Project PRESENTATION

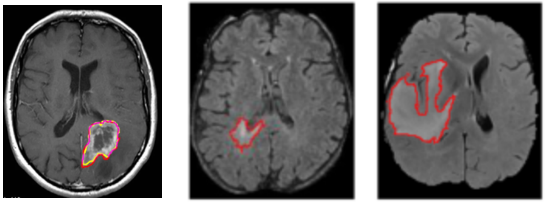
Medical image segmentation based on variational models and machine learning?

Jieyao Wang(Kayla)
Supervisor: Xiaohao Cai
Second Examiner: Fang Xu

OUTLINE

Optimization

1 Introduction



What is image segmentation

Image segmentation is to divide the given image into different regions, where each region has similar properties such as color, texture, brightness, etc.

Why do we need image segmentation

- An essential research directions in medical image processing, especially in image-guided surgery, computer-aided diagnosis, and tumor radiotherapy [1] [2]
- Help doctors make more accurate diagnoses efficiently

Variational method

Category 1: edge-based models

e.g. Snake model

Limitation: not effective for images with weak edge

Category 2: region-based models

e.g. Mumford-Shah mode[3], CV model[4], Multiphase Chan-Vese[5]

Limitation: sensitive to image intensity inhomogeneity and parameter adjustment.

Machine learning method

Neural networks

e.g. Full convolution network, U-net

Supervised learning algorithm:

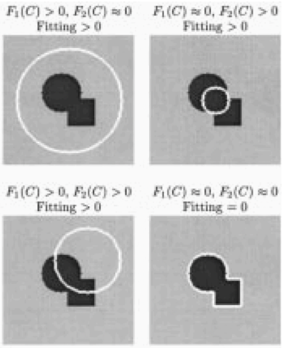
e.g. k-nearest neighbors (k-NN), fuzzy k-nearest neighbors

support vector machine(SVM)

Limitation: result in noise from misclassified pixels, so the results require further post-processing, such as morphological processing, to obtain smooth segmentation boundaries

CV model

-- "Active Contours Without Edges" proposed by Chan and Vese[4]



Suppose an image u_0 is divided into two regions by a boundary of C .

Energy function

$$E(c_1, c_2, C) = \lambda_1 \int_{\mathrm{inside}(C)} |u_0(x,y) - c_1|^2 dxdy + \lambda_2 \int_{\mathrm{outside}(C)} |u_0(x,y) - c_2|^2 dxdy$$
$$+ \mu \cdot \mathrm{Length}(C) + v \cdot \mathrm{Area}(\mathrm{inside}(C))$$

CV model aims to find the optimal solution C to minimize the energy function.

$$\inf_{c_1, c_2, C} E(c_1, c_2, C)$$

Level set method can solve this minimization problem

The boundary C is represented by the zero-crossing level set of a Lipschitz function Ø(x,y,t) written as $C(t) = \{(x,y) | \emptyset(x,y,t) = 0\}$

$\emptyset_0(x,y) = \emptyset(x,y,0)$ denotes the initial contour

CV model

Level set method □ get the optimal solution

$$\frac{\partial \phi}{\partial t} = \delta_\epsilon(\phi)\left[\mu \operatorname{div}\left(\frac{\nabla \phi}{|\nabla \phi|}\right) - \nu - \lambda_1 (u_0 - c_1)^2 + \lambda_2 (u_0 - c_2)^2\right]$$

$$\frac{\partial \phi}{\partial t} = \frac{\phi^{n+1} - \phi^{n}}{dt}$$

Final segmentation result $\emptyset^{n+1}$

$\emptyset^{n+1}$ → the value of $\emptyset$ at timestep n+1

Pseudocode

Algorithm 1 CV segmentation algorithm

Input: $\mathrm{it}_{\max}, v, \mu$

Output: ϕ^n

1: initialise ϕ_0 ;
2: **for** each n in range(1,$\mathrm{it}_{\max}$) **do**
3: Compute c_1, c_2 as the region averages inside
4: and outside the contour;
5: Evolve ϕ^n with one semi-implicit timestep;
6: **end for**

