Seizure detection in EEG using dynamic system analysis

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***Abstract*—An epileptic seizure is the irregularity of brain activity that interferes with normal functions. Electroencephalography (EEG) is widely used for detecting and localizing seizures. Almost all automatic detection methods pursue machine learning algorithms taking various features of EEG recordings as input. Previous studies focused on features in the time domain (e.g. variance), frequency domain (e.g. power spectrum), and time-frequency domains (e.g. wavelets, empirical modes). As an alternative, we present a dynamical system-based detection.**

From the perspective of a dynamic system, EEG is the output of a nonlinear system consisting of a network of neurons. The state of the system can be represented by a point in the phase space. Time evolution of the state creates a unique portrait in the phase space that captures the underlying dynamics of the system. In this paper, we have shown that entropy, as an important characterization of the dynamic system, of the EEG signals can be used to discriminate between the normal and seizure states. Our method differs from other methods in that it provides a biologically interpretable perspective to study the brain in healthy and diseased conditions.

***Index Terms*—seizure detection, EEG, dynamic systems, entropy, time delay embedding, machine learning, phase space.**

I. Introduction

More than 2% of the population worldwide is affected by seizures, and 2.4 million new cases are estimated to occur every year [1]. Electroencephalogram (EEG) recordings are widely used for seizure diagnosis. EEG signal is the readout of collective cortical activity. It is usually obtained by placing electrodes on the surface of the brain intracranially or on the surface of the skull. In healthy humans under normal conditions, the cortical activity displays a specific rhythmic activity depending on the arousal state. In disease states, these rhythms are overridden by aberrant brain activity patterns, such as seizures in epileptic patients. Seizure activity consists of four distinctive phases, namely pre-ictal, ictal, interictal, and postictal [2], [3]. During the ictal phase, the brain wave displays a brief burst of polyspikes and then excessive synchronous discharge [4]. The activity in other phases are less stereotypical and presumably contains features related to the disease.

Manual seizure detection, such as visual inspection, demands expert knowledge, and it's a time-consuming task as the EEG recordings can be tens or hundreds of hours long. That is why developing automatized methods for seizure detection and prevention is crucial. The state-of-the-art techniques are summarized in [5]. All methods follow similar steps (1) signal pre-processing, (2) feature extraction, and (3) classifier model learning and testing. It's essential that the extracted feature is in accordance with the input and requirements of the model. Common EEG features can be categorized into the time domain, frequency domain, wavelet decomposition, and rational transform domain [5]. For example, in the time domain analysis, the seizure onset is detected when the local variance of the signal crosses a designated threshold. In the frequency domain, detection is triggered by changes in the power in specific frequency bands of the spectrum. In a nutshell, those methods convert a time series signal into information in other domains that can be used directly by the classification model. With the advance of AI and powerful computing these days, feature extraction and model learning can be done recursively and simultaneously. However, this might run into the danger of finding features that are not biological or creating models that are not intuitive or interpretable, therefore not adaptable, for the experimenters. On the other hand, time/frequency features might be a poor characterization of this complex system undergoing a transition between normal and abnormal synchronous states.

In this paper, we looked into the EEG signal from the perspective of a dynamical system. We argue that normal and abnormal brain activities can be (1) visualized in the phase space to the best interest of experimenters, (2) manifested in the changes of dynamics, and most importantly, (3) captured by an intrinsic measure, namely entropy, of the system.

The rest of the paper is organized as follows to describe our method in detail. Section II reviews the mathematical concepts used in our method. Section III describes the framework of phase space analysis (our method) and spectrum analysis (alternative as a comparison). Section IV compares the results of the two in different conditions. Section V concludes our findings, points out pitfalls and limitations of this method as well as provides tentative solutions.

II. A primer on phase space analysis

A. Delay embedding and Phase space representation of a time series

A time-varying signal, as is the case with most biomedical systems, carries important information about the dynamics

