434	0.249	1.412	0.844
Standard Value	0.644	0.263	1.330	0.897

IV. Conclusion

A novel way with numerical results is presented to improve the accuracy of the algebraic methods such as ACA and PSA for the VSIE. Due to the different scattering characteristics of conductor and dielectric, the compressed impedance matrix in the traditional algebraic methods may lose the important scattering information of the composite conducting-dielectric materials. By compressing the far-field submatrixes of the four coupling parts separately in the impedance matrix, the accuracy is improved without extra memory and time requirement.

Acknowledgment

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References

[1] C. Lu and W. Chew, "A coupled surface-volume integral equation approach for the calculation of electromagnetic scattering from composite metallic and material targets," *IEEE Transactions on Antennas and Propagation*, vol. 48, no. 12, pp. 1866–1868, 2000.

[2] V. Rokhlin, "Rapid solution of integral equations of scattering theory in two dimensions," *Journal of Computational physics*, vol. 86, no. 2, pp. 414–439, 1990.

[3] J. M. Song and W. C. Chew, "Multilevel fast-multipole algorithm for solving combined field integral equations of electromagnetic scattering," *Microwave and Optical Technology Letters*, vol. 10, no. 1, pp. 14–19, 1995.

[4] K. Zhao, M. N. Vouvakis, and J.-F. Lee, "The adaptive cross approximation algorithm for accelerated method of moments computations of emc problems," *IEEE transactions on electromagnetic compatibility*, vol. 47, no. 4, pp. 763–773, 2005.

[5] Y. Zhang and H. Lin, "Localized pseudo-skeleton approximation method for electromagnetic analysis on electrically large objects," *Progress In Electromagnetics Research*, vol. 57, pp. 103–109, 2015.