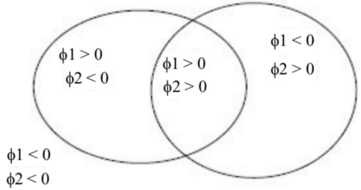
Multiphase Chan–Vese model

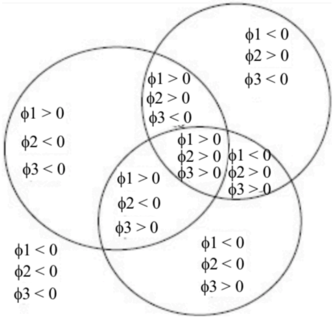
-- A multiphase segmentation model based on the Mumford and Shah model introduced by Chan and Vese[5]

Chan-Vese variational level set region partition principle

n level set functions segment an image into 2^n regions

- two level set functions φ1, φ2 to achieve 4-phase segmentation
- three level set functions φ1, φ2, φ3 to achieve 8-phase segmentation





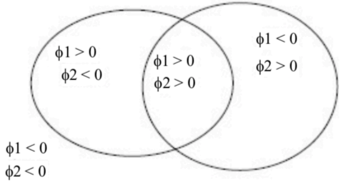
Multiphase Chan–Vese model

Suppose an image u_0 is divided into four regions

Energy function

A 4-phase model

$$
\begin{aligned}
E_4(c,\Phi) = & \int_\Omega (u_0-c_{11})^2 H_\varepsilon(\phi_1) H_\varepsilon(\phi_2)\,\mathrm{d}x\mathrm{d}y + \int_\Omega (u_0-c_{10})^2 H_\varepsilon(\phi_1)\left(1-H_\varepsilon(\phi_2)\right)\mathrm{d}x\mathrm{d}y \\
& + \int_\Omega (u_0-c_{01})^2 \left(1-H_\varepsilon(\phi_1)\right) H_\varepsilon(\phi_2)\,\mathrm{d}x\mathrm{d}y \\
& + \int_\Omega (u_0-c_{00})^2 \left(1-H_\varepsilon(\phi_1)\right)\left(1-H_\varepsilon(\phi_2)\right)\mathrm{d}x\mathrm{d}y \\
& + v\int_\Omega \left|\nabla H_\varepsilon(\phi_1)\right| + v\int_\Omega \left|\nabla H_\varepsilon(\phi_2)\right|,
\end{aligned}
$$



C_{00}, C_{10}, C_{01}, C_{11} → the mean values of u_0 corresponding to the four regions.

υ is a fixed parameter to control the length term in the energy

Multiphase Chan–Vese model

Level set method get the optimal solution

$$\frac{\partial \phi_1}{\partial t} = \delta_\varepsilon(\phi_1)\left\{ v \operatorname{div}\left(\frac{\nabla \phi_1}{|\nabla \phi_1|}\right) - \left[\left((u_0 - c_{11})^2 - (u_0 - c_{01})^2\right) H(\phi_2) + \left((u_0 - c_{10})^2 - (u_0 - c_{00})^2\right)(1 - H(\phi_2))\right]\right\},$$

$$\frac{\partial \phi_2}{\partial t} = \delta_\varepsilon(\phi_2)\left\{ v div\left(\frac{\nabla \phi_2}{|\nabla \phi_2|}\right) - \left[\left((u_0 - c_{11})^2 - (u_0 - c_{10})^2\right) H(\phi_1) + \left((u_0 - c_{01})^2 - (u_0 - c_{00})^2\right)(1 - H(\phi_1))\right]\right\}.$$

$$\frac{\partial \phi_1}{\partial t} = \frac{\phi_1^{n+1} - \phi_1^n}{dt} \qquad \frac{\partial \phi_2}{\partial t} = \frac{\phi_2^{n+1} - \phi_2^n}{dt}$$

Pseudocode

Algorithm 1 Chan-Vese segmentation algorithm

Input: $\mathrm{it}_{\max}, v$

Output: ϕ_1^n, ϕ_2^n

initialise ϕ_1, ϕ_2 ;
for each n in range(1,$\mathrm{it}_{\max}$) **do**
  Compute c_{11}, c_{10}, c_{01} and c_{00} as the region averages;
  Evolve ϕ_1^n, ϕ_2^n with one semi-implicit timestep;
end for

Obtain $\emptyset_1^{n+1}, \emptyset_2^{n+1}$ → the value of $\emptyset_1$ and $\emptyset_2$ at timestep n+1

