Problem Set 6.

Ps 6.4.

a) To the report. If I guess heads, which is equal to $P(s=H \mid r=H) > P(s=T \mid r=H)$

Besides, according to Bayession Rule,

$$P(s=H \mid r=H) = \frac{P(r=H \mid s=H)\, P(s=H)}{P(r=H)}$$

$$= \frac{(1-\theta_1)\cdot \alpha}{\alpha(1-\theta_1)+\theta_2(1-\alpha)}$$

$$P(s=T \mid r=H) = \frac{P(r=H \mid s=T)\, P(s=T)}{P(r=H)}$$

$$= \frac{\theta_2(1-\alpha)}{\alpha(1-\theta_1)+\theta_2(1-\alpha)}$$

when $(1-\theta_1)\alpha > \theta_2(1-\alpha)$

eqyal to $\alpha > \frac{\theta_2}{1-\theta_1+\theta_2}$

Therefore, when $\alpha > \frac{\theta_2}{1-\theta_1+\theta_2}$, the guess is head can based on report head. To tail: $\alpha < \frac{\theta_2}{1-\theta_1+\theta_2}$.

b) when $\theta_1 = \theta_2 = \theta$.

Thus, only when $\alpha > \theta$,

The guess head can be good based on

the report. In other words, guess the head, if the Prior of Head is higher than friend report error probability.

c). If there will be n times report, to guess head correct. for the report, there h time is Head, n−h time is Tail.

Thus. Let $P(S=H \mid r_1 \cdots r_n) > P(S=T \mid r_1 \cdots r_n)$

Then $$P(r_1 \cdots r_n \mid S=H)\, P(S=H) > P(r_1 \cdots r_n \mid S=T)\, P(S=T)$$

$$\theta_1^{n-h}(1-\theta_1)^h \alpha > \theta_2^h (1-\theta_2)^{n-h}(1-\alpha)$$

$$[\theta_1^{n-h}(1-\theta_1)^h + \theta_2^h(1-\theta_2)^{n-h}]\,\alpha > \theta_2^h(1-\theta_2)^{n-h}$$

$$\alpha > \frac{1}{\frac{\theta_1^{n-h}(1-\theta_1)^h}{\theta_2^h(1-\theta_2)^{n-h}} + 1}$$

d) when $\theta_1 = \theta_2 = \theta$. and $n=h$, based on c)

$$\alpha > \frac{1}{\frac{(1-\theta)^n}{\theta^n} + 1}$$

$$\frac{1}{\alpha} < \frac{(1-\theta)^n}{\theta^n} + 1$$

$$\frac{1-\alpha}{\alpha} < \left(\frac{1-\theta}{\theta}\right)^n, \quad \frac{\alpha}{1-\alpha} > \left(\frac{1-\theta}{\theta}\right)^n$$

Thus, when $\frac{\text{Head Prior}}{\text{Tail Prior}} > \left(\frac{\text{correct report posibility}}{\text{mis report posiblity}}\right)^n$

I can always guess Head (since all trail are heads, $n=h$)

What's more when sample is enough large $n \to \infty$

① when $\frac{1-\theta}{\theta} \to 0$, $\theta > \frac{1}{2} \Rightarrow \alpha > 1$

That means when $\theta > \frac{1}{2}$ only when $\alpha > 1$ can guess head

So I can only guess tail reject the head report

② when $\frac{1-\theta}{\theta} \to \infty$ $\theta < \frac{1}{2} \Rightarrow \alpha > 0$

That means when $\theta < \frac{1}{2}$ I can casually guess heads and trust the report.

③ when $1-\theta = \theta$, $\theta = \frac{1}{2} \Rightarrow \alpha > \frac{1}{2}$

That means report correct, wrong are radom the guess can just based on the prior instead of report.

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