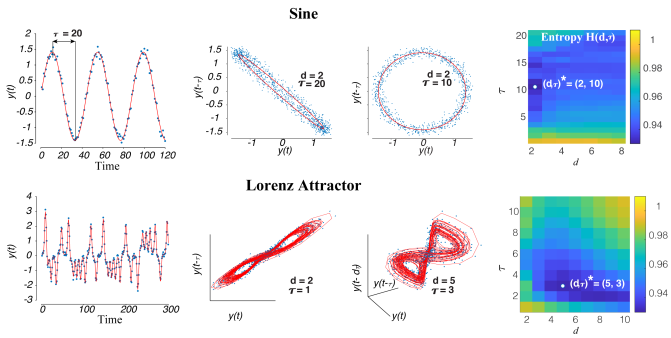

Figure 1. Phase space analyses of two example signals with different complexity. The signal on the top row is a sine wave with white noise added. The bottom row is a one-dimensional readout of the Lorenz attractor, a chaotic dynamical system. By transforming the time series with delayed time embedding, the linear or nonlinear dynamics of the systems can be reconstructed in the phase space.

of the system generating the signal. To uncover the dynamic structure, time-delay embedding has been an established method for reconstructing the underlying dynamics. This was enabled by Taken's theorem formulated in the early 1980s [6], [7]. In the delay embedding, we transform a univariate time series y_t into a set of delayed vectors (DV)

$$\mathbf{x}_t(d,\tau) = [y_t, y_{t-\tau}, \ldots, y_{t-(d-1)\tau}]^T$$

with given embedding dimensionality (d) and time delay (τ). Each DV represents a state of the system which evolves in time, and continuous states form trajectories in the phase space. Two apparently different signals may share a similar phase space portrait, particularly if they originated from systems with the same underlying dynamics. For example, two sinusoidal waves with different spectrum profiles will be similar in their phase space portrait as they are governed by the same underlying dynamics.

In delay embedding, the selection of d and τ are critical for proper representation of the signal dynamics in phase space. Figure 1 demonstrates this point from two examples of different levels of complexity; a sine wave representing a linear system and a Lorentz attractor representing a chaotic system. In the case of sine wave (Figure 1 top row), choosing $\tau = T/4$ (T is the period of the sinusoidal wave) can best reveal its oscillatory dynamics. The embedding dimension (d) indicated the complexity of the system. On the other hand, the Lorenz attractor has a dimensionality ($d > 2$), thus a two-dimensional phase plot would fail to capture the true relationship between the states (Figure 1 middle two columns). Conventionally, the optimal d and τ are obtained using false nearest neighbor and mutual information methods in separate steps [8], [9]. Below we show that optimization of d and τ can be achieved simultaneously by searching for the minimal entropy in its phase space representation [10].

B. Differential Entropy (DE)

Let $\mathbf{x}$ denote all the accessible DVs, and the probability density function $p(\mathbf{x})$ of $\mathbf{x}$ can be estimated. The differential entropy (DE) of the distribution is defined as:

$$H(\mathbf{x}) = -\int p(\mathbf{x}) \ln[p(\mathbf{x})] d\mathbf{x}$$

The Kozachenko-Leonenko (K-L) estimate of the DE provides a practical way of computing the differential entropy [11]:

$$H(\mathbf{x}) = \sum_{j=1}^{N} \ln(N\rho_j) + \ln 2 + Ce$$

N is the total number of states, ρ_j is the Euclidean distance of the jth state to its nearest neighbor (or k nearest neighbors) in the d-dimensional phase space, and $Ce \approx 0.5772$ is the Euler's constant.

In EEG recordings, let $H(\mathbf{x}, d, \tau)$ denote the DE estimated for the embedded time series $\mathbf{x}$ with embedding dimension d and time delay τ. Note that the K-L estimate of DE, like other entropy estimations, is not operationally defined and only a relative value is meaningful. Therefore, we used a relative entropy which is the ratio between $H(\mathbf{x}, d, \tau)$ and $H(\mathbf{x}^s, d, \tau)$ of its surrogates. The surrogates ($\mathbf{x}^s$) were generated randomly by matching the spectral and amplitude distribution of the original time series y_t [12]. Consequently, the ratio between the two DEs, denoted as entropy ratio (ER), was used for our evaluation.

$$ER(\mathbf{x}, d, \tau) = \frac{H(\mathbf{x}, d, \tau)}{< H(\mathbf{x}^s, d, \tau) >} + \frac{d \ln N}{N}$$

in which $< \cdot >$ indicates the average over a pool of randomly generated surrogates, and $\frac{d \ln N}{N}$ is a regularization term that penalizes high embedding dimensions for a short time series (small N). The last column in Figure 1 illustrates the profiles of $ER(d, \tau)$ for the two examples. Optimal d and τ are the values that correspond to the minimum of ER ($minER$). It can be intuitively understood that the proper choice of d and τ can approximately reconstruct the intrinsic manifold of the dynamic system. This reconstructed representation has correct dimensionality and orderly neighboring states, thus the lowest ER, in comparison with the entropy of alternative arrangements with improper d and τ. An $ER = 1$ indicated utter randomness. White noise, for instance, is statistically equivalent to its surrogates resulting in an ER of 1. We then use $minER$ as a general measure of the orderliness of a signal for its simplicity and interpretability.