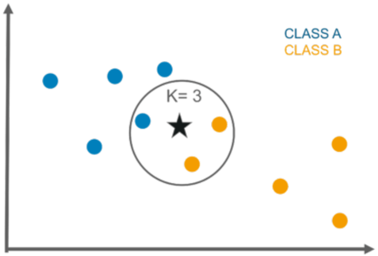
K nearest neighbor (k–NN)

Suppose the training set $X = \{x_i\}_{i=1}^{n}$ belongs to c classes; the label is $Y = \{y_i\}_{i=1}^{n}$;

k : number of nearest neighbors

Classify the query element x'

1. Calculate the Euclidean distances between test data x' and each training data x_i
2. Sort the distances in ascending order
3. Get the top k elements from the sorted distances (k nearest neighbors)
4. Get the most frequent class from k nearest neighbors



Fuzzy k–NN

Suppose the training set $X = \{x_i\}_{i=1}^{n}$ belongs to c classes; k : number of nearest neighbors

Training □ get the membership matrix

$$U = \begin{bmatrix} u_{1,1} & u_{1,2} & \cdots & u_{1,j} \\ u_{2,1} & u_{2,2} & \cdots & u_{2,j} \\ \cdots & \cdots & \cdots & \cdots \\ u_{i,1} & u_{i,2} & \cdots & u_{i,j} \end{bmatrix}$$

i = n □ the number of training samples
j = c □ the number of classes

Membership value formula

$$u_{ij}(x_i) = \begin{cases} 0.51 + (n_j/k) * 0.49, & \text{if } x_i \in C \\ (n_j/k) * 0.49, & \text{if } j \notin C \end{cases}$$

n_j : the number of neighbors found belonging to the jth class

Predict → classify the query element x'

1. Find the k nearest neighbors in the training set, $X_{\text{nei}} = \{x_j\}_{j=1}^{k}$
2. Find the membership values of k samples from U and calculate the weight of each class $(w_1, w_2, .., w_c)$
3. x' belongs to the class with largest weight and the probability of x' is the largest weight.

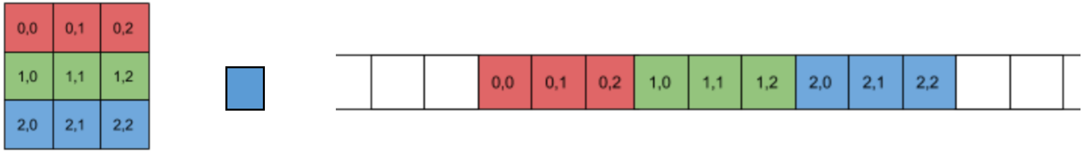
weight formula

$$w_i(x') = \frac{\sum_{j=1}^{k} u_{ij} \left(1/\left\|x' - x_j\right\|^{2/(m-1)}\right)}{\sum_{j=1}^{k} \left(1/\left\|x' - x_j\right\|^{2/(m-1)}\right)}$$

Segmentation method based on machine learning and variational models

Step 1： Preprocess image

➢ Transform 2-d gray-scale images into 1-d pixels



➢ Shuffle 1-d data in random order and split the training set

Step 2 : Classify the pixel intensity

- Train the classifier using the training set
- Classify all 1-d pixels

Obtain the classification values and probabilities

n classes □ the range of classification values is $\{0,1,2, .., n-1\}$

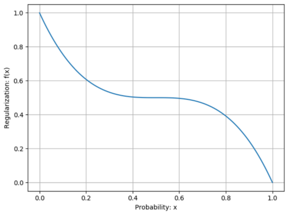
the range of probability is $[0,1]$

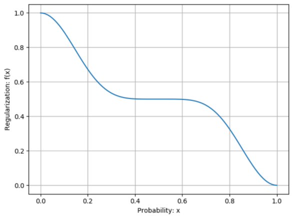
In the experiment use **fuzzy k-NN** and **k-NN**

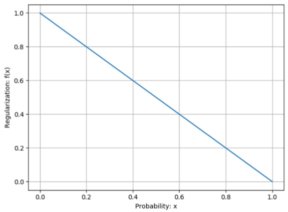
Step 3 : Regularization and score map

- Calculate regularized score : $s = classification\ value + f(p)$

Regularization function f(p)







$$f_1(p) = \frac{1 + (1-2p)^3}{2} \qquad f_2(p) = 0.5(1 - cos^5(\pi(1-p))) \qquad f_3(p) = 1 - p$$

