Homework 4, Math 3000

due on Feb 8, 2022

Before you start, please read the syllabus carefully.

1. Determine whether the following list of vectors are: 1) linearly independent 2) spanning the vector space.
   (a) $\{(1,0,0),(1,1,0),(1,1,1)\}$ in $\mathbb{R}^3$
   (b) $\{1, x-1, (x-1)^2, (x-1)^3\}$ in the vector space of polynomials with degree smaller or equal to 3
   (c) $\{(1,1,0),(1,0,1),(0,1,1)\}$ in the vector space $W := \{(x,y,z) \mid x+y+z=1\}$
   (d) $\{\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}\}$ in the vector space $M_{2\times 2}(\mathbb{R})$.
2. Give a basis of the following vector spaces and determine its dimension:
   (a) $V := \{A\mathbf{x} \mid \mathbf{x} \in \mathbb{R}^3\}$ where
   $$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$
   (b) $V := \{\mathbf{x} \in \mathbb{R}^4 \mid A\mathbf{x} = 0\}$ where
   $$A = \begin{pmatrix} 1 & 1 & 2 & 0 \\ 2 & 0 & 1 & 1 \end{pmatrix}$$
   (c) $V := \{\text{polynomials } P(x) \mid P'(1) = P''(1) = 0, \deg(P(x)) \leq 4\}$.
   (d) V is the intersection of planes $2x+y+z=0$ and $x+2y+z=0$ in $\mathbb{R}^3$.
   (e) V is the set of symmetric matrices in $M_{2\times 2}(\mathbb{R})$. ($A$ is symmetric means $A_{ij} = A_{ji}$ for all i, j)