III. Methods

A. EEG Dataset

We used the widely accepted EEG database provided by the University of Bonn [13]. The dataset consists of five subsets (set A-E, Figure 2). Sets A and B are surface EEG recordings of healthy participants in wakeful states with eyes open (set A) and eyes closed (set B). Set C, D, and E are recorded from epileptic patients. Set E included only the seizure epochs. Set D was recorded within the epileptogenic zone during the interictal phase. Set C is from the hippocampus of the opposite hemisphere during the interictal phase. The sampling rate is 173.61 Hz. Each set consists of 100 recordings, and each recording lasts 23.6 seconds.

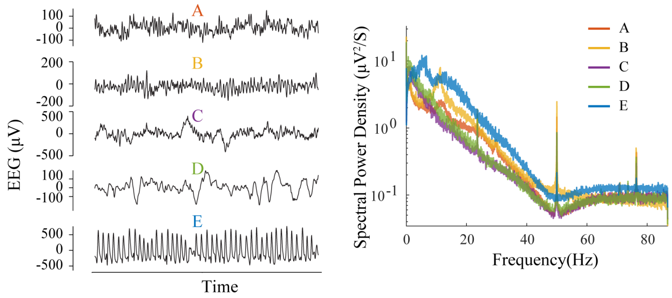

Figure 2. EEG recordings of the Bonn database. Example segments of the EEG recordings were shown for the five example datasets on the left: A, B are from healthy subjects, C, D, and E are from epileptic patients. The averaged spectrum power density was also computed for each dataset.

B. Analysis Procedure

The analysis procedure is illustrated in Figure 3. All data processing and analyses were performed in MATLAB (2020b, MathWorks Inc.).

1) Preprocessing: The Bonn datasets were recorded from different patients under different conditions. To unify the input to the classifier models, we first segmented the EEG data into various time window sizes (1, 2, 3, 4, 5, and 6 seconds). The starting time of each segment was randomly and blindly chosen (Figure 3 top row) to avoid bias. For each dataset and each window size group, 1000 recording segments were used for the following analysis.

2) Feature Extraction: We applied the phase space analysis on the segmented EEG recordings. For a chosen embedding dimension (d) and time delay (τ), a set of states traverses the phase space, and a unique density distribution was obtained. Then, the entropy ratios (ERs) were then estimated, within which we found the optimal choice $d*$ and $\tau*$ corresponding to the minimum of ER ($minER$). In addition, we used spectral power analysis as a counterpart to our method. The spectral power density as the function of frequency ($SPD(f)$) was also computed for each segment. Averaged power intensities for the five characteristic EEG bands, $\delta(.1 - 3.5Hz)$, $\theta(3.5 - 8Hz)$, $\alpha(8 - 14Hz)$, $\beta(14 - 30Hz)$, $\gamma(30 - 80Hz)$, and the spectral power ratios (SPR) between pairs of bands were then obtained [14].

$$SP(i) = \sum_{f \in i} SPD(f), i \in \{\delta, \theta, \alpha, \beta, \gamma\}$$

$$SPR(i,j) = log[SP(i)] - log[SP(j)]$$

In the Bonn dataset, we identified that the SPR between δ and θ as the most prominent feature for the following analysis, as it was the most sensitive SPRs according to the seizure states.

3) Classifier learning and testing: We tested three binary classifiers based on (1) $minER$ as the only, (2) SPR as the only, and (3) both $minER$ and SPR combined. The support vector machine (SVM) algorithm with a polynomial kernel was used as we did not assume correlation amongst features. Each classifier was optimized based on a cross-validation set. We obtained the output for each sample in comparison to the actual seizure states, the regression output for each classifier,

