Propositional Logic

1. Prove or find a counterexample for all statements below

   a. When P is false, $P \wedge \neg P$ is false, so $P \wedge \neg P$ is not valid.

   b. When B is false and A is false, $A \rightarrow B$ can still be true. Thus $A \rightarrow B \models B$ fails.

   c. *Proof.* (A or B) is true, when either A or B is true. When B is true, since $(B \models C)$, C is true. When A is true, since $(A \models B)$, B is true, then C is true. □

   d. *Proof.*

$$\begin{aligned} A \vee \neg(B \wedge \neg C) &= A \vee (\neg B \vee C) \\ &= A \vee C \vee \neg B \end{aligned}$$

$$A \vee C \subseteq A \vee C \vee \neg B$$

$$A \vee \neg(B \wedge \neg C) \models A \vee C$$

□

   e. When A is false, $\neg(A \wedge B) \wedge (A \rightarrow B)$ is true, thus satisfiable. $(\neg A \vee \neg B) \wedge (\neg A \vee B)$

   f. No. $((A \rightarrow \neg B) \wedge A) = (\neg A \vee \neg B) \wedge A$, there is only one model A is true and B is false can make it true. While $(A \wedge (B \vee C))$ has more than one model: e.g. A is ture, B is true and C is ture; A is true, B is false, and C is true.

First-order Logic

2. Translate the following first order logic sentences to English. Predicates and functions carry the obvious meanings.

   a. Northwestern's mascot is Willie Wildcat.

   b. For any x, y and z, if the sum of x and y is odd and z is odd, then the sum of x, y and z is not odd.

3. Translate the following English sentences to first order logic. Name predicates and functions such that their meanings are obvious.

   a. $[student(Greg) \wedge student(John)] \wedge play(Greg, tennis)$

   b. $\forall x, \forall y, [integer(x) \wedge integer(y)] \rightarrow integer(product(x, y))$

4. Security personnel.

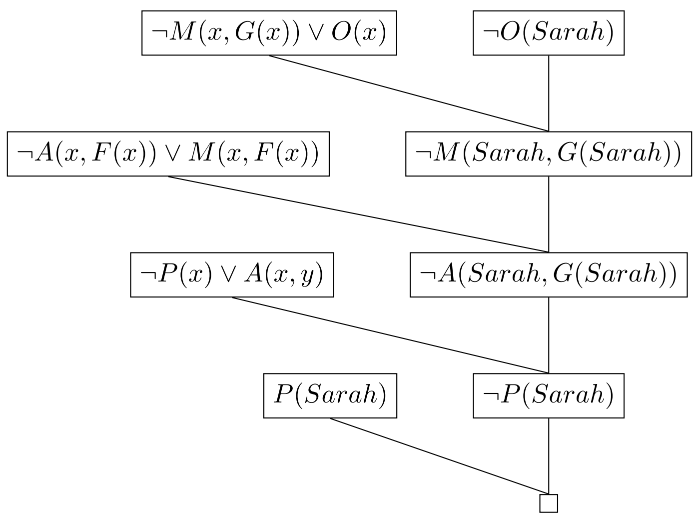
   (i) a. $\forall x, [\exists y, P(x) \rightarrow [S(y) \wedge A(x, y)]]$

   b. $\forall y, [\exists x, S(y) \rightarrow [A(x, y) \wedge P(x)]]$

c. $\forall x, [\forall y, A(x,y) \rightarrow M(x,y)]$
d. $\forall x, \forall y, M(x,y) \rightarrow [O(x) \wedge O(y)]$
e. S(Juan)
f. P(Sarah)
g. O(Sarah)
h. A(Sarah, Juan)

(ii) Convert to CNF:

A	1	$\neg P(x) \vee S(y)$
	2	$\neg P(x) \vee A(x,y)$
B	1	$\neg S(y) \vee A(x,y)$
	2	$\neg S(y) \vee P(x)$
C		$\neg A(x, F(x)) \vee M(x, F(x))$
D	1	$\neg M(x, G(x)) \vee O(x)$
	2	$\neg M(x, G(x)) \vee O(G(x))$
E		$S(Juan)$
F		$P(Sarah)$



(iii) The derived clause: $\neg A(Sarah, G(Sarah))$.
Because we don't know whether there exists advising meeting between Sarah and Juan, i.e. we can't prove M(Sarah, Juan).

Bayes Nets

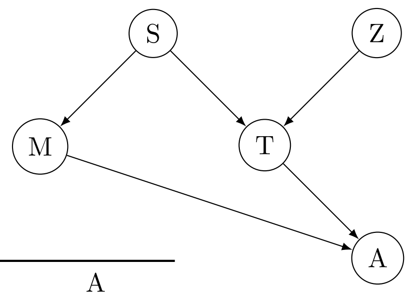
5. a. $1 + 1 + 2 + 4 + 4 = 12$
   b. $2^5 - 1 = 31$
   c. ii, vi, vii, ix.

d. $P(M = 1|S) = 1/2$
$P(S = 1)P(Z = 1)P(M = 1|S)P(T = 1|S, Z)P(A = 1|M, T) = 1/50$
$\rightarrow P(T = 1|S, Z) * P(A = 1|M, T) = 16/25$

S	
T	F
0.25	0.75

Z	
T	F
0.25	0.75

	M	
S	T	F
F	0.5	0.5
T	0.5	0.5



		T	
S	Z	T	F
F	F	0.8	0.2
F	T	0.8	0.2
T	F	0.8	0.2
T	T	0.8	0.2

		A	
M	T	T	F
F	F	0.8	0.2
F	T	0.8	0.2
T	F	0.8	0.2
T	T	0.8	0.2

e. 9.