# **Efficient Approximate PageRank**

Assignment 4 - Report Hao Wang (haow2)

#### Q1:

We know di is the degree of vertx u in the ith push, which is defined by the architecture of the graph, so we can know that  $di \ge 0$ .

So we can know that

$$\sum_{i=1}^{T} d_i = T \overline{d_i} \quad \text{and} \quad 0 \leq \overline{d_i} \leq \infty$$

Because:  $\sum_{i=1}^{T} d_i \leq \frac{1}{\epsilon \alpha}$  so we can know:  $T\overline{d_i} \leq \frac{1}{\epsilon \alpha}$ 

Then we know,  $T \le \frac{1}{\varepsilon \alpha \overline{d_i}}$ 

So T is finite, and the algorithm is guaranteed to finish after finite number of push operations.

### Q2:

$$R_{\alpha} = \alpha \sum_{t=0}^{\infty} (1 - \alpha)^{t} W^{t} = \alpha \left( I + \sum_{u=1}^{\infty} (1 - \alpha)^{u} W^{u} \right)$$

So we can see:

$$sR_{\alpha} = \alpha s + s \left(\alpha \sum_{t=1}^{\infty} (1-\alpha)^t W^t \right) = \alpha s + (1-\alpha) s \left(\alpha I + \alpha \sum_{t=1}^{\infty} (1-\alpha)^t W^t \right) W = \alpha s + (1-\alpha) s R_{\alpha} W = s \left[\alpha + (I-\alpha) R_{\alpha} W \right]$$

So  $pr(\alpha, s)$  is linear to s

#### Q3:

$$R_{\alpha} = \alpha \sum_{t=0}^{\infty} (1-\alpha)^t W^t = \alpha \bigg( I + \sum_{{\scriptscriptstyle u=1}}^{\scriptscriptstyle \infty} (1-\alpha)^{\scriptscriptstyle u} W^{\scriptscriptstyle u} \bigg)$$

So we can see:

$$sR_{\alpha} = \alpha s + s \left( \alpha \sum_{t=1}^{\infty} (1 - \alpha)^{t} W^{t} \right) = \alpha s + (1 - \alpha) s \left( \alpha I + \alpha \sum_{t=1}^{\infty} (1 - \alpha)^{t} W^{t} \right) W = \alpha s + (1 - \alpha) s R_{\alpha} W = \alpha s + (1 - \alpha) R_{\alpha} s W$$
$$= \alpha s + (1 - \alpha) pr(\alpha, sW)$$

## **Q4**:

Because 
$$pr(\alpha,r) = pr(\alpha,r-r_u) + pr(\alpha,r_u)$$
 and  $pr(\alpha,s) = \alpha s + (1-\alpha)pr(\alpha,sW)$ 

$$pr(\alpha, r) = pr(\alpha, r-r_u) + \alpha r_u + (1-\alpha) pr(\alpha, r_u W)$$

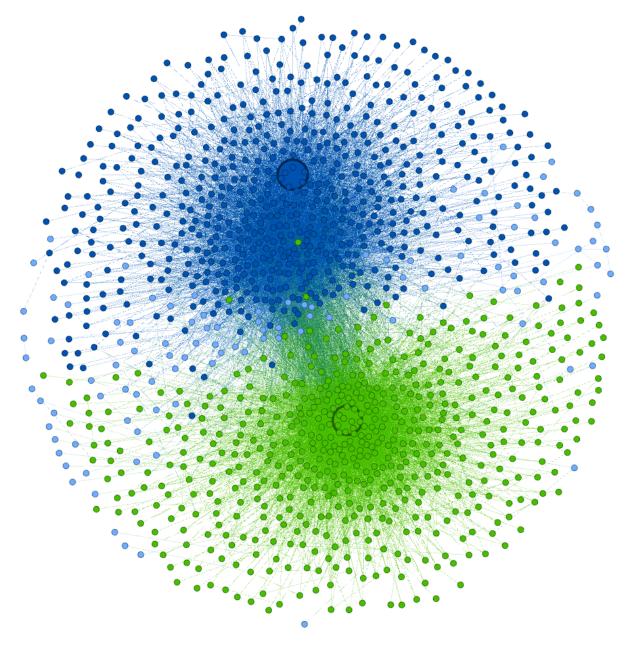
$$= pr(\alpha, r-r_u) + \alpha r_u + pr(\alpha, (1-\alpha)r_u W)$$

$$= pr(\alpha, r - r_u + (1-\alpha)r_u W) + \alpha r_u$$

$$= pr(\alpha, r') + p' - p$$

So: 
$$p' + pr(\alpha, r') = p + pr(\alpha, r)$$

# Q5:



## Q6:

- Did you receive any help whatsoever from anyone in solving this assignment? Yes / No. If you answered 'yes', give full details: No.
- Did you give any help whatsoever to anyone in solving this assignment? Yes / No. If you answered 'yes', give full details: No.