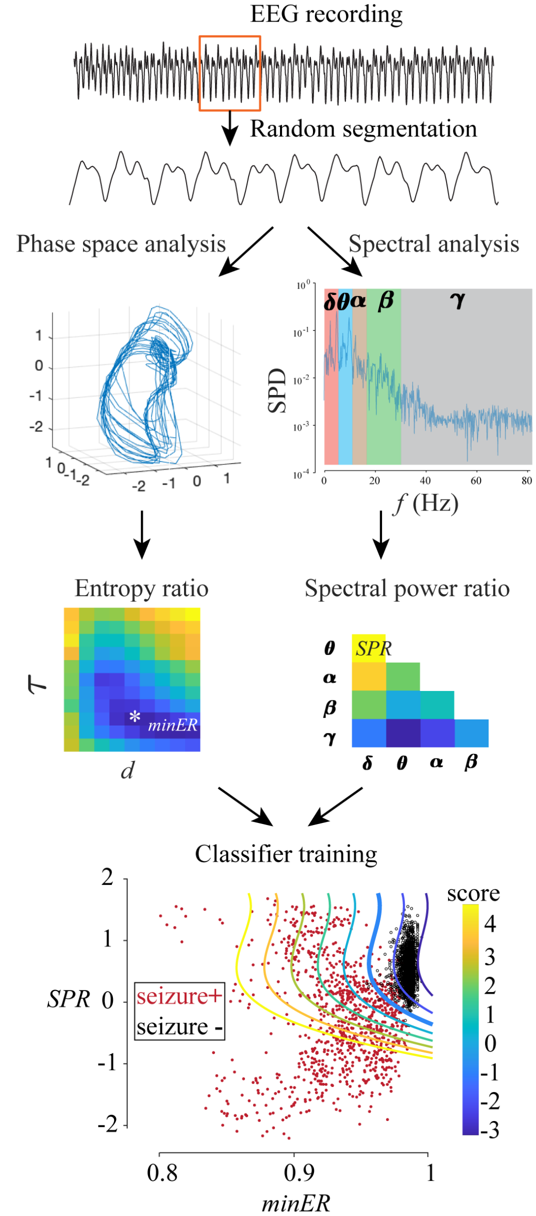

Figure 3. Analysis pipeline.

and a contour plot of the output score was shown in Figure 3 bottom row.

4) Performance Measures: We used detection accuracy to evaluate the performance of three methods. The accuracy was defined as

$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN}$$

where TP (true positive) is the number of seizure segments correctly detected by the classifier, and TN (true negative) is the number of segments correctly detected as normal. FP is the false positive, and FN is the false negative. Detection sensitivity,

$$\text{Sensitivity} = \frac{TP}{TP + FN}$$

and specificity,

$$\text{Specificity} = \frac{TN}{TN + FP}$$

were also used as performance measures.

IV. Results

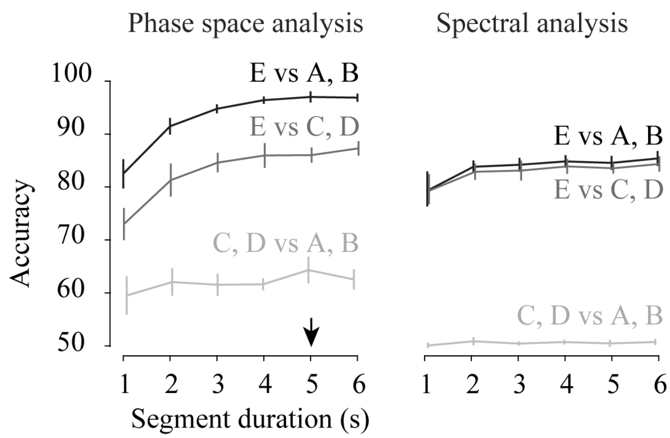

Figure 4. Comparison of the classification accuracy of the two methods. The phase space analysis performs significantly better than the spectral analysis in all condition groups and segment durations.

Compared with the classification result using spectral power ratio, our method using phase space analysis produced significantly better accuracy (Figure 4). The performance of our proposed method was evaluated across patients condition group (E vs A, B and C, D vs A, B), within patients condition group (E vs C, D), respectively. The method can well discriminate between healthy subjects and patients (across-condition groups). It can also perform reasonably well for interictal and postictal situations (within-condition group). In the across condition group (C, D vs A, B), note that our method performed better than spectral analysis, which was at chance level (∼50%). This suggests that the phase state may be a relevant level of description[15]. This description helps explain and understand the system generating abnormal brain activity.

