

Learning Notes of Apr 2019

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stage 1

1. The model of the satellite with solar panels.
2. The model of the prismatic deployable trusses.
3. The dynamics of the prismatic deployable trusses.

1 Lie Group and Lie Algebra

Group: Set G with composition rule satisfying 4 axioms.

1. closure: $\forall g_1, g_2 \in G, g_1 g_2 \in G$
2. associativity: $\forall g_1, g_2, g_3 \in G, (g_1 g_2) g_3 = g_1 (g_2 g_3)$
3. identity: $\exists e \in G, \forall g \in G, ge = eg = g$
4. invertibility: $\forall g \in G, \exists g^{-1} \in G, gg^{-1} = g^{-1}g = e$

Lie group: groups with continuity.
矩阵指数函数

$$\begin{aligned} e^{At} &= I + At + \frac{(At)^2}{2!} + \frac{(At)^3}{3!} + \cdots + \frac{(At)^n}{n!} + \cdots \\ \frac{d}{dt}(e^{At}) &= A + A^2 t + \frac{A^3 t^2}{2!} + \cdots + \frac{A^n t^{n-1}}{(n-1)!} + \cdots \\ &= A(I + At + \frac{(At)^2}{2!} + \cdots + \frac{(At)^{n-1}}{(n-1)!} + \cdots) = Ae^{At} \end{aligned}$$

Orthogonal group: $\{M \in \mathbb{R}^{n \times n} | MM^T = I, \det(M) = \pm 1\}$

Special Orthogonal group $SO(3) = \{\mathbf{R} \in \mathbb{R}^{3 \times 3} | \mathbf{R}^T \mathbf{R} = \mathbf{I}, \det(\mathbf{R}) = 1\}$

Special Euclidean group $SE(3) = \left\{ \mathbf{T} = \begin{bmatrix} \mathbf{R} & \mathbf{x} \\ \mathbf{0}^T & 1 \end{bmatrix} \in \mathbb{R}^{4 \times 4} | \mathbf{R} \in SO(3), \mathbf{x} \in \mathbb{R}^3 \right\}$

Lie Algebra

$$so(3) = \{\phi \in \mathbb{R}^3\}$$

$$se(3) = \{\xi \in \mathbb{R}^6\}$$