Homework 2, Math 3000

due on Jan 25, 2022

Before you start, please read the syllabus carefully.

1. Denote the vectors $\mathbf{u} = (2, 1, 0)$, $\mathbf{v} = (1, 2, 1)$ and $\mathbf{w} = (0, 1, 2)$ in $\mathbb{R}^3$. Compute the following:

   (a) $2\mathbf{u} + 3\mathbf{v}$;

   (b) $\mathbf{u} \cdot (\mathbf{u} + \mathrm{v})$;

   (c) The length of $\mathbf{u}$, v.

   (d) The side length of the triangle formed by $\mathbf{u}$ and $\mathbf{v}$ with two of the sides;

   (e) The projection of $\mathbf{u}$ along $\mathbf{v}$;

   (f) Decompose $\mathbf{u}$ into a sum of two vectors $\mathbf{u}_\perp$ and $\mathbf{u}_\parallel$ with respect to $\mathbf{v}$ (i.e. $\mathbf{u} = \mathbf{u}_\perp + \mathbf{u}_\parallel$ and $\mathbf{u}_\perp \perp \mathbf{v}$ and $\mathbf{u}_\parallel \parallel \mathbf{v}$).

   (g) The area of the triangle formed by $\mathbf{u}$ and $\mathbf{v}$ with two of the sides;

   (h) Find all x, y and z such that the vector $(1, 2, 3) = x\mathbf{u} + y\mathbf{v} + z\mathbf{w}$;

   (i) Find all vectors $\mathbf{q}$ such that $\mathbf{q}$ is perpendicular to both $\mathbf{u}$ and $\mathbf{v}$.

   (j) Write down the equation of the plane that is perpendicular to $\mathbf{u}$ and passing through the origin $(0, 0, 0)$ in $\mathbb{R}^3$.

2. For the following matrix, determine whether they are in its row echelon form.

   (a)
   $$\begin{pmatrix} 2 & 3 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

   (b)
   $$\begin{pmatrix} 2 & 3 & 4 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

   (c)
   $$\begin{pmatrix} 0 & 3 & 4 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

   (d)
   $$\begin{pmatrix} 2 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

(e)

$$\begin{pmatrix} 2 & 3 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3. For each linear system in Ex. 1 in HW 1, write it in the matrix form $A\mathbf{x} = \mathbf{b}$. And for each matrix A, determine its reduced row echelon form.