Problem Set 9

9.1

a) Acwording to problem, there's restriant:

$f(x) = w^T x + b = 0$ for x_a subject

$$\mathcal{L}(x, \lambda) = \|x - x_a\|^2 - \lambda(w^T x + b)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = -(w^T x + b) = 0 \qquad \frac{\partial \mathcal{L}}{\partial x} = 2(x - x_a) - \lambda w = 0$$

Thus,

$$2w^T(x - x_a) = \lambda w^T w$$

$$2(w^T x + b - b) - w^T x_a) = \lambda \|w\|^2$$

$$-2f(x_a) = \lambda \|w\|^2$$

$$\lambda = \frac{-2f(x_a)}{\|w\|^2}$$

and

$$(x - x_a) = \frac{-f(x_a)}{\|w\|^2} w$$

$$\|x - x_a\| = \sqrt{\frac{f(x_a)^2}{\|w\|^2}} = \frac{f(x_a)}{\|w\|}$$

this is the distance between hyperplane to x_a.

b) to the origin: $x_0 = 0$

$$\|x - x_0\| = \|x\| = \frac{f(x_0)}{\|w\|} = \frac{|b|}{\|w\|}$$

is the distance between hyperplane to x_0

Since $(x - x_a) = \frac{-f(x_a)}{||w||^2} w$

x_a project to hyperplane x.

$$x_p = x_a + (x - x_a) = x_a - \frac{f(x_a)}{||w||^2} w$$

P8.5

a) Since the soft-margin restriant: $y_i(w^T x_i + b) \geq 1 - \xi_i$

$\xi_i \geq 1 - y_i(w^T x_i + b)$ and $\xi_i \geq 0$.

Thus $\xi_i \geq \max(0, 1 - y_i(w^T x_i + b))$

And when the optimal: constriant is active

$y_i(w^T x_i + b) = 1 - \xi_i$, $\xi_i = \max(1 - y_i(w^T x_i + b), 0)$

b) Since $E(w, b) = \frac{1}{2}||w||^2 + C\sum_{i=1}^{n} \xi_i$

and since a): $\xi_i = \max(0, 1 - y_i(w^T x_i + b))$

$$E(w, b) = \frac{1}{2}||w||^2 + C\sum_{i=1}^{n} \max(0, 1 - y_i(w^T x_i + b))$$

So $E(w, b) \propto \frac{1}{2C}||w||^2 + \sum_{i=1}^{n} \max(0, 1 - y_i(w^T x_i + b))$

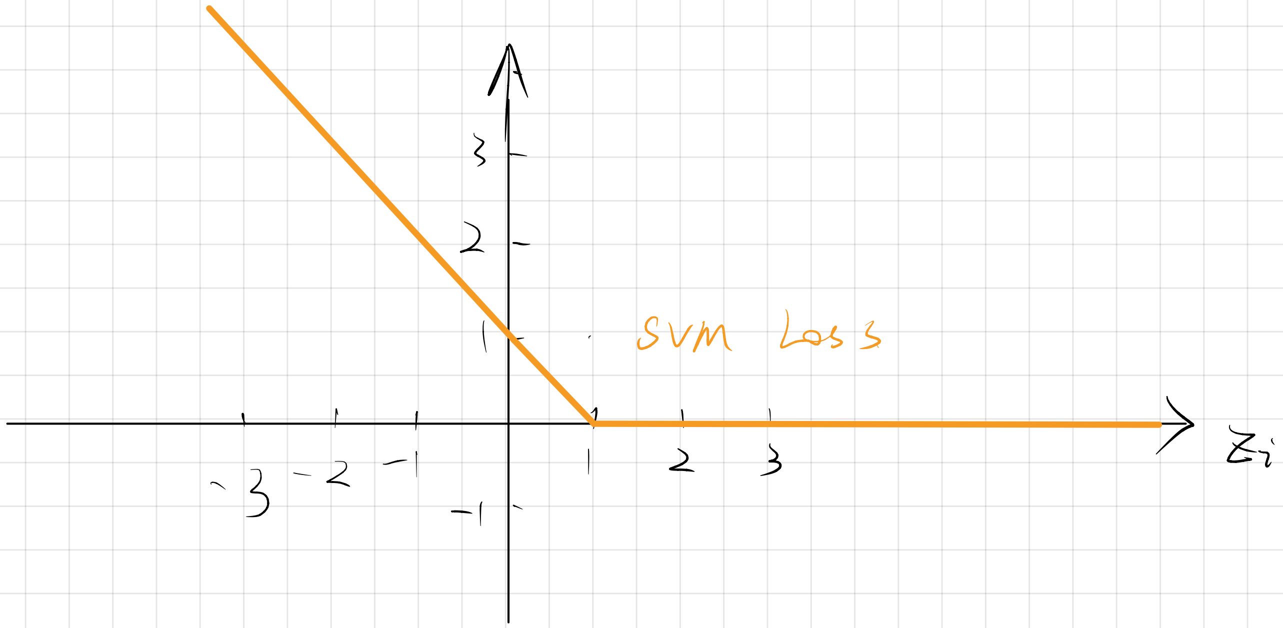
can let $\lambda = \frac{1}{2C}$.

and to min Loss function:

it is: $\min E(w, b) = \min\left[\sum_{i=1}^{n} \max(0, 1 - y_i(w^T x_i + b)) + \lambda||w||^2\right]$

The first term is $\max(0, 1-z_i)$

where $z_i = y_i(w^T x_i + b)$, $L_{svm} = \max(0, 1-z_i)$



To $z_i < 0$ SVM's penalty is the coefficient to distance of point from boundary

$0 \leq z_i \leq 1$ SVM give the margin penalty

$z_i > 1$ there's no penalty if point is out of the margin.

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