Common properties

1. $f(p)$ lies in [0,1]
2. Decreasing monotonically
3. The equations hold

$$\lim_{p\to 0} f(p) = 1$$
$$\lim_{p\to 0.5} f(p) = 0.5$$
$$\lim_{p\to 1} f(p) = 0$$

- Generate a score map (the same size as the image)

Regularized : 1-d regularized score → 2-d score map, range : $[0, c]$

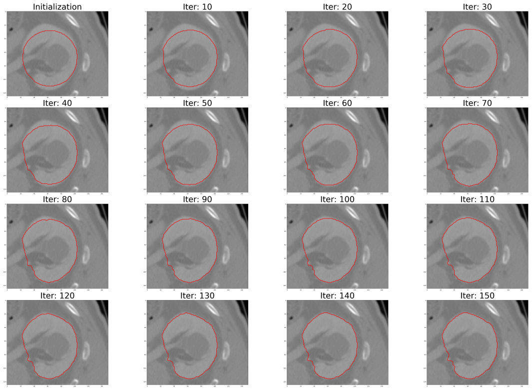
No regularized: 1-d classification values → 2-d score map, range: $\{0,1,..,c-1\}$

Step 4： Segment the score map using variational model

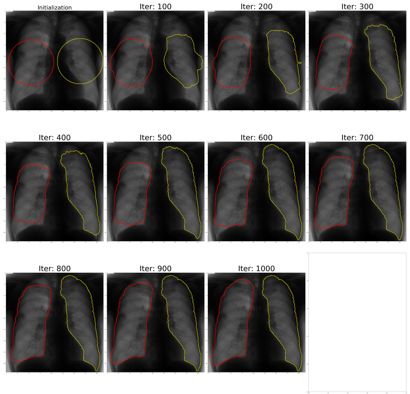
automatically select a model based on the number of classes in the image

the number of classes = 2 □ CV model

the number of classes > 2 □ multiphase Chan-Vese model



CV segmentation process



Multiphase Chan-Vese segmentation process

4 Optimization

Optimize searching k nearest neighbors

K-d tree

Time complexity : simplest brute force algorithm O(n) □ K-d tree O(log n)

Matrix operation

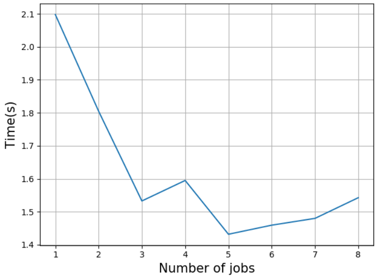
How much superior Numpy matrix is compared to loop operations?

The 'ndarray' in Numpy, is a fast and space-saving multidimensional array.

Python is an interpreted language. The code of Numpy is written in C language, so it avoids the general cost of loops in Python, element-by-element dynamic type checking and pointer indirection.

Parallelization

The initial number of jobs is set to 4 because the system has four processors available.



F1. Number of jobs and running time

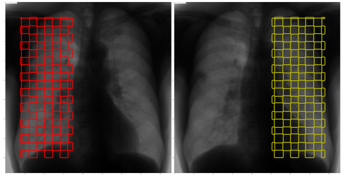
Dataset

A small Iris dataset in the UCI responsibility, contains a set of 150 instances under four attributes.

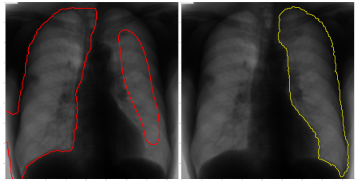
Fuzzy k-NN	Before optimization	After optimization
Time(second)	About 20	0.003

Optimize multiphase segmentation model

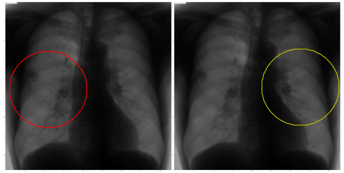
Yellow and red lines represent two level set functions in the multiphase model.



