Computer Exercise on Systems' Simulation / Phase Portraits

Nonlinear Control Systems

Summer Semester 2021

1. Consider the pendulum in Figure 1 with friction.

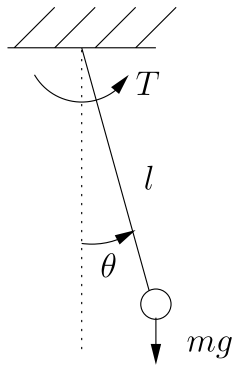

Abbildung 1: Pendulum.

The equation of motion is:

$$\begin{aligned} m\bar{a} &= -mg\sin(\theta) + \frac{1}{l}T - kl\dot{\theta} \\ \bar{a} &= l\ddot{\theta}. \end{aligned}$$

After rearranging you obtain:

$$\ddot{\theta} = -\frac{g}{l}\sin(\theta) + \frac{1}{ml^2}T - \frac{k}{m}\dot{\theta}.$$

With the choice of the state variables $x_1 = \theta$ and $x_2 = \dot{\theta}$, the following state-space model results

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -ax_2 - b\sin x_1 + cu \end{aligned}$$

with

$$a = \frac{k}{m}, \quad b = \frac{g}{l}, \quad c = \frac{1}{ml^2}, \quad u = T.$$

The following values are assumed: $a = 1$, $b = 2$ and $u = 0$.

Tasks:

(a) Linearise the system at the equilibrium point ($x_1^0 = 0$, $x_2^0 = 0$) and determine a linear state-space model that describes the system near the equilibrium point.

(b) Simulate the non-linear original system and the linearised system simultaneously in Matlab and/or Simulink. Investigate the system behaviour of both systems for the initial angles $5°$, $20°$ and $45°$ and the initial angular velocity $x_2(0) = 0$.

(c) Create a single plot with all your simulation results and save it as a PDF file.

2. Create phase portraits and vector field diagrams using Matlab for the following systems:

$$\text{System 1)}\quad \begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= x_1 - 2\arctan(x_1 + x_2)\end{aligned}$$

$$\text{System 2)}\quad \begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -x_1 + x_2(1 - 3x_1^2 - 2x_2^2)\end{aligned}$$

$$\text{System 3)}\quad \begin{aligned}\dot{x}_1 &= 2x_1 - x_1 x_2\\ \dot{x}_2 &= 2x_1^2 - x_2\end{aligned}\;.$$

Discuss the qualitative behaviour of the systems.

3. Simulate the control loop with $K = 1$ shown in Figure 2 in Simulink with and without initial pulse-shaped disturbance d at the system output. Plot the output signal y and control signal u in a common diagram over time. Describe the system behaviour qualitatively and quantitatively.

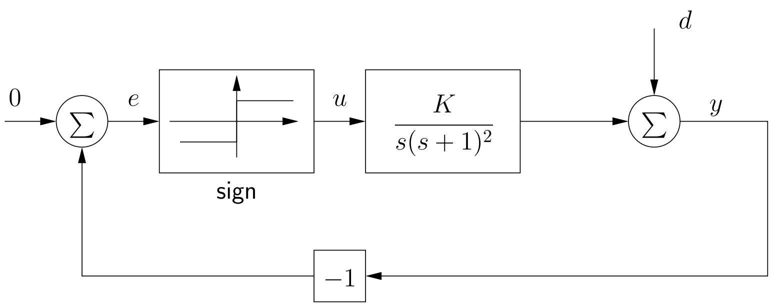

Abbildung 2: Nonlinear control system.