For any signal detection algorithm, there is a trade-off between accuracy and response time. It's of great importance for a method to respond promptly to seizure onsets in real-life situations. Previously reported median response latency can vary greatly (from 10's seconds to minutes) depending on the recording configurations and detection methods [16], [17]. We found our detection accuracy asymptotes at 95% when the segment duration was around 5 seconds (Figure 4). Therefore, we chose only the window width of 5 seconds for the later tests.

The classifier using the SPR was not sensitive to the duration of EEG segments larger than 2 seconds (Figure 4 right). The accuracy did not differ significantly between the two groups. Overall, we found that the classifier with the phase space method performs better than the one with the spectral method. In addition, we trained a classifier using both

	SPR	ER	Combined
E vs. A, B	65.5 [58.9 , 73.5]	85.6 [75.3, 93.7]	93.8 [89.1, 97.7]
	97.2 [92.1, 99.2]	99.4 [97.8, 99.8]	99.5 [98.0, 99.8]
E vs. C, D	61.3 [56.6, 68.6]	69.2 [55.5, 84.3]	89.5 [79.4, 96.2]
	97.9 [94.3, 99.0]	92.2 [86.9, 94.8]	95.3 [91.3, 97.8]
A, B vs. C, D	20.8 [12.8, 39.9]	48.0 [33.2, 67.0]	48.6 [33.2, 67.0]
	73.9 [55.4, 87.6]	98.2 [85.0, 99.5]	81.5 [61.2, 91.2]

Table I

Performance Benchmark. The detection sensitivity (upper) and specificity (lower) of using SPR alone or ER alone and their combination were compared. The numbers were estimated at the probability levels of 50% [27%, 73%], corresponding to the SVM output score of 0, -1, and 1.

features as input. The bottom plot in Figure 3 showed the distribution of the model output score in the feature space. The contour line at score = 0 illustrated the boundary of two well separated clusters for normal and seizure states. Combining the SPR and ER improved the performance also indicated these two are complementary features that should be exploited in parallel. Table I provides the summary of the performance metrics tested on all three classifiers.

V. Conclusion

We have shown the feasibility of using dynamic system analysis on EEG to discriminate seizure states. So far, we have used the entropy of the system as the only input feature to the classifier. Other classification algorithms in combination with other features [9], [18], [19], including but not limited to artificial neural networks (ANN) using correlation and empirical modes of the EEG, are valuable and should be incorporated.

The datasets used in this paper were isolated single-channel recordings, which is suitable for detecting the focal seizure. Our method can also be applied to patients with generalized seizures, where both the onset and traveling of epileptic activity can be detected via combining the phase space analysis from multiple channels.

Using entropy as the core feature, we can detect seizure and non-seizure (E vs A, B) cases with greater accuracy than that for seizure and between-seizure (E vs C, D) cases. However, with spectral analysis, such a difference was less prominent (Figure 4). The reason is that pre-ictal activity (D) and ictal activity (E) can have drastically different phase space portraits, even though they might share a similar spectrum profile. This implies that the phase space representation reveals the underlying dynamics which might closely relate to the generation of seizure activity.

For noisy EEG recordings, our phase state representation can be obscured by large noise. In the examples using sine and Lorenz attractor, we introduced a white noise contributing to 10% of data variance to the original signals (the red lines in Figure 1). Our simulation has shown that large noise (>15-20% of total variance) can cause erroneous and uncertain results in analyzing the phase space and estimating the entropy. The experimenters should use caution in applying this method. One should always examine the recordings, and make an effort to eliminate the sources of noise during data collection.

We also estimated the amount of computation time for both analysis methods. We found that ours is 17 times slower than using the spectral power method. Besides the fact that there are efficient algorithms established for spectrum calculation, a main reason is due to the optimization processes in entropy calculation that search among a range of allowable d and τ at the same time. We know, for instance, that the sine wave is generated by a linear dynamic system with low dimensionality ($d = 2$). Similarly, by correlation analysis of the EEG signal, one can set a reasonable range of τ values. Such prior knowledge of the structure of data would greatly constrain the phase space and improve efficiency.

Overall, we have explored the phase space representation for the EEG signals under normal and abnormal seizure states. Unlike methods in the time-frequency domains, our method uses entropy, a measure of global orderliness of the signal. Ours is a non-parametric method, meaning that the feature extraction can avoid potential issues associated with subjective selection of fitting functions and parameters. It's an heuristic approach that studies of time series signals in other biomedical fields can benefit from.

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