F1. checkerboard initialization

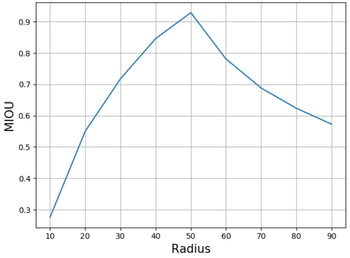


F2. Segmentation result after 800 iterations,MIOU : 0.82

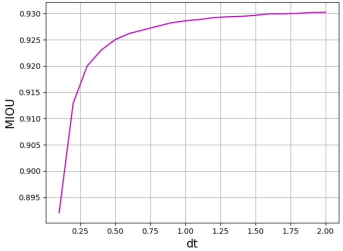


F3. circle initialization

F4. Segmentation result after 800 iterations, MIOU : 0.92



F5. Radius and segmentation accuracy



F6. Timestep dt and segmentation accuracy

➢ **Initialization of a level set function**

The circle initialization converges faster(F1-4). The segmentation accuracy reaches a peak when radius is 50.(F5)

➢ **Time step dt selection**

The time step of the variational model is set to 1.

5 Experiments

Data set

Dataset 1: 20 kidney CT images from the Grand Challenge platform

resolution 125 × 150

number of classes : 2 (background and kidney)

Dataset 2: 20 chest radiographs from the JSRT database

resolution 210×205

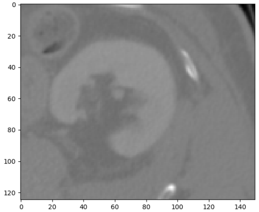
number of classes : 3 (left lung, right lung, background)

Environment

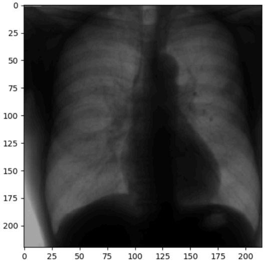
runs on a Macbook pro with a 2.9ghz CPU and 8GB RAM

Predefined parameters

- CV model: μ=0.6, ν=0, λ_1= λ_2 =1, dt=0.2, maximum iterations = 150
- Chan-Vese model : ν=0.7, dt=1, maximum iterations = 1000
- k-NN, fuzzy k-NN : k=9, parallel jobs = 4, ratio of training set = 0.5



Kidney image



Chest radiograph

5 Experiments

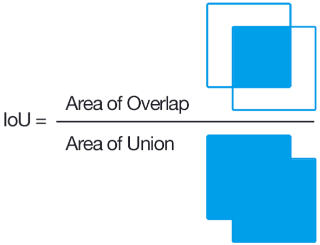
Evaluation metrics

1. Intersection over union (IOU)

IOU is the area of overlap between the predicted segmentation and the ground truth divided by the area of union between the predicted segmentation and the ground truth.

$$\mathrm{IOU} = \left|\frac{A \cap B}{A \cup B}\right|$$

A : segmentation results; B : the ground truth



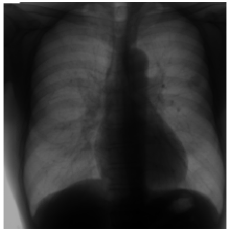
2. Mean IOU

The mean IOU of the image is calculated by taking the IOU of each class and averaging them.

$$\text{mean IOU} = \frac{1}{n}\sum_{i=1}^{n} IOU_i$$

5 Experiments

Score map – Lung image

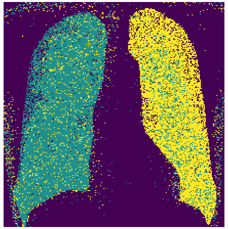

(1) Original image

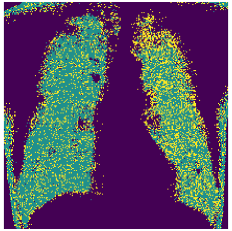

(2) the ground truth

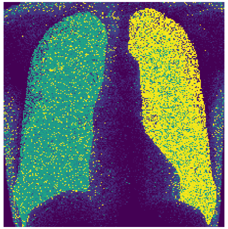
The regularization mainly affects elements at the intersection of different classes.

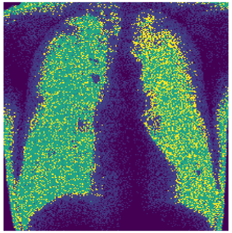
It can identify ambiguous pixels at the edge of the lungs, which are marked as new classes(the blue dots), thus improving the classification of the model.

The classification of fuzzy k-NN outperforms that of k-NN.

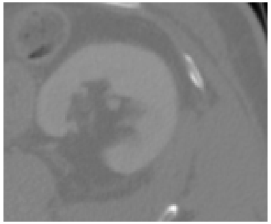

(3) Fuzzy k-NN, no regularized


(4) k-NN, no regularized


(5) Fuzzy k-NN, regularized


(6) k-NN, regularized

Score map – Kidney image

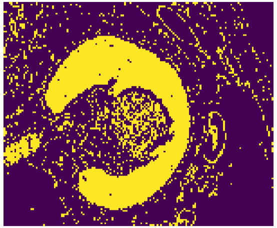


(1) Original image

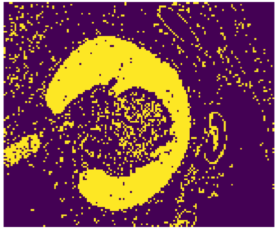
(2) the ground truth

Because of the small difference in pixel values of kidney images, the generated score maps are greatly different from the ground truth.

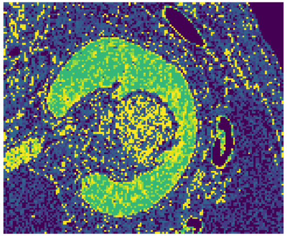
There is no obvious difference between fuzzy K-NN algorithm and K-NN algorithm in CT image classification.



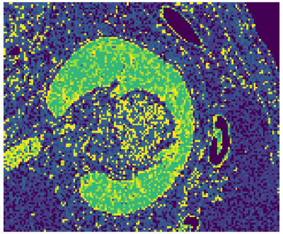
(3) Fuzzy k-NN, no regularized



(4) k-NN, no regularized



(5) Fuzzy k-NN, regularized

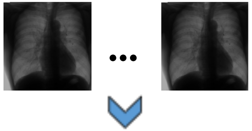


(6) k-NN, regularized

5 Experiments

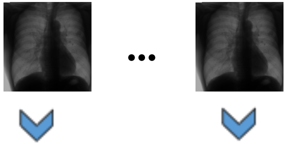
Two schemes: using one classifier or multiple classifiers

Single classifier scheme



One classifier

Multi-classifier scheme



classifier 1 … classifier n

Exp.	Images	One classifier(Mean IOU)		Multiple classifiers(Mean IOU)	
		fuzzy k-NN	k-NN	fuzzy k-NN	k-NN
1	Kidney	0.8697	0.8713	0.8771	0.877
2	Lungs	0.8654	0.7831	0.9252	0.7776

T1. Segmentation accuracy using one classifier and multiple classifiers

Exp.	Images	One classifier(Time:s)		Multiple classifiers(Time: s)	
		fuzzy k-NN	k-NN	fuzzy k-NN	k-NN
1	Kidney	5.99	29.75	4.84	15.87
2	Lungs	5.95	28.73	3.98	27.68

T2. Time (in second) for generating score maps using one classifier and multiple classifiers

The method based on multiple classifiers is more accurate than that based on single classifier.

Compared with single classifier, the multi-classifier scheme can significantly save the time of generating the score map.

Multi-classifier scheme outperforms single classifier scheme.

5 Experiments

Result

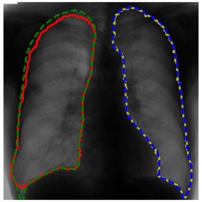
- The accuracy of proposed method is higher than that of the traditional variational method.
- The fuzzy k-NN method is superior to the k-NN method in mean IOU.

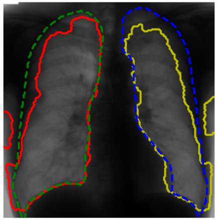
Exp.	Images	Proposed method: Mean IOU fuzzy k-NN	Mean IOU k-NN	Mean IOU CV/Chan-Vese
1	Kidney (2 classes)	0.8756	0.8754	0.7783
2	Lungs (3 classes)	0.9247	0.77	0.533

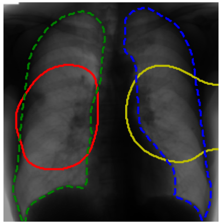
T1. The segmentation accuracy

Exp.	Images	Classes	Proposed method: IOU fuzzy k-NN	IOU k-NN	IOU Chan-Vese
2	Lungs(3 classes)	Left lung	0.95	0.75	0.533
		Right lung	0.89	0.78	0.778

T2. The segmentation accuracy of the left and right lungs on chest radiographs


F1. Proposed method using fuzzy k-NN


F2. Proposed method using k-NN

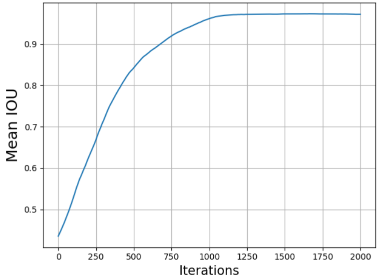

F3. multiphase Chan-Vese

Result -- Time

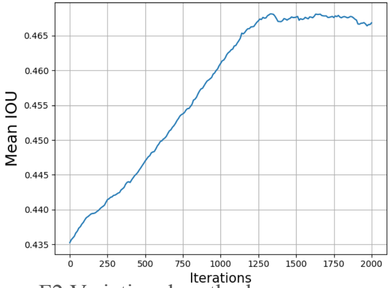
- ➢ The proposed method converges faster than the traditional method. (F1, F2)
- ➢ Based on the same number of iterations, our method takes longer to segment images than the variational method.
- ➢ Fuzzy k-NN method takes less time than k-NN method. It is more efficient.

Exp.	Images	fuzzy k-NN			k-NN			CV/Chan-Vese
		Total(s)	SM(s)	VM(s)	Total(s)	SM(s)	VM(s)	Total/VM(s)
1	Kidney	11.63	4.99	6.61	23.21	16.84	6.34	6.61
2	Lungs	429	6.6	422	444	32	411	482

T1. Computational time. SM : the time in generating score maps. VM : the time of the variational model



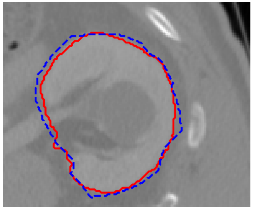
F1. Proposed method using fuzzy k-NN, converge after 1000 iterations



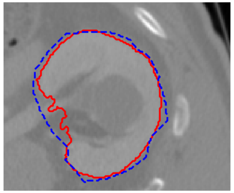
F2 Variational method, converge after 1300 iterations

Result -- Regularization

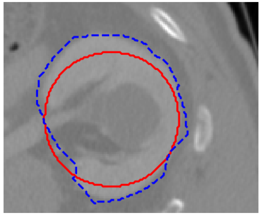
- The method using regularization has higher accuracy than method without regularization.
- Methods using nonlinear functions are superior to linear methods.
- Proposed method is not sensitive to the choice of nonlinear functions



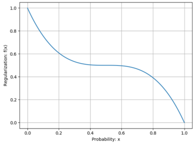
F1. Proposed method (fuzzy k-NN)



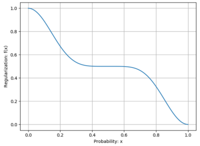
F2. No regularized method (fuzzy k-NN)



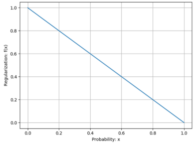
F3. CV method (No regularization)



F4. Func(3.6)



F5. Func(3.7)



F6. Func(3.8)

Exp.	Images	Func (3.6)		Func (3.7)		Func (3.8)		No Regularization	
		Fk-NN	k-NN	Fk-NN	k-NN	Fk-NN	k-NN	Fk-NN	k-NN
1	Kidney	0.8767	0.8754	0.8765	0.8758	0.8752	0.8747	0.8360	0.8375
2	Lungs	0.9247	0.77	0.92514	0.77	0.906	0.77	0.9138	0.75

T1. Segmentation accuracy using different regularization functions

6 Conclusion

1. Multi-classifier scheme outperforms single classifier scheme.
2. The proposed method has higher segmentation accuracy than the traditional variational method, and converge faster.
3. Fuzzy k-NN method outperforms k-NN method in accuracy and computational time.
4. Regularization can improve segmentation accuracy to some extent.

Reference

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THANK YOU